Electromagnetism and Gravity with Continuous Spin

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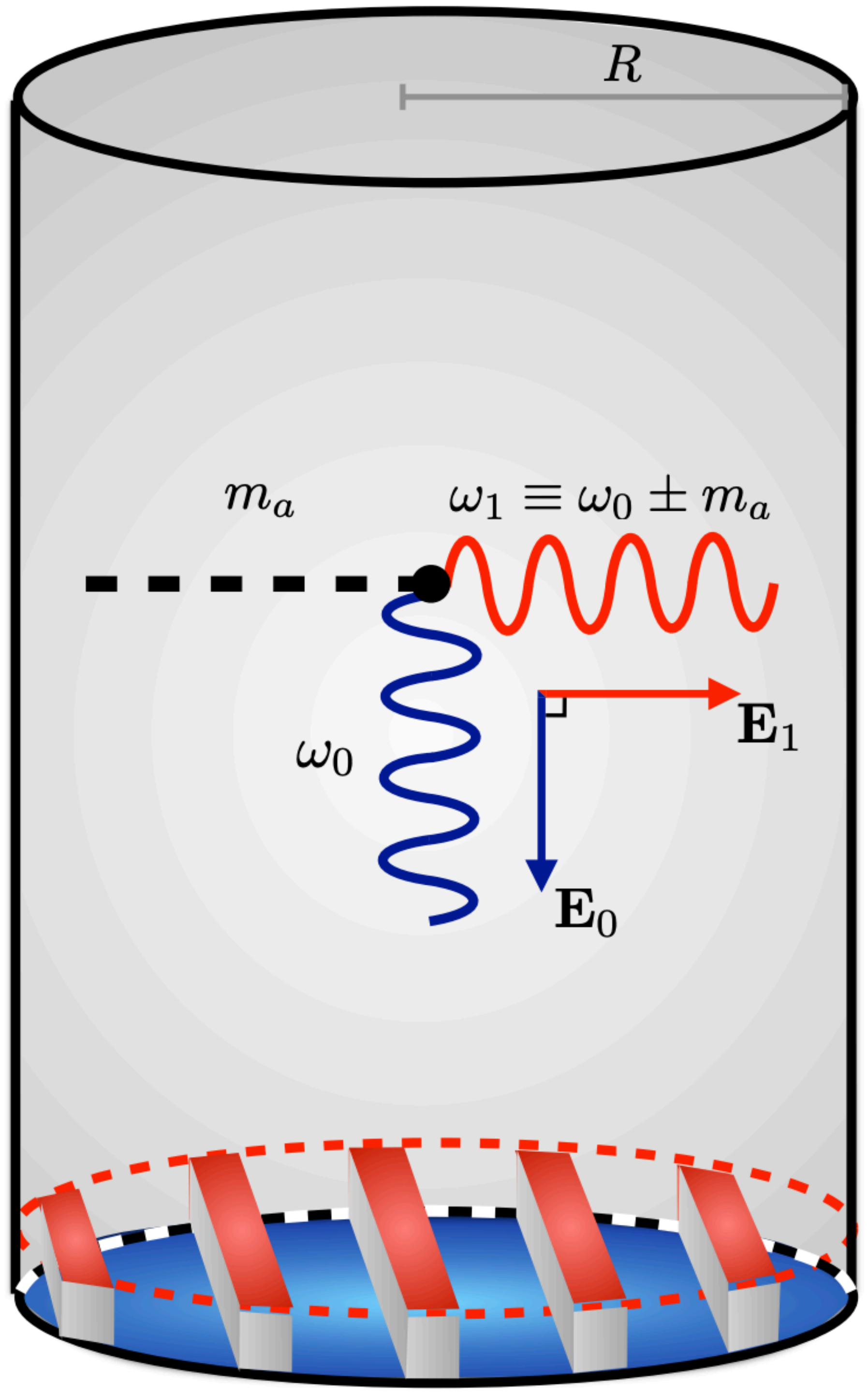
Stanford Pheno Seminar — February 3, 2023

arXiv:2302.xxxxx, with Philip Schuster and Natalia Toro

My main focus is new experimental methods to search for dark matter.



axion upconversion

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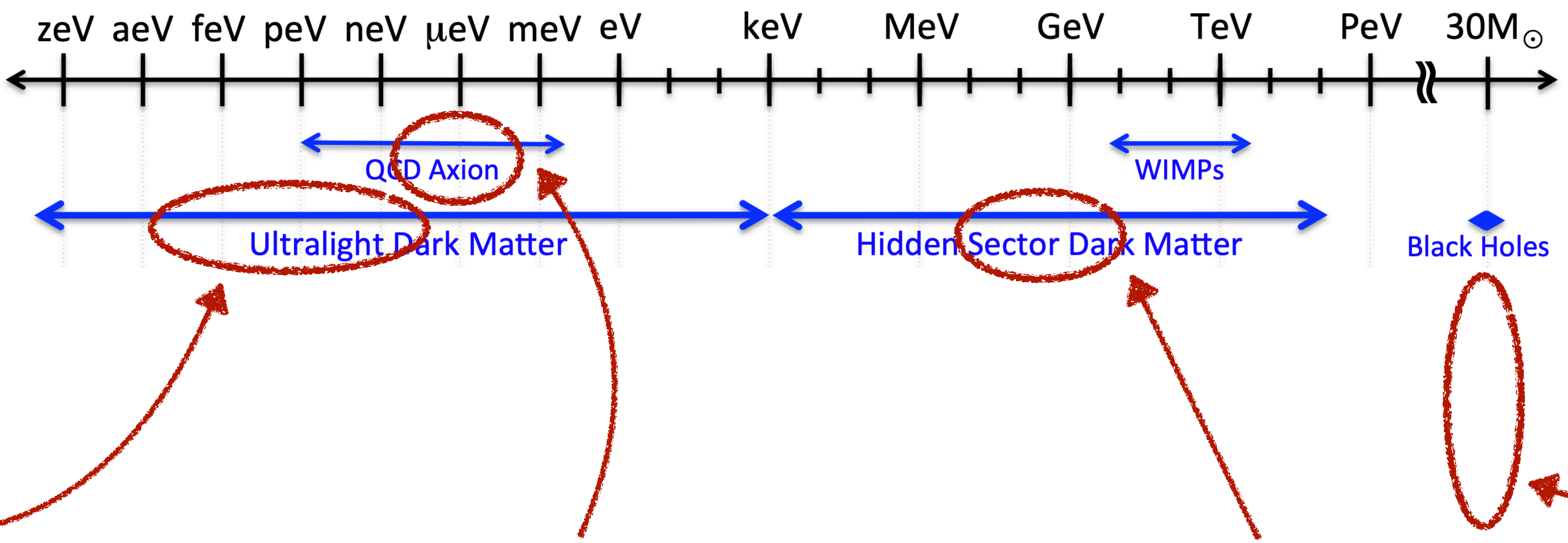

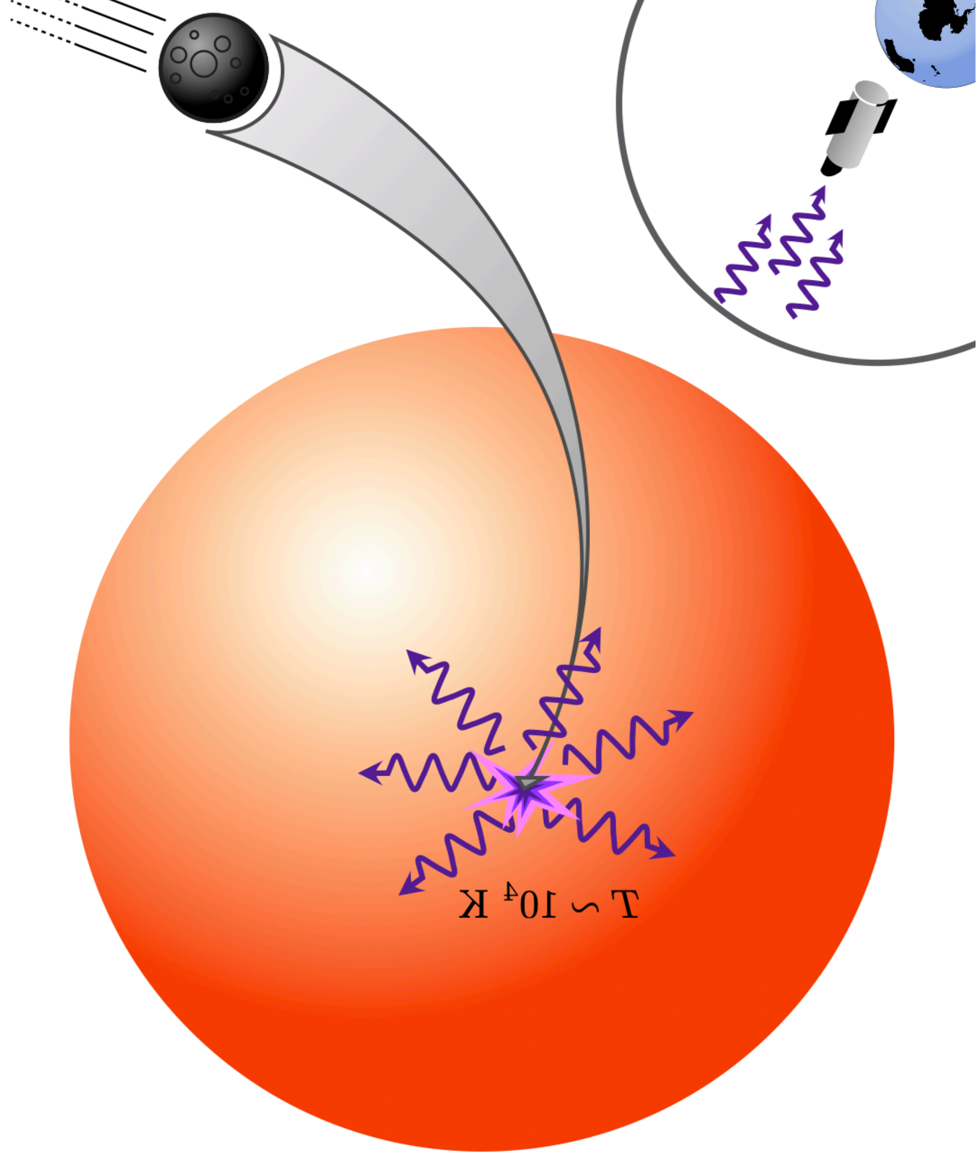

stellar shock transients

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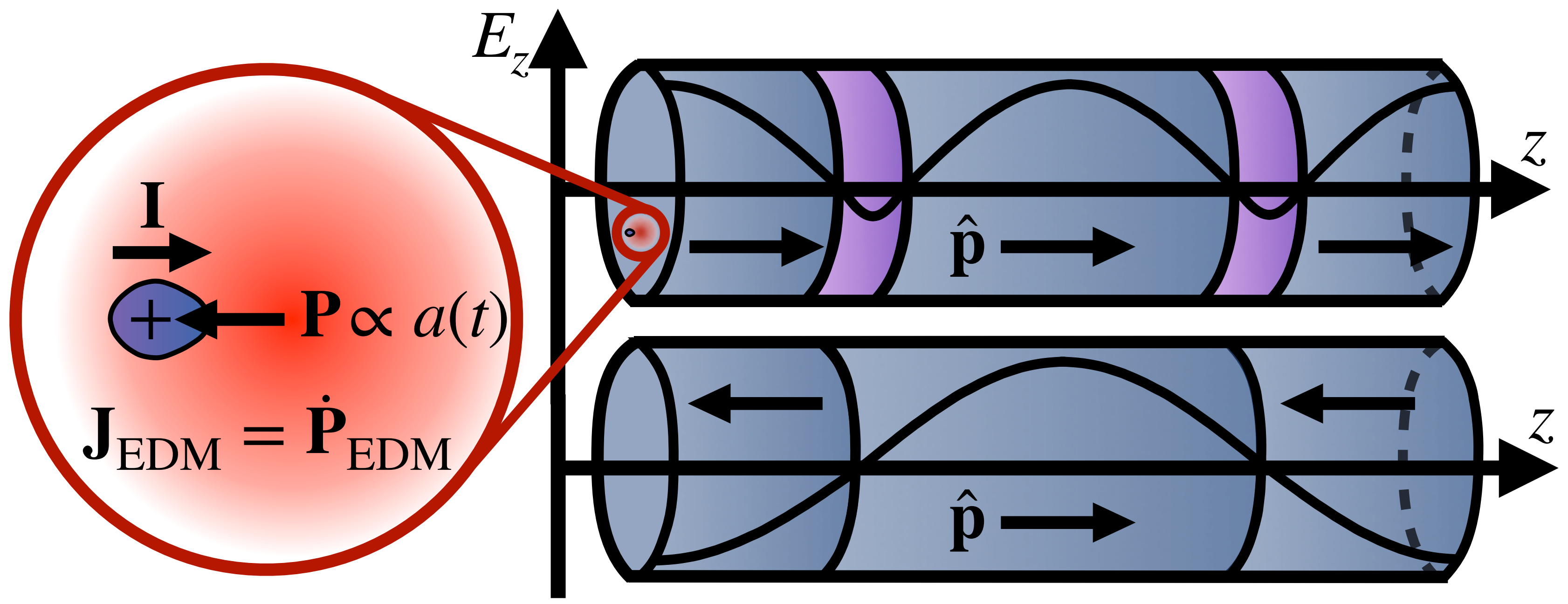
polarization haloscopes



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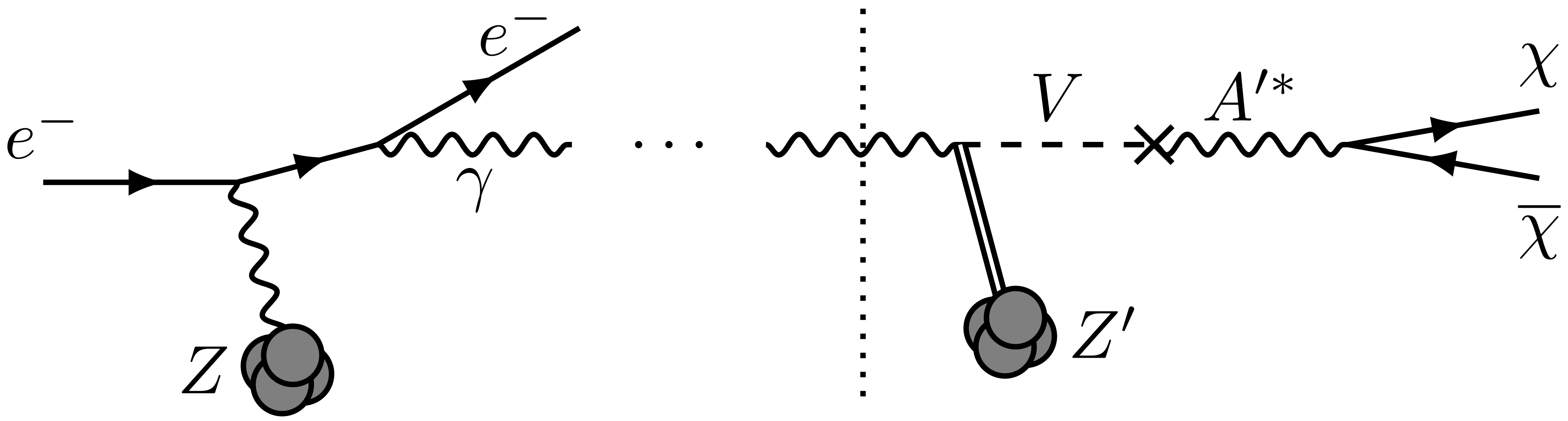
invisible meson decays



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Today’s talk is **not** about dark matter; it’s about the foundational question of what long-range forces can exist in the universe.

But treated with the same spirit of staying close to experiment and observation!

Classifying Particles by Mass and Spin Scale

States transform under translations P^μ and rotations/boosts $J^{\mu\nu}$

Particle states with definite momentum obey $P^\mu \,|\, k, \sigma\rangle = k^\mu \,|\, k, \sigma\rangle$

Little group transformations $W^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}J_{\nu\rho}k_\sigma$ affect only internal state σ

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Different types of particles classified by $P^2 = m^2$ and $W^2 = -\rho^2$

What is the physical meaning of the spin scale ρ?

Classifying Particles by Mass and Spin Scale

For $m^2 > 0$, representations are spin S massive particles

States are $|k, h\rangle$ for helicity $h = -S, \ldots, S$, which is not Lorentz invariant

Boosts mix helicities by amount determined by $\rho = m\sqrt{S(S+1)}$

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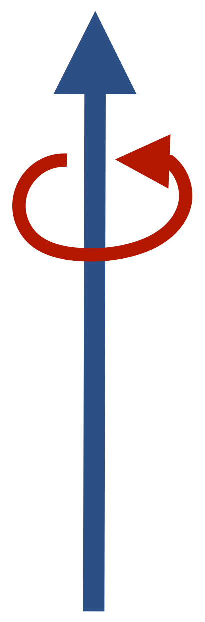
Boosts mix helicities by amount determined by $\rho = m\sqrt{S(S+1)}$

For $m^2 = 0$, states are still indexed by helicity $|k, h\rangle$

Spin scale again determines how helicity varies under boosts

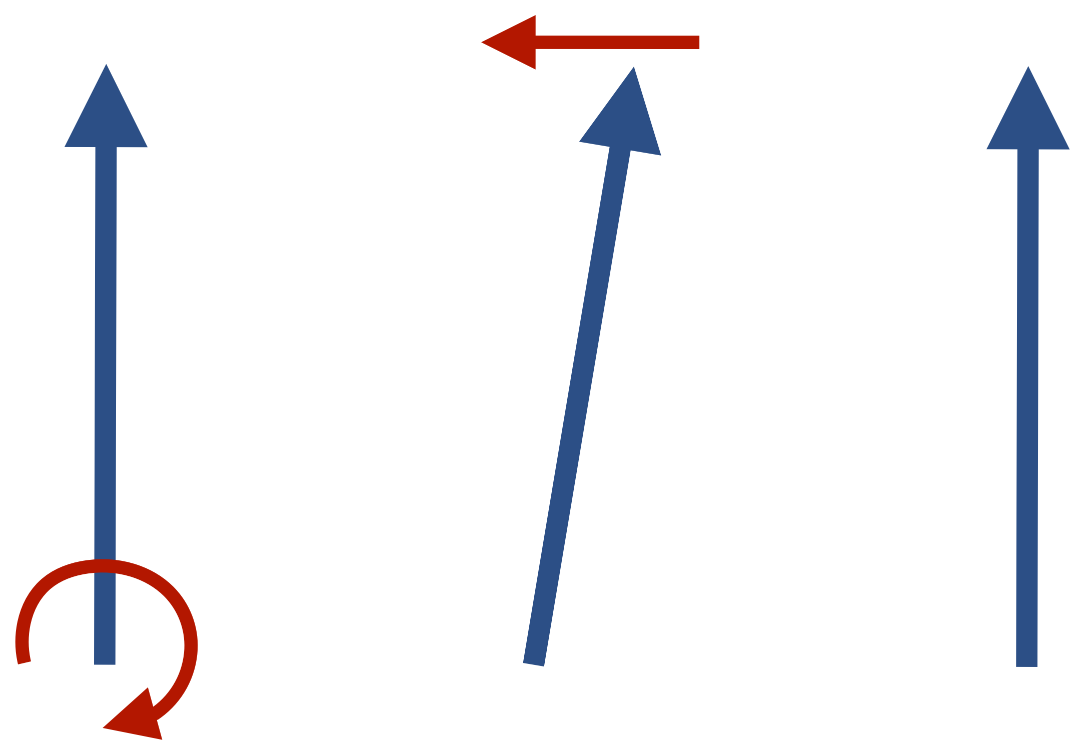
The Massless Little Group

For a massless particle, $k^\mu = (\omega, 0, 0, \omega)$, little group generators are



$$R = J_z$$

$$R\,|\,k, h\rangle = h\,|\,k, h\rangle$$



$$T_1 = J_x + K_y$$

$$T_2 = J_y - K_x$$

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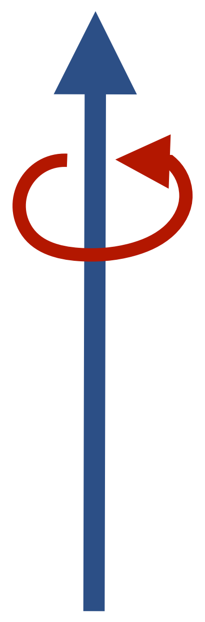

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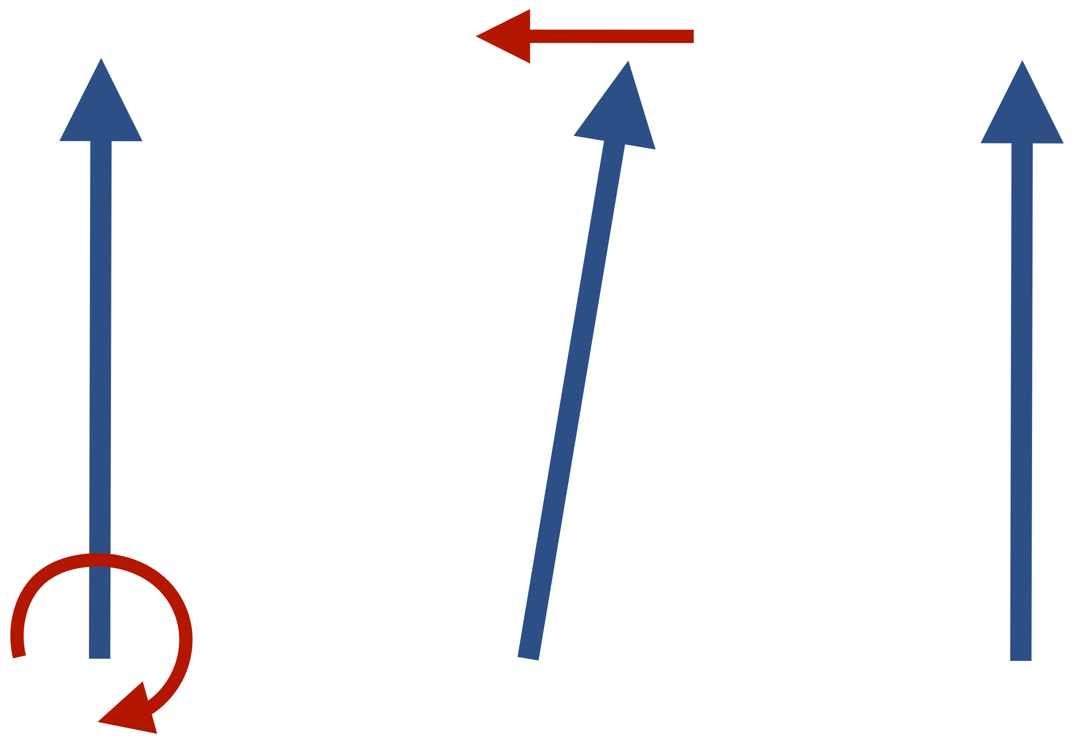

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Defining $T_\pm = T_1 \pm iT_2$, commutation relations imply

$$T_\pm\,|\,k, h\rangle = \rho\,|\,k, h \pm 1\rangle$$

Generic result is an **infinite** ladder of integer-spaced helicities!

Allowed Helicities for Massless Particles

Generic massless particle representation has continuous spin scale ρ

Since h is always integer or half-integer, gives two options

bosonic continuous spin

$\cdots$

$h = 2$

$h = 1$

$h = 0$

$h = -1$

$h = -2$

$\cdots$

fermionic continuous spin

$\cdots$

$h = 3/2$

$h = 1/2$

$h = -1/2$

$h = -3/2$

$\cdots$

Allowed Helicities for Massless Particles

If we set $\rho = 0$, recover a single helicity h (related to $-h$ by CPT symmetry)

Focus on bosonic case, which can mediate long-range forces

$h = 0$

$|h| = 1$

$|h| = 2$

$|h| = 3$

$\ldots$

Allowed Helicities for Massless Particles

If we set $\rho = 0$, recover a single helicity h (related to $-h$ by CPT symmetry)

Focus on bosonic case, which can mediate long-range forces

$h = 0$ massless scalar (requires fine-tuning)

$|h| = 1$ photon (minimal coupling to conserved charge)

$|h| = 2$ graviton (minimal coupling to stress-energy)

$|h| = 3$ higher spin (no minimal couplings allowed)

$\dots$

Role of each $|h|$ in nature well-understood from general arguments

Why Not Consider Continuous Spin?

Ruled out by Weinberg soft theorems?

Theorems assume Lorentz invariant h

Generalize to good soft factors for $\rho \neq 0$

Schuster and Toro, JHEP (2013) 104/105

Incompatible with field theory?

Simple free gauge theory found

Schuster and Toro, PRD (2015)

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Infinite h leads to infinities in scattering/cosmology/astrophysics/Hawking/Casimir/…?

Smooth $\rho \to 0$ limit where all but one $|h|$ decouples

Often results even stay finite for $\rho \to \infty$ due to nontrivial analytic features, reminiscent of string theory

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Force in a radiation background: $$\frac{\mathbf{F}}{q} = \mathbf{E} + \mathbf{v} \times \mathbf{B} + \left(\frac{\rho}{\omega}\right)^2 \left(\frac{\mathbf{v}_\perp(\mathbf{v}_\perp \cdot \mathbf{E})}{4} - \frac{v_\perp^2 \mathbf{E}}{8}\right) + \ldots$$

Corrections at low frequencies and long distances

Well-behaved for large ρ

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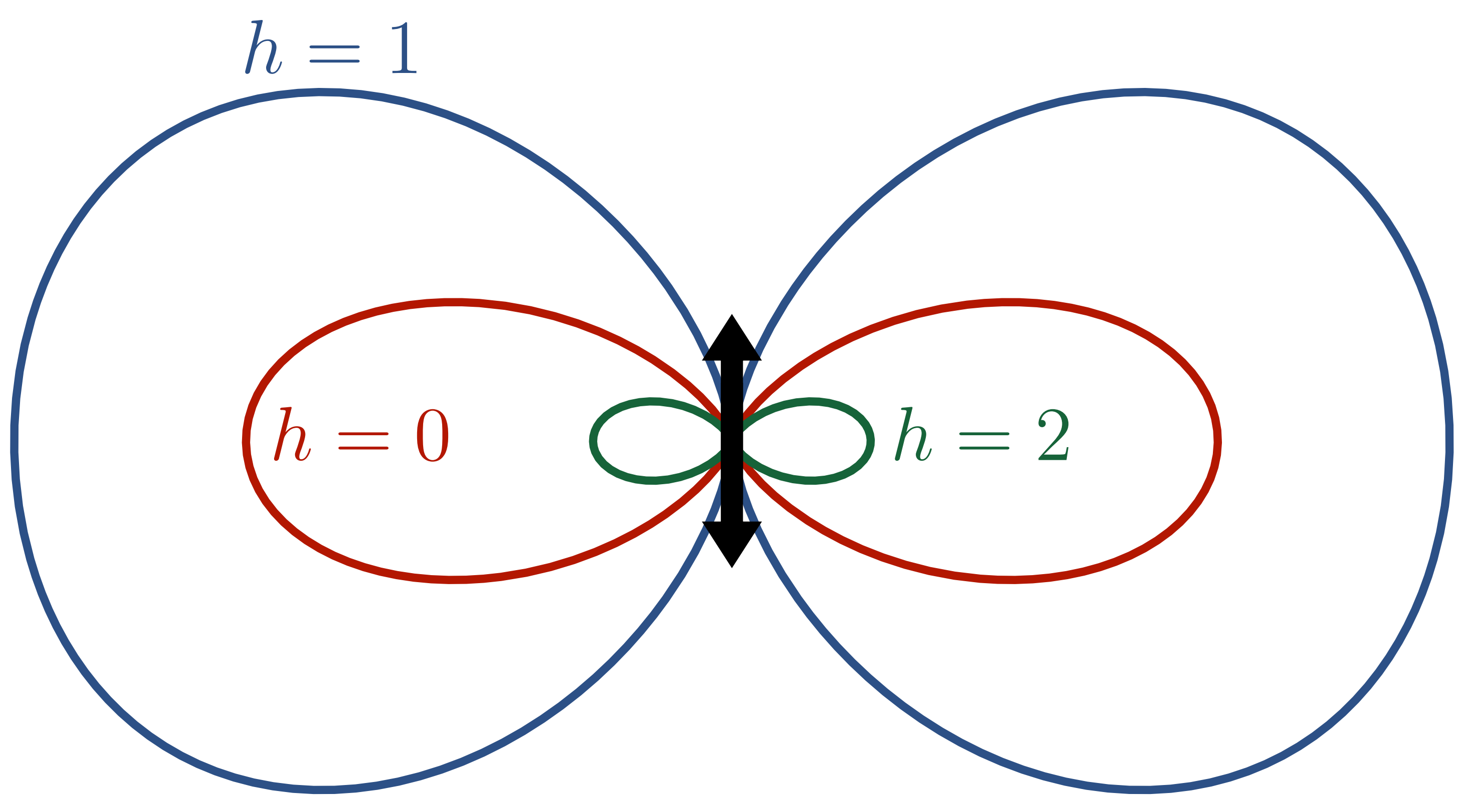
Radiation from an oscillating particle:



$$P = \frac{q^2 \langle a^2 \rangle}{6\pi} \times \begin{cases} (\rho\ell)^2/40 & h = 0 \\ 1 - 3(\rho\ell)^2/20 & h = \pm 1 \\ (\rho\ell)^2/80 & h = \pm 2 \end{cases} + \ldots$$

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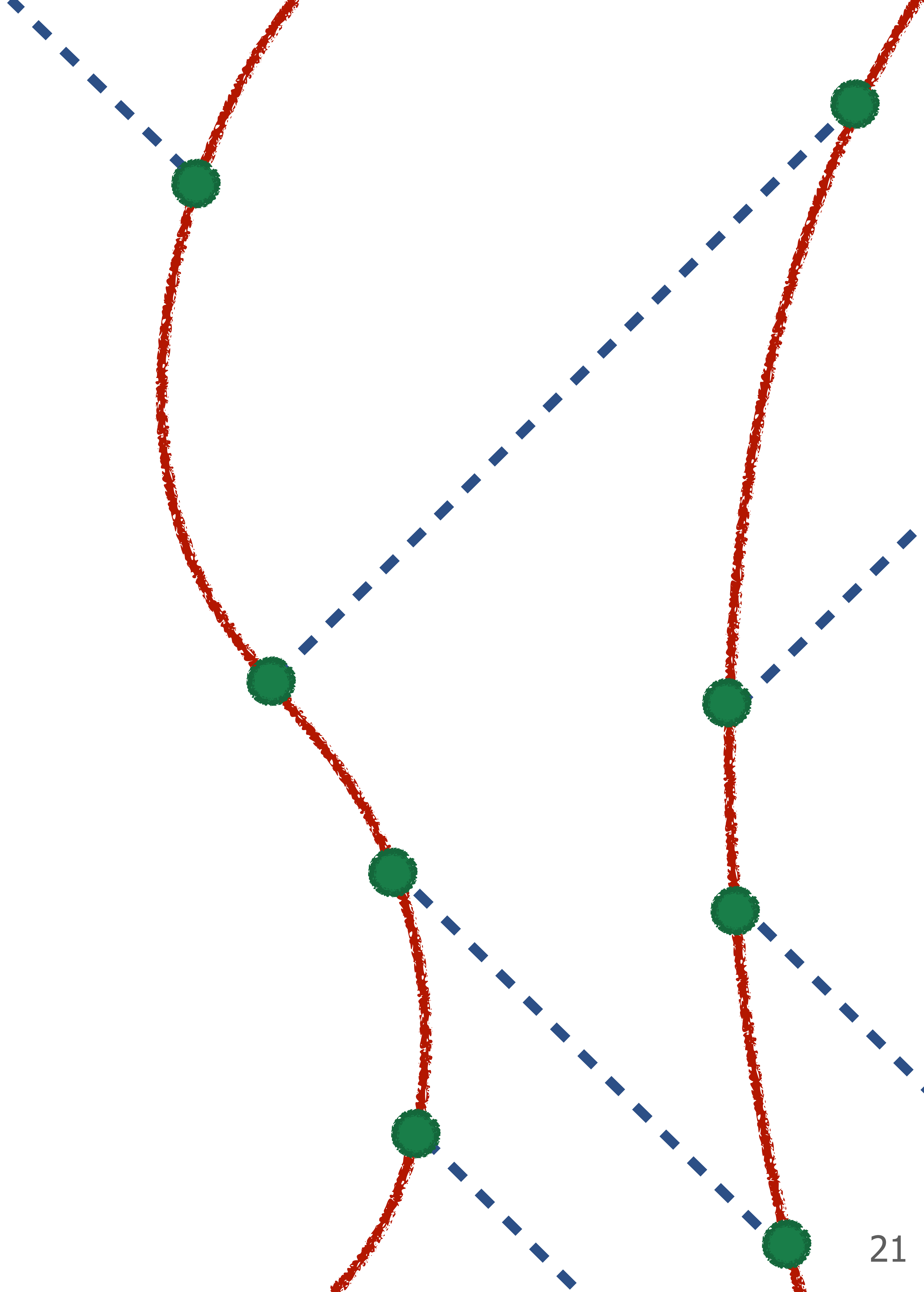
For the model builder: “because it’s novel”

A new infrared deformation of gauge theories, which may shed light on long-distance physics (dark matter, cosmic acceleration)

A new type of spacetime symmetry based on a bosonic superspace, possibly relevant for tuning problems (hierarchy, cosmological constant)

Outline

- **Free continuous spin fields**
- Coupling matter particles
- Physics with continuous spin



Free Fields for Massless Particles

Tricky even for $\rho = 0$, by mismatch of field and particle degrees of freedom

scalar $h = 0$ — scalar field ϕ, no extra components

photon $h = \pm 1$ — vector field A_μ, $4 - 2 = 2$ extra components

must use action with gauge symmetry $\delta A_\mu = \partial_\mu \alpha$

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photon $h = \pm 1$	vector field A_μ, $4 - 2 = 2$ extra components
	must use action with gauge symmetry $\delta A_\mu = \partial_\mu \alpha$
graviton $h = \pm 2$	sym. tensor field $h_{\mu\nu}$, $10 - 2 = 8$ extra components
	must use action with gauge symmetry $\delta h_{\mu\nu} = \partial_{(\mu} \xi_{\nu)}$
higher spin $\lvert h \rvert > 2$	sym. tensor field $\phi_{\mu_1 \ldots \mu_h}$, many extra components

Given complexity of higher h, constructing a continuous spin field seems intractable!

Introducing Vector Superspace

A field in "vector superspace" (x^μ, η^μ) has tensor components of all ranks

$$\Psi(\eta, x) = \phi(x) + \sqrt{2}\,\eta^\mu A_\mu(x) + (2\eta^\mu\eta^\nu - g^{\mu\nu}(\eta^2+1))\,h_{\mu\nu}(x) + \dots .$$

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Simple expression has free Lagrangian for each tensor field simultaneously!

$$\mathscr{L}[\Psi] = \frac{1}{2} \int_\eta \delta'(\eta^2 + 1)(\partial_x \Psi)^2 + \frac{1}{2}\, \delta(\eta^2 + 1)(\Delta\Psi)^2 \qquad \Delta = \partial_x \cdot \partial_\eta$$

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Integration produces tensor contractions

$$\int_\eta \delta(\eta^2+1) = \int_\eta \delta'(\eta^2+1) \equiv 1$$

$$\int_\eta \delta(\eta^2+1)\,\eta^\mu\eta^\nu = -\frac{1}{4}g^{\mu\nu}$$

$$\int_\eta \delta'(\eta^2+1)\,\eta^\mu\eta^\nu = -\frac{1}{2}g^{\mu\nu}$$

Recovering Familiar Actions

$$\mathscr{L}[\phi] = \frac{1}{2} \int_\eta \delta'(\eta^2+1)(\partial_x \Psi)^2 + \frac{1}{2}\,\delta(\eta^2+1)(\partial_x \cdot \partial_\eta \Psi)^2 \bigg|_{\Psi=\phi}$$

Recovering Familiar Actions

$$\mathscr{L}[\phi] = \frac{1}{2} \underbrace{\int_\eta \delta'(\eta^2+1)}_{\text{gives } 1} \underbrace{(\partial_x \Psi)}_{\partial_x \phi}{}^2 + \frac{1}{2}\, \delta(\eta^2+1)(\partial_x \cdot \underbrace{\partial_\eta \Psi}_{\partial_\eta \phi = 0})^2 \Big|_{\Psi=\phi} = \frac{1}{2} (\partial_x \phi)^2$$

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$$\mathscr{L}[A_\mu] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)(\partial_x \Psi)^2 + \frac{1}{2}\delta(\eta^2+1)(\partial_x \cdot \partial_\eta \Psi)^2 \bigg|_{\Psi=\sqrt{2}\eta^\mu A_\mu}$$

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$$\mathscr{L}[A_\mu] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)\underbrace{(\partial_x\Psi)^2}_{(\sqrt{2}\,\eta_\mu\partial_x A^\mu)^2} + \frac{1}{2}\,\delta(\eta^2+1)\underbrace{(\partial_x\cdot\partial_\eta\,\Psi)^2}_{(\sqrt{2}\,\partial_\mu A^\mu)^2}\Bigg|_{\Psi=\sqrt{2}\eta^\mu A_\mu} = -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial_\mu A^\mu)^2$$

More generally, we recover the linearized Einstein-Hilbert action, and higher-rank Fronsdal actions, with no mixing

Recovering Familiar Dynamics

One equation of motion contains Maxwell, linearized Einstein, Fronsdal:

$$\delta'(\eta^2 + 1)\, \partial_x^2 \Psi - \frac{1}{2} \Delta \left(\delta(\eta^2 + 1)\, \Delta\Psi\right) = 0$$

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One gauge transformation contains $U(1)$ gauge transformations, diffeomorphisms, …

$$\delta \Psi = (\eta \cdot \partial_x - \frac{1}{2}(\eta^2 + 1)\Delta)\, \epsilon(\eta, x)$$

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$$\delta\Psi = (\eta \cdot \partial_x - \frac{1}{2}(\eta^2 + 1)\Delta)\, \epsilon(\eta, x)$$

One mode expansion contains scalar modes, photon modes, graviton modes, …

$$\Psi_{k,h} = e^{-ik\cdot x}\, (\eta \cdot \epsilon_{\pm})^{|h|}$$

Turning on the Spin Scale

All previous results can be generalized to arbitrary ρ by defining $\Delta = \partial_x \cdot \partial_\eta + \rho$

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Still get one mode of each helicity, but now the action, equation of motion, gauge symmetric, and plane waves all mix tensor ranks, e.g.

$$\mathscr{L} \supset \frac{\rho}{\sqrt{2}} \phi \, \partial_\mu A^\mu$$

$$\Psi_{k,h} = e^{-ik\cdot x} \, e^{-i\rho\eta\cdot q} \, (\eta \cdot \epsilon_\pm)^{|h|} \qquad\qquad q \cdot k = 1$$

Because of mixing, tensor expansion is complicated and physically opaque, while vector superspace description remains simple

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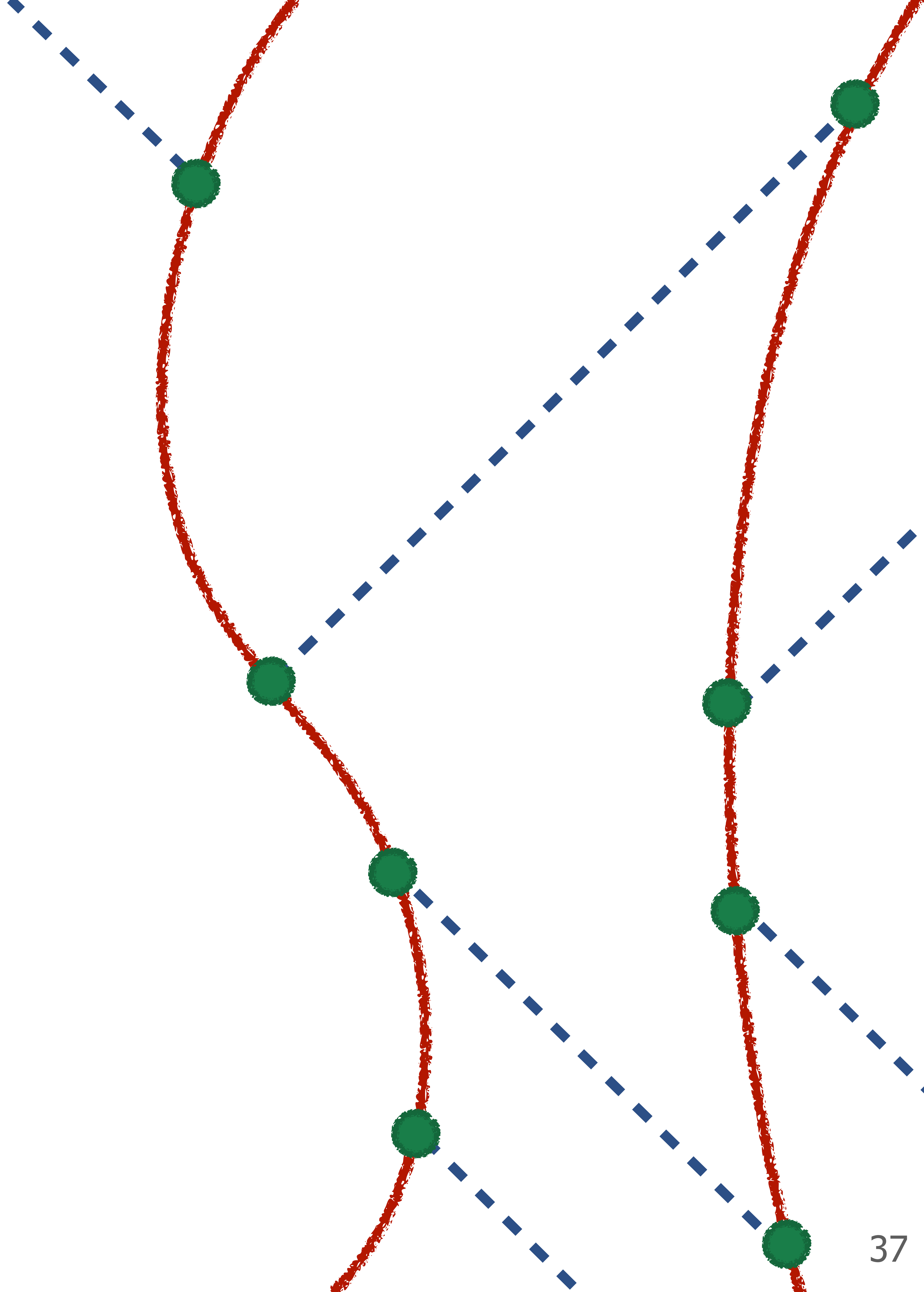

Coupling Currents to Fields

Couple the continuous spin field to a current by

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Recover familiar results by tensor decomposition

$$J(\eta, x) = J(x) - \sqrt{2}\, \eta^\mu J_\mu(x) + (2\eta^\mu \eta^\nu + g^{\mu\nu})\, T_{\mu\nu}(x) + \ldots$$

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Gauge invariance of the coupling gives a "continuity condition"

$$\delta(\eta^2+1)\Delta J = 0 \longrightarrow \begin{aligned} \partial_\mu J^\mu &= -\rho J \\ \partial_\mu T^{\mu\nu} &= -\rho J^\nu \end{aligned}$$

Currents From Matter Particles

In familiar theories, the current from a matter particle is local to its worldline $z^\mu(\tau)$

$$J(x) = g \int d\tau\, \delta^4(x - z(\tau))$$

$$J^\mu(x) = e \int d\tau\, \delta^4(x - z(\tau)) \frac{dz^\mu}{d\tau}$$

$$T^{\mu\nu}(x) = m \int d\tau\, \delta^4(x - z(\tau)) \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau}$$

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scalar-like current $\longrightarrow$ $$J(x) = g \int d\tau \, \delta^4(x - z(\tau))$$

vector-like current $\longrightarrow$ $$J^\mu(x) = e \int d\tau \, \delta^4(x - z(\tau)) \frac{dz^\mu}{d\tau}$$

tensor-like current $\longrightarrow$ $$T^{\mu\nu}(x) = m \int d\tau \, \delta^4(x - z(\tau)) \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau}$$

We define families of continuous spin currents by their $\rho \to 0$ limits

Of course, non-minimal couplings also possible, but less interesting

Tensor case more subtle: for free particles conservation requires $\ddot{z}^\mu = 0$

Simple Scalar-Like Currents

The current and (strong) continuity condition in Fourier space are

$$J(\eta, k) = \int d\tau\, e^{ik\cdot z(\tau)} f(\dot{z}, k, \eta) \qquad\qquad (-ik\cdot\partial_\eta + \rho) f = 0$$

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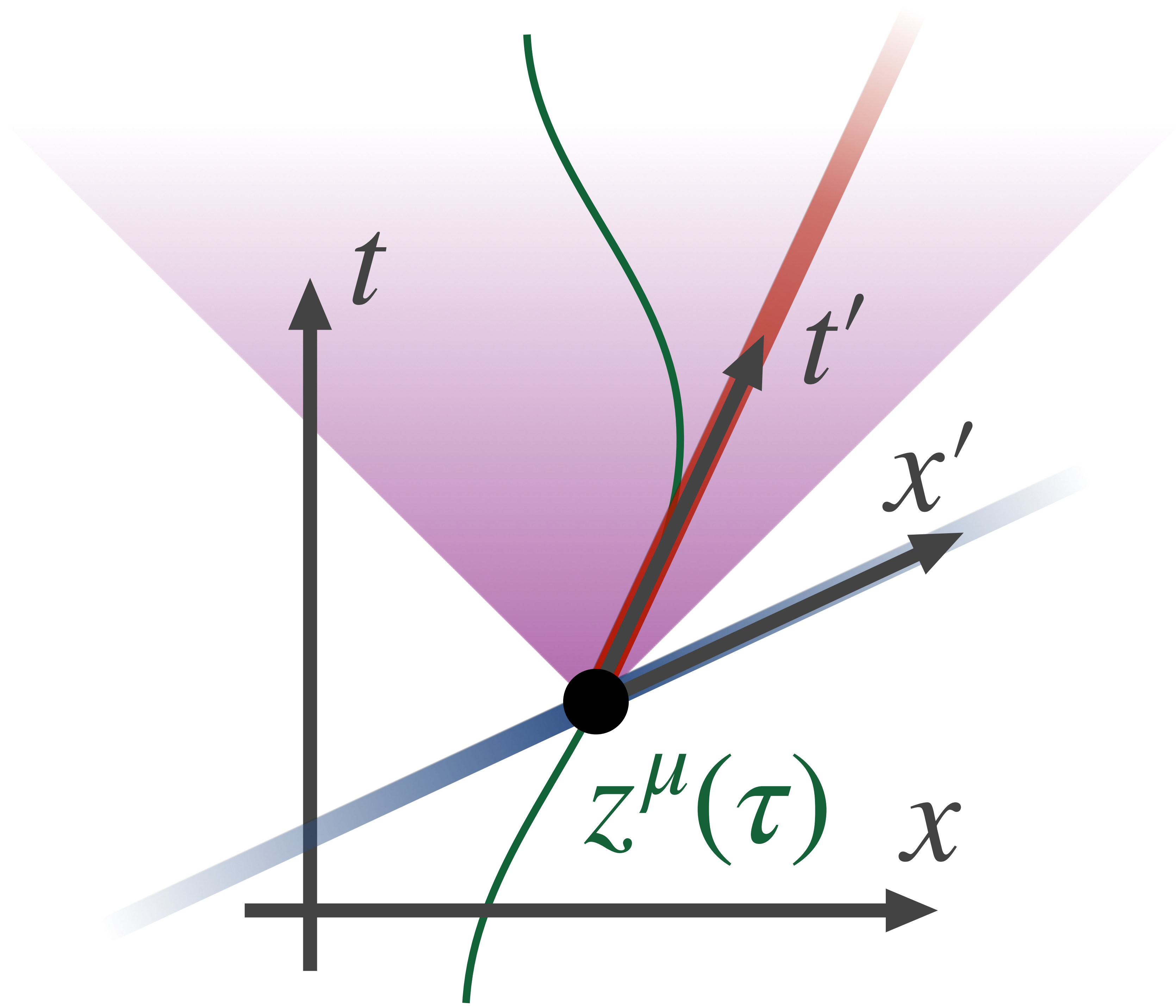
We know the complicated general solution, but one simple family of f is

$$f = e^{-i\rho\eta\cdot V/k\cdot V} \qquad\qquad V = \begin{cases} \dot{z} & \text{temporal} \\ k - (k\cdot\dot{z})\dot{z} & \text{spatial} \\ k + \beta\,\dot{z} & \text{inhomogeneous} \end{cases}$$

Dependence on k implies these currents are **not** local to particle's worldline!

Localization Properties

The current of a worldline element is spread in spacetime:



Spatial current spread over plane $t' = 0$

Temporal current on ray $\mathbf{x}' = 0,\, t' \geq 0$

Inhomogeneous current in forward light cone

"Causal" currents confined in forward light cone can emerge from local theories

"Static" Fields

Continuous spin field of a particle at rest (in a gauge where $\partial^2 \Psi = J$) is simply

$$\psi = \frac{1}{r} \times \begin{cases} J_0\left(2\sqrt{\rho\,\eta^0 t}\right) & \text{temporal current} \\ J_0\left(\sqrt{2\rho\,(|\overrightarrow{\eta}||\vec{r}| - \overrightarrow{\eta}\cdot\vec{r})}\right) & \text{spatial current} \end{cases}$$

$$J_0(\sqrt{z}) = 1 - z/4 + \ldots$$

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Generally, deviations appear in infrared: distance/time scales greater than $1/\rho$

$$J \sim \delta^3(\mathbf{r}) - \frac{\rho\,|\vec{\eta}\,|}{r^2} J_2\left(\sqrt{2\rho\,(|\vec{\eta}||\vec{r}| - \vec{\eta}\cdot\vec{r})}\right) \quad \text{spatial current}$$

$$J_2(\sqrt{z}) = z/8 + \dots$$

As $\rho \to 0$, the delocalized part of the current gets **less** local, but also decouples!

A Universality Result

Our key technical result: **all** currents can be decomposed as

$$f = \exp\left(-i\rho\frac{\eta\cdot\dot{z}}{k\cdot\dot{z}}\right)h(k\cdot\dot{z}) + k^2(\ldots) + (\text{gauge variation})$$

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$$f = \exp\left(-i\rho\frac{\eta\cdot\dot z}{k\cdot\dot z}\right) h(k\cdot\dot z) + k^2(\ldots) + (\text{gauge variation})$$

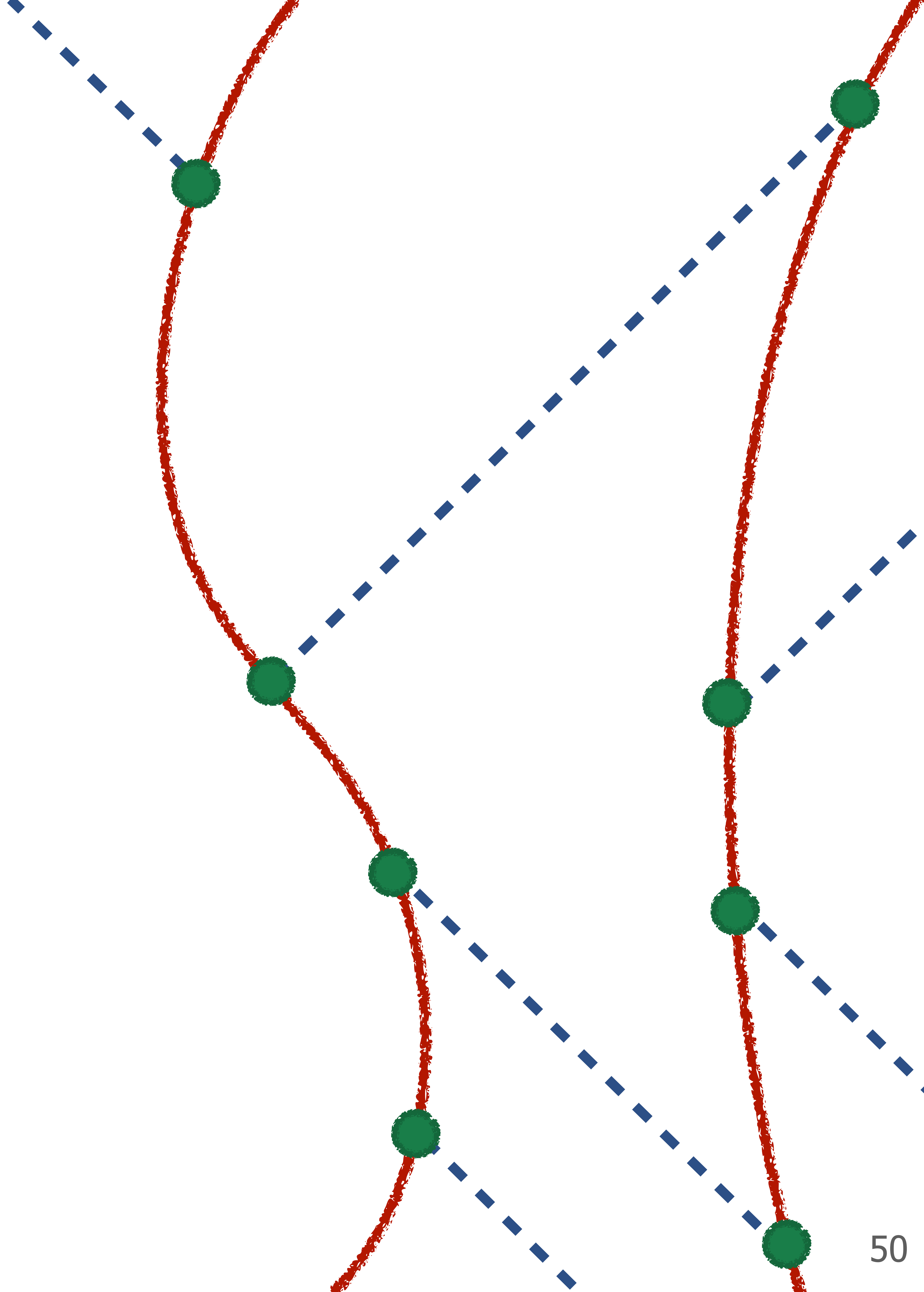
Only the first term contributes to processes involving radiation ($k^2 = 0$), and

$$h = \begin{cases} g & \text{scalar-like current} \\ e\,k\cdot\dot z & \text{vector-like current} \\ m\,(k\cdot\dot z)^2 + \ldots & \text{tensor-like current} \end{cases}$$

Many observable results are independent of detailed current, depend only on ρ

Outline

- Free continuous spin fields
- Coupling matter particles
- **Physics with continuous spin**



Extracting the Physics

From the action $S[\Psi, z_i^\mu(\tau)]$ we can compute any desired classical observable:

“Integrate out” Ψ
(plug solution into action)

Matter interaction potential

$V(\mathbf{r}_i, \mathbf{v}_i, \ldots)$

Depends on current

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with given Ψ

$\downarrow$

Matter forces in background
$$\mathbf{F}(\Psi)$$

Universal for radiation

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$\mathbf{F}(\Psi)$

Universal for radiation

Solve Ψ equation of motion
with given trajectories

$\downarrow$

Radiation emission
$dP_h/d\Omega$

Universal

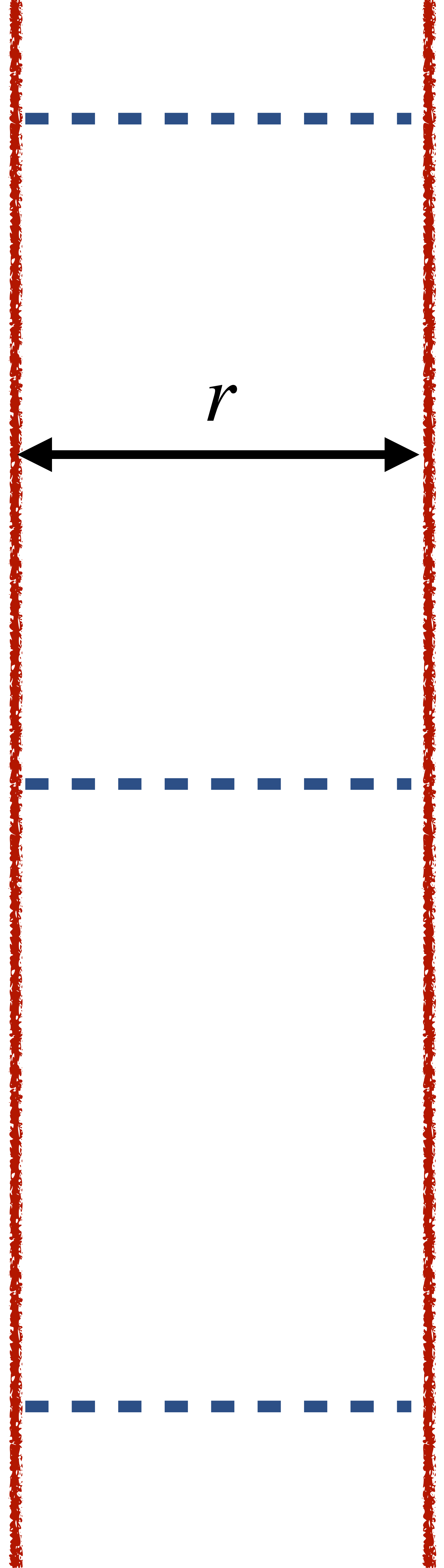
Static Forces

For particles with separation r and scalar-like currents:

$$V(r) = \frac{g^2}{4\pi r} \times \begin{cases} 1 & \text{spatial current} \\ 1 & \text{temporal current} \\ 1 - c_1\sqrt{\rho\beta}\,r + c_2\rho\beta r^2 + \ldots & \text{inhomogeneous current} \end{cases}$$

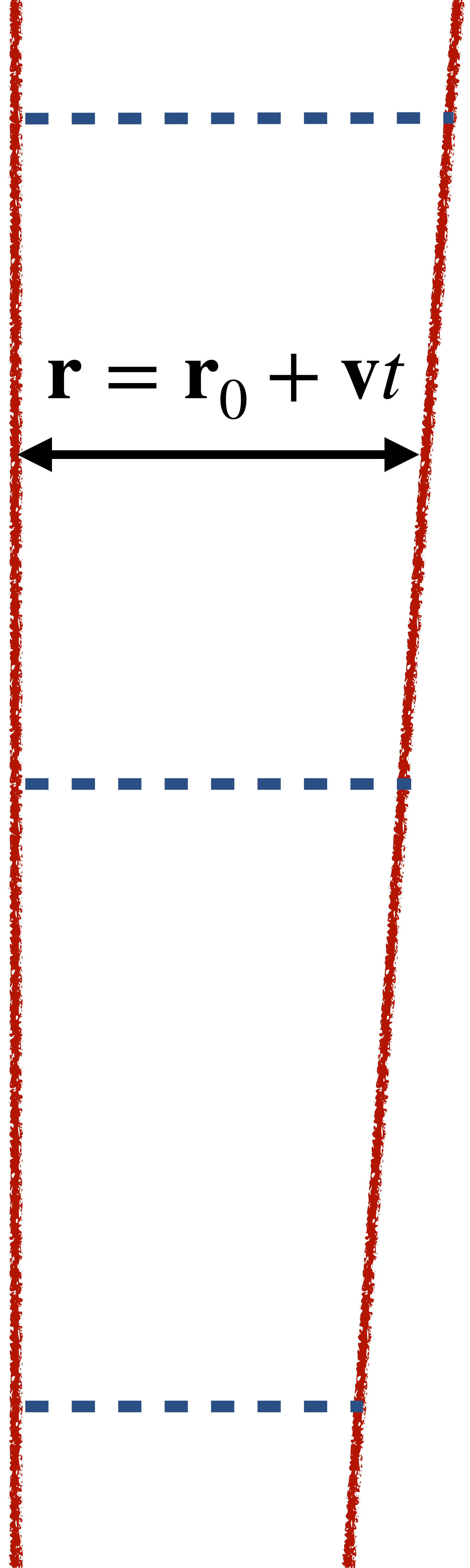
Always unchanged at small r, varies at large r

Qualitatively similar results for vector-like currents



Velocity-Dependent Forces

Consider moving particles to extract velocity-dependent potential (same procedure yields magnetic forces in electromagnetism)



For the spatial scalar-like current:

$$V(\mathbf{r}, \mathbf{v}) = \frac{g^2}{4\pi r} I_0\left(\sqrt{\rho(vr - \mathbf{v}\cdot\mathbf{r})}\right) J_0\left(\sqrt{\rho(vr - \mathbf{v}\cdot\mathbf{r})}\right)$$

Corrections appear in a series in $\rho v r$

Radiation Forces

Force on particle with vector-like current in background of frequency ω, helicity h:

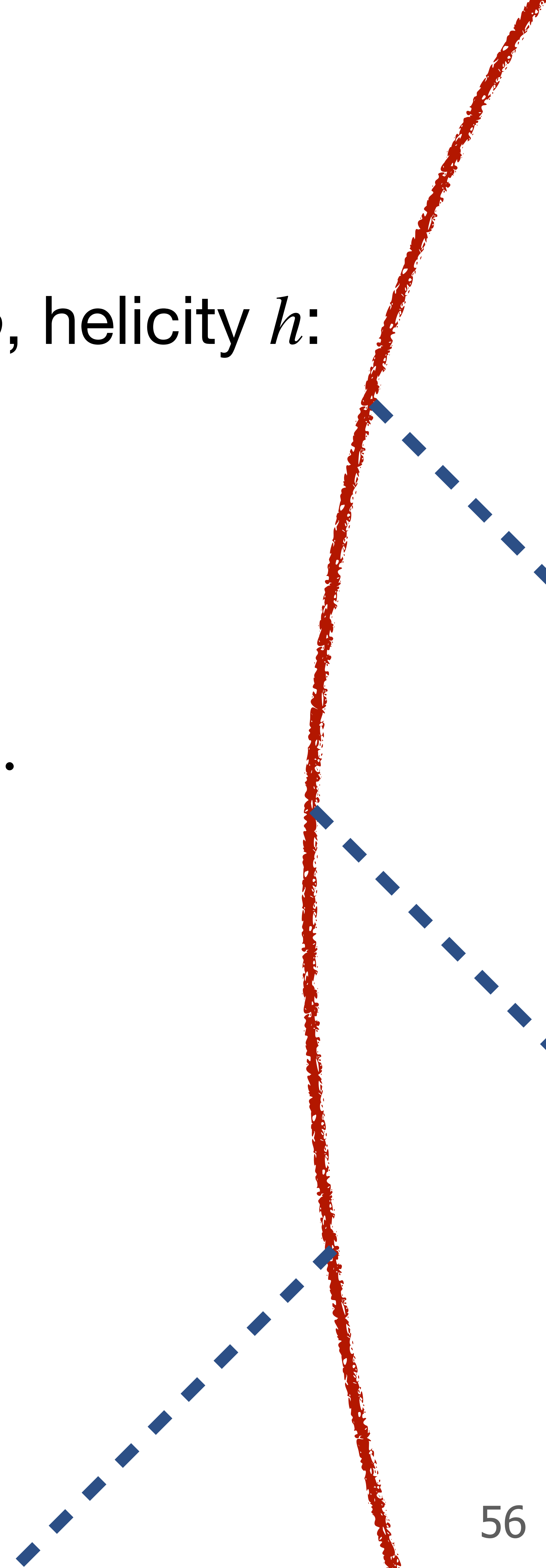
$$\frac{\mathbf{F}_{h=0}}{q} = \frac{\rho}{\omega}\,\frac{\dot{\phi}\,\mathbf{v}_\perp}{2} + \ldots$$

$$\frac{\mathbf{F}_{h=\pm 1}}{q} = \mathbf{E} + \mathbf{v}\times\mathbf{B} + \left(\frac{\rho}{\omega}\right)^2\left(\frac{\mathbf{v}_\perp(\mathbf{v}_\perp\cdot\mathbf{E})}{4} - \frac{v_\perp^2\,\mathbf{E}}{8}\right) + \ldots$$

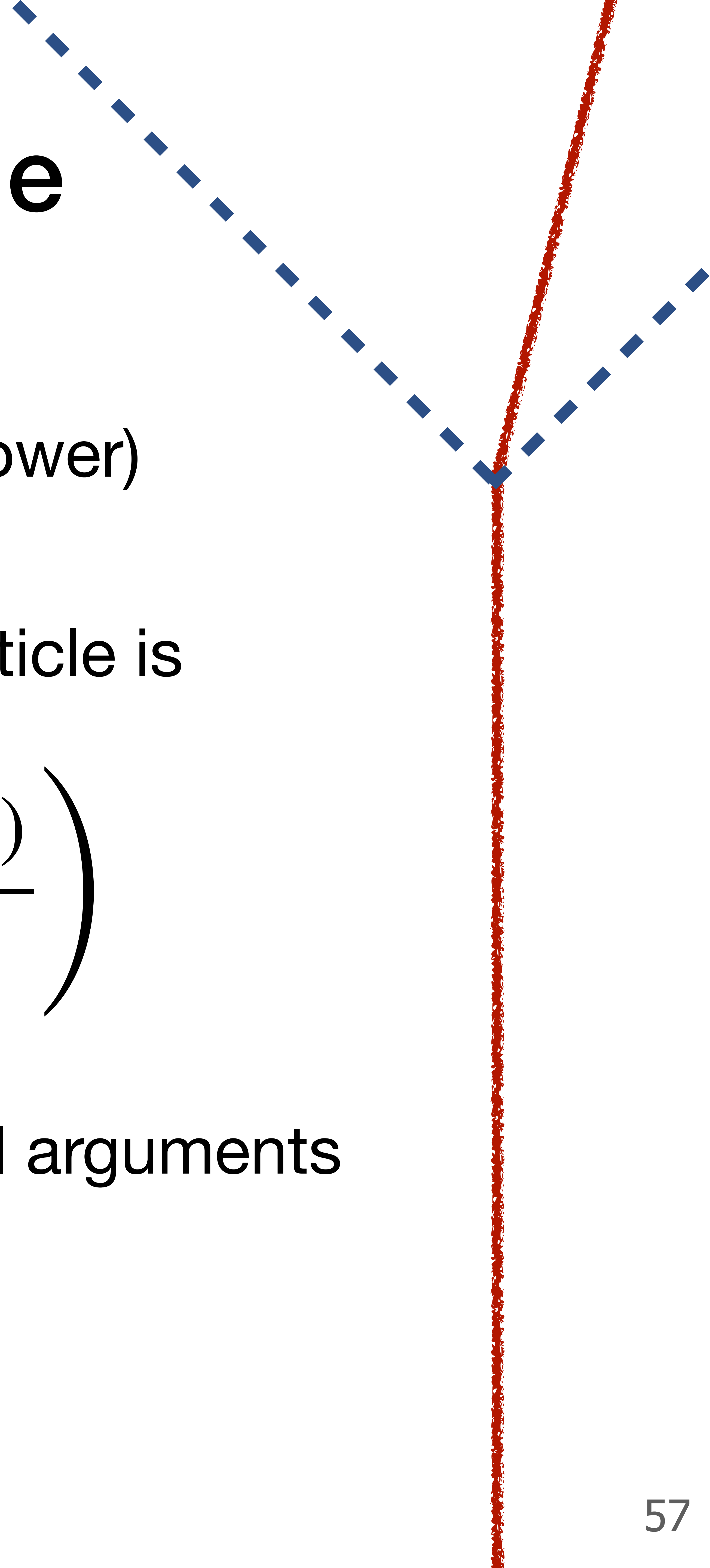
$$\frac{\mathbf{F}_{h=\pm 2}}{q} = \frac{\rho}{\omega}\,\frac{\dot{h}_+\,(v_x\,\hat{\mathbf{x}} - v_y\,\hat{\mathbf{y}})}{4} + \ldots$$

All corrections are transverse and velocity-dependent

Full expressions are Bessel functions, convergent at large arguments



Radiation From Kicked Particle



Computing radiation emission involves some subtleties
(defining the radiation field, time-averaging for gauge invariant power)

For any scalar-like current, radiation amplitude from a kicked particle is

$$a_{h,k} \propto g \left(\frac{\tilde{J}_h(\rho \,|\, \epsilon_- \cdot p / k \cdot p \,|)}{k \cdot p} - \frac{\tilde{J}_h(\rho \,|\, \epsilon_- \cdot p' / k \cdot p' \,|)}{k \cdot p'} \right)$$

which exactly matches soft emission amplitudes fixed by general arguments

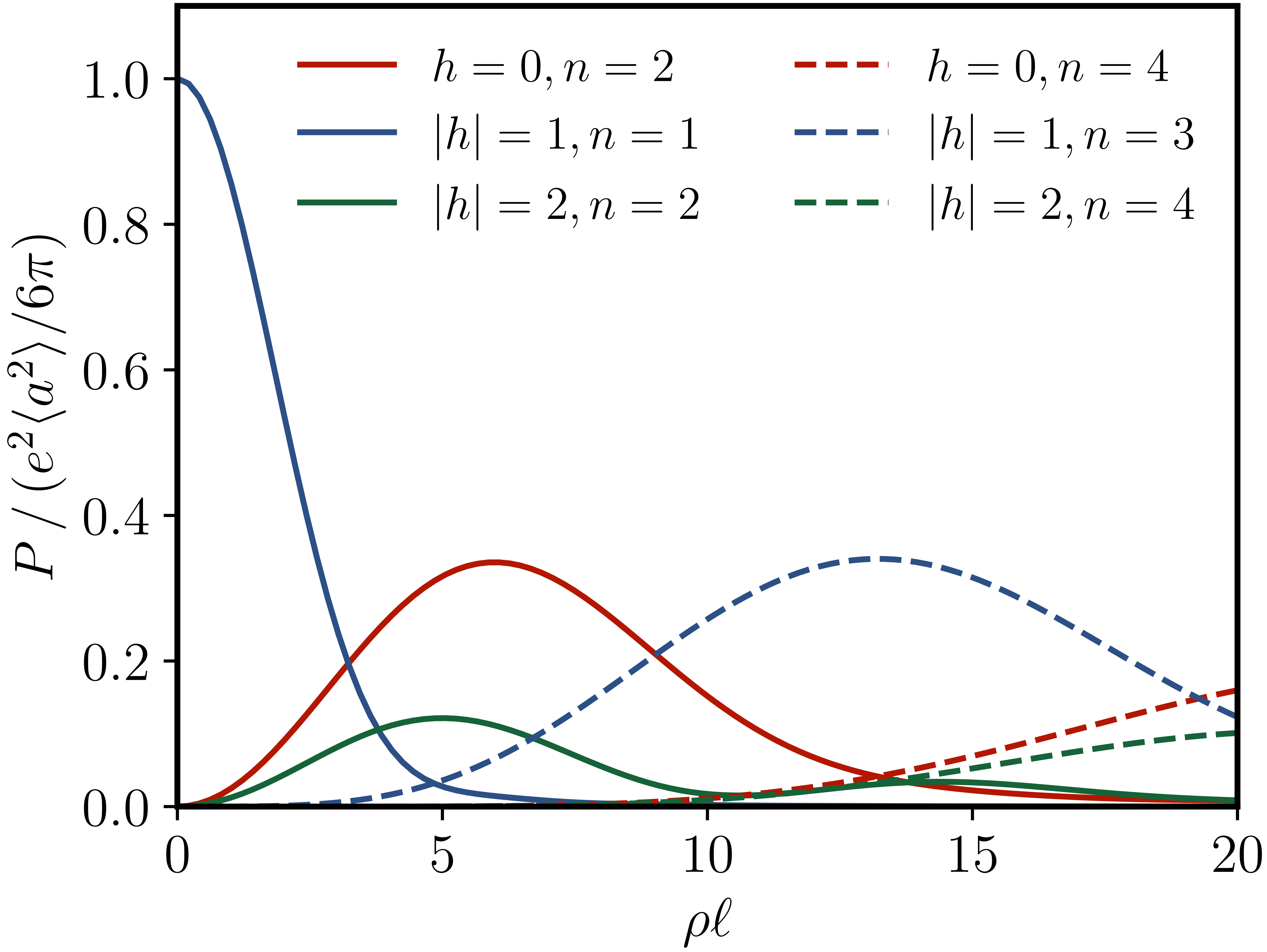
Same agreement for vector-like currents

Radiation From Oscillating Particle

Consider oscillating particle with amplitude ℓ, vector-like current

Radiation emitted in all helicities, harmonics $\omega_n = n\omega_0$

At large $\rho\ell$ many harmonics and helicities contribute…



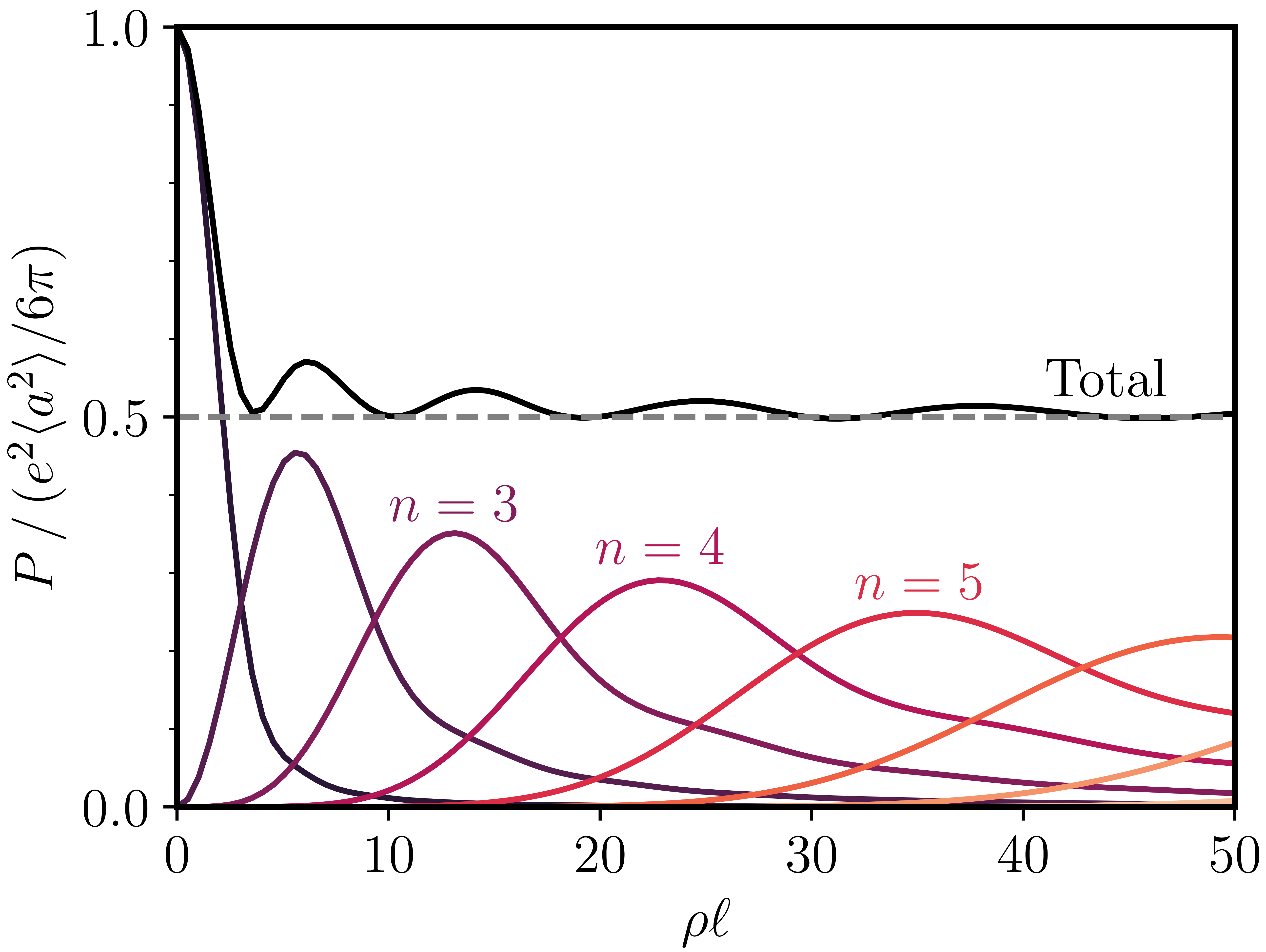
Radiation From Oscillating Particle

Consider oscillating particle with amplitude ℓ, vector-like current

Radiation emitted in all helicities, harmonics $\omega_n = n\omega_0$

At large $\rho\ell$ many harmonics and helicities contribute…

…but the total power radiated is still finite!



The Spin Scale of the Photon

A clear next step, which can be addressed with only existing results

Light shining through walls?

Precision Coulomb tests?

Helioscopes?

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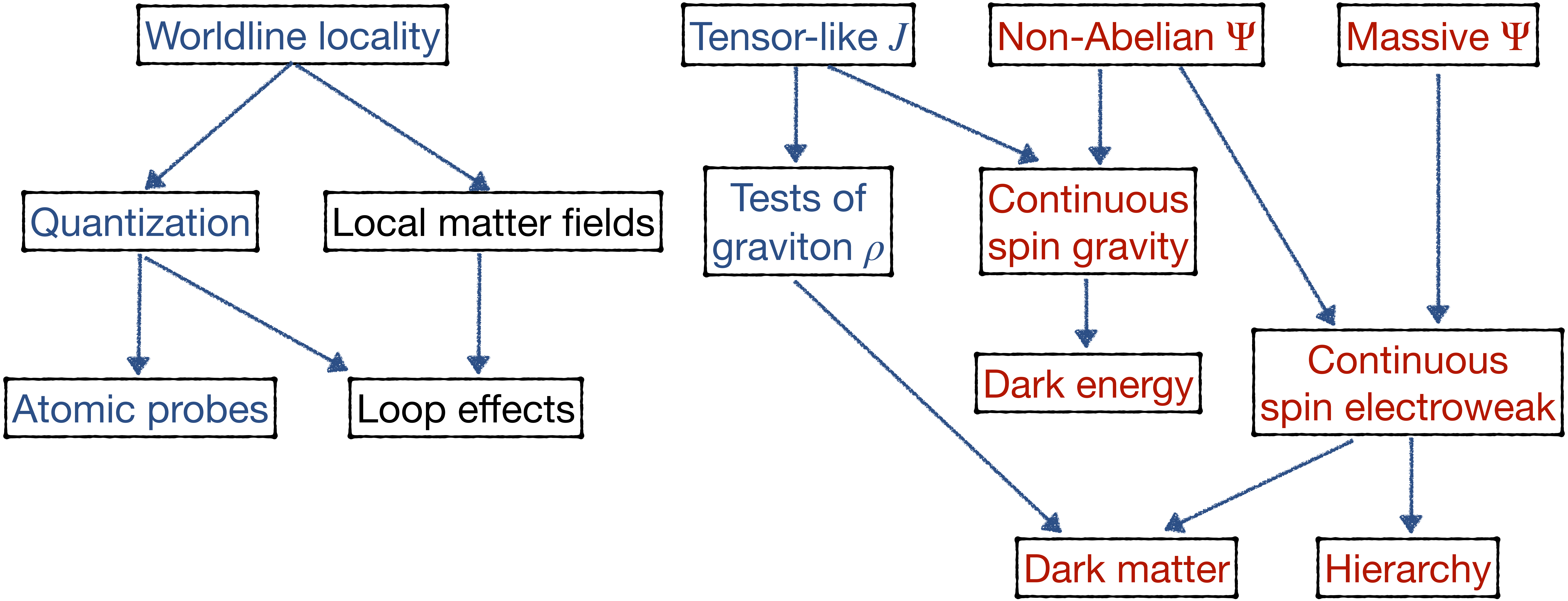
Impact on early universe thermodynamics?

Relic radiation in partner helicities?

Vast number of possible directions, many of which we haven’t even thought of yet!

Responsible Steps and Higher Speculations



Responsible Steps and Higher Speculations

Worldline locality

Quantization

Local matter fields

Tensor-like J

Non-Abelian Ψ

Massive Ψ

Tests of graviton ρ

Continuous spin gravity

Atomic probes

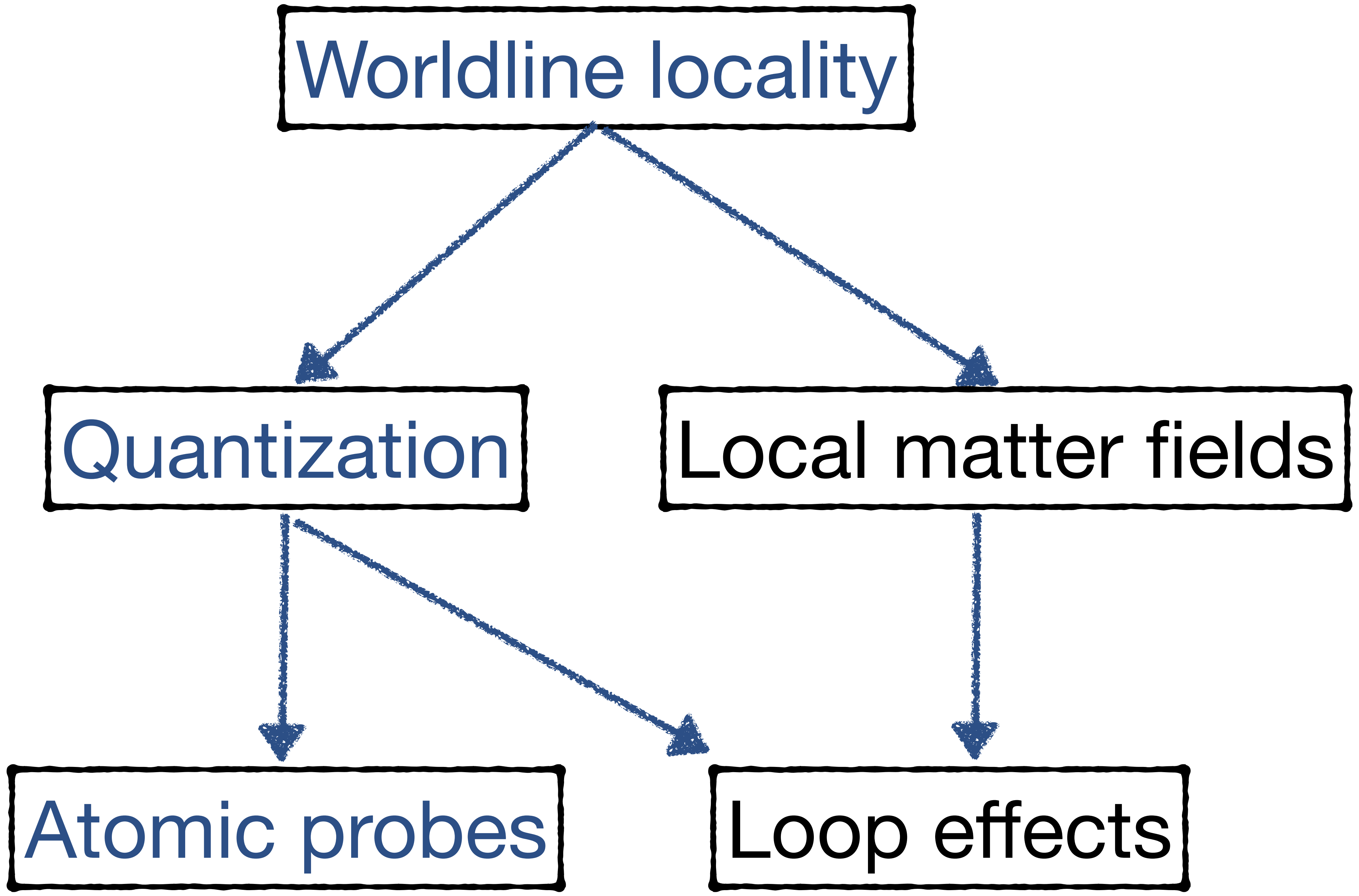
Continuous spin electroweak

The form of the currents suggest they emerge from extended objects, with internal degrees of freedom

We already have a local, causal description of some currents using worldline fields

Formulating quantum matter fields requires more work, not needed for quantization

Responsible Steps and Higher Speculations



Quantization proceeds by usual techniques

Quantum description allows probes using precision techniques like spectroscopy, atom interferometry, atomic clocks, spin precession…

But may not be superior as “classical” probes are naturally at lower frequency, longer distance

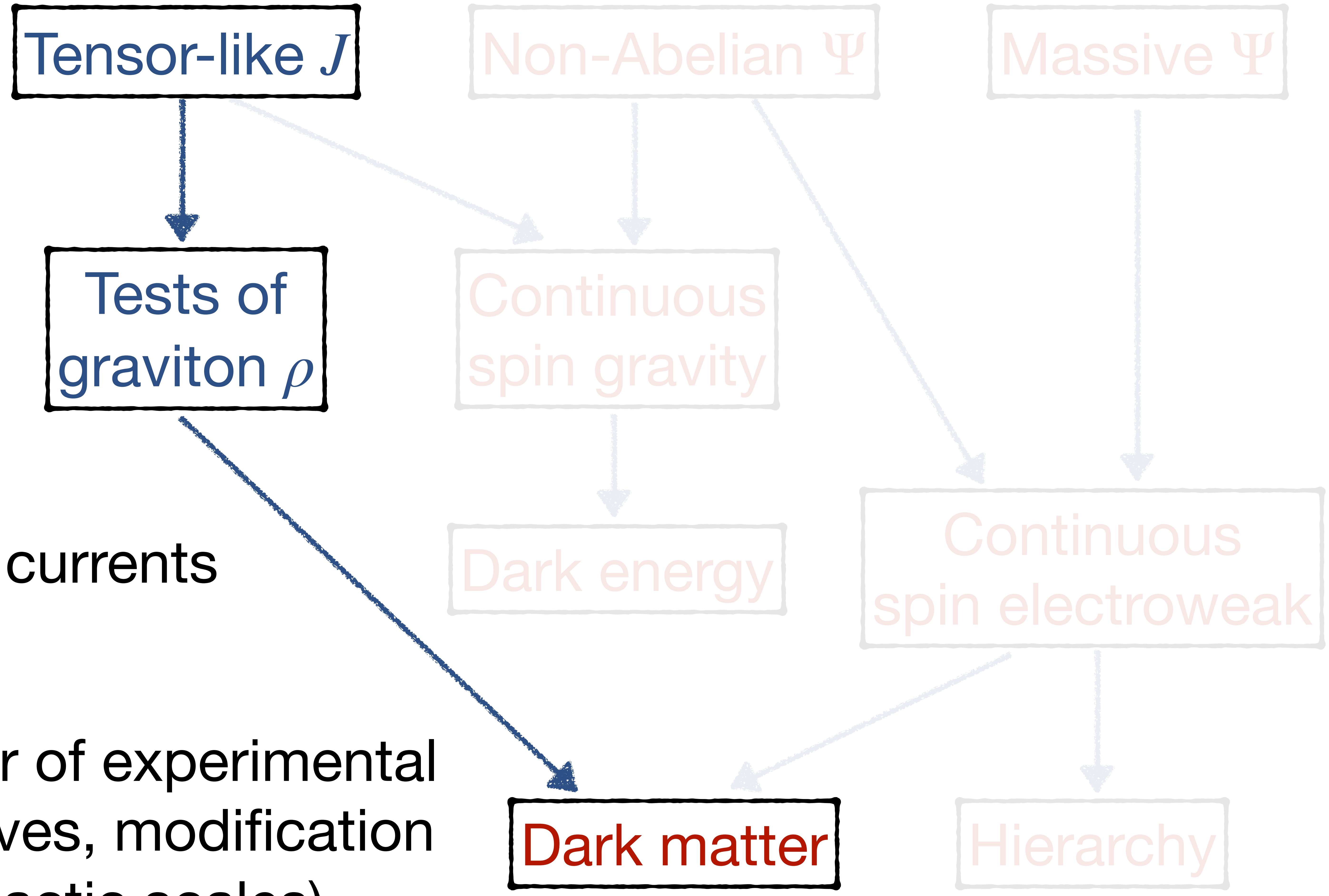
Consistency at loop level is an interesting formal question, but loop effects not clearly relevant to phenomenology

Responsible Steps and Higher Speculations

Even in linearized gravity, can't couple particles without considering force that keeps them in orbit

Similar fix needed to define tensor-like currents

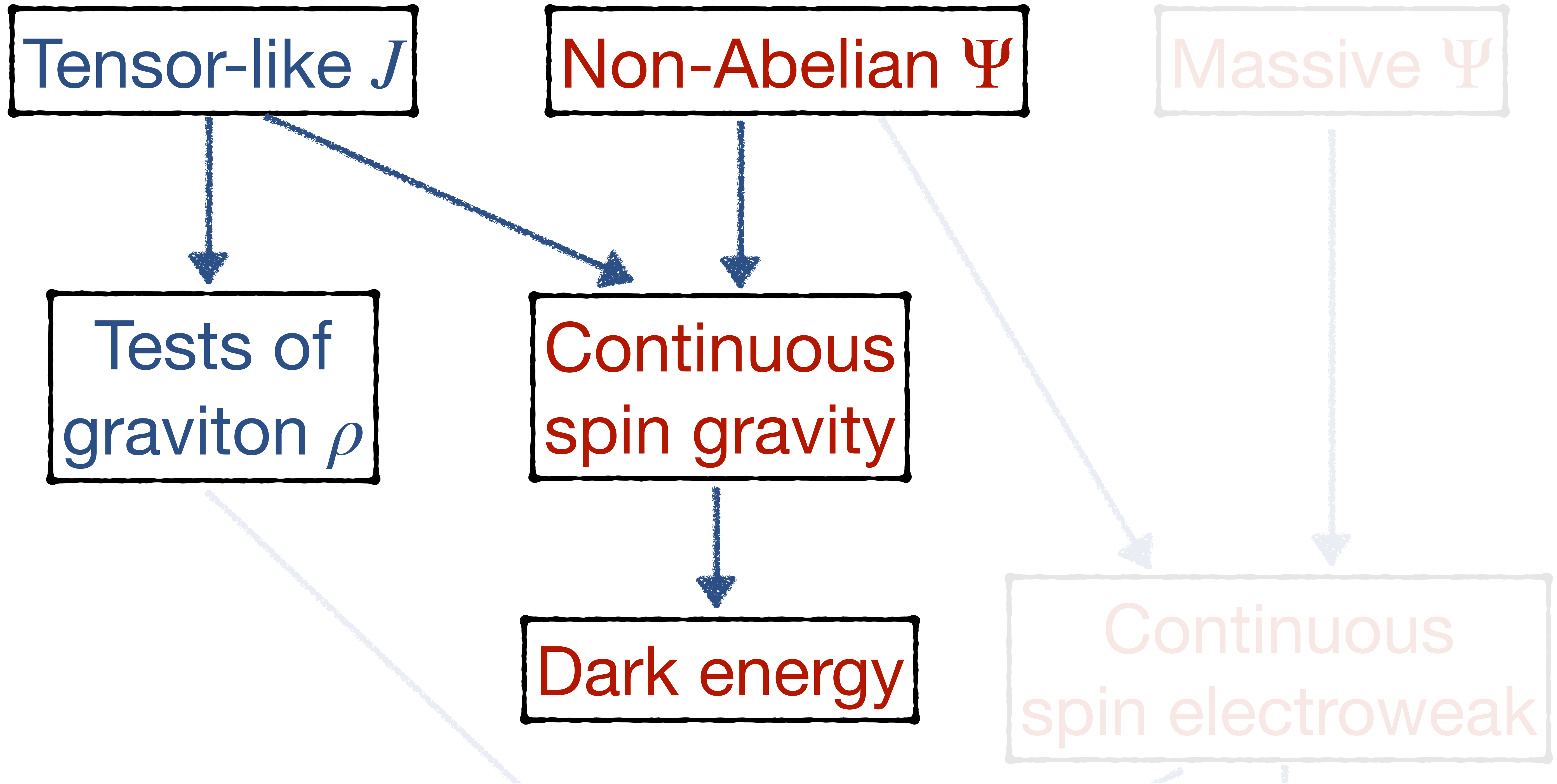
Once done, opens door to vast number of experimental probes (also including gravitational waves, modification of gravitational force for $1/\rho$ on galactic scales)



Responsible Steps and Higher Speculations

Linearized gravity with sources can only be made fully consistent by promoting to general relativity

Analogue for continuous spin requires non-Abelian case



Atomic probes

Loop effects

Continuous spin gauge symmetry appears to forbid cosmological constant $\mathscr{L} \sim \Lambda g^{\mu\nu} h_{\mu\nu}$

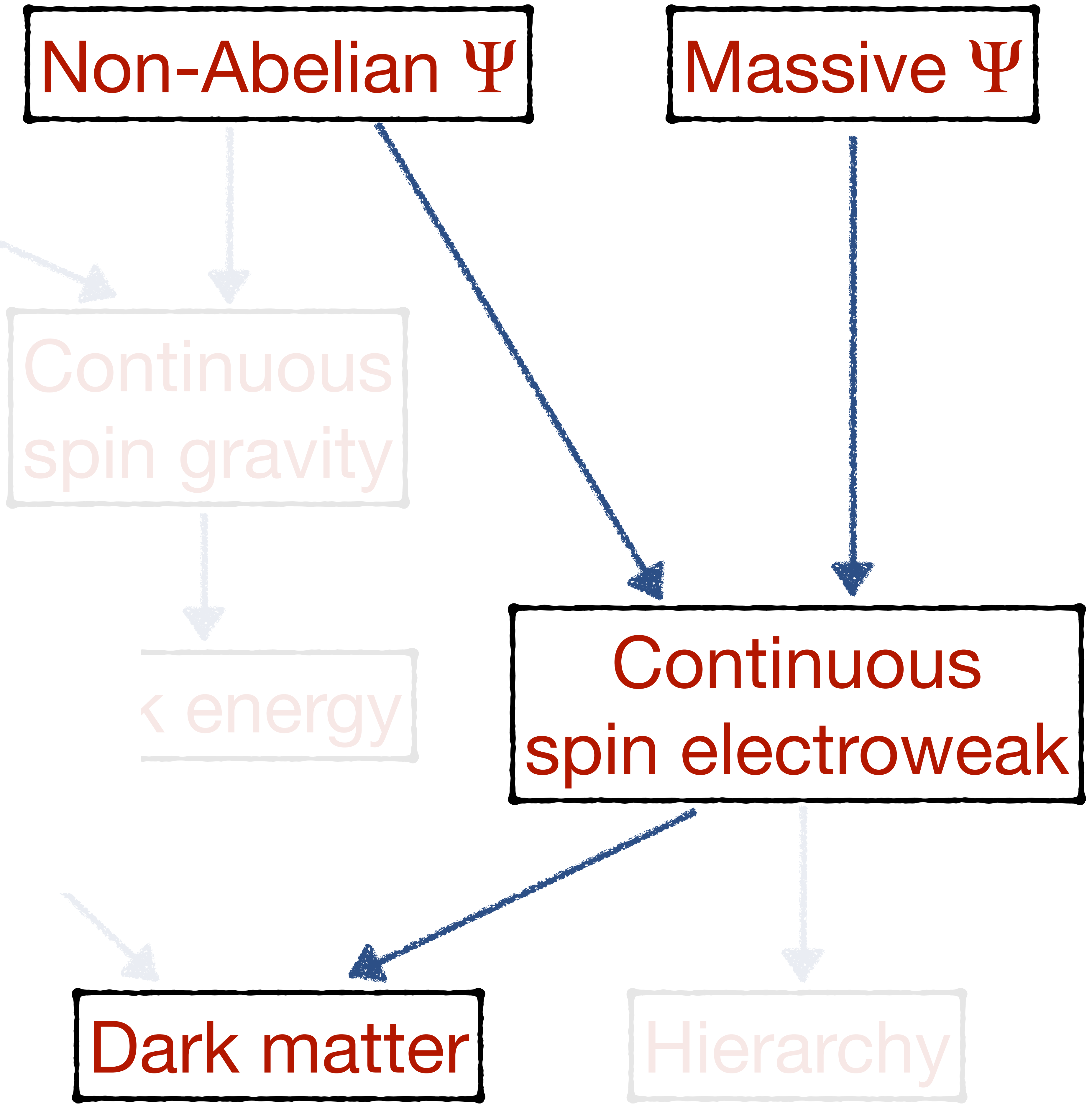
ierarchy

For $1/\rho$ on cosmological scales, may affect universe's expansion

Responsible Steps and Higher Speculations

Giving mass would yield an ordinary massive field with weakly-interacting massive “partner” fields

In SM, naturally produces DM candidates, like MSSM

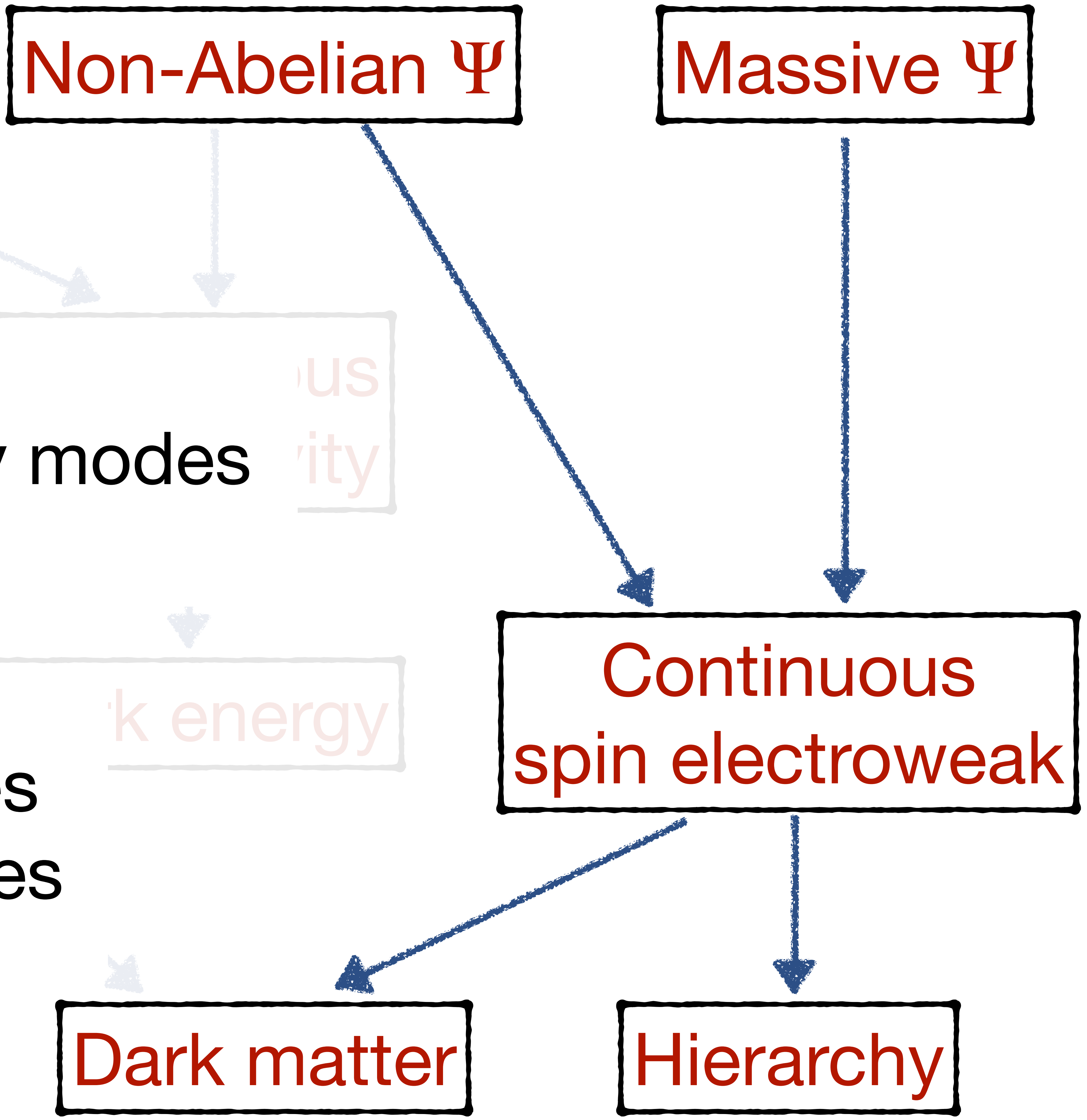


Responsible Steps and Higher Speculations

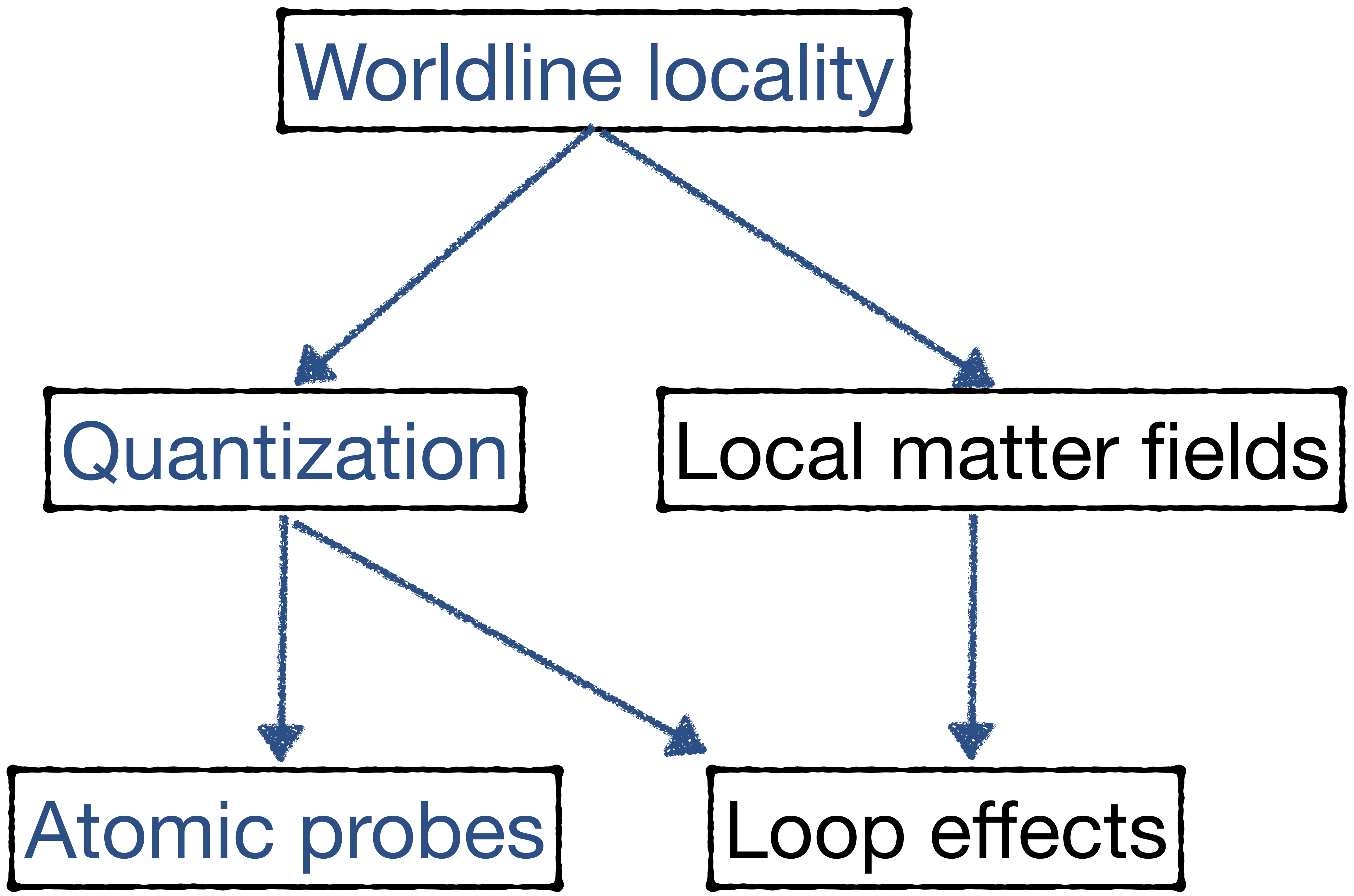
Continuous spin gauge symmetry protects the mass of scalar $h = 0$ mode, by relating it to $h \neq 0$ modes!

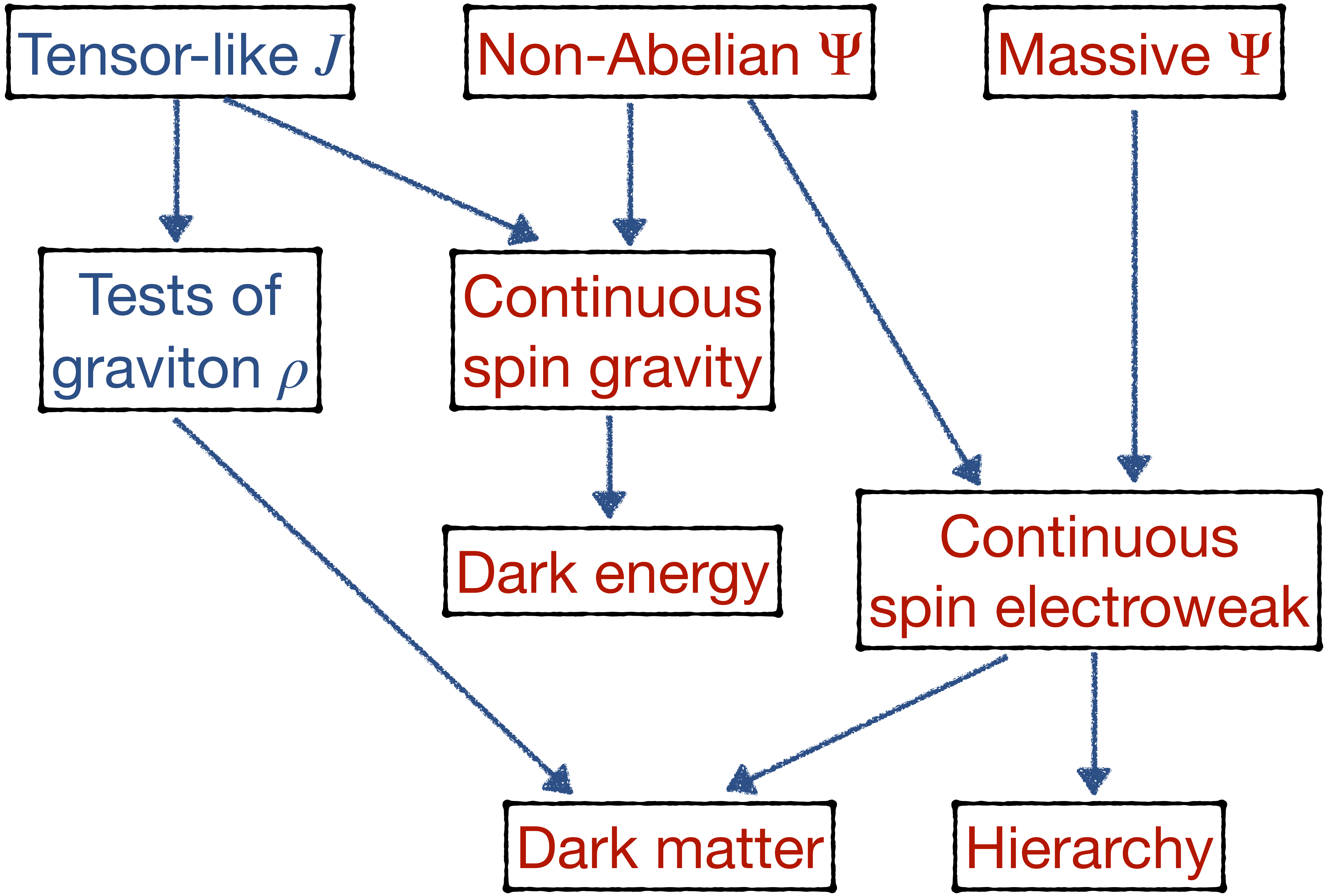
Related to conserved charge $N^{\mu\nu} = i\eta_{[\mu}\partial^x_{\nu]}$ mixing helicity modes

Bosonic analogue of supersymmetry translation, evades Coleman-Mandula by virtue of infinite number of helicities



Responsible Steps and Higher Speculations





Much work to do.

Thanks for listening!