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Logarithm Rules

Mike Estela

5–6 minutes

Rules or Laws of Logarithms

In this lesson, you'll be presented with the common rules of logarithms, also known as the "log rules". These seven (7) log rules are useful in expanding logarithms, condensing logarithms, and solving logarithmic equations. In addition, since the inverse of a logarithmic function is an exponential function, I would also recommend that you go over and master the exponent rules. Believe me, they always go hand in hand.

If you're ever interested as to why the logarithm rules work, check out my lesson on proofs or justifications of logarithm properties.

But if you think you have a good grasp of the concept, you can simply check out the practice problems below to test your knowledge.

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Logarithm Rules Practice Problems

Rules of Logarithms

Rule 1: $\log_b(M \cdot N) = \log_b M + \log_b N$

Rule 2: $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$

Rule 3: $\log_b\left(M^k\right) = k \cdot \log_b M$

Rule 4: $\log_b(1) = 0$

Rule 5: $\log_b(b) = 1$

Rule 6: $\log_b\left(b^k\right) = k$

Rule 7: $b^{\log_b(k)} = k$

Where:

$b > 0$ but $b \neq 1$, and M, N, and k are real numbers but M and N must be positive!

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Descriptions of Logarithm Rules

Rule 1: **Product Rule**

$$\log_b (M \cdot N) = \log_b M + \log_b N$$

The logarithm of the product is the sum of the logarithms of the factors.

Rule 2: **Quotient Rule**

$$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$$

The logarithm of the ratio of two quantities is the logarithm of the numerator minus the logarithm of the denominator.

Rule 3: **Power Rule**

$$\log_b \left(M^k\right) = k \cdot \log_b M$$

The logarithm of an exponential number is the exponent times the logarithm of the base.

Rule 4: **Zero Rule**

$$\log_b(1) = 0$$

The logarithm of 1 to any base is always equal to zero. As long as b is positive but $b \neq 1$.

Rule 5: **Identity Rule**

$$\log_b(b) = 1$$

The logarithm of the argument (inside the parenthesis) wherein the argument equals the base is equal to 1.

Rule 6: **Inverse Property of Logarithm**

$$\log_b\left(b^k\right) = k$$

The logarithm of an exponential number where its base is the same as the base of the log is equal to the exponent.

Rule 7: **Inverse Property of Exponent**

$$b^{\log_b(k)} = k$$

Raising the logarithm of a number to its base is equal to the number.

Examples of How to Apply the Log Rules

Example 1: Evaluate the expression below using Log Rules.

$\log_2 8 + \log_2 4$

Express 8 and 4 as exponential numbers with a base of 2. Then, apply Power Rule followed by Identity Rule. After doing so, you add the resulting values to get your final answer.

$$\begin{aligned}\log_2 8+\log_2 4 &= \log_2 2^3+\log_2 2^2\\ &= 3\log_2 2+2\log_2 2\\ &= 3(1)+2(1)\\ &= 3+2\\ \log_2 8+\log_2 4 &= 5\end{aligned}$$

So the answer is 5.

Example 2: Evaluate the expression below using Log Rules.

$\log_3 162 - \log_3 2$

We can’t express 162 as an exponential number with base 3. It appears that we’re stuck since there are no rules that can be applied in a direct manner.

The Logarithm Rules can be used in reverse, though! Observe that by using the Quotient Rule reversed, the log expression may be written as a single logarithmic number.

$$\begin{aligned}\log_3 162-\log_3 2 &= \log_3\left(\frac{162}{2}\right)\\ &= \log_3(81)\\ &= \log_3 3^4\\ &= 4\log_3 3\end{aligned}$$

$$= 4(1)$$

$$\log_3 162 - \log_3 2 = 4$$

We did it! By applying the rules in reverse, we generated a single log expression that is easily solvable. The final answer here is 4.

Example 3: Evaluate the expression below.

$$\log_5 500 - 2\log_5 2 + \log_4 32 + \log_4 8$$

There appear to be many things going on at the same time. First, see if you can simplify each of the logarithmic numbers. If not, start thinking about some of the obvious logarithmic rules that apply.

By observation, we see that there are two bases involved: 5 and 4. We can start this out by combining the terms that have the same base. Let's simplify them separately.

For log with base 5, apply the Power Rule first followed by Quotient Rule. For log with base 4, apply the Product Rule immediately. Then get the final answer by adding the two values found.

$$\begin{aligned}\log_5 500 - 2\log_5 2 + \log_4 32 + \log_4 8 &= \log_5 500 - \log_5 2^2 + \log_4 32 + \log_4 8\\ &= \log_5 500 - \log_5 4 + \log_4 32 + \log_4 8\\ &= \log_5\left(\frac{500}{4}\right) + \log_4(32\cdot 8)\\ &= \log_5(125) + \log_4(256)\\ &= \log_5\left(5^3\right) + \log_4\left(4^4\right)\\ &= 3\cdot\log_5 5 + 4\cdot\log_4(4)\\ &= 3(1) + 4(1)\\ &= 3 + 4\\ &= 7\end{aligned}$$

Yep, the final answer is 7.

Example 4: Expand the logarithmic expression below.

$\log_3\left(27x^2y^5\right)$

A product of factors is contained within the parenthesis. Apply the Product Rule to express them as a sum of individual log expressions. Make an effort to simplify numerical expressions into exact values whenever possible. Use Rule 5 (Identity rule) as much as possible because it can help to simplify the process.

$$\begin{aligned}\log_3\left(27x^2y^5\right) &= \log_3(27)+\log_3\left(x^2\right)+\log_3\left(y^5\right)\\ &= \log_3\left(3^3\right)+\log_3\left(x^2\right)+\log_3\left(y^5\right)\\ &= 3\log_3(3)+2\log_3(x)+5\log_3(y)\\ &= 3(1)+2\log_3(x)+5\log_3(y)\end{aligned}$$

$$\log_3\left(27x^2y^5\right)=3+2\log_3(x)+5\log_3(y)$$

I must admit that the final answer appears “unfinished.” But we shouldn’t be concerned as long as we know we followed the rules correctly.

Example 5: Expand the logarithmic expression.

$$\log_7\left(\frac{49m^6}{k^3}\right)$$

The approach is to apply the Quotient Rule first as the difference of two log expressions because they are in fractional form. Then utilize the Product Rule to separate the product of factors as the sum of logarithmic expressions.

$$\begin{aligned}\log_7\left(\frac{49m^6}{k^3}\right) &= \log_7\left(49m^6\right) - \log_7\left(k^3\right)\\ &= \log_7(49) + \log_7\left(m^6\right) - \log_7\left(k^3\right)\\ &= \log_7\left(7^2\right) + \log_7\left(m^6\right) - \log_7\left(k^3\right)\\ &= 2\log_7(7) + 6\log_7(m) - 3\log_7(k)\\ &= 2(1) + 6\log_7(m) - 3\log_7(k)\end{aligned}$$

$$\log_7\left(\frac{49m^6}{k^3}\right) = 2 + 6\log_7(m) - 3\log_7(k)$$

Example 6: Expand the logarithmic expression.

$$\log_2\left(\frac{12w^5}{\sqrt{y}}\right)$$

This one has a radical expression in the denominator. Remember that the square root symbol is the same as having a **power of** $\frac{1}{2}$. Express the radical denominator as $y^{\frac{1}{2}}$. Just like problem #5, apply the Quotient Rule for logs and then use the Product Rule.

$$\log_2\left(\frac{12w^5}{\sqrt{y}}\right) = \log_2\left(\frac{12w^5}{y^{\frac{1}{2}}}\right)$$

$$= \log_2\left(12w^5\right) - \log_2\left(y^{\frac{1}{2}}\right)$$

$$= \log_2(12) + \log_2\left(w^5\right) - \frac{1}{2}\log_2(y)$$

$$= \log_2(4 \cdot 3) + 5\log_2(w) - \frac{1}{2}\log_2(y)$$

$$= \log_2(4) + \log_2(3) + 5\log_2(w) - \frac{1}{2}\log_2(y)$$

$$\log_2\left(\frac{12w^5}{\sqrt{y}}\right) = 2 + \log_2(3) + 5\log_2(w) - \frac{1}{2}\log_2(y)$$

Example 7: Expand the logarithmic expression.

$$\log_3\left[\frac{18(x+2)^2}{(x-2)^3(x+5)^2}\right]$$

A problem like this may cause you to doubt if indeed you arrived at the correct answer because the final answer can still look "unfinished". However, as long as you applied the log rules properly in every step, there's nothing to worry about.

You might notice that we need to apply the Quotient Rule first because the expression is in fractional form.

$$\log_3\left[\frac{18(x+2)^2}{(x-2)^3(x+5)^2}\right]$$

$$= \log_3\left[18(x+2)^2\right] - \log_3\left[(x-2)^3(x+5)^2\right]$$

$$= \log_3(18) + \log_3(x+2)^2 - \left[\log_3(x-2)^3 + \log_3(x+5)^2\right]$$
$$= \log_3(18) + \log_3(x+2)^2 - \log_3(x-2)^3 - \log_3(x+5)^2$$
$$= \log_3(9\cdot 2) + 2\log_3(x+2) - 3\log_3(x-2) - 2\log_3(x+5)$$
$$= \log_3(9) + \log_3(2) + 2\log_3(x+2) - 3\log_3(x-2) - 2\log_3(x+5)$$
$$= 2 + \log_3(2) + 2\log_3(x+2) - 3\log_3(x-2) - 2\log_3(x+5) \checkmark$$

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