Multimaterial Simulations using the Ghost Fluid Method

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Abstract

The unsteady, compressible Euler equations for multimaterial flow in one dimension have been solved numerically by employing a level set method and two versions of the Ghost Fluid Method.

1. Introduction

Literature review. Write about the usefulness of solving Euler eqs and CFD in general. What has happened historiccaly in the understanding of these? Different solvers, especially considerations of multi-material cases and use of level set methods.

The Euler equations govern adiabatic and inviscid flow of a fluid. In one dimension, with density ρ, velocity u, total energy E and pressure p, they are given by

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} = 0\,, \tag{1}$$

where the vectors of conserved quantities $\mathbf{U}$ and their fluxes $\mathbf{F}(\mathbf{U})$ are given by

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ E \end{bmatrix}\,, \quad \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ u(E+p) \end{bmatrix}\,.$$

It is sometimes convenient to work in terms of the primitive variables $\mathbf{W} = \begin{pmatrix} \rho, & u, & p \end{pmatrix}^T$. The total energy is the sum of the kinetic and potential energy of the system, i.e.

$$E = \frac{1}{2}\rho u^2 + \rho e\,, \tag{2}$$

where e is the internal energy, related to the other variables through the equation of state. For an ideal gas, the equation of state is

$$e = \frac{p}{(\gamma - 1)\rho}\,, \tag{3}$$

where γ denotes the ratio of specific heats for the gas. Several other fluids can be approximated by a so-called stiffened ideal gas equation of state,

$$e = \frac{p + \gamma p_\infty}{(\gamma - 1)\rho}\,. \tag{4}$$

Here, a material-dependent stiffening parameter p_∞ has been introduced. Note that Eq. (4) reduces to Eq. (3) for materials with $p_\infty = 0$.

Which special considerations need to be taken into account for multimaterial flow? Differences in EoS means we need to think carefully about material interfaces.

The numerical methods which have been employed are explained in section 2, before several test cases and their results are discussed in section 3. Section 4 concludes the report.

2. Numerical methods

2.1. Numerical solutions for the Riemann Problem

Given a conservation equation and two sets of piecewise constant states separated by a single discontinuity, the initial value problem of evolving this system in time is called a Riemann problem. It is very useful in the study of the Euler equations for two reasons. Firstly, it allows for exact (up to an arbitrary accuracy) solutions for systems obeying Eq. (1) which have a single contact discontinuity. Secondly, the discretization of space which is inevitable in computational schemes for solving differential equations, allows for precise solvers of conservation equations based on the solutions of many local Riemann problems.

For the Euler equations and initial conditions

$$\mathbf{W}(x, t=0) = \begin{cases} \mathbf{W}_L, & x \leq 0 \\ \mathbf{W}_R, & x > 0 \end{cases}\,,$$

Figure 1 shows typical states the system can have, in addition to characteristics for waves propagating in space-time. There are two types of resultant waves that propagate through space, in addition to the contact wave (dashed). The first is a shock wave, depicted as a thick line on the left, while the second is a rarefaction wave, shown as several gradually decaying (in strength) lines on the left. Any combination of shock and rarefaction waves can occur on the left and right sides of the contact discontinuity, and the result is only dependent on $\mathbf{W}(x, 0)$.

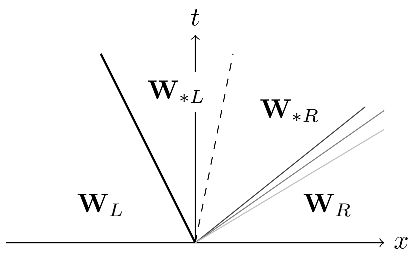

Figure 1: Possible wave configurations for the Riemann problem for Euler's equations in one dimension.

History of solving Riemann problems. Mention different approximate solvers.

For this project, the exact solver has been implemented.

Each of the four states separated by the waves are constant. Additionally, pressure and velocity is constant for the star states. How much more should be said about this?

2.2. Schemes for the Euler equations

Explain the order of schemes and the difference between centered and RP-based schemes.

2.2.1. Slope-Limiting Centered (SLIC)

2.2.2. Weighted-Average Flux (WAF)

2.3. Level-set method

2.4. Ghost Fluid Methods

2.4.1. Original Ghost Fluid Method

2.4.2. Riemann Ghost Fluid Method

3. Results

3.1. Moving contact discontinuity

3.2. Simple ghost fluid tests

3.3. Multimaterial shock tubes for gases

3.4. Water-gas shock tube test

4. Conclusions

Acknowledgements

Thanks Steve.

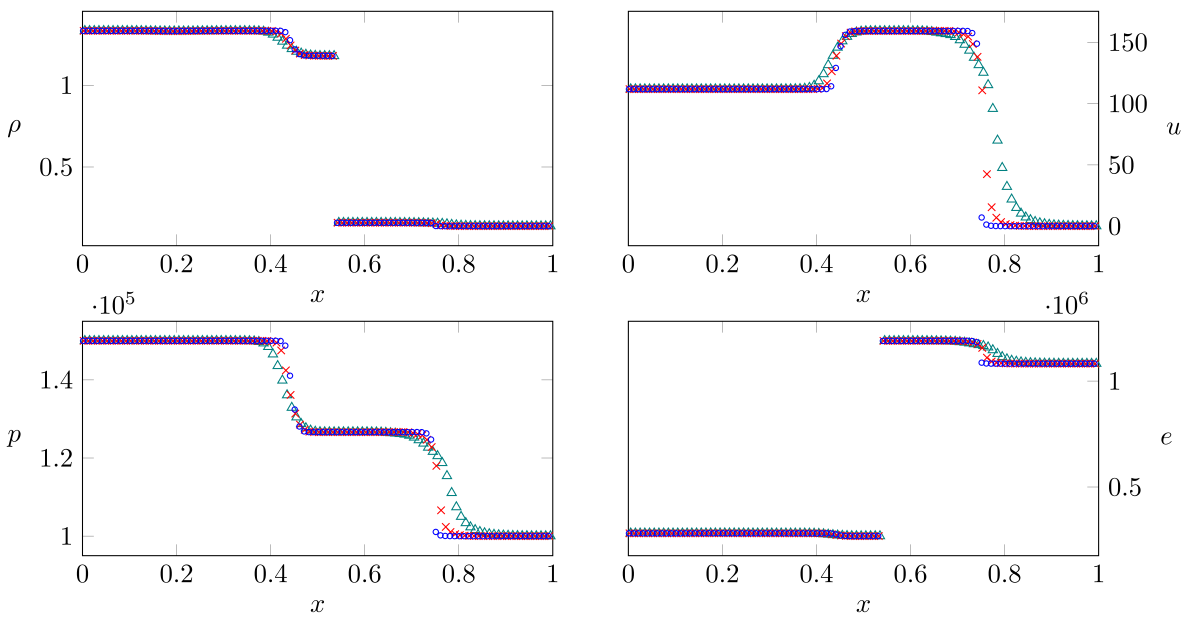

Figure 2: Original Ghost Fluid method for test C. △$N = 100$ ×$N = 200$ ○$N = 400$

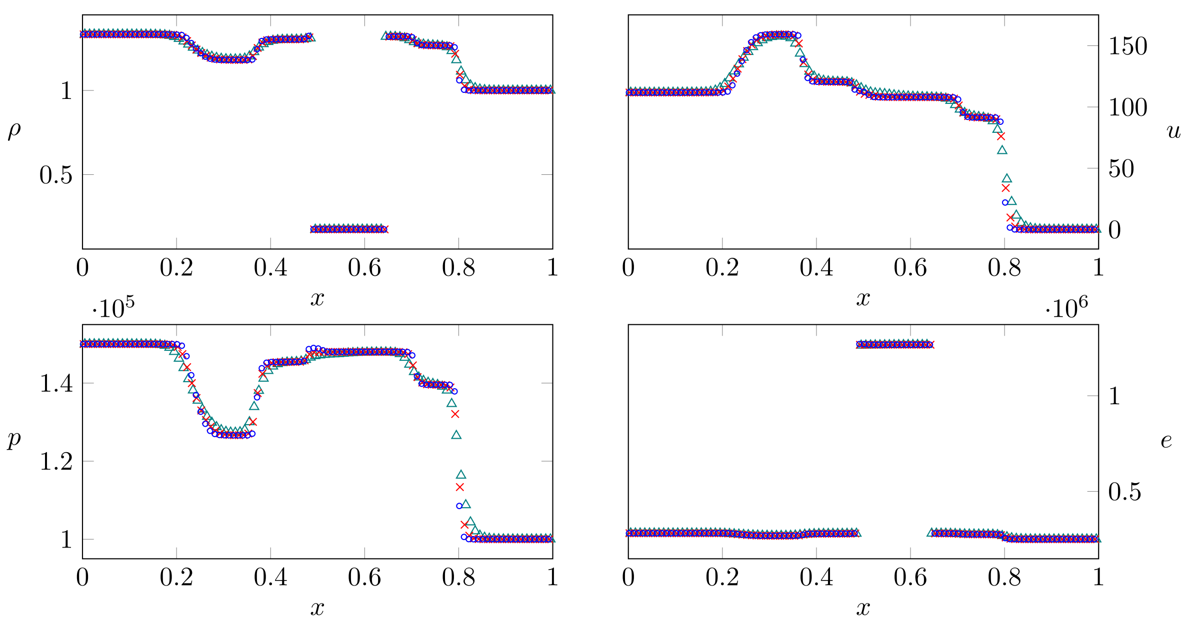

Figure 3: Riemann Ghost Fluid method for test C. △$N = 100$ ×$N = 200$ ○$N = 400$

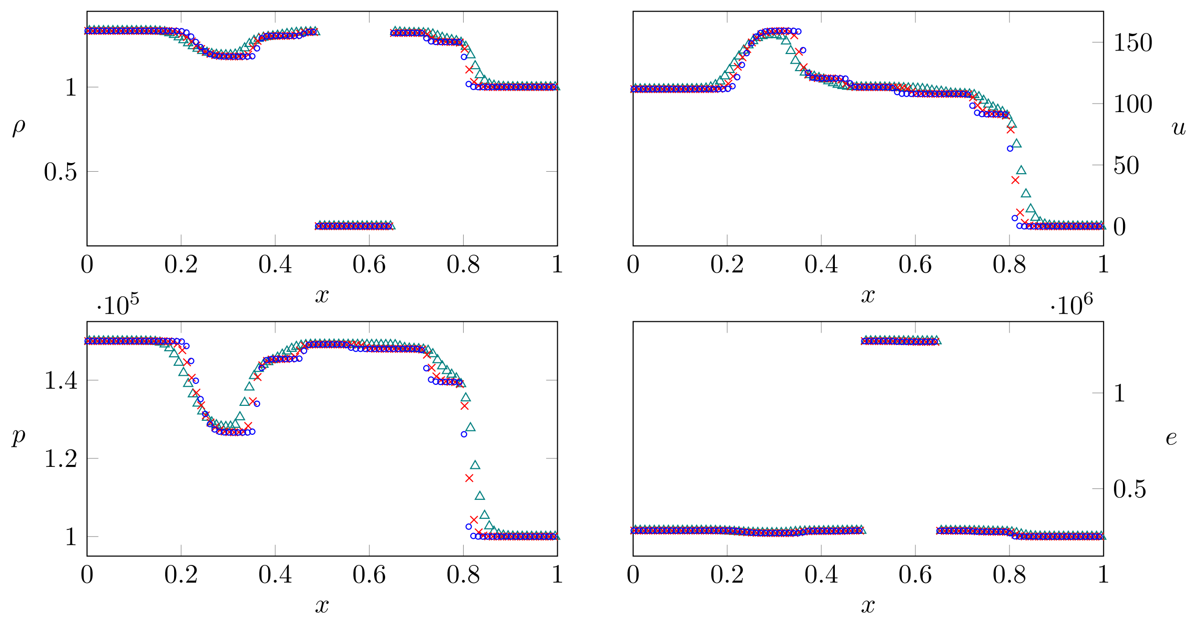

Figure 4: Original Ghost Fluid method for test D. △$N = 100$ ×$N = 200$ ∘$N = 400$

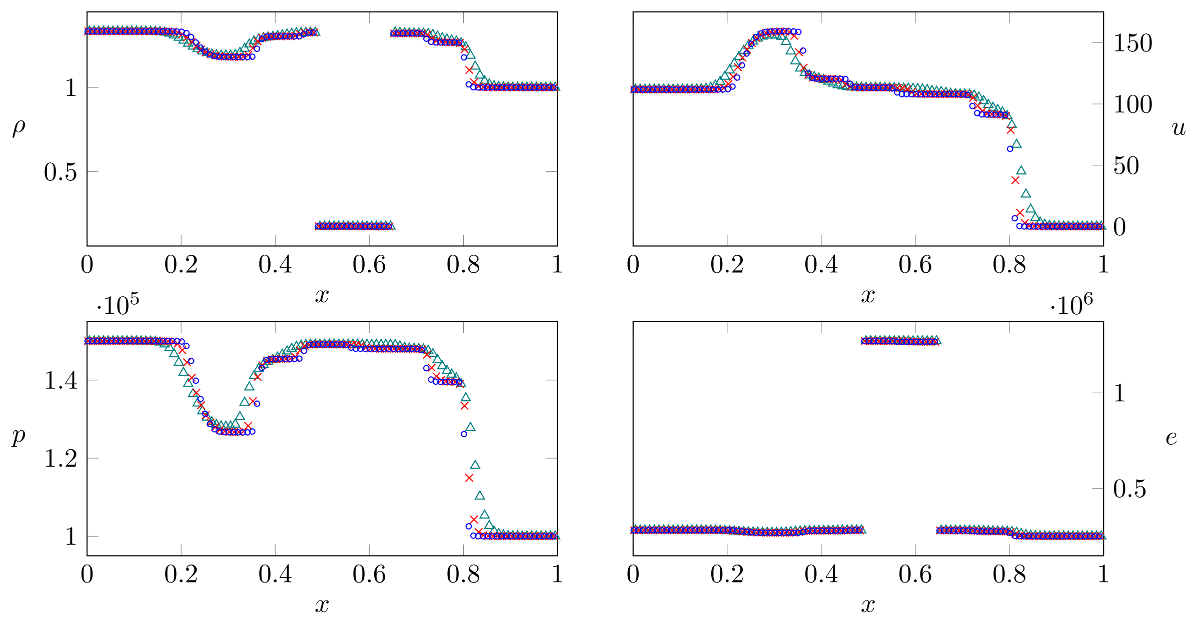

Figure 5: Riemann Ghost Fluid method for test D. △$N = 100$ ×$N = 200$ ∘$N = 400$

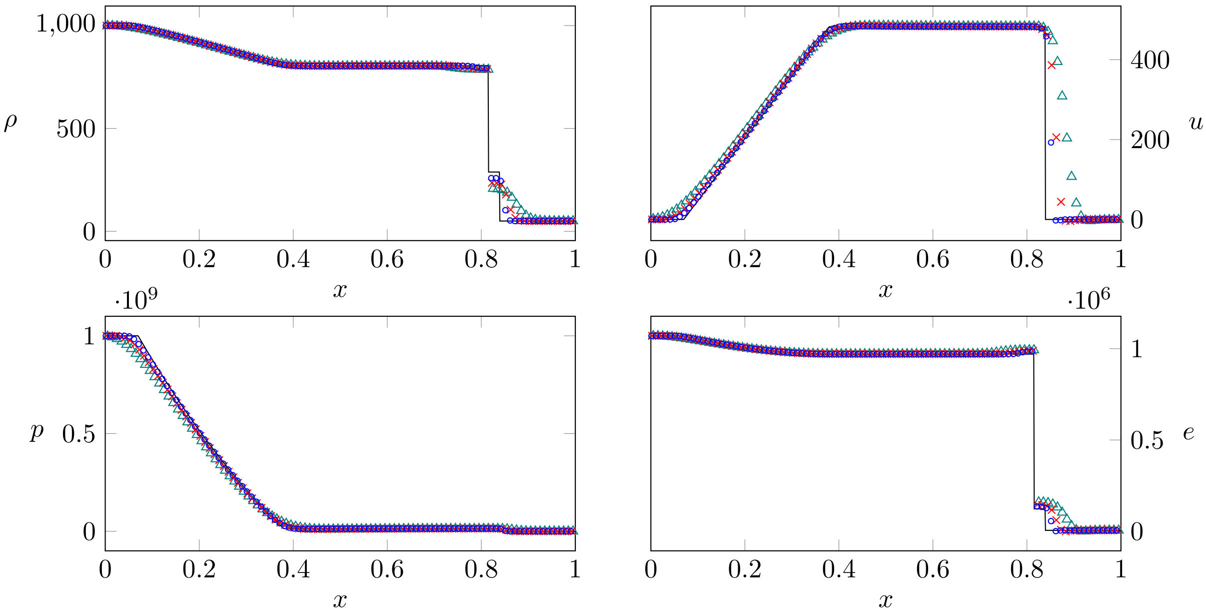

Figure 6: Riemann Ghost Fluid method for test E. △$N = 100$ ×$N = 200$ ○$N = 400$

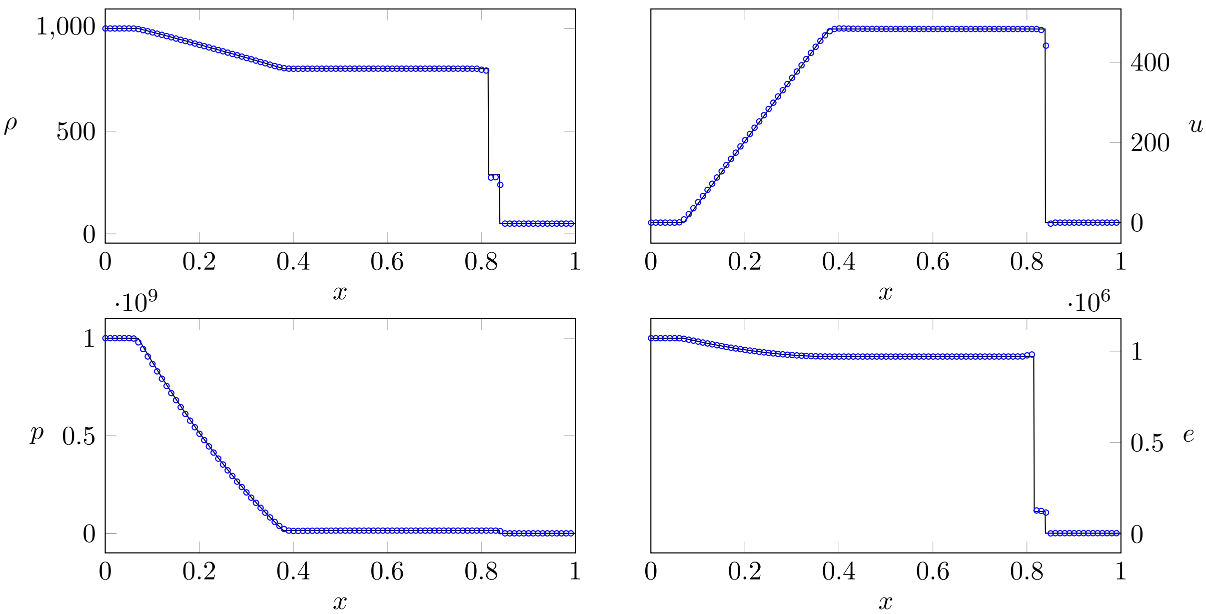

Figure 7: Riemann Ghost Fluid method for test E with higher accuracy ($N = 1000$).