prb5-2_errorCos

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if $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots = \sum_{k=0}^{\infty} (-1)^n \cdot \frac{x^{2n}}{(2n)!}$

and approx. $\cos(\frac{1}{n}) \approx 1 - \frac{1}{(2n^2)}$, then

order of error $\cos(\frac{1}{n}) \approx O(?)$

$e \cos\theta \approx 1$ @ 0.1408 rad $\approx 8.07°$

$$\boxed{\text{error} = \cos\left(\tfrac{1}{n}\right) - \text{approx } \cos\left(\tfrac{1}{n}\right)}$$

$$\cos\left(\text{approx}\cos\left(\tfrac{1}{n}\right)\right) = 1 - \frac{\left(\frac{1}{n}\right)^2}{2!} + \frac{\left(\frac{1}{n}\right)^4}{4!} - \frac{\left(\frac{1}{n}\right)^6}{6!}$$

$$\cos\left(\text{approx}\cos\left(\tfrac{1}{n}\right)\right) = 1 - \frac{\left(1 - \frac{1}{(2n^2)}\right)^2}{2!} + \frac{\left(1 - \frac{1}{(2n^2)}\right)^4}{4!} - \frac{\left(1 - \frac{1}{(2n^2)}\right)^6}{6!}$$

$$= 1 - \frac{\left(\cancel{1}^{2n^2} - \frac{1}{2n^2}\right)^2}{2!} + \frac{\left(\cancel{1}^{2n^2} - \frac{1}{(2n^2)}\right)^4}{4!}$$

$$= 1 - \frac{\left(\frac{2n^2-1}{2n^2}\right)^2}{2} + \frac{\left(\frac{2n^2-1}{2n^2}\right)^4}{24}$$

$$= 1 - \frac{(2n^2-1)^2}{4n^4}\left(\frac{1}{2}\right) + \frac{(2n^2-1)^4}{16n^8}\left(\frac{1}{24}\right)$$

$$= 1 - \frac{(2n^2-1)^2}{8n^4} + \frac{(2n^2-1)^4}{384n^8}$$

$$\text{approx}\cos\left(\tfrac{1}{n}\right) = 1 - \frac{1}{(2n^2)}$$

$$\boxed{\text{approx}\cos\left(\tfrac{1}{n}\right) = \frac{2n^2-1}{2n^2}}$$

$$\left(1 - \frac{1}{2n^2}\right)^2$$

$$\frac{1}{2n^2}\left(1 - \tfrac{1}{1}\right)^2 + \left(1 - \tfrac{1}{1}\right)^4 - \left(1 - \tfrac{1}{1}\right)^6$$

$$0^2 + 0^4 - 0^6$$

error =

$$\text{error}\cos\left(\tfrac{1}{n}\right) = -\frac{\frac{1}{n^3}}{3!} + \frac{\frac{1}{n^5}}{5!} - \frac{\frac{1}{n^7}}{7!}$$

Order of error is the leading or largest polynomial of degree, d.

$$\boxed{\begin{gathered} p(n) = O\left(\tfrac{1}{n^d}\right) \\ d = 3 \end{gathered}}$$