Projection Method for Incompressible Flow

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Abstract

1. Introduction

- Introduction to the Navier-Stokes equations: history, difficulties, prize money.
- Show how they simplify under the assumption of incompressibility. Nondimensionalize the equations.
- Define stream function and vorticity.
- Talk about viscosity, and the two different representations we consider (Newtonian fluid with μ = const, Non-Newtonian fluid examplified by Bingham plastic).
- Introduce the lid-driven cavity test problem, explain what has been done in the literature.

2. Numerical methods

2.1. Newtonian fluid

- (Fractional step) projection methods
- Spatial discretization: the finite volume method
- Remember staggered grid!
- Boundary conditions
- Choice of time step for stability
- Solving the systems of equations with Eigen
- Checking if steady-state has been reached
- Transforming results to vorticity streamline formulation

2.2. Bingham plastic fluid

- Transient solution possible, cite appropriate paper and explain difficulty in discretizing the viscuous term
- Reynolds number zero leads to removal of time-dependency because of alternative nondimensionalization of pressure
- Treatment of singularity in effective viscosity: regularization
- Finite volume method w/o need for staggered grid, discretization of viscuous term
- Solution of steady-state system: SIMPLE and its extensions

3. Results

3.1. Transient behaviour

- Impulsively started
- What happens as a function of time?
- Results for different Re?

3.2. Steady-state solution

All results for Re=100,400,1000,3200. . . . .

- 1D slices in the geometric center, including Ghia's results
- Stram lines and velocity vector fields
- Vorticity
- (Pressure field)

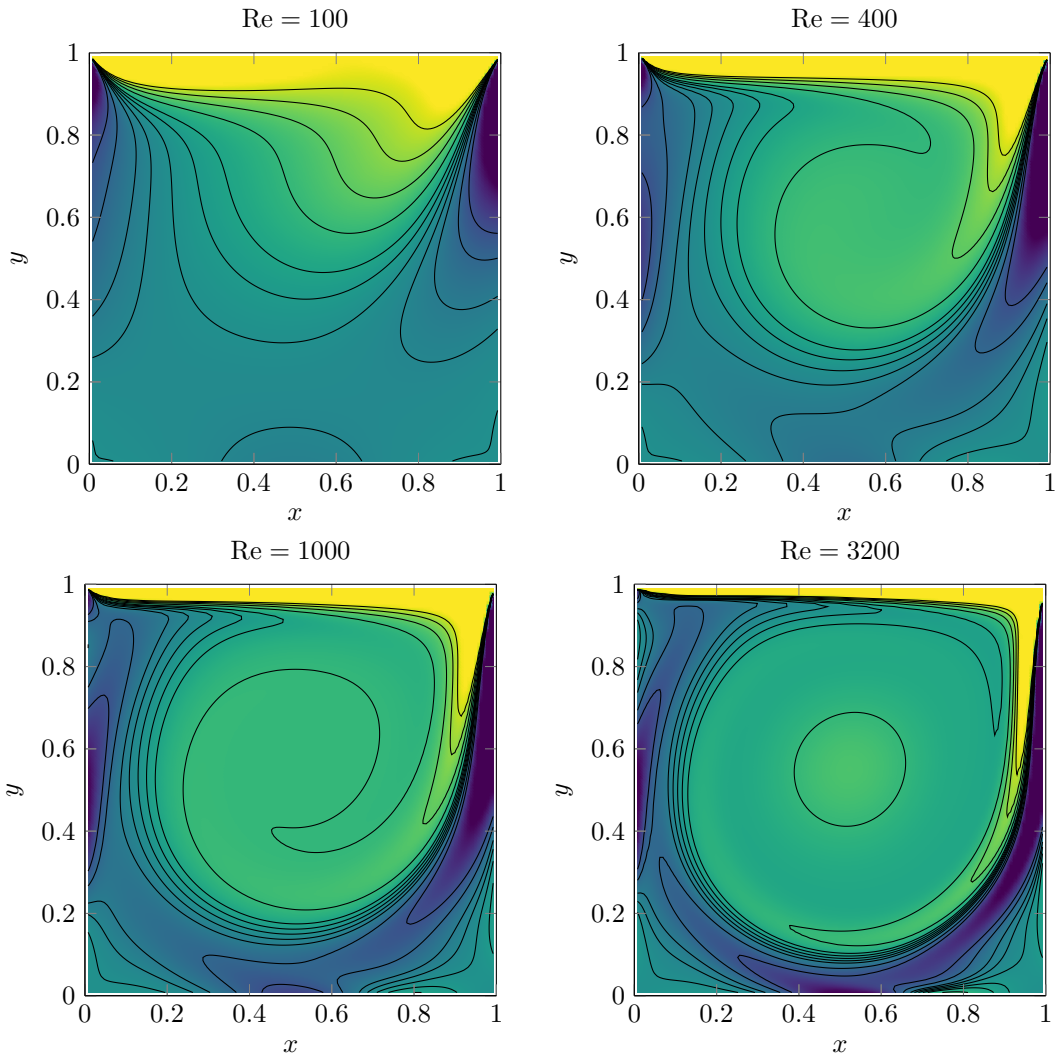

Figure 1: Vorticity for different Reynold's numbers

3.3. Computational efficiency

- Computational complexity of the linear systems
- Runtime (and no of time steps) as a function of Re and N
- Plots of Δt vs. N for different Re

4. Discussion

- Everything works, results exactly as in literature
- Transient method is slow for high Re, SIMPLE could be better
- Other improvements include Hockney algorithm and multigrid methods
- Discuss stability and computational efficiency of

5. Conclusions

References