p6_fibonacciRecurVsNonRec

Monday, October 30, 2023 11:16 PM

FibAray (int n) $n = [46, 50]$

```
int array [n+1]        47

array [0] = 0
array [1] = 1
for ( int i=2; i<=n; i++){     2<=46
    array [i] = array [i-1] + array [i-2]
}
return array[n]    returns array [46]
```

$O_b \quad O_i$

$O_b + \sum_{i=2}^{N} (O_i + O_{s1} + + O_{s2})$

n loop in main()   op on i   subtract i-1   subtract i-2

$O_b + \sum_{i=2}^{N} O_i + \sum O_{s1} + \sum O_{s2}$

$\sum O_{is1} = \sum_{m=i=2}^{N} 1 = (N-m)+1$
$= N-m+1$ ← constant
$= (N-(2)+1) O_{is1}$
$= (N-2+1) O_{is1}$
$= (N-1) O_{is1}$ [i-1]

SumofN = C(n-m+1)

$\sum O_{is2} = \sum_{m=i=2}^{N} = (N-m)+1$
$= (N-(m+1))+1$
$= (N-2-1+1)$
$= (N-2) O_{is2}$ [i-2]

$\sum_{i=m=2}^{n} a_i = a_m + a_{m+1} + a_{m+2} + a_{m+3} \ldots a_{n-1} + a_n$

$\sum_{i=2}^{n} a_2 = a_2 + a_{2+1} + a_{2+2} + a_{2+3} + \ldots a_{N-1} + a_N$

Formal defined recursively:

$\sum_{i=a}^{b} g(i) = 0, \quad b<a$

$\sum_{i=a}^{b} g(i) = g(b) + \sum_{i=a}^{b-1} g(i)$, for $b \geq a$

$\sum_{i=2}^{N} O_i = \frac{N(N+1)}{2}$

if $T(0) = T(1) = 0$

else $(n > 1)$

$T(n) = T(n-1) + T(n-2) + 1$

★ $T(n-2) \approx T(n-1)$ backward subst.

$T(n) = T(n-1) + T(n-1) + 1$

backward subst

$T(n) = 2T(n-1) + 1 \longrightarrow T(n-1) = 2 * T(n-1) + 1$

$= 2 * [2 * T(n-1) + 1] + 1$

$= 2[2T(n-1) + 2] + 1$

$= 4T(n-2) + 3$ ★ $T(n-2) = 2 * T(n-3) + 1$

$= 2 * [2 * [2 * T(n-3) + 1]$

$= 2 * [2 * [2T(N-3) + 1] + 1] + 1$

$= 8T(n-3) + \cancel{2+2+2+1}$ 7

★ $T(n-3) = 2 * T(n-4) + 1$

$= 2 * [2 * [2 * [2 * T(n-4) + 1] + 1] + 1] + 1$

$= 16T(n-4) + 15$?

$= T(n) = 2^k \cdot T(n-k) + (2^k + 1), \quad k \in R$

Induction to prove T(n).

Subs) $T(0) = 1$

$n-k = 0$
$k = n$

$T(n) = 2^n * T(0) + (2^n + 1)$
$= 2^n + 2^n - 1$
$T(n) = O(2^n)$

★ approx $T(n-2) \approx T(n-1)$,

since $T(n-2) \leq T(n-1)$

Tight upper bound

$T(n) = O(\theta^n)$

$\theta = \frac{(1+\sqrt{5})}{2}$