Algorithm Analysis Lec 9-18-23 around 25 min

Mark Sort

```
for(int i=0; i<N-1; i++){
    for(int j=i+1; j<N; j++){
        If(a[i] > a[j]){
            temp = a[i]
            a[i] = a[j]
            a[j] = temp;
        }
    }
}
```

ops. before O_b ops on i O_i ops. on j O_j PO_s Probability + its constant → flip coin 50% heads

$$O_b + \underbrace{\sum_{i=0}^{N-2}\left(O_i + \overbrace{\sum_{j=i+1}^{N-1}\left(O_j + \overset{\text{swap}}{PO_s}\right)}^{\text{j loop}}\right)}_{\text{i loop}}$$

O represents operations which equates $T(O)$ → clock cycles

* Lab Appndx Sum of N proved $\Sigma = c(n-m+1)$

J loop :

$$\sum_{j=x=i+1}^{y=n-1} 1 = (y-x)+1$$

Let $O_j + PO_s = O_{js}$ ← c constant

$$\text{So} \rightarrow \left(\overset{y}{(N-1)} \overset{-}{-} \overset{x}{(i+1)} \overset{+1}{+1}\right) O_{js}$$

$$(N-i-1)\,O_{js}$$

$$O_b + \sum_{i=0}^{N-2} \left(O_i + (N-1-i) \, O_{+5} \right)$$

How to rewrite sum of i

$$\sum_{i=0}^{x} i = x \cdot (x+1)/2$$

$$\sum_{i=0}^{x} \left((i+1)^2 - i^2 \right) = (\cancel{1^2} - 0^2) + (\cancel{2^2} - \cancel{1^2}) + (3^2 - \cancel{2^2}) + \vdots + (\cancel{x^2} - (\cancel{x-1})^2) + ((x+1)^2 - \cancel{x^2})$$

$$\sum_{i=0}^{x} \left(\cancel{i^2} + 2i + 1 - \cancel{i^2} \right) = (x+1)^2$$

$$\sum_{i=0}^{x} (2i+1)$$

$$2\sum_{i=0}^{x} i + \sum_{i=0}^{x} 1$$

$$\sum_{i=0}^{x} i = \frac{(x+1)^2 - (x+1)}{2}$$

$$= \frac{x^2 + 2x + 1 - x}{2}$$

$$= \frac{x^2 + x + 1}{2}$$

$$= \frac{(x+1)(x+1-1)}{2}$$

$$\sum_{i=0}^{x} i = (x+1)\,x/2$$

N-2

$n = n-2$

$N-2 = x$

$$O_b + \sum_{i=0}^{N-2} \left(O_i + (N-1-i)\,O_{ds}\right)$$

$n-2+1 = n-1$

sum of these 2 terms
wr. $i=0$ & $n-2$, then

$0 \;\; -1 \;\; -2$

here

$$O_b + \sum_{i=0}^{N-2} \left(O_i + (N-1)\,O_{ds} - i\,O_{ds}\right)$$

$$O_b + (N-1)\left(O_i + (N-1)\,O_{ds}\right) - O_{ds}\sum_{i=0}^{N-2} i$$

$$(N-1)^2 O_{ds} + (N-1)\,O_i + O_b - \frac{(N-2)(N-1)}{2}\,O_{ds}$$

$f(N) =$ second order polynomial in N

$$\left(O_{ds} - \frac{O_{ds}}{2}\right)N^2 + \left(-2O_{ds} + O_i + \tfrac{1}{2}O_{ds}\right)N + \left(2O_{ds} - O_i + O_b\right)$$

$$c''N^2 + c'N + c$$

$c'' = (O_{ds}/2) \qquad c' = (O_i - \tfrac{3}{2}O_{ds})$

$c = (2O_{ds} - O_i + Ob)$