

# Conclusion

This final problem set wraps up some loose ends and points out a way forward.

## 1 Partial Derivatives

In **T2**, we mentioned a few more advanced rules for using partial derivatives. In this section we'll consider some problems that require them.

- [2] **Problem 1.** For a function  $f(x, y)$  with differential

$$df = f_x dx + f_y dy$$

prove that the identity

$$\left. \frac{\partial f_x}{\partial y} \right|_x = \left. \frac{\partial f_y}{\partial x} \right|_y$$

must be satisfied. Conversely, if you are given a differential that doesn't satisfy this, you can be sure that it's not the differential of *any* function. For an ideal gas, write down the differential  $dQ$  in terms of  $dT$  and  $dV$ , and use it to prove that  $Q$  is not a state function.

- [2] **Problem 2.** Starting from the differential  $dU = -P dV + T dS$ , show that

$$-\left. \frac{\partial P}{\partial S} \right|_V = \left. \frac{\partial T}{\partial V} \right|_S.$$

This is an example of a Maxwell relation.

- [3] **Problem 3** (Blundell 12.2). Consider a gas with the equation of state  $PV = f(T)$ . Show that

$$\left. \frac{\partial P}{\partial T} \right|_V = \frac{1}{V} \frac{df}{dT}, \quad \left. \frac{\partial V}{\partial T} \right|_p = \frac{1}{p} \frac{df}{dT}.$$

Also show that

$$\left. \frac{\partial Q}{\partial V} \right|_P = C_P \left. \frac{\partial T}{\partial V} \right|_P, \quad \left. \frac{\partial Q}{\partial P} \right|_V = C_V \left. \frac{\partial T}{\partial P} \right|_V.$$









In an adiabatic process, we have  $dQ = 0$ . By taking the differential with respect to  $dp$  and  $dV$ , show that  $PV^\gamma$  is constant, thus generalizing the rule for adiabatic processes for ideal gases.

- [3] **Problem 4** (Kardar). We know from kinetic theory and the equipartition theorem that the energy of an ideal gas depends only on the temperature. However, this can also be deduced from pure thermodynamics. Starting from  $dE = T dS - P dV$ , show that  $PV = Nk_B T$  implies that  $E$  can only depend on  $T$ . You will have to use a Maxwell relation.

- [3] **Problem 5.**  [INPhO 2020, problem 1.](#)

## 2 Extra Problems

The following questions contain applied physics topics. They were not included in the main problem sets mostly because they used a variety of topics about equally, but they're great review practice.

- [5] **Problem 6.**  IPhO 2013, problem 2. A rather long but straightforward problem, with many numerical calculations, illustrating a real practical setup in detail. Use the data sheet of physical constants provided in the folder.
- [5] **Problem 7.**  IPhO 2013, problem 3. A solid question on an ice sheet, using thermo and data analysis. Use the data sheet of physical constants provided in the folder.
- [5] **Problem 8.**  IZhO 2022, problem 2. A practical problem on estimating the greenhouse effect.
- [5] **Problem 9.**  IPhO 2018, problem 3. Some models in biophysics, using fluids and thermo.
- [5] **Problem 10.**  IPhO 2021, problem 1. The basics of geophysics, using fluids and waves.
- [4] **Problem 11.**  GPhO 2022, problem 2. Some simple estimates involving renewable energy.
- [5] **Problem 12.**  APhO 2014, problem 1. A very hard problem on atmospheric convection.
- [5] **Problem 13.**  APhO 2012, problem 3. A somewhat technical problem that combines polarization and interference, to derive a neat effect called a “geometric phase”.

### 3 Extra Theoretical Exams

If you’ve made it this far, you’re probably training for the IPhO. I’ve reserved questions from a few Olympiads you can take as full-length exams for practice. Note that NBPhO exams are usually 2 days long, with 5 hours per day. However, they include experimental questions that you can’t do at home, and they’re somewhat easier than the IPhO, APhO, or EuPhO, so I’ve adjusted the time limits accordingly.

- [8] **Problem 14.** 🕒 USAPhO 2003.
- [8] **Problem 15.** 🕒 USAPhO 2005.
- [10] **Problem 16.** 🕒 USAPhO 2001.
- [10] **Problem 17.** 🕒 USAPhO 2000.
- [12] **Problem 18.** 🕒 [NBPhO 2021](#).
- [12] **Problem 19.** 🕒 [NBPhO 2019](#).
- [12] **Problem 20.** 🕒 [GPhO 2019](#).
- [15] **Problem 21.** 🕒 IPhO 2015.
- [15] **Problem 22.** 🕒 [NBPhO 2022](#).
- [18] **Problem 23.** 🕒 [EuPhO 2020](#).
- [18] **Problem 24.** 🕒 APhO 2017.
- [20] **Problem 25.** 🕒 IPhO 2019.
- [20] **Problem 26.** 🕒 [EuPhO 2021](#).
- [20] **Problem 27.** 🕒 APhO 2019.

Once you finish this and get your IPhO medal, you’ve definitely outgrown Olympiad physics. The stuff covered in Olympiads *is* real physics – the rough estimates and calculations a physicist would do when exploring a new idea actually resemble what you see in IPhO and APhO problems. But the IPhO and APhO are restricted by a syllabus that assumes very little mathematical background. There is a whole lot more beautiful, fascinating stuff you can do once you pick up vector calculus and linear algebra, the two most important pieces of the physicist’s toolkit. For more advice on what you can do next, see my [second advice file](#). You have a lot of options, and you’ve earned it.