#5 $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

Approximation $\sin\left(\frac{1}{N}\right) \approx \frac{1}{N}$

Error $\sin\left(\frac{1}{N}\right) = \sin\left(\frac{1}{N}\right) -$ Approximation $\sin\left(\frac{1}{N}\right)$

$$\sin\left(\frac{1}{N}\right) = \frac{1}{N} - \frac{\left(\frac{1}{N}\right)^3}{3!} + \frac{\left(\frac{1}{N}\right)^5}{5!} - \frac{\left(\frac{1}{N}\right)^7}{7!} + \dots$$

$$= \frac{1}{N} - \frac{\frac{1}{N^3}}{3!} + \frac{\frac{1}{N^5}}{5!} - \frac{\frac{1}{N^7}}{7!}$$

Approximation $\sin\left(\frac{1}{N}\right) \approx \frac{1}{N}$

$$\sin\left(\frac{1}{N}\right) - \text{Approx} \sin\left(\frac{1}{N}\right) = \frac{1}{N} - \frac{\frac{1}{N^3}}{3!} + \frac{\frac{1}{N^5}}{5!} - \frac{\frac{1}{N^7}}{7!} - \frac{1}{N}$$

$$= -\frac{\frac{1}{N^3}}{3!} + \frac{\frac{1}{N^5}}{5!} - \frac{\frac{1}{N^7}}{7!} \Rightarrow \frac{\frac{1}{N^3}}{3!} - \frac{\frac{1}{N^5}}{5!} + \frac{\frac{1}{N^7}}{7!}$$

$$\text{Error } \sin\left(\frac{1}{N}\right) = \frac{\frac{1}{N^3}}{3!} - \frac{\frac{1}{N^5}}{5!} + \frac{\frac{1}{N^7}}{7!}$$

As N gets larger the subsequent terms get smaller and smaller. For Big O, we only care about the leading term and coefficient. Therefore the order of the error for

$$\text{error } \sin\left(\frac{1}{N}\right) = O\left(\frac{1}{3!\,N^3}\right)$$