MPhil in scientific computing
Written assignment
Multimaterial simulations using the Ghost Fluid Method
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This project focuses on solving the compressible multimaterial unsteady Euler equations in one spatial dimension, with two versions of the ghost fluid method.

The report should be based on the following format:

- Introduction to the uses of the Euler equations, the numerical solutions of the equations, and a brief literature review of the ghost fluid and level set methods
- Explanation of the numerical methods used in providing the results detailed in this project
- Description of each of the tests carried out, describing both initial data and the results, with accompanying figures incorporating, at least, those required by the tests described below
- All information obtained from external sources should be appropriately cited, and a reference list included.

Numerical methods and plotting information

Throughout this project, two versions of the ghost fluid method should be used:

1. The basic ghost fluid method should be applied as described in Fedkiw, Aslam, Merriman and Osher "A Non-oscillatory Eulerian Approach to Interfaces in Multimaterial Flows (the Ghost Fluid Method)", (1999)
2. The Riemann ghost fluid method is outlined in Sambasivan and UdayKumar "Ghost Fluid Method for Strong Shock Interactions Part 1: Fluid-Fluid Interfaces", (2009).

For the fluid evolution, any suitable second-order numerical method may be used, and the level set update may be attempted by any scheme (first-order is appropriate here).

When presenting results, three different resolutions should be considered for each test case, in order to investigate convergence behaviour (100, 200 and 400 cells are good choices). Plots should be made for density, velocity, pressure and specific internal energy; all resolutions can be shown on a single plot. In cases of very sharp features, it may be necessary to also consider a very high resolution test case.

Moving contact discontinuity

The first objective is to investigate how ghost fluid methods can be used to provide a sharp description of a moving contact discontinuity (interface). These tests can be compared to an exact solution, which is simply the movement of the contact discontinuity with speed v. In this case, only density plots need be provided, and for a single suitable choice of resolution. The tests are:

1. Use the initial data on a domain $x \in [0, 1]$:

	ρ	v	p	γ
$x \leq 0.25$	1	0.5	1	1.4
$x > 0.25$	0.138	0.5	1	1.4

   with a final time of $t = 1$.

   Run this test for both the standard (single material) Euler solver, and ghost fluid method versions, where the initial level set function will be defined by

   $$\phi = x - 0.25$$

2. Use the initial data, again on a domain $x \in [0, 1]$:

	ρ	v	p	γ
$x \leq 0.25$	1	0.5	1	1.4
$x > 0.25$	0.138	0.5	1	1.67

   with a final time of $t = 1$.

   Run this test using ghost fluid methods only.

Simple ghost fluid tests

The second objective is to use the Ghost Fluid Method in tests in which there is some behaviour across an interface, but still achieves a known solution. For this, use the tests in Toro's book "Riemann Solvers and Numerical Methods for Fluid Dynamics", Table 4.1 in section 4.3.3. The left and right states should be separated using a level set function with ghost fluid boundary conditions applied, results can be compared to the exact solution.

These tests are designed to test the limits of numerical methods, therefore perfect results should not be expected in all cases. In cases where there is discrepancy, is there a physical or numerical reason for this?

Multimaterial shock tubes for gases

The third objective is to show the interaction of a shock wave with an interface between two different materials (air and helium).

Use the Ghost Fluid Method to simulate the following test ("Test B" from Fedkiw, Aslam, Merriman and Osher "A Non-oscillatory Eulerian Approach to Interfaces in Multimaterial Flows (the Ghost Fluid Method),(1999)) which uses a domain $x \in [0, 1]$, and initial data:

	ρ	v	p	γ
$x \leq 0.05$	1.3333	$0.3535\sqrt{10^5}$	1.5×10^5	1.4
$0.05 < x \leq 0.5$	1	0	1×10^5	1.4
$x > 0.5$	0.1379	0	1×10^5	1.67

This test uses physically realistic variables (i.e. they can be measured in SI units), and is run to a final time $t = 0.0012$s. Plot the results for this test for at least three different resolutions, for the density, velocity, pressure and specific internal energy.

The second shock tube test considers a small region of helium, again in a domain $x \in [0, 1]$, and has initial data:

	ρ	v	p	γ
$x \leq 0.05$	1.3333	$0.3535\sqrt{10^5}$	1.5×10^5	1.4
$0.05 < x \leq 0.4$	1	0	1×10^5	1.4
$0.4 < x \leq 0.6$	0.1379	0	1×10^5	1.67
$x > 0.6$	1	0	1×10^5	1.4

This test is comparable to 'Test B' from Wang *et al.*, "A thermodynamically consistent and fully conservative treatment of contact discontinuities for compressible multicomponent flows", but with physically realistic variables. The final time should be $t = 0.0014$ such that the wave pattern shown in this paper is visible. For this test, care must be taken to ensure that the extrapolation of variables is applied in the correct direction.

Water-gas shock tube test

The final objective is to consider a test between water and air, for which there is a large difference in the equation of state; water obeys a stiffened gas equation of state, $p = (\gamma - 1)\,\rho e + \gamma p_\infty$. In this test case, the basic ghost fluid method is not expected to deal well with the different materials, it is sufficient to use only the Riemann ghost fluid method.

The initial data for this test is taken from Chinnayya, Daniel and Saurel, "Modelling detonation waves in heterogeneous energetic materials", (2003), shown in Fig. 12 (note there is a typo in the initial data for the water pressure in the text of the paper). It may be necessary to run a high-resolution test to capture the difference between the shock and contact discontinuity.

	ρ	v	p	γ	p_∞
$x \leq 0.7$	1000	0	10^9	4.4	6×10^8
$x > 0.7$	50	0	10^5	1.4	0