Workshop 2: Control flow and list comprehensions

FIE463: Numerical Methods in Macroeconomics and Finance using Python

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See GitHub repository for notebooks and data:

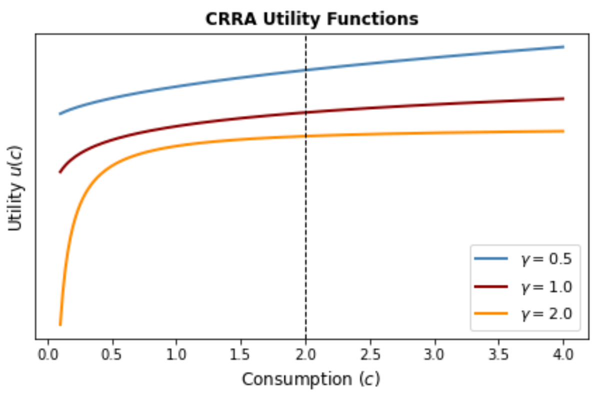
https://github.com/richardfoltyn/FIE463-V26

Exercise 1: CRRA utility function

The CRRA utility function (constant relative risk aversion) is the most widely used utility function in macroeconomics and finance. It is defined as

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(c) & \text{else} \end{cases}$$

where c is consumption and γ is the (constant) risk aversion parameter, and $\log(\bullet)$ denotes the natural logarithm. The following figure illustrates this utility function for various values for γ:



1. You want to evaluate the utility at $c = 2$ for various levels of γ.
    1. Define a list `gammas` with the values 0.5, 1, and 2.
    2. Loop over all elements in `gammas` and evaluate the corresponding utility. Use an `if` statement to correctly handle the two cases from the above formula.

       Hint: Import the `log` function from the `math` module to evaluate the natural logarithm:

       ```
       from math import log
       ```

       Hint: To perform exponentiation, use the `**` operator (see the list of operators).
    3. Store the utility in a dictionary, using the values of γ as keys, and print the result.

2. **[Advanced]** Can you solve the exercise using a single list comprehension to create the result dictionary?

   Hint: You will need to use a conditional expression we covered in the lecture.

Solution.

Part 1 — Compute utilities

[1]:
```python
# Define list of gammas that should be considered
gammas = [0.5, 1.0, 2.0]
```

[2]:
```python
# Import log function from math module
from math import log

# Create empty dictionary to store gamma & utility values
utils = {}

# Consumption level at which to evaluate utility
cons = 2.0

for gamma in gammas:
    if gamma == 1.0:
        # Handle log case
        u = log(cons)
    else:
        # Handle general CRRA case
        u = cons**(1.0-gamma) / (1.0 - gamma)

    # Store resulting utility level in dictionary
    utils[gamma] = u

# Print utility levels
utils
```

[2]:
```
{0.5: 2.8284271247461903, 1.0: 0.6931471805599453, 2.0: -0.5}
```

Part 2 — List comprehension

It is possible to compress the entire loop into a single list comprehension. We need to use a conditional expression within the list comprehension to correctly handle the two CRRA cases.

[3]:
```python
# Compute utilities in a single list comprehension
utils = {
    gamma: log(cons) if gamma == 1.0 else cons**(1.0-gamma) / (1.0-gamma)
    for gamma in gammas
}
```

[4]:
```python
# Print utility levels
utils
```

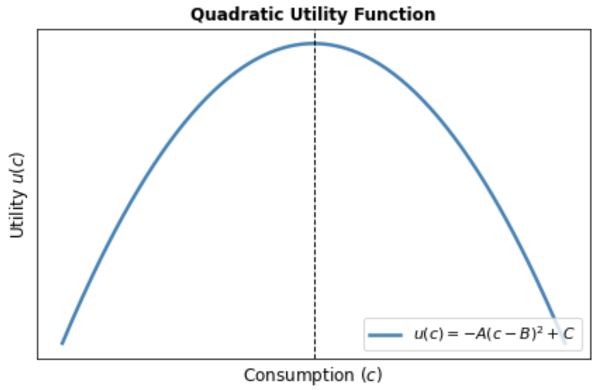
[4]:
```
{0.5: 2.8284271247461903, 1.0: 0.6931471805599453, 2.0: -0.5}
```

Exercise 2: Maximizing quadratic utility

Consider the following quadratic utility function

$$u(c) = -A(c - B)^2 + C$$

where $A > 0$, $B > 0$ and C are parameters, and c is the consumption level. The following figure illustrates this utility function:



In this exercise, you are asked to locate the consumption level which delivers the maximum utility for the parameters $A = 1$, $B = 2$, and $C = 10$.

1. Find the maximum using a loop:
    1. Create an array `cons` of 51 candidate consumption levels that are uniformly spaced on the interval $[0, 4]$.
    2. Loop through all candidate consumption levels, and compute the associated utility. If this utility is larger than the previous maximum value `u_max`, update `u_max` and store the associated consumption level `cons_max`.
    3. Print `u_max` and `cons_max` after the loop terminates.
2. Repeat the exercise, but instead use vectorized operations from NumPy:
    1. Compute and store the utility levels for *all* elements in `cons` at once (simply apply the formula to the whole array).
    2. Locate the index of the maximum utility level using `np.argmax()`.
    3. Use the index returned by `np.argmax()` to retrieve the maximum utility and the corresponding consumption level, and print the results.

Solution.

Part 1 — Loop to maximize utility

[5]:
```
# Parameters
A = 1.0
B = 2.0
C = 10.0
```

[6]:
```
# Import NumPy
import numpy as np
```

```
# Candidate consumption levels
cons = np.linspace(0.0, 4.0, 51)
```

[7]:
```
# Initialize max. utility level at the lowest possible value
u_max = -np.inf

# Consumption level at which utility is maximized
cons_max = None

# Evaluate utility for each candidate consumption level, update maximum
for c in cons:
    u = - A * (c - B)**2.0 + C
    if u > u_max:
        # New maximum found, update values
        u_max = u
        cons_max = c

# Print maximum and maximizer
print(f'Utility is maximized at c={cons_max} with u={u_max}')
```

```
Utility is maximized at c=2.0 with u=10.0
```

Part 2 — Vectorized grid search

[8]:
```
# Evaluate all utilities at once using vectorized NumPy operations
util = -A * (cons - B)**2.0 + C

# Print utility levels
util
```

[8]:
```
array([ 6.    ,  6.3136,  6.6144,  6.9024,  7.1776,  7.44  ,  7.6896,
        7.9264,  8.1504,  8.3616,  8.56  ,  8.7456,  8.9184,  9.0784,
        9.2256,  9.36  ,  9.4816,  9.5904,  9.6864,  9.7696,  9.84  ,
        9.8976,  9.9424,  9.9744,  9.9936, 10.    ,  9.9936,  9.9744,
        9.9424,  9.8976,  9.84  ,  9.7696,  9.6864,  9.5904,  9.4816,
        9.36  ,  9.2256,  9.0784,  8.9184,  8.7456,  8.56  ,  8.3616,
        8.1504,  7.9264,  7.6896,  7.44  ,  7.1776,  6.9024,  6.6144,
        6.3136,  6.    ])
```

[9]:
```
# Locate the index of the maximum
imax = np.argmax(util)

# Recover the utility and the consumption level at the maximum
u_max = util[imax]
cons_max = cons[imax]

print(f'Utility is maximized at c={cons_max} with u={u_max}')
```

```
Utility is maximized at c=2.0 with u=10.0
```

Exercise 3: Summing finite values

In this exercise, we explore how to ignore non-finite array elements when computing sums, i.e., elements which are either NaN ("Not a Number", represented by `np.nan`), $-\infty$ (`-np.inf`) or ∞ (`np.inf`). Such situations arise if data for some observations is missing and is then frequently encoded as `np.nan`.

1. Create an array of 1001 elements which are uniformly spaced on the interval $[0, 10]$. Set every second element to the value `np.nan`.

   Hint: You can select and overwrite every second element using `start:stop:step` array indexing.

   Using `np.sum()`, verify that the sum of this array is NaN.

2. Write a loop that computes the sum of finite elements in this array. Check that an array element is finite using the function `np.isfinite()` and ignore non-finite elements.

   Print the resulting sum of finite elements.

3. Since this use case is quite common, NumPy implements the function `np.nansum()` which performs exactly this task for you.

   Verify that `np.nansum()` gives the same result and benchmark it against your loop-based implementation.

   Hint: You'll need to use the `%%timeit` cell magic (with two %) if you want to benchmark all code contained in a cell.

Solution.

Part 1 — Create NaN array

[10]:
```python
import numpy as np

# Create uniformly spaced data
data = np.linspace(0, 10, 1001)

# Set every second element to NaN
data[1::2] = np.nan

# Print first 10 elements to illustrate pattern
data[:10]
```

[10]:
```
array([0.  ,  nan, 0.02,  nan, 0.04,  nan, 0.06,  nan, 0.08,  nan])
```

[11]:
```python
# Check that "standard" summation returns NaN
np.sum(data)
```

[11]:
```
np.float64(nan)
```

Part 2 — Sum finite elements

To implement a loop summing all finite elements, we proceed as follows:

[12]:
```python
# Initialize sum to zero
s = 0.0

# Loop through elements and only add them if they are finite
for x in data:
    if np.isfinite(x):
        # Add finite element
        s += x

print(f'The sum of finite values is {s}')
```

```
The sum of finite values is 2505.0000000000005
```

Part 3 — Benchmark `nansum()`

To benchmark the loop, we copy the code from above but remove the `print()` statement as we are not interested in benchmarking that part. Note that we need to use the `%%timeit` cell magic with *two* leading % to benchmark the entire cell, not just the first line.

[13]:
```
%%timeit
# Initialize sum to zero
s = 0.0

# Loop through elements and only add them if they are finite
for x in data:
    if np.isfinite(x):
        # Add finite element
        s += x
```

```
562 µs ± 2.42 µs per loop (mean ± std. dev. of 7 runs, 1,000 loops each)
```

We can compare this to NumPy's `np.nansum()` function:

[14]:
```
%timeit np.nansum(data)
```

```
5.68 µs ± 13.3 ns per loop (mean ± std. dev. of 7 runs, 100,000 loops each)
```

As you can see, `np.nansum()` is approximately 80 times faster (the exact value depends on your hardware and software).

Exercise 4: Approximating the sum of a geometric series

Let $\alpha \in (-1, 1)$. The sum of the geometric series $(1, \alpha, \alpha^2, \dots)$ is given by

$$\sigma = \sum_{i=0}^{\infty} \alpha^i = \frac{1}{1-\alpha}$$

In this exercise, you are asked to approximate this sum using the first N values of the sequence, i.e.,

$$\sigma \approx s_N = \sum_{i=0}^{N} \alpha^i$$

where N is chosen to be sufficiently large.

1. Assume that $\alpha = 0.9$. Write a `while` loop to approximate the sum σ by computing s_N for an increasing N. Terminate the computation as soon as an additional increment α^N is smaller than 10^{-10}. Compare your result to the exact value σ.
2. Now assume that $\alpha = -0.9$. Adapt your previous solution so that it terminates when the *absolute value* of the increment is less than 10^{-10}. Compare your result to the exact value σ.

   Hint: Use the built-in function `abs()` to compute the absolute value.

Solution.

Part 1 — Approximate sum with $\alpha = 0.9$

[15]:
```
alpha = 0.9

# True sum
sigma = 1 / (1 - alpha)

# Termination tolerance
tol = 1e-10

# Variable to store the running sum
s_N = 0.0
# Iteration counter
N = 0

while True:
    # Increment to be added to sum
    increment = alpha**N

    # Check whether increment satisfies termination criterion
    if increment < 1e-10:
        break

    # Increment sum and loop counter
    s_N += increment
    N += 1

print(f'Approximation: {s_N:.10f}')
print(f'Exact value:   {sigma:.10f}')
print(f'Difference: {sigma-s_N:.8e}')
print(f'Number of iterations: {N}')
```

```
Approximation: 9.9999999990
Exact value:   10.0000000000
Difference: 9.53034984e-10
Number of iterations: 219
```

Part 2 — Approximate sum with $\alpha = -0.9$

[16]:
```
alpha = -0.9

# True sum
sigma = 1 / (1 - alpha)

# Variable to store the running sum
s_N = 0.0
# Iteration counter
N = 0

while True:
    # Increment to be added to sum
    increment = alpha**N

    # Check whether increment satisfies termination criterion
    if abs(increment) < 1e-10:
        break

    # Increment sum and loop counter
    s_N += increment
    N += 1
```

```python
print(f'Approximation: {s_N:.10f}')
print(f'Exact value:   {sigma:.10f}')
print(f'Difference: {sigma-s_N:.8e}')
print(f'Number of iterations: {N}')
```

```
Approximation: 0.5263157895
Exact value:   0.5263157895
Difference: -5.01603203e-11
Number of iterations: 219
```