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## Preface

Many of computer graphics classes in colleges are focused on real-time rendering and animation. However, it is not easy to find an appropriate textbook, which presents the state of the art in interactive graphics, is balanced between theory and practicality, and is of a proper length to be covered in a semester. This book is written for answering the need and presents the *must-know* in interactive graphics. This book fits the advanced undergraduate or beginning graduate classes for 'Computer Graphics' and 'Game Programming.'

Another primary reader group of this book may be composed of game developers, who have experience in graphics APIs and shader programming but have felt lack of theoretical background in 3D graphics. A lot of programming manual-like books can be found in the bookstore, but they do not provide a sufficient level of mathematical background for the game developers. Assuming that the readers have minimal understanding of vectors and matrices, this book provides an opportunity to combine their experiences with the background theory of computer graphics.

At the core of contemporary interactive graphics is the evolution of GPU. The content of this book is organized around GPU programming. The GPU is partitioned into programmable stages and hard-wired stages. This book presents a variety of algorithms for the programmable stages, and the indispensable knowledge required to configure the hard-wired stages.

The organization and presentation of this book have been carefully designed so as to enable the readers to easily understand the key aspects of interactive graphics. Over the chapters, a lot of 3D presentations are provided in order to help the readers quickly grasp the complicated topics. An important organizational feature of this book is that theoretical or technical details are presented in separate notes (in shaded boxes) and in optional sections (marked by asterisk). They can be safely skipped without any difficulty in understanding the main stream of the book.

Two well-known graphics APIs are Direct3D and OpenGL. This book is API-neutral but presents the sample programs bit by bit in many places. They help the readers understand how the interactive graphics algorithms are implemented. However, the sample programs are located at the optional part and can be safely skipped.

If the optional parts are not taught, the content of this book would be appropriate for being covered in a semester for the undergraduate class. If needed, additional choices may be made among the required parts. For example, Sections 6.3 and 7.4 may be skipped.