

Mark Sort Proof $O(N^2)$

$$O_b + O_i + O_j + PO_s$$

$$O_b + \sum_{i=0}^{N-2} \left(O_i + \sum_{j=1+i}^{N-1} (O_j + PO_s) \right) \quad \text{where}$$

$O_j + PO_s = O_{js}$
 $O_{js} = \text{constant}$

$$O_b + \sum_{i=0}^{N-2} \left(O_i + \sum_{j=1+i}^{N-1} (O_{js}) \right)$$

$$\sum_{j=1+i}^{N-1} (O_{js}) = O_{js} \sum_{j=1+i}^{N-1} 1 = (N-1 - (1+i) + 1) = N-1-1-i+1$$

$$= N-i-1$$

$$O_b + \sum_{i=0}^{N-2} (O_i + O_{js} (N-i-1))$$

$$O_b + \sum_{i=0}^{N-2} O_i + O_{js} \left[\sum_{i=0}^{N-2} N - \sum_{i=0}^{N-2} i - \sum_{i=0}^{N-2} 1 \right]$$

$$O_b + \sum_{i=0}^{N-2} O_i + O_{js} \left[\sum_{i=0}^{N-2} (N-1) - \sum_{i=0}^{N-2} i \right]$$

$$\sum_{i=0}^{N-2} i = \frac{x(x+1)}{2} = \frac{(N-2)(N-1)}{2}$$

$$O_b + \sum_{i=0}^{N-2} O_i + O_{js} \left[\sum_{i=0}^{N-2} (N-1) - \frac{(N-2)(N+1)}{2} \right]$$

$$O_b + \sum_{i=0}^{N-2} O_i + O_{js} \sum_{i=0}^{N-2} \left((N-1) - \left(\frac{(N-2)(N+1)}{2} \right) \right) O_{js}$$

$$O_b + \sum_{i=0}^{N-2} \left(O_i + O_{js}(N-1) \right) - \frac{(N-2)(N+1)}{2} O_{js}$$

$$\sum_{i=0}^{N-2} O_i = O_i \rightarrow \sum_{i=0}^{N-2} 1 = N-m+1 \quad \begin{matrix} N = N-2 \\ m = 0 \end{matrix}$$

$$N-2-0+1 = N-1$$

$$O_b + O_i(N-1) + O_{js} \sum_{i=0}^{N-2} N-1 - \frac{(N-2)(N+1)}{2} O_{js}$$

$$O_b + (N-1) \left[O_i + O_{js} \sum_{i=0}^{N-2} 1 \right] \quad \begin{matrix} \sum_{i=0}^{N-2} 1 = N-m+1 = N-2+1 \\ = N-1 \end{matrix}$$

$$O_b + (N-1) \left(O_i + O_{js}(N-1) \right) - \frac{(N-2)(N+1)}{2} O_{js}$$

$$O_b + (N-1)O_i + (N-1)^2 O_{js} - \frac{(N-2)(N+1)}{2} O_{js}$$

$$O_b + (N-1)O_i + (N^2 - 2N + 1)O_{js} - (N-2)(N+1) \frac{O_{js}}{2}$$

$$O_b + NO_i - O_i + O_{js} N^2 - 2NO_{js} - (N^2 - N - 2) \frac{O_{js}}{2}$$

$$O_b + \underbrace{NO_i}_{\uparrow} - \underbrace{O_i}_{\uparrow} + \underbrace{N^2 O_{js}} - \underbrace{2NO_{js}} - \underbrace{\frac{N^2 O_{js}}{2}} + \underbrace{\frac{NO_{js}}{2}} + \underbrace{\frac{2O_{js}}{2}}_{\uparrow}$$

$$N^2 \left(O_{js} - \frac{O_{js}}{2} \right) + N \left(O_i - 2O_{js} + \frac{1}{2} O_{js} \right)$$

$$+ (O_b - O_i + O_{js})$$

$$\Rightarrow N^2 \left(\frac{O_{js}}{2} \right) + N \left(O_i - \frac{3}{2} O_{js} \right) + (O_b - O_i + O_{js})$$

$$C_2 = \frac{O_{js}}{2} \quad C_1 = O_i - \frac{3}{2} O_{js} \quad C_0 = O_b - O_i + O_{js}$$

$$C_2 N^2 + C_1 N + C_0$$

$F(N) = 2^{\text{nd}} \text{ degree polynomial!}$