Truthmakers and Information States

Inclusion, Containment, Duality

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ILLC and Philosophy, University of Amsterdam

Prague, November 18, 2025

Workshop on Truthmakers, Possibilities, and Information States

Plan for the talk

I’ll discuss a cluster of observations on points of contact between truthmaker and information semantics. These fall under three connected themes:

- Information states (à la BSML) and Containment.
- Truthmakers and Inclusion.
- Translations.

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In BSML, like in inquisitive semantics, formulas are evaluated at sets of valuations ('teams') $t \subseteq \{v \mid v : \mathbf{At} \to \{0, 1\}\}$, $\quad t \vDash \varphi$.

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Observation 2: Telltale of containment logics

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1. Points of contact between BSML and truthmaker semantics.
2. BSML-style information semantics for containment logics.

Semantics for containment logics.

Containment and relevance

Containment logics obey the the proscriptive principle:

$$\varphi \vdash \psi \qquad \text{implies} \qquad \mathbf{At}(\varphi) \supseteq \mathbf{At}(\psi).$$

Strong form of variable sharing:

$$\varphi \vdash \psi \qquad \text{implies} \qquad \mathbf{At}(\varphi) \cap \mathbf{At}(\psi) \neq \varnothing.$$

Signature invalidities:

1. $p \wedge \neg p \nvdash q$ [like relevant logics]
2. $p \nvdash q \vee \neg q$ [like relevant logics]
3. $p \nvdash p \vee q$ [like BSML]

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Angell's Analytic Entailment (AC)

One prominent containment logic is Angell's analytic entailment AC.
AC is, as shown by Ferguson (2016) and Fine (2016), the containment fragment of FDE:

$$\varphi \vdash_{AC} \psi \qquad \text{iff} \qquad \varphi \vdash_{FDE} \psi \text{ and } \mathbf{Lit}(\varphi) \supseteq \mathbf{Lit}(\psi).$$

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- It is a containment logic.
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First goal: BSML-style semantics for AC.

BSML and explosion

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Problem: $p \wedge \neg p \models q$.

BSML and explosion

Equivalent **BSML semantics:** for $t \in \mathcal{P}(\{v \mid v : \mathbf{At} \to \{0,1\}\})$ we define

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BSML and explosion

Equivalent BSML semantics: for $t \in \mathcal{P}(X)$ and a valuation $V^+ : \mathbf{At} \to \mathcal{P}\mathcal{P}(X)$ such that

- $V^+(p)$ is a non-empty (principal) ideal;
- $V^-(p)$ is the non-empty ideal $\downarrow (\bigcup V(p))^c$,

we define

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BSML-style semantics for AC

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AC **semantics:** Given $\mathcal{P}(X)$ and $V^+, V^- : \mathbf{At} \to \mathcal{P}\mathcal{P}(X)$ s.t.

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Theorem (soundness and completeness)

$\varphi \vDash \psi$ if and only if $\varphi \vdash_{AC} \psi$.

Follow-ups

We obtained a complete semantics for AC.

[1] Daniels (1990); Priest (2010).

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Question: As AC is characterized by

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can we also give semantics for the logic characterized by

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Proof. As before (note available).

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Proof. As before (note available). [Proofs work for distr. lattices too.]

[1] Daniels (1990); Priest (2010).

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We obtained a complete semantics for AC.

Question: As AC is characterized by

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- What other containment logics arise by varying the frames (lattices, semilattices, distributive semilattices, etc.) or valuations?
- For instance, can we obtain a complete semantics for Correia's (2016) logic of factual equivalence?

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Recall

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Inferential patterns:

$$p \not\models p \vee q$$

$$p \wedge q \models p$$

Observation 1: Mirror image of truthmaker entailment

Observation 2: Telltale of containment logics

And recall the two guiding themes:

1. Points of contact between BSML and truthmaker semantics.
2. BSML-style semantics for containment logics. ✓

Truthmakers and Inclusion.

Replete truthmaker entailment

Write $\varphi \Vdash \psi$ for replete truthmaker preservation.

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Replete truthmaker entailment

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Theorem[2]

Replete truthmaker entailment is the inclusion fragment of FDE; i.e.,

$$\varphi \Vdash \psi \qquad \text{iff} \qquad \varphi \vdash_{FDE} \psi \text{ and } \mathbf{Lit}(\varphi) \subseteq \mathbf{Lit}(\psi).$$

[2] I imagine this is known, but I haven't found it stated.

A sample of corollaries

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Likewise, duals of Fine's (2016) valence/partial-truth accounts of AC characterize replete truthmaker entailment (as FDE is equivalently defined as reflection of falsity).

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Before we proceed, two further remarks on truthmakers and inclusion.

Maxim: *Exactify!*

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But what does it mean to exactify? When is a semantics *exact*?

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Remark 1: On what it means for a semantics to be *exact*

When is a semantics *exact*?

- Say that $\models$ satisfies the inclusion principle if

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– Caveat: How about explosion and its dual? Perhaps inclusion *modulo* explosion and its dual?[3]

[3] The signature invalidities of 'inclusion logics' include explosion and its dual, but maybe exactness should only generalize the invalidity of simplification (think counterfactuals, modalities, etc.).

Remark 2: On replete entailment and wholly relevance

A-B Analysis: Replete Entailment and Wholly Relevance

Recall Anderson and Belnap's (1962) tautological entailments:

1) For A_i a conjunction of literals, and B_j a disjunction of literals, let

$$A_i \vdash_T B_j \qquad \text{:iff} \qquad \mathbf{Lit}(A_i) \cap \mathbf{Lit}(B_j) \neq \varnothing.$$

2) Lift it as follows:

$$\bigvee A_i \vdash_T \bigwedge B_j \qquad \text{:iff} \qquad \forall i, j : A_i \vdash_T B_j.$$

3) For arbitrary φ, ψ with normal forms $\varphi \equiv \bigvee A_i, \psi \equiv \bigwedge B_j$, define

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1) For A_i a conjunction of literals, and B_j a disjunction of literals, let

$$A_i \vdash_T B_j \qquad \text{:iff} \qquad \mathbf{Lit}(A_i) \cap \mathbf{Lit}(B_j) \neq \varnothing.$$

2) Lift it as follows:

$$\bigvee A_i \vdash_T \bigwedge B_j \qquad \text{:iff} \qquad \forall i, j : A_i \vdash_T B_j.$$

3) For arbitrary φ, ψ with normal forms $\varphi \equiv \bigvee A_i, \psi \equiv \bigwedge B_j$, define

$$\varphi \vdash_T \psi \qquad \text{:iff} \qquad \bigvee A_i \vdash_T \bigwedge B_j.$$

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Follow-ups and future work

Follow-ups I'd like to think about:

1. Like replete entailment, can other truthmaker entailments be given a double-barreled analysis?
2. For instance, can (non-)inclusive entailment be captured by stronger inclusion principles?
3. Can (or has) a truthmaker semantics been given for

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5. Can other logics, like intuitionistic logic or LP, be given contrapositive semantics? Is it trivial, is it worthwhile?
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Translations.

Source logic: BSML with NE and ⧫

Fix a non-empty finite set of propositional variables $\mathbf{At}$, and define:

$$\varphi ::= \bot \mid \text{NE} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \blacklozenge\varphi.$$

Definition

For $t \subseteq \{v \mid v : \mathbf{At} \to \{0,1\}\}$, we have

$t \models \text{NE}$	iff	$t \neq \varnothing$
$t \mathrel{=\!\!\!\mid} \text{NE}$	iff	$t = \varnothing$
$t \models \blacklozenge\varphi$	iff	$\exists\, s \subseteq t$ such that $\varnothing \neq s \models \varphi$
$t \mathrel{=\!\!\!\mid} \blacklozenge\varphi$	iff	$\forall s \subseteq t$: $s \mathrel{=\!\!\!\mid} \varphi$
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Target logic: modal information logic

Target logic is the modal logic in the language with two modalities,

$$\varphi ::= \bot \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle \mathrm{sup} \rangle \varphi\varphi \mid \langle \mathsf{s}^* \rangle \varphi,$$

for $p \in \mathbf{At}_\pm := \{p_+, p_- \mid p \in \mathbf{At}\}$, interpreted over distributive semilattices $(S, \curlyvee)$, where

$s \Vdash \langle \mathrm{sup} \rangle \varphi\psi$ iff $\exists\, t, u$ s.t. $t \Vdash \varphi,\ u \Vdash \psi,$ and $s = t \curlyvee u$.

$s \Vdash \langle \mathsf{s}^* \rangle \varphi$ iff $\exists\, s_1, \ldots, s_n$ s.t. each $s_i \Vdash \varphi$ and $s = s_1 \curlyvee \cdots \curlyvee s_n$.

Objective: Define translation pair $\cdot^+, \cdot^-$ s.t. for all φ, ψ:

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$$\Gamma := \{\mathsf{H}(\mathsf{NE}^+ \vee \mathsf{NE}^-), \bigwedge_{p \in \mathbf{At}} \langle \mathrm{sup} \rangle p^+ p^- \},$$

and define $\cdot^+, \cdot^-$ via the double-recursive clauses:

$$\begin{array}{rclrcl}
\bot^+ & := & \mathsf{NE}^- & \bot^- & := & \top \\
\mathsf{NE}^+ & := & \bigwedge_{p \in \mathbf{At}} \neg(p^+ \wedge p^-) & \mathsf{NE}^- & := & \bigwedge_{p \in \mathbf{At}} (p^+ \wedge p^-) \\
p^+ & := & \mathsf{H}\langle \mathsf{s}^* \rangle p_+ & p^- & := & \mathsf{H}\langle \mathsf{s}^* \rangle p_- \\
(\neg\varphi)^+ & := & \varphi^- & (\neg\varphi)^- & := & \varphi^+ \\
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Theorem

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Translating BSML

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BSML translation contra truthmaker translation

Translation clauses for BSML:

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Translating inquisitive logic

For the case of inquisitive logic, translate $\mathbin{\mathbb{v}}$, $\to$ as follows:

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Theorem (translation of Inq)

$$\varphi \vDash \psi \qquad \text{iff} \qquad \Gamma, \varphi^+ \Vdash \psi^+.$$

Remark

The translation can be extended to other propositional team logics too, including all fragments of the grammar:

$$\begin{aligned}\varphi ::= & \bot \mid \mathsf{NE} \mid p \mid \neg\alpha \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \blacklozenge\varphi \mid \varphi \mathbin{\mathbb{v}} \varphi \mid \varphi \to \varphi \mid {\sim}\varphi \mid \\ & {=}(\vec{\alpha};\vec{\alpha}) \mid \vec{\alpha} \perp_{\vec{\alpha}} \vec{\alpha} \mid \vec{\alpha} \subseteq \vec{\alpha} \mid \vec{\alpha} | \vec{\alpha} \mid \vec{\alpha} \Upsilon \vec{\alpha}.\end{aligned}$$

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Thank you!

References i

New connectives

On connectives:

- *Fact 1:* Team semantics for $\{\neg, \wedge, \vee\}$ gives us classical logic.
- *Fact 2:* In classical logic, $\{\neg, \wedge, \vee\}$ is famously functionally complete: all other connectives are definable by these.
- *Fact 3:* In team semantics, $\{\neg, \wedge, \vee\}$ can only capture a fraction of the expressible connectives. For example, $\mathbin{\rotatebox[origin=c]{-90}{$\vDash$}}$ is not definable using $\{\neg, \wedge, \vee\}$.
- *Consequence:* We have a semantic framework for expressions beyond classical assertions, such as questions.

Take-away: *Teams provide for ways to express meanings not readily expressible in single-valuation semantics;* and thus for considering new connectives!

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(Propositional) team logics: propositionhood

- Given any condition-based semantics, we obtain a notion of propositionhood defined as a set of conditions. *Slogan:* Proposition = a set of conditions.
- In team semantics, conditions are teams.
- So, propositions are sets of teams. **Caveat:** The standard terminology is not 'propositions' but 'properties'.

Since our meaning space now has structure (as powersets), we can consider natural restrictions on what a proposition is. Or what different kinds of propostions/meanings there are! For instance, assertions contra questions. (Note the analogy with generalized quantifiers.)

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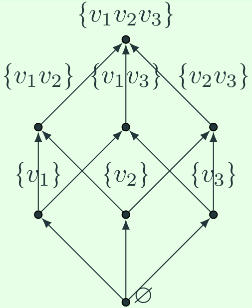

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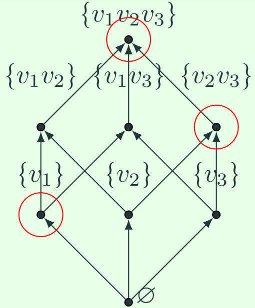

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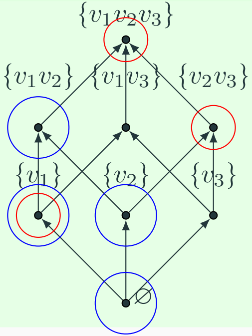

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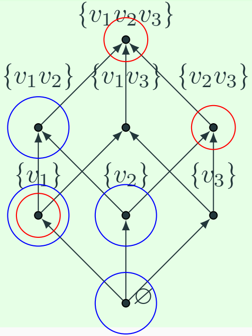

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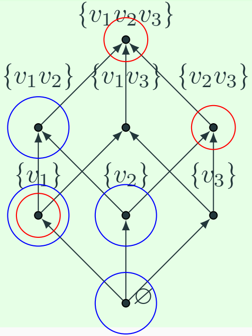

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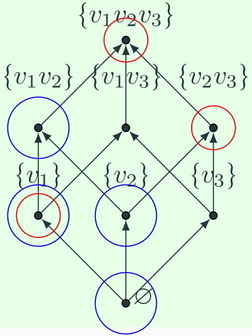

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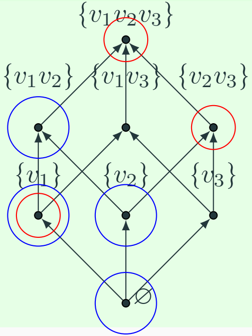

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Notions of propositionhood (closure properties)

Take-away: *Teams provide for ways to express meanings not readily expressible in single-valuation semantics; and thus for considering new notions of propositionhood!*

Definition (some restrictions on propositionhood)

ϕ is *downward closed*:	$[s \vDash \phi$ and $t \subseteq s] \implies t \vDash \phi$
ϕ is *union closed*:	$[s \vDash \phi$ f.a. $s \in S \neq \varnothing] \implies \bigcup S \vDash \phi$
ϕ has the *empty team property*:	$\varnothing \vDash \phi$
ϕ is *flat*:	$s \vDash \phi \iff \{v\} \vDash \phi$ for all $v \in s$
ϕ is *convex*:	$[s \vDash \phi, u \vDash \phi$ and $s \subseteq t \subseteq u] \implies t \vDash \phi$

Convexity generalizes downward closure:

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Interface of connectives and propositionhood

The choice of connectives and the corresponding notion of propositionhood are closely connected. Here are some examples:

- Classical formulas are flat (so union closed) [i.e., classical assertions are flat]
- Formulas with $\mathbin{\mathbb{v}}$ need not be union closed [i.e., questions are not union closed]
- Consider the *epistemic might* operator $\blacklozenge$, defined as

$$s \models \blacklozenge\phi \iff \exists t \subseteq s : t \neq \varnothing \ \&\ t \models \phi.$$

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