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Short communication

Flow of Bingham plastics in a lid-driven square cavity

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The present work is concerned with the benchmark problem of flow in a lid-driven square cavity. The geometry offers a typically simple two-dimensional flow, for which accurate solutions exist for Newtonian fluids [1], while for non-Newtonian *viscoelastic* fluids numerical solutions have also appeared recently [2]. An important class of non-Newtonian materials exhibits a yield stress, which must be exceeded before significant deformation can occur. The models presented for such so-called *viscoplastic* materials include the Bingham, Herschel–Bulkley and Casson [3]. Several researchers have studied these materials in non-trivial flows (see [4–8]). For the benchmark problem at hand and to the authors' best knowledge, the only numerical *viscoplastic* work was done by Bercovier and Engelman [9] with a regularized Bingham model. These authors included inertia ($Re = 1$) and studied four values of the yield stress (2.5, 5, 7.5, 10) showing in an elementary way the yielded/unyielded regions. At that early time, only 100 (10×10) elements were used for the computations.

To model the stress-deformation behavior of materials with a yield stress, the Bingham constitutive equation has been modified by Papanastasiou [5]. In simple shear flow it takes the form

$$\tau = \tau_{\mathrm{y}}[1 - \exp(-m\dot{\gamma})] + \mu\dot{\gamma}, \tag{1}$$

where τ is the shear stress, $\dot{\gamma}$ the shear rate, $\tau_{\mathrm{y},}$ the yield stress, μ is a constant plastic viscosity, and m is the stress growth exponent, which has been introduced to avoid the discontinuity inherent in any viscoplastic model (also called a "regularization" parameter). In this way, the equation is valid for both the yielded and unyielded areas. Note that when the shear stress τ falls below $\tau_{\mathrm{y},}$ a solid structure is formed (unyielded). This equation mimics the ideal Bingham plastic (for $m \geq 200\,\mathrm{s}$ or by dimensionless arguments when $m\tau_y/\mu \geq 200$ [8]). For flow simulations, a dimensionless Bingham number *Bn*, is defined respectively by

$$Bn = \frac{\tau_{\mathrm{y}} H}{\mu V}, \tag{2}$$

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where H is a characteristic length (the channel width or height) and V is a characteristic speed taken as the velocity of the lid. In all cases, the Newtonian fluid corresponds to $Bn = 0$. However, at the other extreme of an unyielded solid, $Bn \to \infty$.

In full tensorial form the constitutive Eq. (1) of the Bingham plastic is then written as

$$\bar{\bar{\tau}} = \eta \bar{\bar{\dot{\gamma}}}, \tag{3}$$

where η is the apparent viscosity given by

$$\eta = \mu + \frac{\tau_{\mathrm{y}}}{|\dot{\gamma}|}[1 - \exp(-m|\dot{\gamma}|)] \tag{4}$$

and $|\dot{\gamma}|$ is the magnitude of the rate-of-strain tensor $\bar{\bar{\dot{\gamma}}} = \nabla \bar{v} + \nabla \bar{v}^T$, which is given by

$$|\dot{\gamma}| = \sqrt{\frac{1}{2}\mathrm{II}_{\dot{\gamma}}} = \left[\frac{1}{2}\{\bar{\bar{\dot{\gamma}}} : \bar{\bar{\dot{\gamma}}}\}\right]^{1/2}, \tag{5}$$

where $\mathrm{II}_{\dot{\gamma}}$ is the second invariant of $\bar{\bar{\dot{\gamma}}}$. To track down yielded/unyielded regions, we employ the criterion that the material flows (yields) only when the magnitude of the extra stress tensor $|\tau|$ exceeds the yield stress τ_{y}, i.e.

yielded:

$$|\tau| = \sqrt{\frac{1}{2}\mathrm{II}_{\tau}} = \left[\frac{1}{2}\{\bar{\bar{\tau}} : \bar{\bar{\tau}}\}\right]^{1/2} > \tau_{\mathrm{y}}, \tag{6a}$$

unyielded:

$$|\tau| \leq \tau_{\mathrm{y}}. \tag{6b}$$

The above constitutive equation is solved together with the conservation equations using the finite element method (FEM) as explained previously [6]. Two finite element grids have been used with 400 (MESH1) and 1600 (MESH2) elements, and the singularity is handled at the lid comer as shown in Fig. 1. This is the usual way of dealing with the singularity, while a more detailed study has been given in [2]. We note that because of the use of the Papanastasiou modification in the Bingham constitutive equation, the flow is essentially viscous, and it does not present any extra difficulties around the singularities, as is the case with viscoelastic fluids.

We have set μ, H, V as unity (therefore $Bn = \tau_y$), and changed the yield stress values from $\tau_y = 0$ (Newtonian fluid) to $\tau_y = 500{,}000$. Thus, the whole range of Bingham numbers $0 \leq Bn < \infty$ could be studied. Two values of m were used: 200 s and 1000 s. The results for the progressive growth of the unyielded zone (shaded) for the lid-driven flow of Bingham plastics in a square cavity are shown in Fig. 2 as obtained with MESH2 and $m = 1000$ s. As the Bn number increases the unyielded region increases and occupies more of the cavity but always leaves a yielded region close to the driven lid. Qualitatively, similar results were also reached in the very early work by Bercovier and Engelman [9].

It is instructive to look at the streamlines in these cases. This is done in Fig. 3, where the streamlines obtained through integration of the velocity field with boundary conditions for the stream function $\psi = 0$ at the periphery, are shown together with the yielded/unyielded regions. Ten equally-spaced streamlines are shown between the zero value at the boundary and the minimum value at the eye of the vortex. Strictly

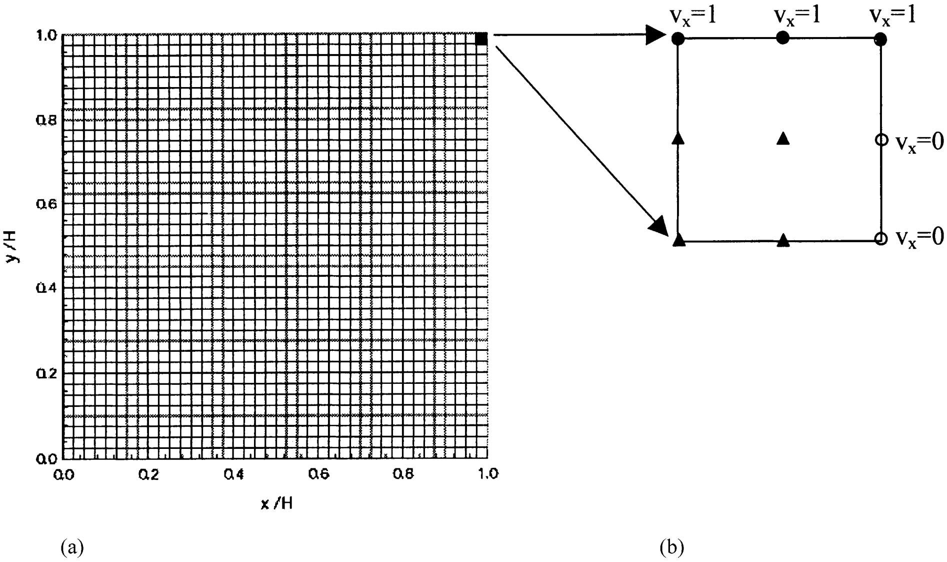

Fig. 1. (a) Finite element grids used in the simulations. The original grid MESH1 contains 400 elements (20 × 20) and has been further subdivided into 4 × 4 giving a denser grid MESH2 with 1600 elements (40 × 40); (b) Handling of the singular corner points in the upper right-corner element.

speaking, the unyielded zone should be completely quiescent, or else an Eulerian steady flow cannot exist. However, because the viscoplastic model used is essentially viscous, there are streamlines crossing inside the unyielded island, indicating that these are really apparently unyielded regions (AUR), where the deformation is extremely small. A particle going through these regions would take an extremely long time to close its streamline contour.

Regarding the pressure distribution along the lid, the results for 3 different *Bn* numbers are shown in Fig. 4. The pressure is set to zero at a reference point (here the mid-point at the lower wall) and takes very high values at the corner singularities, which grow higher with *Bn* number (see Table 1). Again, because of the essential viscous behavior of the model, no particular problems were found with the pressure, except its very high values.

Other quantities of interest in this problem are the vortex intensity (value of the stream function in the eye of the vortex, $-\psi^*_{\mathrm{min}}$) and its location. Because of symmetry due to creeping flow ($Re = 0$), its location

Table 1
Maximum pressure values for different *Bn* number (MESH2, $m = 1000$ s)

Bn	Pressure (P_{max})
0	227.31
2	243.93
20	417.47
200	2905.5
1000	14304.0

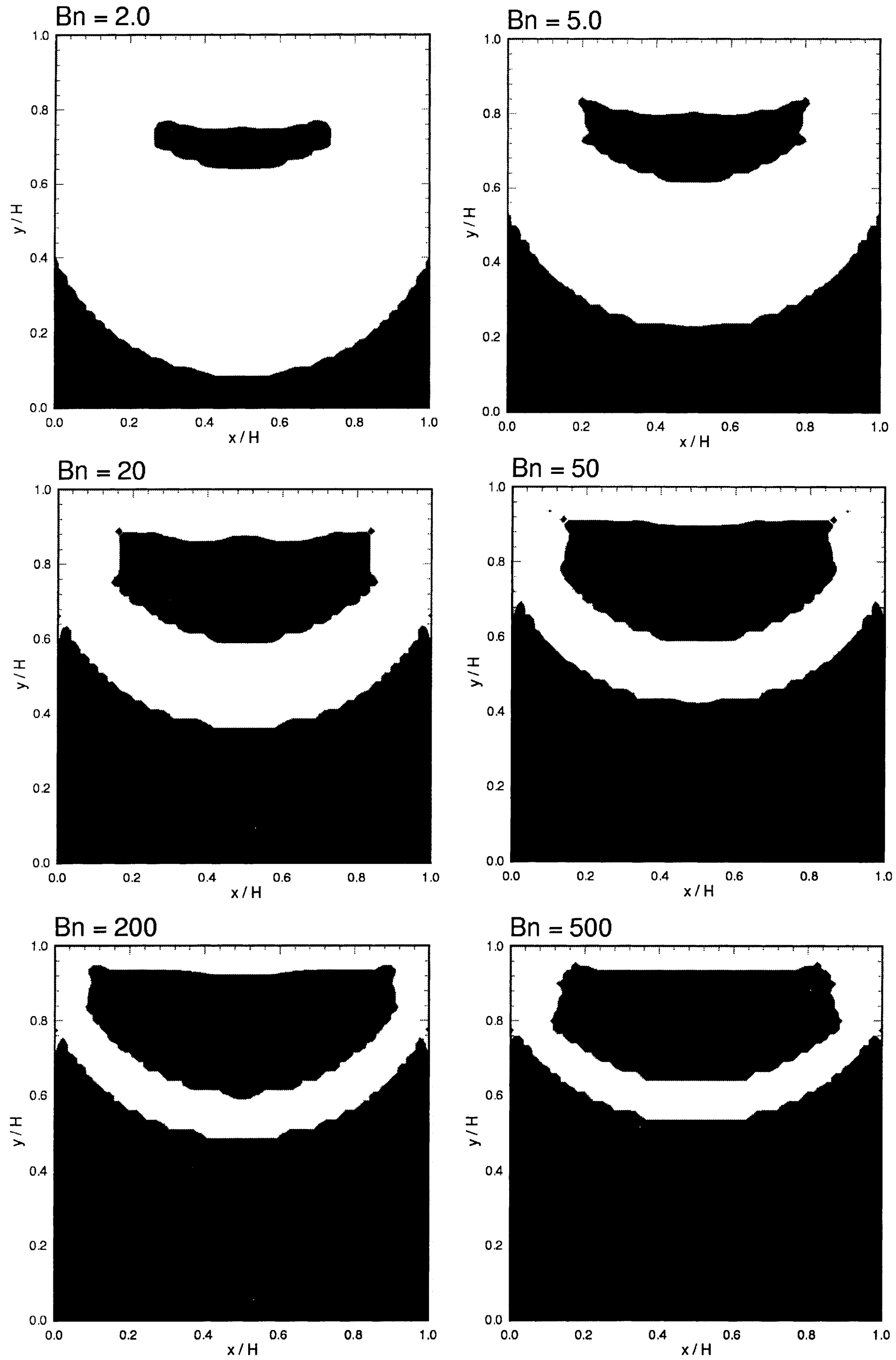

Fig. 2. Progressive growth of the unyielded zone (shaded) for lid-driven flow of Bingham plastics in a square cavity (MESH2, $m = 1000\,\mathrm{s}$).

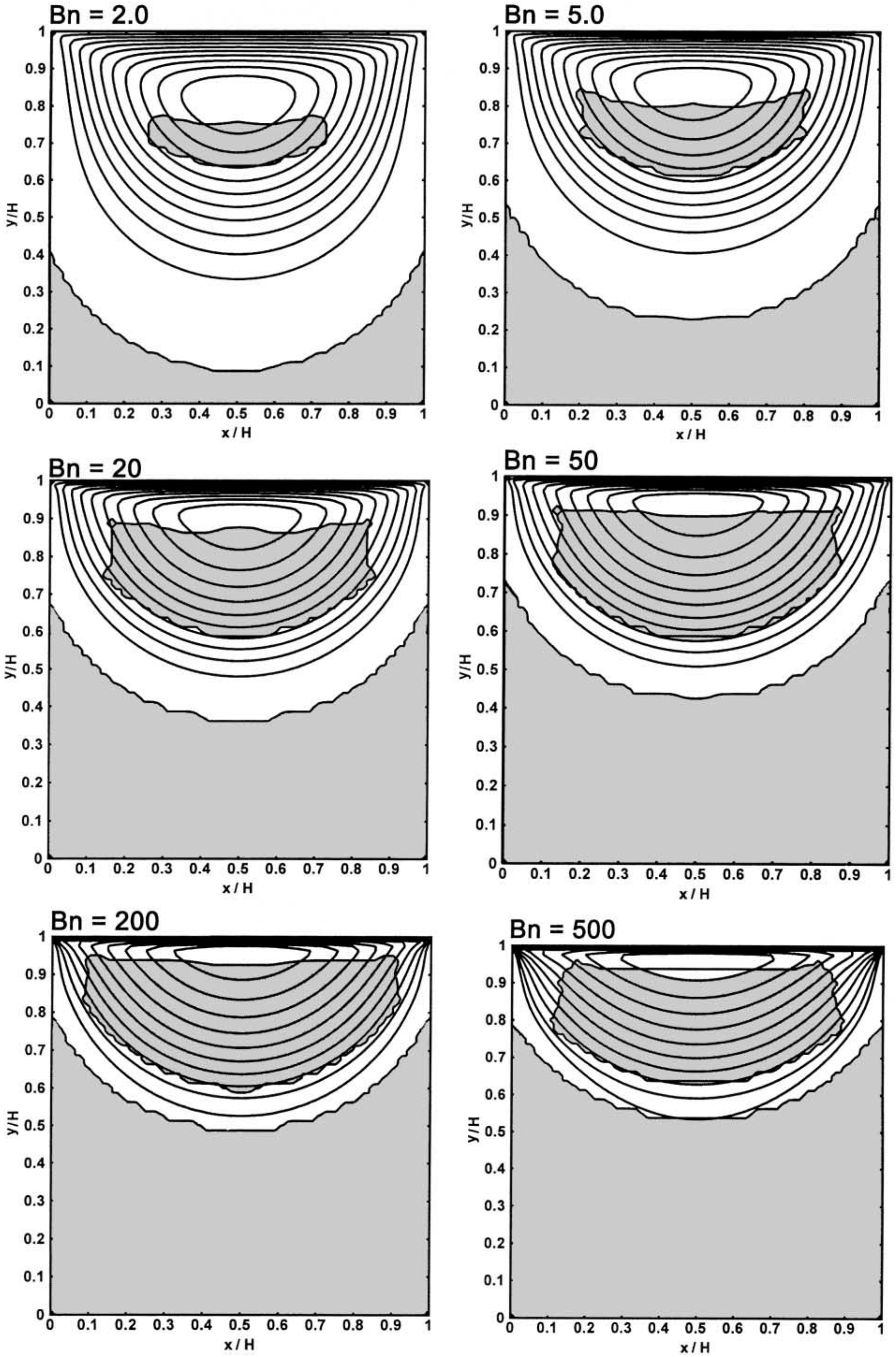

Fig. 3. Streamline patterns together with the yielded/unyielded zones for lid-driven flow of Bingham plastics in a square cavity (MESH2, $m = 1000\,\mathrm{s}$).

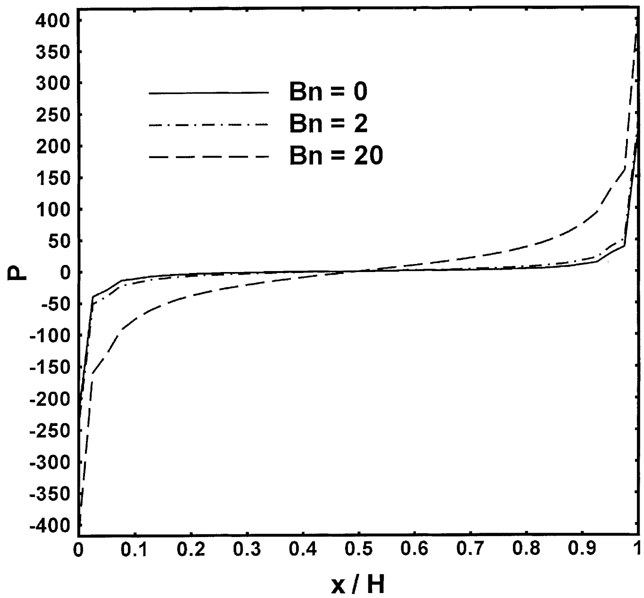

Fig. 4. Dimensionless pressure distribution along the lid for different values of *Bn* (MESH2, $m = 1000\,\mathrm{s}$).

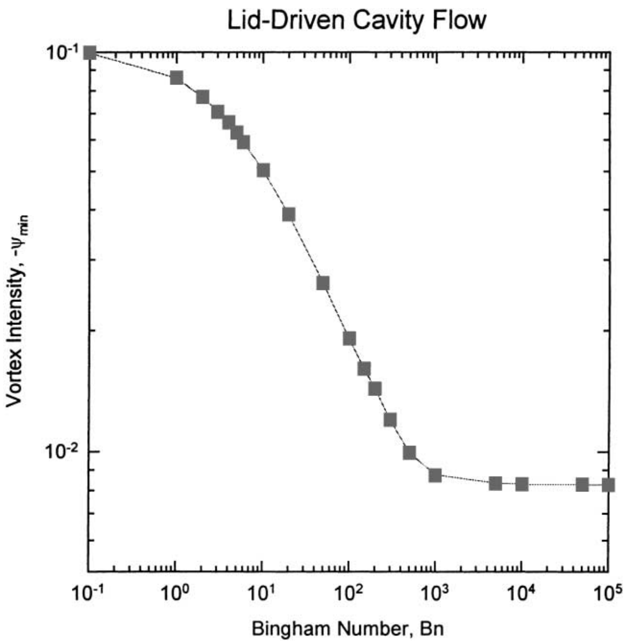

Fig. 5. Vortex intensity vs. *Bn* (MESH2, $m = 1000\,\mathrm{s}$).

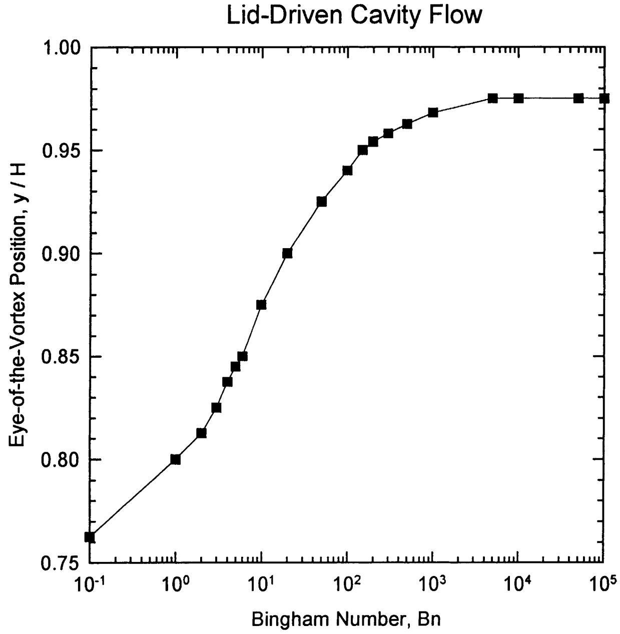

Fig. 6. Vertical eye-of-the-vortex position *Yv* vs. dimensionless Bingham number *Bn*. The horizontal coordinate is always $x = 0.5$ (MESH2, $m = 1000$ s).

is always at $x = 0.5$, but the y-value changes (it moves closer to the lid as *Bn* increases). The Newtonian values are $-\psi^*_{\mathrm{min,N}} = 0.0995$ and $y_N = 0.7625$, in very good agreement with the results of 0.0999 and 0.7643, respectively, given in [2]. The results for the range of *Bn* numbers are shown in Figs. 5 and 6, respectively. It is seen that both the quantities follow a sigmoidal form, and after $Bn = 1000$, they do not change appreciably. The present results indicate that increasing the yield stress decreases substantially the vortex intensity and moves the eye of the vortex closer to the lid.

Two issues are raised when using the Papanastasiou modification to the ideal Bingham model. One is the mesh dependence of the results and the other is the value of the parameter *m*, beyond which the results are independent of its value. Both issues have been successfully answered in the paper by Burgos and Alexandrou [7] and have been confirmed in the present work. High values of *m* combined with local numerical errors ϵ make the term $e^{-m(\dot{\gamma}\pm\varepsilon)}$ in the Papanastasiou constitutive equation a non-smooth function, giving non-smooth yield lines, as evidenced in Figs. 2 and 3. Higher values of *m* render the exponent more troublesome producing more zig-zagging behavior. On the other hand, more elements do not affect the results very much, they just render the zig-zagging smaller.

Acknowledgements

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