

$$\begin{aligned}
K &= W_K X \\
k_j^i &= w_r^i x_j^r \\
q_j^i &= v_r^i x_j^r \\
\frac{\partial k_j^i}{\partial w_b^a} &= \delta_{rb}^{ia} x_j^r \\
&= \delta^{ia} x_j^b \\
\frac{\partial q_j^i}{\partial v_b^a} &= \delta_{rb}^{ia} x_j^r \\
&= \delta^{ia} x_j^b
\end{aligned}$$

$$\begin{aligned}
S &= K^T Q \\
s_j^i &= k_i^r q_j^r \\
\frac{\partial s_j^i}{\partial q_b^a} &= k_i^r \delta_{jb}^{ra} \\
&= k_i^a \delta_{jb}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial s_j^i}{\partial v_b^a} &= \frac{\partial s_j^i}{\partial q_d^c} \frac{\partial q_d^c}{\partial v_b^a} \\
&= k_i^c \delta_{jd} \delta^{ca} x_d^b \\
&= k_i^a x_j^b
\end{aligned}$$

$$\begin{aligned}
K &= WX \\
k_d^c &= w_r^c x_d^r \\
\frac{\partial k_d^c}{\partial w_b^a} &= \delta_{rb}^{ca} x_d^r = \delta^{ca} x_d^b \\
S &= K^T Q \\
s_j^i &= k_i^r q_j^r \\
\frac{\partial s_j^i}{\partial k_b^a} &= q_j^r \delta_{ib}^{ra} \\
&= q_j^a \delta_{ib}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial s_j^i}{\partial w_b^a} &= \frac{\partial s_j^i}{\partial k_d^c} \frac{\partial k_d^c}{\partial w_b^a} \\
&= q_j^c \delta_{id} \delta^{ca} x_d^b \\
&= q_j^a x_i^b
\end{aligned}$$

$$\begin{aligned}
Y &= f(S) \\
y_j^i &= s_{ij}/t_j \\
\frac{\partial y_j^i}{\partial s_b^a} &= \delta^{kjb} d^{iak} = y_j^i (\delta_b^a - y_b^a) \delta_{jb} \\
\frac{\partial l}{\partial s_b^a} &= \frac{\partial l}{\partial y_j^i} \frac{\partial y_j^i}{\partial s_b^a} \\
\frac{\partial l}{\partial s_b^a} &= \Delta_j^i \delta^{jbk} d^{iak} \\
\frac{\partial l}{\partial s_b^a} &= \sum_{i,j,k} \Delta_j^i \delta^{jbk} d^{iak} \\
\frac{\partial l}{\partial s_b^a} &= \Delta_b^i d^{iab}
\end{aligned}$$