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CS 2235 Data Structures and Algorithms

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(2 Points each) Using the definition of big-oh notation, prove or disprove the following assertions (i.e., you must provide the c > 0 and $n_0 \ge 1$ that fulfills the definition, or give a formal argument for why the assertion is false).

1.
$$2n^3 - 7n^2 + 100n - 36$$
 is in $O(n^3)$
$$|2n^3 - 7n^2 + 100n - 36| \le 2n^3 + |7n^2| + 100n + |36|$$

$$\le 2n^3 + 7n^3 + 100n^3 + 36n^3$$

$$\le 145n^3$$
 Thus, $2n^3 - 7n^2 + 100n - 36 \le 145n^3$ for all $n \ge 1$ So, $2n^3 - 7n^2 + 100n - 36$ is in $O(n^3)$

2.
$$10n + 3 \log(n)$$
 is in O(n)

$$\begin{split} |10n+3\,\log(n)| &\leq 10n+3\,\log(n)\\ &\leq 10n+3\log(n)n\\ &\leq 13\log(n)n \end{split}$$

Thus, $10n + 3 \log(n) \le 13 \log(n)n$ for all $n \ge 1$

So, $10n + 3 \log(n)$ is in O(n)

3. n/1000 is in O(1)

$$|n/1000| \le (1/1000)n$$

 $\le (1/1000)n$

Thus,
$$n/1000 \le (1/1000)n$$
 for all $n \ge 1$

The assertion is false; n/1000 is in O(n)

4. $\log(n)^2 + \log(n)/30$ is in $O(\log(n)^2)$

$$\begin{split} |log(n)^2 + log(n)/30| &\leq log(n)^2 + (1/30)log(n) \\ &\leq log(n)^2 + (1/30)log(n)^2 \\ &\leq (31/30)\ log(n)^2 \end{split}$$

Thus,
$$log(n)^2 + log(n)/30 \le (31/30) log(n)^2$$
 for all $n \ge 1$

So,
$$log(n)^2 + log(n)/30$$
 is in $O(log(n)^2)$

5.
$$n^2/\log(n) + 3n$$
 is in $O(n^2)$

$$\begin{split} |n^2/\log(n) + 3n| & \leq (1/log(n))n^2 + 3n \\ & \leq (1/log(n))n^2 + 3n^2 \\ & \leq (3/log(n))n^2 \end{split}$$

Thus,
$$n^2/\log(n) + 3n \le (3/\log(n))n^2$$
 for all $n \ge 1$

So,
$$n^2/\log(n) + 3n$$
 is in $O(n^2)$

(2 Points each) For each function below, provide the "tightest" big-oh bound. You can do this from the definition of big-oh if you are unsure of the answer, or you can use shortcuts and just provide the big-oh value.

```
6. 36n is in O(n)
7. n²/2 + 15n is in O(n²)
8. (n²/4)(8/n) is in O(n)
9. n + 10 log(n) is in O(n)
10. 87262 is in O(1)
```

(4 Points each) For each method below, provide the tightest big-oh running time.

```
public int m1FindLargest(int[] array) {
    if (array.length != 0) {
        int value = array[0];
        for (int i = 1; i < array.length; i++) {
            if (array[i] > value) {
                value = array[i];
            }
        }
        return value;
    }
    return -1;
}
```

The algorithm is in O(n) (worst case scenario).

```
public void m2PrintTriangle(int size) {
    for (int i = 1; i <= size; i++) {
        for (int j = 1; j <= 1; j++) {
            System.out.print("*");
        }
        System.out.println();
    }
}</pre>
```

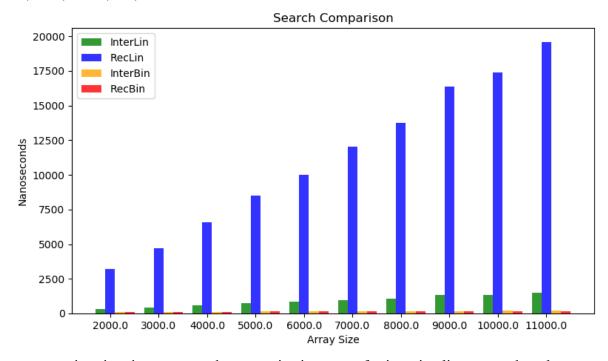
The algorithm is in $O(n^2)$.

```
public void m3PrintBooks(String books[], int[] stars) {
    if (books.length == stars.length) {
        for (int i = 0; i < books.length; i++) {
            System.out.print(books[i] + "'s stars: ");
            for (int j = 0; j < stars[i]; j++) {
                 System.out.print("*");
            }
            System.out.println();
        }
    }
}</pre>
```

The algorithm is in $O(n^2)$ (worst case scenario).

Part 2 - Binary Search

N, IterLin, RecLin, IterBin, RecBin 2000,321,3199,115,120 3000,435,4722,115,106 4000,573,6570,119,117 5000,726,8531,139,144 6000,831,10002,142,140 7000,949,12027,143,138 8000,1074,13757,164,150 9000,1313,16376,170,174 10000,1329,17414,190,167 11000,1511,19610,186,168



The computation time increases as the array size increases for iterative linear search and recursive linear search. The computation time is relatively constant for iterative binary search and recursive binary search while slightly increasing. Recursive linear search has the greatest computation time. Iterative binary search and recursive binary search have the least computation time. Benchmarking was performed on a 4-core processor with 4.0 GHz core frequency.