Math 240: Midterm 2 Q4

Daniel Ko

Spring 2020

§1 a

Prove that $2x^3 + 3x$ is $O(x^4)$.

Proof. We need to choose $C, k > 0$ such that for all $x > k$, $2x^3 + 3x \leq Cx^4$.
Choose $C = 5$ and $k = 1$. Then for all $x > 1$,

$$\begin{aligned} 5x^4 &= 2x^4 + 3x^4 \\ &> 2x^3 + 3x^4 && x > 1 \\ &> 2x^3 + 3x && x^3 > 1 \end{aligned}$$

as desired □

§2 b

Prove that $x^4 + 3x$ is not $O(2x^3 + 3x)$.

Proof. Given $C, k > 0$, we need to choose $x > k$ such that $x^4 + 3x > C \cdot (2x^3 + 3x)$.
Choose $x = max\{5C, k\} + 1$. Then $x > k$ and

$$\begin{aligned} x^4 + 3x &> 5Cx^3 && x > 5C \text{ and } x > 0 \\ &= 2Cx^3 + 3Cx^3 \\ &> 2Cx^3 + 3Cx && x > 0 \text{ and } C > 0 \\ &= C \cdot (2x^3 + 3x) \end{aligned}$$

as desired. □