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(35) $\mathbb{E}\left[(Y-c)^2 \mid X=x\right] =$

$$= \mathbb{E}\left[Y^2 \mid X=x\right] - 2c\,\mathbb{E}\left[Y \mid X=x\right] + c^2 = g(c)$$

$$g'(c) = 2\left(c - \mathbb{E}\left[Y \mid X=x\right]\right) \overset{set}{=} 0$$

$$g''(c) = 2 > 0$$

$$\Rightarrow f^*(x) = c^* = \mathbb{E}\left[Y \mid X=x\right].$$

$$R(f^*) = \mathbb{E}\left[Y^2 \mid X=x\right] - \mathbb{E}\left[Y \mid X=x\right]^2 =$$

$$= \mathrm{Var}\left[Y \mid X=x\right]$$

(36) a) $g(c) = \mathbb{E}\left[\,|Y-c| \mid X=x\right] =$

$$= \int_{-\infty}^{+\infty} dF_{\eta}(y|x)\,|y-c| =$$

$$= \int_{c}^{+\infty} dy\, f_{\eta}(y|x)(y-c) + \int_{-\infty}^{c} dy\, f_{\eta}(y|x)(c-y) =$$

$$= \underbrace{\int_{c}^{+\infty} dy\, f_{\eta}(y|x)\, y - \int_{-\infty}^{c} dy\, f_{\eta}(y|x)\, y}_{= \mathbb{E}[Y|X=x] - 2\int_{-\infty}^{c} dy\, f(y|x)} - c\left(1 - F_{\eta}(c|x)\right) +$$

$$+ c\,F_{\eta}(c|x)$$

$g'(c) = -2\cancel{f_\eta(c|x)}c - (1-F_\eta(c|x)) +$

$+ \cancel{c f_\eta(c|x)} + F_\eta(c|x) + \cancel{c f_\eta(c|x)} =$

$= 2F_\eta(c|x) - 1 \overset{\text{set}}{=} 0$

$g''(c) = 2 f_\eta(c|x) \geq 0$

$F_\eta(c^*|x) = 0{,}5 \Rightarrow$

это квантиль пор-ка 0,5:

$\frac{1}{2}$ эл-тов $< c^*$, $\frac{1}{2}$ эл-тов $\geq c^*$

$\Rightarrow f^*(x) = \text{median}(Y|X=x)$

37 Это просто.

$L(y', y) = -\delta(y - y')$

Тогда $g(c) = \mathbb{E}[\delta(Y-c)|X=x] =$

$= -f_\eta(c|x) \Rightarrow c^* = \arg\max_c f_\eta(c|x)$

т.е. это мода.

(38) $L(y', y)$:

$y' \backslash y$	0	1
0	0	l_0
1	l_1	0

$$f^*(x) = \arg\min_y \mathbb{E}_Y[L(Y, y) \mid X = x] =$$

$$= \arg\min_y \underbrace{\left(\delta_y^0 \cdot l_1 P_M(y|x) + \delta_y^1 \cdot l_0 \cdot P_M(y|x)\right)}_{(\delta_y^0 l_1 + \delta_y^1 l_0) P_M(y|x)} =$$

$$= \arg\min_y \begin{cases} l_1 P_M(0|x), & y = 0 \\ l_0 (1 - P_M(0|x)), & y = 1 \end{cases} =$$

$$= \begin{cases} 0, & P_M(0|x) \le \frac{l_0}{l_1 + l_0} \\ 1, & P_M(0|x) > \frac{l_0}{l_1 + l_0} \end{cases}$$

$$\arg\max_y l_y P_M(y|x) = \arg\max_y \begin{cases} l_0 P_M(0|x), & y = 0 \\ l_1 (1 - P_M(0|x)), & y = 1 \end{cases} =$$

$$= \begin{cases} 0, & P_M(0|x) \le \frac{l_0}{l_1 + l_0} \\ 1, & P_M(0|x) > \frac{l_0}{l_1 + l_0} \end{cases}$$

39 $L(y', y) = l_{y'y}$

$$f^*(x) = \arg\min_{y'} \mathbb{E}(L(y', Y) \mid X = x) =$$

$$= \arg\min_{y'} \sum_{y'=1}^{k} l_{y'y} P_M(y \mid x)$$