

QQ

Monday, October 30, 2023 12:07 AM

□ What's your definition
of derive? #5 + b

- spreadsheet
- c++
- solve it algebraically

#6 recursion vs non recursion

w/ timing or operational analysis
or mathematical.

1145
original dominant term

take limit when you get dom. term \rightarrow ad becomes constant

#8 bit vector 90% full
5 bits

Vector Size = ?

Bigger sample size increases the
odds?

Why is the coin analogy biased?

11:36

#12 only rewrite make function

Reactive

array and form
and size array

and know

sd = 0 null
|

+ 1 for many multiple size

short code

W sets and maps

1. sorts

2. counts - changes resets counter,
loops finds max freq

then counts max freq

num nodes, then
choose array.

- shock # nodes and
freq, and modes

$n + 2$

loop
stucks in important
info.

Shrink code

* didn't make us do a
timings for studying
w/r. to maps and sets ↗

wrt. to maps and sets

do you save time with
here

order of map

by hand

#(Big D x f(n))

getting order

fb loop

h fib-amy

fb rev

1143

2D Bios comb Cpp

probabilityCards_1pair

Saturday, October 28, 2023 10:04 PM

Problem 8

Given 4 cards with
13 possible face values,
calculate the probability of

1 pair ?

2 pair ?

3 of a Kind ?

4 of a Kind ?

sample size :

4 suits
 \times 13 values per suit
n Cards = 52 cards

sample size : $|52 \times 51|$

sample size = $|2652|$

	1	2	3	4	5	6	7	8	9	10	11	12	13
♣	2	3	4	5	6	7	8	9	10	J	Q	K	A
♦	2												
♥	2												
♦	2												

2S 2C 2D 2H
C D H X D H S C H S C H
2 2 1 0

C D H S D H S C 11 D 4 4

$$\begin{array}{r} 3 \\ + 2 \\ + 1 \\ + 0 \\ \hline \end{array}$$

6 possibilities for one suit value.

$\times 13$ face values

78 ways to get a pair of any kind of face value

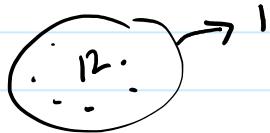
n ways to get 1 pair of 2
sample size

=

get 1 card $\rightarrow \frac{1}{13}$



get 2 cards $= \frac{1}{12}$



$$\frac{1}{13} \times \frac{1}{12} = \frac{1}{156} =$$

$$n C_r \binom{n}{r} = \frac{n!}{r!(n-r)!} = C(n, n-r)$$

unique #'s
selection

$$C \binom{13}{2} = \frac{13!}{1!(13-1)!} \times C(4) \frac{4!}{2!2!}$$

$$\begin{aligned}
 C\left(\frac{13}{2}\right) &= \frac{13!}{1!(13-1)!} \times C\left(\frac{4}{2}\right) \frac{4!}{2!2!} \\
 &= \frac{13 \cdot 12 \cdot 11 \cdot 10 \dots}{1 \cdot 12 \cdot 11 \cdot 10 \dots} \times \frac{4 \cdot 3 \cdot 2 \cdot 1}{4} \\
 &= 13 \times 6
 \end{aligned}$$

$$nCr = 78 \text{ combinations for 1 face value card}$$

$$\begin{aligned}
 &= 78 \times 13 \text{ types of face cards} \\
 &= 1014 \text{ ways to get a pair} \\
 \text{sample size} &= |52 \times 51| = 2652
 \end{aligned}$$

$$\frac{1014}{2652} \times 100 = 38\%$$

① pick 1 card value from the 13 diff face values

$$\frac{\# \text{ of diff values}}{\# \text{ used!} \# \text{ unused!}}$$

$$\frac{13!}{1!12!} = \boxed{13 \text{ ways to pick one type of face value card}}$$

② 4 2's cards

$$= \frac{\text{total num of 2's}}{\text{used! not used!}}$$

$$= \frac{4! \# 2's}{2! 2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 1 \cdot 2} = \frac{24}{4}$$

6 ways to get one pair #2's

(3)

$$\text{Part 1} \times \text{Part 2} = 13 \times 6 = \boxed{78 \text{ ways to get one pair}}$$

(4)

78 ways to get 1 pair of an. type of card
 $\times 13$ diff face value cards

1014 ways to get 1 pair in a deck

(5)

total ways to get 1 pair from deck

Sample size

$$| 52 \times 51 |$$

$$\frac{1014}{52 \times 51} = \frac{1014}{2652} \times 100 =$$

38.2%
of getting
1 pair

2pair3_4ofKind

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2 pair

$$\text{sample size} = |52 \cdot 51 \cdot 50 \cdot 49| = |6,497,400|$$

$${}_{13}C_1 \times {}_4C_2 + {}_{13}C_1 + {}_4C_2$$

$$\left(\frac{13!}{1!12!} \times \frac{4!}{2!2!} \right)^2$$

$$\left(\frac{1014}{2652} \right)^2$$

$$\frac{1014^2}{2652^2} \times 100 = \boxed{14.6\% \text{ of getting} \\ 2 \text{ pairs}}$$

3 of a kind

$$\text{sample size: } |52 \cdot 51 \cdot 50| = |132,600|$$

$${}_{13}C_1 \binom{13}{1} \frac{13!}{1!(13-1)!} \times {}_4C_3 \binom{4}{3} \frac{4!}{3!1!}$$

$$\frac{13 \cdot 12 \cdot 11}{1 \cdot 12 \cdot 11} \times \frac{4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1}$$

$13 \times 4 =$ 52 ways to get
3 of a kind

$$\frac{52}{132,600} \times 100 =$$
 0.0392 %

Four of a Kind:

sample size: $| 52 \cdot 51 \cdot 50 \cdot 49 | = 6,497,400$

$$13C_1 \times 4C_4$$

$$\binom{13}{1} \frac{13!}{1!(13-1)!} \times \binom{4}{4} \frac{4!}{4!}$$

$13 \times 1 = 13$ ways to get
four of a kind

$$\frac{13}{6497400} \times 100 =$$
 0.00020008 %
of getting
"four of a kind"

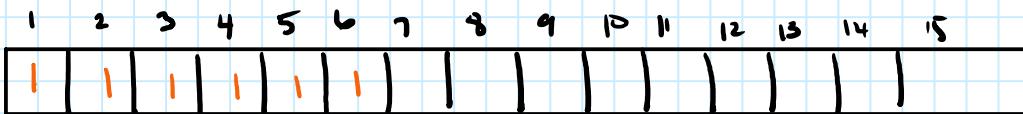
BitVector

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$$n = 15$$

$$\text{filled} = n \times 40\% = 6 \text{ filled}$$

Probability 5 bits all fall within the filled section?



$$C \left(\begin{smallmatrix} 15 \\ 5 \end{smallmatrix} \right)$$

$$C \left(\begin{smallmatrix} 40 \\ 5 \end{smallmatrix} \right) = \frac{40!}{5! 35!}$$

$$= \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36}{5!} \cancel{\cdot 35!} \cdot \cancel{34!} \dots$$

$$= \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36}{5!}$$

$$= \frac{3.079 \times 10^9}{120}$$

possible ways to fill the 40% filled section of the vector

$$= 2.567 \times 10^7$$

problem5_errorSinCos

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$$\text{if } \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

and approx. $\sin(\frac{1}{n}) \approx \frac{1}{n}$, then

order of error $\sin(\frac{1}{n}) \approx O(?)$

$$\boxed{\text{error} = \sin\left(\frac{1}{n}\right) - \text{approx } \sin\left(\frac{1}{n}\right)}$$

$$\text{error} = \frac{\frac{1}{n} - \frac{1}{n^3} - \frac{1}{3!} + \frac{1}{n^5} - \frac{1}{5!} + \frac{1}{n^7} - \frac{1}{7!}}{-\frac{1}{n}}$$

$$\text{error } \sin\left(\frac{1}{n}\right) = -\frac{1}{n^3} + \frac{1}{n^5} - \frac{1}{n^7}$$

Order of error is the leading

Order of error is the leading or largest polynomial of degree, d .

$$p(n) = O\left(\frac{1}{n^d}\right)$$

$$d = 3$$

FibArray (int n) $n = [4b, 5b]$

```
int array [n+1]
array [0] = 0
array [1] = 1
for (int i=2; i<=n; i++) {
    array [i] = array [i-1] + array [i-2]
}
return array[n] returns array [4b]
```

$0_b \quad 0_i$

$$\begin{aligned} 0_b + \sum_{i=2}^n (0_i + 0_{i-1} + \dots + 0_{i-2}) \\ 0_b + \sum_{i=2}^n 0_i + \sum_{i=2}^n 0_{i-1} + \dots + \sum_{i=2}^n 0_{i-2} \\ \sum_{i=2}^n 0_i = \sum_{m=1}^n 1 = (N-m)+1 \\ = N-m+1 \\ = (N-(2+1))0_{i-1} \\ = (N-2+1)0_{i-1} \\ = (N-1)0_{i-1} \quad [i-1] \\ \sum_{i=2}^n 0_i = \sum_{m=1}^n 1 = (N-m)+1 \\ = (N-(m+1))+1 \\ = (N-2-1+1) \\ = (N-2)0_{i-1} \quad [i-2] \end{aligned}$$

$$\begin{aligned} \sum_{i=m+2}^n a_i &= a_m + a_{m+1} + a_{m+2} + a_{m+3} \dots a_{m+1} + a_n \\ \sum_{i=2}^n a_i &= a_2 + a_{3,1} + a_{3,2} + a_{3,3,1} \dots a_{3,1} + a_n \end{aligned}$$

Formal defined recursively:

$$\sum_{i=a}^b g(i) = 0, \quad b < a$$

$$\sum_{i=a}^b g(i) = g(b) + \sum_{i=a}^{b-1} g(i), \quad \text{for } b \geq a$$

$$\sum_{i=2}^n 0_i = \frac{N(N+1)}{2}$$

if $T(0) = T(1) = 0$

$\exists (n > 1)$

$$T(n) = T(n-1) + T(n-2) +$$

$\star T(n-2) \approx T(n-1)$ backward subst.

$$T(n) = T(n-1) + T(n-1) +$$

$$T(n) = 2T(n-1) + \quad \rightarrow T(n-1) = 2T(n-1) + 1$$

$$= 2 \times [2T(n-1) + 1] + 1$$

$$= 2 \times [2T(n-1) + 2] + 1$$

$$= 4T(n-2) + 3 \quad \star T(n-2) = 2T(n-3) + 1$$

$$= 2 \times [2 \times [2T(n-3) + 1]]$$

$$= 2 \times [2 \times [2T(n-3) + 1] + 1] + 1$$

$$= 8T(n-3) + 2 + 2 + 2 + 1 \quad \star T(n-3) = 2T(n-4) + 1$$

$$= 2 \times [2 \times [2 \times [2T(n-4) + 1]] + 1] + 1$$

$$= 16T(n-4) + 15$$

$$= T(n) = 2^k \cdot T(n-k) + (2^k + 1), \quad k \in \mathbb{R}$$

Induction to prove $T(n)$.

Subst $T(0) = 1$

$$n-k=0$$

$$k=n$$

$$\begin{aligned}T(n) &= 2^n \cdot T(0) + (2^n - 1) \\&= 2^n + 2^n - 1 \\T(n) &= O(2^n)\end{aligned}$$

* approx $T(n-2) \approx T(n-1)$,
since $T(n-2) \leq T(n-1)$

Tight upper bound

$$\begin{aligned}T(n) &= O(\vartheta^n) \\ \vartheta &= \frac{(1+\sqrt{5})}{2}\end{aligned}$$

Fib

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for (0 = n < = nloop)

fibAry(n) [0, nloop] ^{4b}

return s[i:n]				
2 <= i <= n				
n	i <= n	i-1	i-2	
0	$[0] = 0$			
1	$[1] = 1$			
2	$[2] = 1$	$1 + 0$	1	
2	$[2] = 1$	$2^0 \cdot 1 = 1$	$2^0 \cdot 1 = 1$	1
3	$[3] = ?$ 3	$2^1 \cdot 1 = 1$	$2^1 \cdot 1 = 1$	2
i = 2	$[2] = 1$	$[2-1] = 1$ $[1] = 1$ $[0] = 0$	$[2-2] = 1$ $[1] = 1$ $[0] = 0$	$= [1] + [1] = 1$
i = 3	$[3] = 2$	$[3-1] = 1$ $[2] = 1$ $[1] = 1$ $[0] = 0$	$[3-2] = 1$ $[2] = 1$ $[1] = 1$ $[0] = 0$	$= [2] + [1] = 1 + 1 = 2$
i = 4	$[4] = ?$ 3	$[4-1] = 2$ $[3] = 2$ $[2] = 1$ $[1] = 1$ $[0] = 0$	$[4-2] = 2$ $[3] = 1$ $[2] = 1$ $[1] = 1$ $[0] = 0$	$= [3] + [2] = 2 + 1 = 3$

0, 1, 1, 2, 3, 5

$$f(n) = (n-1) + (n-2) , \quad x \geq 2$$