Mod/Div Turing Machine

Individual Coursework
F29FB, Spring 2022

Submitted by

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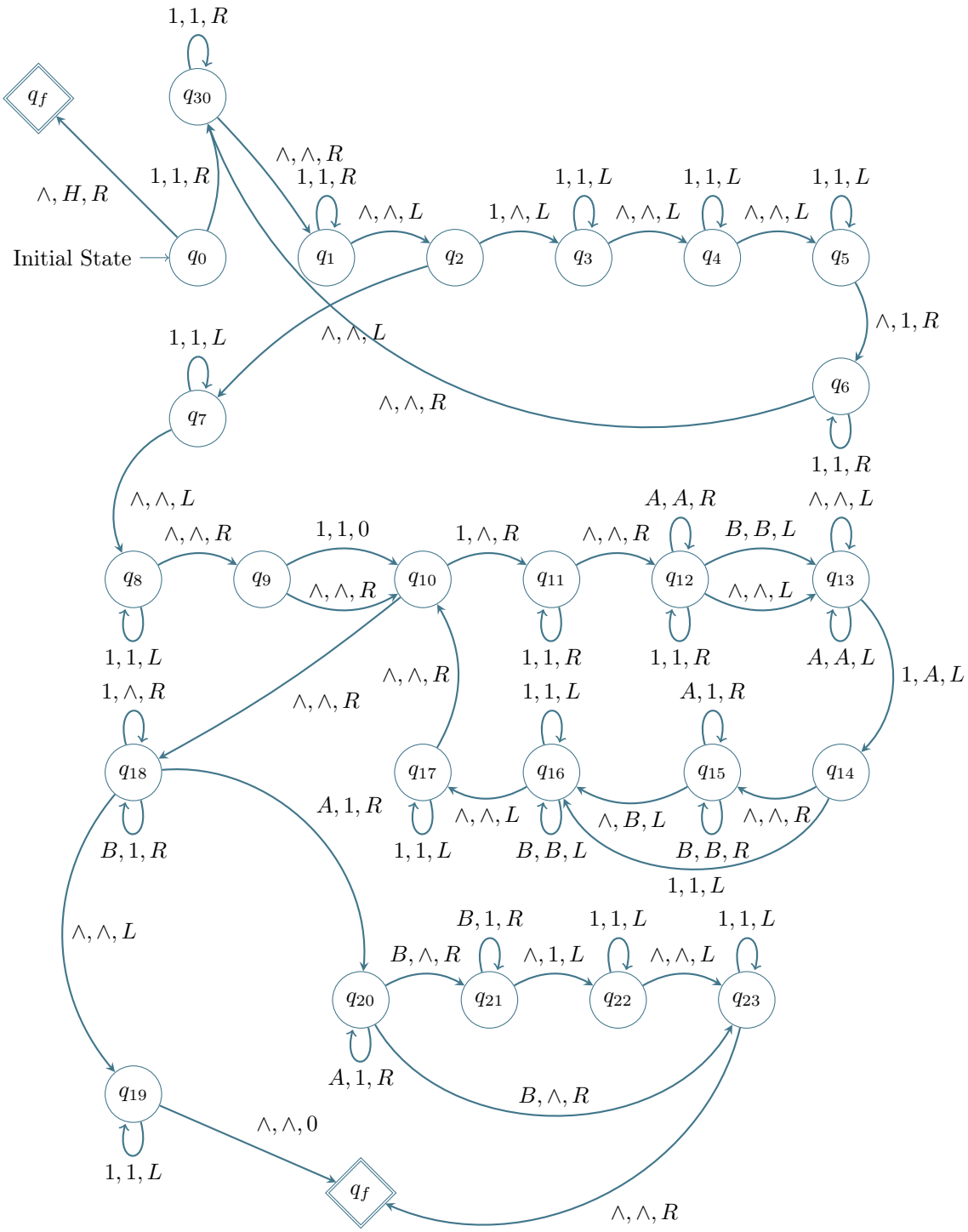
1 Graph


q_f
q_{30}
$1,1,R$
$\wedge,H,R$
$1,1,R$
$\wedge,\wedge,R$
$1,1,R$
Initial State
q_0
q_1
$\wedge,\wedge,L$
q_2
$1,\wedge,L$
q_3
$\wedge,\wedge,L$
q_4
$\wedge,\wedge,L$
q_5
$\wedge,1,R$
q_6
$1,1,R$
$\wedge,\wedge,R$
$1,1,L$
$\wedge,\wedge,L$
q_7
$\wedge,\wedge,L$
q_8
$\wedge,\wedge,R$
q_9
$1,1,0$
$\wedge,\wedge,R$
q_{10}
$1,\wedge,R$
q_{11}
$\wedge,\wedge,R$
q_{12}
A,A,R
B,B,L
q_{13}
$\wedge,\wedge,L$
$1,1,L$
$1,1,R$
$1,1,R$
A,A,L
$1,A,L$
$1,\wedge,R$
$\wedge,\wedge,R$
$\wedge,\wedge,R$
$1,1,L$
$A,1,R$
q_{18}
q_{17}
q_{16}
q_{15}
q_{14}
$A,1,R$
$\wedge,\wedge,L$
$\wedge,B,L$
$\wedge,\wedge,R$
$B,1,R$
$1,1,L$
B,B,L
B,B,R
$1,1,L$
$\wedge,\wedge,L$
$B,1,R$
$1,1,L$
$1,1,L$
$B,\wedge,R$
$\wedge,1,L$
$\wedge,\wedge,L$
q_{20}
q_{21}
q_{22}
q_{23}
$A,1,R$
$B,\wedge,R$
q_{19}
$\wedge,\wedge,0$
$1,1,L$
q_f
$\wedge,\wedge,R$

2 Mathematical Notation

$s_0 \equiv \wedge, s_1 \equiv 1, s_2 \equiv A, s_3 \equiv B, s_4 \equiv H$

$$
\begin{aligned}
&M_g = \{\\
&((q_0, s_0) \to (q_f, s_4, 0)),\\
&((q_0, s_1) \to (q_{30}, s_1, R)),\\
&((q_1, s_1) \to (q_1, s_1, R)),\\
&((q_1, s_0) \to (q_2, s_0, L)),\\
&((q_{30}, s_1) \to (q_{30}, s_1, R)),\\
&((q_{30}, s_0) \to (q_1, s_0, R)),\\
&((q_2, s_0) \to (q_7, s_0, L)),\\
&((q_2, s_1) \to (q_3, s_0, L)),\\
&((q_3, s_1) \to (q_3, s_1, L)),\\
&((q_3, s_0) \to (q_4, s_0, L)),\\
&((q_4, s_1) \to (q_4, s_1, L)),\\
&((q_4, s_0) \to (q_5, s_0, L)),\\
&((q_5, s_1) \to (q_5, s_1, L)),\\
&((q_5, s_0) \to (q_6, s_1, R)),\\
&((q_6, s_1) \to (q_6, s_1, R)),\\
&((q_6, s_0) \to (q_{30}, s_0, R)),\\
&((q_7, s_1) \to (q_7, s_1, L)),\\
&((q_7, s_0) \to (q_8, s_0, L)),\\
&((q_8, s_1) \to (q_8, s_1, L)),\\
&((q_8, s_0) \to (q_9, s_0, R)),\\
&((q_9, s_1) \to (q_{10}, s_1, 0)),\\
&((q_9, s_0) \to (q_{10}, s_0, R)),\\
&((q_{10}, s_1) \to (q_{11}, s_0, R)),\\
&((q_{10}, s_0) \to (q_{18}, s_0, R)),\\
&((q_{11}, s_1) \to (q_{11}, s_1, R)),\\
&((q_{11}, s_0) \to (q_{12}, s_0, R)),\\
&((q_{12}, s_1) \to (q_{12}, s_1, R)),\\
&((q_{12}, s_2)) \to (q_{12}, s_2), R)),\\
&((q_{12}, s_3)) \to (q_{13}, s_3), L)),\\
&((q_{12}, s_0) \to (q_{13}, s_0, L)),\\
&((q_{13}, s_0) \to (q_{13}, s_0, L)),\\
&((q_{13}, s_2)) \to (q_{13}, s_2), L)),\\
&((q_{13}, s_1) \to (q_{14}, s_2), L)),\\
&((q_{14}, s_0) \to (q_{15}, s_0, R)),\\
&((q_{14}, s_1) \to (q_{16}, s_1, L)),\\
&((q_{15}, s_2)) \to (q_{15}, s_1, R)),\\
&((q_{15}, s_3)) \to (q_{15}, s_3), R)),\\
&((q_{15}, s_0) \to (q_{16}, s_3), L)),\\
&((q_{16}, s_1) \to (q_{16}, s_1, L)),\\
&((q_{16}, s_3)) \to (q_{16}, s_3), L)),\\
&((q_{16}, s_0) \to (q_{17}, s_0, L)),\\
&((q_{17}, s_1) \to (q_{17}, s_1, L)),\\
&((q_{17}, s_0) \to (q_{10}, s_0, R)),\\
&((q_{18}, s_1) \to (q_{18}, s_0, R)),\\
&((q_{18}, s_3)) \to (q_{18}, s_1, R)),\\
&((q_{18}, s_2)) \to (q_{20}, s_1, R)),\\
&((q_{18}, s_0) \to (q_{19}, s_0, L)),\\
&((q_{19}, s_1) \to (q_{19}, s_1, L)),\\
&((q_{19}, s_0) \to (q_f, s_0, 0)),\\
&((q_{20}, s_2)) \to (q_{20}, s_1, R)),\\
&((q_{20}, s_3)) \to (q_{21}, s_0, R)),\\
&((q_{20}, s_0) \to (q_{23}, s_0, L)),\\
&((q_{21}, s_3)) \to (q_{21}, s_1, R)),\\
&((q_{21}, s_0) \to (q_{22}, s_1, L)),\\
&((q_{22}, s_1) \to (q_{22}, s_1, L)),\\
&((q_{22}, s_0) \to (q_{23}, s_0, L)),\\
&((q_{23}, s_1) \to (q_{23}, s_1, L)),\\
&((q_{23}, s_0) \to (q_f, s_0, R)),\\
&\}
\end{aligned}
$$

3 Input (3, 5)

(i) The TM starts by checking if the head starts atA∧ (blank) then the divisor is 0 and the ticket is invalidated and the TM halts. Otherwise the divisor isAnatural number and goes to the next state.

q_0: ∧ ∧ ∧@111∧11111∧ ∧ ∧

(ii) Go to the rightmost of the unary until we reachAblank.

q_{30}: ∧ ∧ ∧1@11∧11111∧ ∧ ∧
q_{30}: ∧ ∧ ∧11@1∧11111∧ ∧ ∧
q_{30}: ∧ ∧ ∧111@∧11111∧ ∧ ∧

(iii) Check if there isAunary and go to the rightmost of the said unary. Otherwise (if blank, ∧) then go to DIV/MOD part of TM.

q_1: ∧ ∧ ∧111∧@11111∧ ∧ ∧
q_1: ∧ ∧ ∧111∧1@1111∧ ∧ ∧
q_1: ∧ ∧ ∧111∧11@111∧ ∧ ∧
q_1: ∧ ∧ ∧111∧111@11∧ ∧ ∧
q_1: ∧ ∧ ∧111∧1111@1∧ ∧ ∧
q_1: ∧ ∧ ∧111∧11111@∧ ∧ ∧

(iv) Once at the rightmost of the unary and check if there isAunary number, move it to the leftmost of the divisor and go back to (ii). Otherwise (if blank) copying is done and go to the leftmost of the divisor then starts the mod/div operation.

q_2: ∧ ∧ ∧111∧1111@1∧ ∧ ∧
q_3: ∧ ∧ ∧111∧111@1∧ ∧ ∧
q_3: ∧ ∧ ∧111∧11@11∧ ∧ ∧
q_3: ∧ ∧ ∧111∧1@111∧ ∧ ∧
q_3: ∧ ∧ ∧111∧@1111∧ ∧ ∧
q_3: ∧ ∧ ∧111@∧1111∧ ∧ ∧
q_4: ∧ ∧ ∧11@1∧1111∧ ∧ ∧
q_4: ∧ ∧ ∧1@11∧1111∧ ∧ ∧
q_4: ∧ ∧ ∧@111∧1111∧ ∧ ∧
q_4: ∧∧@∧111∧1111∧ ∧ ∧
q_5: ∧@∧∧111∧1111∧ ∧ ∧
q_6: ∧1@∧111∧1111∧ ∧ ∧

(v) Once copying is done(determined by iv.), then go to the leftmost of the divisor and start mod/div operation.

q_2: ∧∧11111∧111@∧ ∧ ∧
q_7: ∧∧11111∧11@1∧ ∧ ∧
q_7: ∧∧11111∧1@11∧ ∧ ∧
q_7: ∧∧11111∧@111∧ ∧ ∧
q_7: ∧∧11111@∧111∧ ∧ ∧
q_8: ∧∧1111@1∧111∧ ∧ ∧
q_8: ∧∧111@11∧111∧ ∧ ∧
q_8: ∧∧11@111∧111∧ ∧ ∧
q_8: ∧∧1@1111∧111∧ ∧ ∧
q_8: ∧∧@11111∧111∧ ∧ ∧
q_8: ∧@∧11111∧111∧ ∧ ∧
q_9: ∧∧@11111∧111∧ ∧ ∧

(vi) Subtract -1 from the leftmost side of the dividend, go to the rightmost side of the divisor and keep track of the subtraction using the character A.

q_{10}: ∧∧@11111∧111∧∧
q_{11}: ∧ ∧ ∧@1111∧111∧∧
q_{11}: ∧ ∧ ∧1@111∧111∧∧
q_{11}: ∧ ∧ ∧11@11∧111∧∧
q_{11}: ∧ ∧ ∧111@1∧111∧∧
q_{11}: ∧ ∧ ∧1111@∧111∧∧
q_{12}: ∧ ∧ ∧1111∧@111∧∧
q_{12}: ∧ ∧ ∧1111∧1@11∧∧
q_{12}: ∧ ∧ ∧1111∧11@1∧∧
q_{12}: ∧ ∧ ∧1111∧111@∧∧
q_{13}: ∧ ∧ ∧1111∧11@1∧∧
q_{14}: ∧ ∧ ∧1111∧1@1A∧∧

(vii) Go to the leftmost of the dividend and repeat to (vi) until the divisor is only A.

q_{16}: ∧ ∧ ∧1111∧@11A∧∧
q_{16}: ∧ ∧ ∧1111@∧11A∧∧
q_{17}: ∧ ∧ ∧111@1∧11A∧∧
q_{17}: ∧ ∧ ∧11@11∧11A∧∧
q_{17}: ∧ ∧ ∧1@111∧11A∧∧
q_{17}: ∧ ∧ ∧@1111∧11A∧∧
q_{17}: ∧∧@∧1111∧11A ∧∧

(viii) Once the divisor is replaced with A's we know it fits once in the dividend, therefore we replace it with1s again and addAB at the rightmost point of the divisor (indicates our quotient) then we go to the leftmost of the dividend and repeat the process (back to vi) until we can no longer subtract.

q_{14}: ∧∧11@∧AAA∧∧
q_{15}: ∧∧11∧@AAA∧∧
q_{15}: ∧∧11∧1@AA∧∧
q_{15}: ∧∧11∧11@A∧∧
q_{15}: ∧∧11∧111@∧∧
q_{16}: ∧∧11∧11@1B∧∧
q_{16}: ∧∧11∧1@11B∧∧
q_{16}: ∧∧11∧@111B∧∧
q_{16}: ∧∧11@∧111B∧∧
q_{17}: ∧∧1@1∧111B∧∧
q_{17}: ∧∧@11∧111B∧∧
q_{17}: ∧@∧11∧111B

(ix) We then discard of any left over 1s, and convert any As to ones, then we take the leftmost B, turn it into a blank and move it to the rightmost as a 1, while moving to the rightmost we turn any B's to 1s.

(x) We then move the leftmost of the reminder.

End result:

q_f: ∧@1 1 ∧ 1

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(A) rmoddiv$(s, d) = (r, q)$ is accomplished by subtracting the dividend (d) and keeping track of the subtractions in the divisor, once a divisor is full of subtractions we know it fits in the dividend at least once, this is repeated until it is no longer possible to subtract, giving us the reminder (r) and quotient (q). Output is then formatted to match the specification of ($r \wedge q$).

(B) $\wedge$ indicates a blank, a blank is used for identifying unarys, and if we are inbeetween unarys, leftmost, and or rightmost of an unary.
1 indicates a natural number in a unary format (i.e. $3 \equiv 111$).
A is used to indicate reminders in a unary format.
B is used to indicate the quotient in a unary format.

(C) loop q_{30} to q_6 is our copying machines flipping the input rmoddiv(s, d) to rmoddiv(d, s), this allows us to think of the problem in a more human natural way (i.e. $\frac{\text{dividend}}{\text{divisor}}$).

loop q_{10} to q_7 is our subtraction and reminder tracker loop, allowing us to to increase our quotient and reminder when appropriate (for reminder with each subtraction, for quotient when the divisor is full of reminders).

(D) This TM implementation fails when the dividend is blank, as there are no checks for such a situation, as a result calculations such as rmoddiv$(2, 0)$ (i.e. $\frac{0}{2}$) results in 1, as subtractions have no were to increase the reminder and therefore get lost.

(E) It was decided to flip the input in order for the design to make more intuitive sense when coming up with a solution, as $\frac{\text{dividend}}{\text{divisor}}$ is the classical way of expressing division.