Probabilities and Counting

Terminology

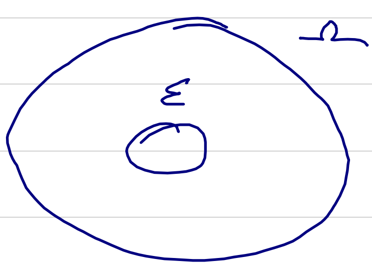
$p \rightarrow$ probability

$\mathcal{E} \rightarrow$ event

$m \rightarrow$ the count

$\Omega \rightarrow$ sample space

Diagram - Venn



$$p(\mathcal{E}) = \frac{m(\mathcal{E})}{m(\Omega)} \rightarrow 0 \le p(\mathcal{E}) \le 1$$

Ways to count

$$N! = \prod_{i=1}^{N} i = 1 \cdot 2 \cdot 3 \cdots N$$

$$\log_e N! = \log_e \prod_{i=1}^{N} i = \sum_{i=1}^{N} \log_e i$$

$$N! = e^{\log_e N!} = e^{\sum_{i=1}^{N} \log_e i}$$

Let P = permutation → ordered
C = combination → unordered
r = with replacement allowed

Define

$${}_{N}P_{m}^{r} = N^{m} \qquad {}_{N}C_{m}^{r} = \frac{(N+m-1)!}{(N-1)!\,m!}$$

$${}_{N}P_{m} = \frac{N!}{(N-m)!} \qquad {}_{N}C_{m} = \frac{N!}{(N-m)!\,m!}$$

Permutations with and without replacement

Combinations with and without replacement

Example problems

1) Calculate how many pin numbers are possible using 3 digits base 8 digits $\{0, 1, 2, \ldots 7\}$

$${}_{N}P_{m}^{r} = N^{m} = 8^{3} \rightarrow \{000, 001, \ldots 776, 777\}$$

$$\underline{\underline{512}}$$

2) Using 1) how many pin numbers have no duplicate digits

$$_{N}P_{m} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = \underline{\underline{336}}$$

3) What is probability of receiving a pin number from 2) & 1) where no numbers are repeated

$$P(E) = \frac{n(E)}{n(\Omega)} = \frac{N!/(N-m)!}{N^m} = \frac{336}{512} \approx \underline{\underline{66\%}}$$

4) If you have 3 gallons of ice cream, a gallon each of chocolate, vanilla, and strawberry. How many different triple cones are possible where order doesn't matter. List the combinations.

$$_{N}C^{r}_{m} = \frac{5!}{2!\,3!} = 10$$

cvs, ccv, ccs, vvc, vvs, ssc, ssv, ccc, vvv, sss.

5) Power ball lottery

5 numbers of 69 in any order
and 1 number of 26.

a) How many different ways to pull
a number

$$n(\Omega) = {}_{69}C_5 \; {}_{26}C_1$$

b) What is probability of winning

$$P(\varepsilon) = \frac{n(\varepsilon)}{n(\Omega)} = \frac{{}_5C_5 \; {}_1C_1}{{}_{69}C_5 \; {}_{26}C_1}$$

or odds $= 1 : 1/P(\varepsilon) = 1 : {}_{69}C_5 \; {}_{26}C_1$

c) What is probability or odds of 4 numbers

$$P(\varepsilon) = \frac{{}_5C_4 \; {}_{64}C_1 \; {}_{25}C_1}{{}_{69}C_5 \; {}_{26}C_1} = 1 : \frac{{}_{69}C_5 \; {}_{26}C_1}{{}_5C_4 \; {}_{64}C_1 \; {}_{25}C_1}$$