QQ

Monday, October 30, 2023 12:07 AM

☐ what's your definition of derive ? #5 + 6

- spread sheet
- c++
- solve it algrabeically

#6 recursion vs non recursion

w/ timing or operational analysis or mathematical.

1145

originated dominant term

take limit then you get dom. term —▷ and becomes constant

#8 bit vector 40% full
5 bits

Vector Size = ?

Bigger sample size increases the odds ?

Why is the coin analogy biased ?

11:36

#12 only rewrite mode function

rewrite

change a mid term
and size array

and know

id = 0 null
1

+1 < array multip size

shrink code
w/ sets and maps

1. sorts
2. counts - if changes resets counter,
loops finds max freq

then counts max freq

num mods, then
create array.
• stick # modes and
freq, and modes

n+2

Loop
sticks in important
info.

Shrink code

* didn't make us do a
timing / op study
w/r. to maps and sets

w/r. to maps and sets

do you save time with these

order ~~A~~ Ecmor

by hand

#6 BigO x f(n)

getting order

~~A~~ Fib-array

Fib recur

Fib loop

1143

2D Bios combo cpp

Problem 8

Given 4 cards with
13 possible face values,

calculate the probability of

1 pair ?
2 pair ?
3 of a kind ?
4 of a kind ?

sample size:

$$\begin{array}{rl} & 4 \text{ suits} \\ \times & \underline{13} \text{ values per suit} \\ \text{nCards} = & 52 \text{ cards} \end{array}$$

sanple size : $|52 \times 51|$

$$\boxed{\text{sample size} = |2652|}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13
♠	2	3	4	5	6	7	8	9	10	J	Q	K	A
♣	2												
♦	2												
♥	2												

2S	2C	2D	2H
C D H	~~S~~ D H	~~S~~ ~~C~~ H	~~S~~ ~~C~~ ~~D~~
3	2	1	0

C D H ~~S~~ D H ~~S~~ ~~C~~ " ~~S~~ ~~C~~ ~~D~~ ~~H~~

3 + 2 + 1 + 0

6 possibitites for one suit value.

x 13 face values

78 ways to get a pair of any kind of face value

$$\frac{\text{nWays to get 1 pair of 2}}{\text{sample size}}$$

=

get 1 card → $\frac{1}{13}$

13 → 1

get 2 cards = $\frac{1}{12}$

12 → 1

$$\frac{1}{13} \times \frac{1}{12} = \frac{1}{156} =$$

unique #'s

$$_nC_r \binom{n}{r} = \frac{n!}{r!(n-r)!} = C(n, n-r)$$

selection

$$C\binom{13}{2} = \frac{13!}{1!(13-1)!} \times C\binom{4}{2} \quad \frac{4!}{2!2!}$$

$$C\binom{13}{2} = \frac{13!}{1!\,(13-1)!} \times C\binom{4}{2}\ \frac{4!}{2!\,2!}$$

$$= \frac{13\cdot\cancel{12}\cdot\cancel{11}\cdot\cancel{10}\cdots}{1\ \cancel{12}\cdot\cancel{11}\cdot\cancel{10}\cdots} \times \frac{\cancel{4}\cdot 3\cdot 2\cdot 1}{\cancel{4}}$$

$$= 13 \times 6$$

$$\boxed{{}_nC_r = 78 \text{ combinations for 1 face value card}}$$

$= 78 \times 13$ types of face cards

$= 1014$ ways to get a pair

sample size $= |52 \times 51| = 2652$

$\frac{1014}{2652} \times 100 = 38\%$

---

① pick 1 card value from the 13 diff face values

$$\frac{\#\text{ of diff values}}{\#\text{ used}!\ \#\text{ unused}!}$$

$$\frac{13!}{1!\,12!} = \boxed{13 \text{ ways to pick one type of face value card}}$$

② 4 2's cards

$$= \frac{\text{total num of 2's}}{\text{used}!\ \text{not used}!}$$

$$= \frac{4!}{2!\,2!}\ \#2\text{'s} = \frac{4\,3\,2\,1}{1\cdot2\cdot1\cdot2} = \frac{24}{4}$$

$\boxed{\text{6 ways to get one pair \#2's}}$

(3)

Part 1 × Part 2 = 13 × 6 = $\boxed{\text{78 ways to get one pair}}$

(4)

78 ways to get 1 pair of an. type of card
x 13 diff face value cards

$\boxed{\text{1014 ways to get 1 pair in a deck}}$

(5)

$$\frac{\text{total ways to get 1 pair from deck}}{\text{sample size}}$$

|52 × 51|

$$\frac{1014}{52 \times 51} = \frac{1014}{2652} \times 100 = \boxed{\text{38.2\% of getting 1 pair}}$$

prb7b_2pair3_4ofKind

Sunday, October 29, 2023 12:37 AM

2 pair

sample size $= |52 \cdot 51 \cdot 50 \cdot 49| = |6,497,400|$

$_{13}C_1 \times {}_4C_2 + {}_{13}C_1 + {}_4C_2$

$$\left(\frac{13!}{1!\,12!} \times \frac{4!}{2!\,2!}\right)^2$$

$$\left(\frac{1014}{2652}\right)^2$$

$$\frac{1014^2}{2652^2} \times 100 = \boxed{14.6\% \text{ of getting 2 pairs}}$$

3 of a kind

sample size: $|52 \cdot 51 \cdot 50| = |132,600|$

$$_{13}C_1 \binom{13}{1} \frac{13!}{1!\,(13-1)!} \times {}_4C_3 \binom{4}{3} \frac{4!}{3!\,1!}$$

$$\frac{13 \cdot \cancel{12 \cdot 11}}{1 \cdot \cancel{12 \cdot 11}} \times \frac{4 \cdot \cancel{3 \cdot 2} \cdot 1}{\cancel{3 \cdot 2} \cdot 1 \cdot 1}$$

$13 \times 4 = \boxed{\text{52 ways to get 3 of a kind}}$

$$\frac{52}{132{,}600} \times 100 = \boxed{0.0392\%}$$

Four of a Kind:

sample size: $|52 \cdot 51 \cdot 50 \cdot 49| = |6{,}497{,}400|$

$${}_{13}C_1 \times {}_4C_4$$

$$\binom{13}{1} \frac{13!}{1!(13-1)!} \times \binom{4}{4} \frac{4!}{4!}$$

$13 \times 1 = 13$ ways to get four of a kind

$$\frac{13}{6497400} \times 100 = \boxed{0.00020008\% \text{ of getting "Four of a Kind"}}$$

BitVector

Monday, October 30, 2023 12:09 AM

$n = 15$

$\text{filled} = n \times 40\% = 6 \text{ filled}$

Probability 5 bits all fall within the filled section?

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1									

$C\binom{15}{6}$

$$C\binom{40}{5} = \frac{40!}{5!\,35!}$$

$$= \frac{40\ 39\ 38\ 37\ 36\ \cancel{35!}\ \cancel{34}\ldots}{5!\ \cancel{35}!}$$

$$= \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36}{5!}$$

$$= \frac{3.079 \times 10^9}{120}$$

possible ways to fill the 40% filled section of the vector $\boxed{= 2.567 \times 10^7}$

prb5_errorSinCos

Monday, October 30, 2023 12:29 AM

if $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

and approx. $\sin(\frac{1}{n}) \approx \frac{1}{n}$, then

order of error $\sin(\frac{1}{n}) \approx O(?)$

$$\boxed{\text{error} = \sin\left(\frac{1}{n}\right) - \text{approx } \sin\left(\frac{1}{n}\right)}$$

$$\text{error} = \begin{array}{l} \cancelto{0}{\frac{1}{n}} - \frac{\frac{1}{n^3}}{3!} + \frac{\frac{1}{n^5}}{5!} - \frac{\frac{1}{n^7}}{7!} \\ -\frac{1}{n} \\ \hline \end{array}$$

$$\text{error } \sin\left(\frac{1}{n}\right) = -\frac{\frac{1}{n^3}}{3!} + \frac{\frac{1}{n^5}}{5!} - \frac{\frac{1}{n^7}}{7!}$$

Order of error is the leading

Order of error is the leading or largest polynomial of degree, d.

$$\boxed{\begin{gathered} p(n) = O\left(\frac{1}{n^d}\right) \\ d = 3 \end{gathered}}$$

FibAray(int n) n= [46,50]

```
int array [n+1]

array [0] = 0
array [1] = 1

for ( int i=2; i<=n; i++){
    array [i] = array [i-1] + array [i-2]
}
return array[n]
```

47

2<=46

returns array [46]

O_b O_i

n loop in main()

op on i

subtract i-1

subtract i-2

$$O_b + \sum_{i=2}^{N} \left(O_i + O_{is1} + + O_{is2} \right)$$

$$O_b + \sum_{i=2}^{N} O_i + \sum O_{is1} + \sum O_{is2}$$

$$\sum O_{is1} = \sum_{m=i=2}^{N} 1 = (N-m)+1$$
$$= N-m+1$$
$$= (N-(2)+1)\, O_{is1}$$
$$= (N-2+1)\, O_{is1}$$
$$= (N-1)\, O_{is1} \quad [i-1]$$

constant

SumofN = c(n-m+1)

$$\sum O_{is2} = \sum_{m=i=2}^{N} = (N-m)+1$$
$$= (N-(m+1))+1$$
$$= (N-2-1+1)$$
$$= (N-2)\, O_{is2} \quad [i-2]$$

$$\sum_{i=m=2}^{n} a_i = a_m + a_{m+1} + a_{m+2} + a_{m+3} \dots a_{n-1} + a_n$$

$$\sum_{i=2}^{n} a_n = a_2 + a_{2+1} + a_{2+2} + a_{2+3} + \dots a_{N-1} + a_N$$

Formal defined recursively:

$$\sum_{i=a}^{b} g(i) = 0, \quad b<a$$

$$\sum_{i=a}^{b} g(i) = g(b) + \sum_{i=a}^{b-1} g(i), \text{ for } b \geq a$$

$$\sum_{i=2}^{N} O_i = \frac{N(N+1)}{2}$$

if $T(0) = T(1) = 0$

else $(n > 1)$

$$T(n) = T(n-1) + T(n-2) + 1$$

$\star\, T(n-2) \approx T(n-1)$ backward subst.

$$T(n) = T(n-1) + T(n-1) + 1$$

backward subst

$$T(n) = 2T(n-1) + 1 \longrightarrow T(n-1) = 2 \times T(n-1) + 1$$

$$= 2 \times \left[2 \times T(n-1) + 1 \right] + 1$$

$$= 2\left[2T(n-1) + 2 \right] + 1$$

$$= 4T(n-2) + 3$$

$\star\, T(n-2) = 2 \times T(n-3) + 1$

$$= 2 \times \left[2 \times \left[2 \times T(n-3) + 1 \right] \right]$$

$$= 2 \times \left[2 \times \left[2T(N-3) + 1 \right] + 1 \right] + 1$$

$$= 8T(n-3) + \cancel{2+2+2+1}\ 7$$

$\star\, T(n-3) = 2 \times T(n-4) + 1$

$$= 2 \times \left[2 \times \left[2 \times \left[2 \times T(n-4) + 1 \right] + 1 \right] + 1 \right] + 1$$

$$= 16\, T(n-4) + 15\,?$$

$$= T(n) = 2^K \cdot T(n-K) + (2^K + 1), \quad K \in R$$

Induction to prove T(n).

Subst $T(0) = 1$

$n-k=0$
$k=n$

$T(n) = 2^n \times T(0) + (2^n + 1)$
$= 2^n + 2^n - 1$
$T(n) = O(2^n)$

* approx $T(n-2) \approx T(n-1)$,
since $T(n-2) \leq T(n-1)$

<u>Tight upper bound</u>
$T(n) = O(\theta^n)$
$\theta = \frac{(1+\sqrt{5})}{2}$

Fib

Monday, October 30, 2023 11:48 PM

for (0=n<= nLoop) 10

fibAry(n) [0, nLoop] 46

returns a[n]

2=i<=n

0, 1, 1, 2, 3, 5

n	i<=n	i-1	i-2	
0	[0] =0			
1	[1] = 1			
2	[2] = 1	1 +	0	1
2	[2] = 1	2-1= [1]=1 +	2-2=0 [0]=0	1
3	[3] = ~~?~~ 2	3-1= [2]=1	3-2= [1]=1	2
i = 2	[2] =1	[2-1]= [1]=1 +	[2-2] [0]=0	=[1] + [0] = 1
i = 3	[3] = 2	[3-1]= [2]=1	[3-2]= [1]=1	[2] + [1] = 1 + 1 = 2
i = 4	[4] = ~~?~~ 3	[4-1]= [3]=2	[4-2]= [2]=1	= [3] + [2] 2 + 1 = 3

$$f(n) = (n-1) + (n-2) \quad , \quad x \geq 2$$