

# Mod/Div Turing Machine

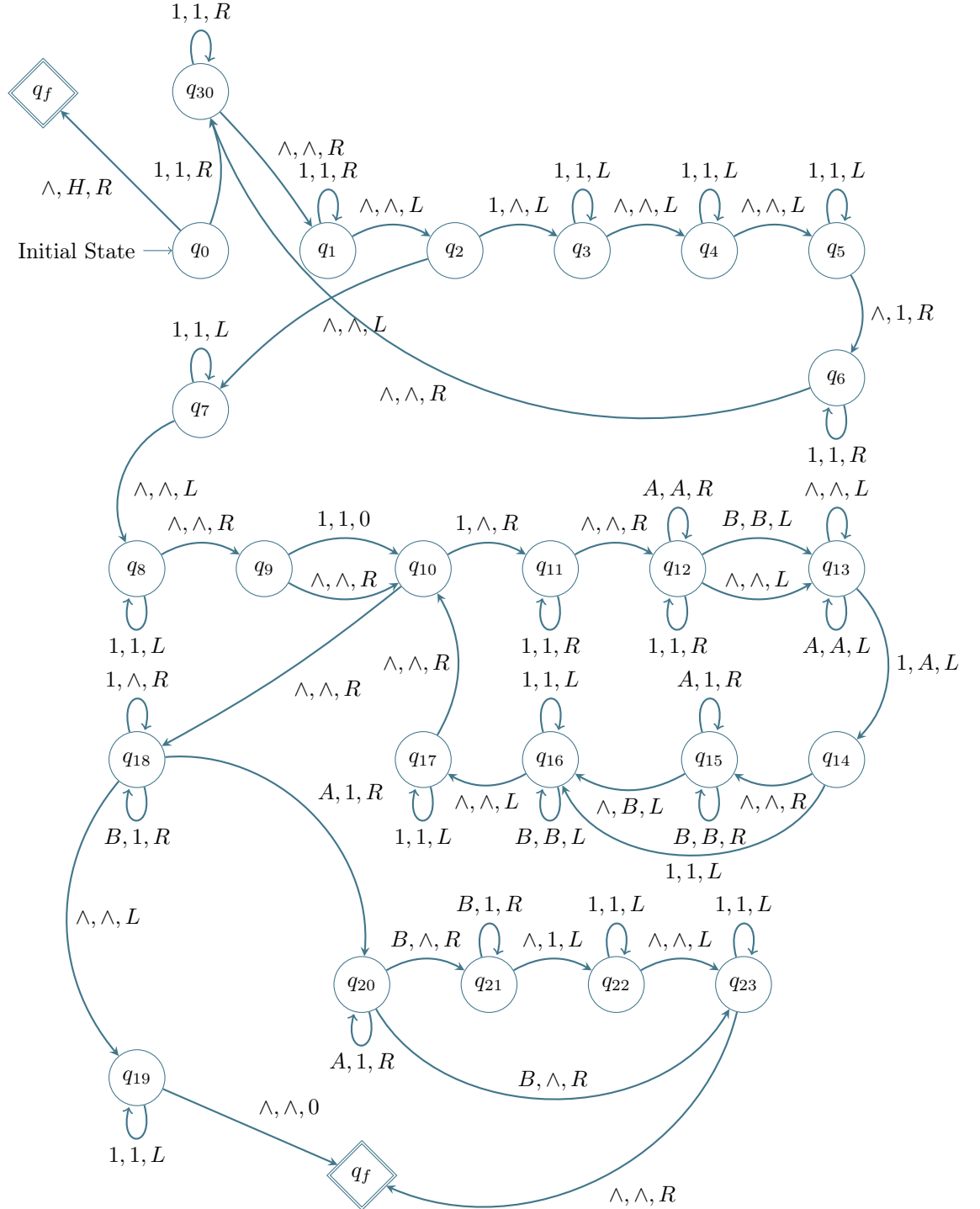
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INDIVIDUAL COURSEWORK  
F29FB, SPRING 2022

SUBMITTED BY

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# 1 Graph



## 2 Mathematical Notation

$$s_0 \equiv \wedge, s_1 \equiv 1, s_2 \equiv A, s_3 \equiv B, s_4 \equiv H$$

$$M_g = \{ \begin{array}{l} ((q_0, s_0) \rightarrow (q_f, s_4, 0)), \\ ((q_0, s_1) \rightarrow (q_{30}, s_1, R)), \\ ((q_1, s_1) \rightarrow (q_1, s_1, R)), \\ ((q_1, s_0) \rightarrow (q_2, s_0, L)), \\ ((q_{30}, s_1) \rightarrow (q_{30}, s_1, R)), \\ ((q_{30}, s_0) \rightarrow (q_1, s_0, R)), \\ ((q_2, s_0) \rightarrow (q_7, s_0, L)), \\ ((q_2, s_1) \rightarrow (q_3, s_0, L)), \\ ((q_3, s_1) \rightarrow (q_3, s_1, L)), \\ ((q_3, s_0) \rightarrow (q_4, s_0, L)), \\ ((q_4, s_1) \rightarrow (q_4, s_1, L)), \\ ((q_4, s_0) \rightarrow (q_5, s_0, L)), \\ ((q_5, s_1) \rightarrow (q_5, s_1, L)), \\ ((q_5, s_0) \rightarrow (q_6, s_1, R)), \\ ((q_6, s_1) \rightarrow (q_6, s_1, R)), \\ ((q_6, s_0) \rightarrow (q_{30}, s_0, R)), \\ ((q_7, s_1) \rightarrow (q_7, s_1, L)), \\ ((q_7, s_0) \rightarrow (q_8, s_0, L)), \\ ((q_8, s_1) \rightarrow (q_8, s_1, L)), \\ ((q_8, s_0) \rightarrow (q_9, s_0, R)), \\ ((q_9, s_1) \rightarrow (q_{10}, s_1, 0)), \\ ((q_9, s_0) \rightarrow (q_{10}, s_0, R)), \\ ((q_{10}, s_1) \rightarrow (q_{11}, s_0, R)), \\ ((q_{10}, s_0) \rightarrow (q_{18}, s_0, R)), \\ ((q_{11}, s_1) \rightarrow (q_{11}, s_1, R)), \\ ((q_{11}, s_0) \rightarrow (q_{12}, s_0, R)), \\ ((q_{12}, s_1) \rightarrow (q_{12}, s_1, R)), \\ ((q_{12}, s_2) \rightarrow (q_{12}, s_2, R)), \\ ((q_{12}, s_3) \rightarrow (q_{13}, s_3, L)), \\ ((q_{12}, s_0) \rightarrow (q_{13}, s_0, L)), \\ ((q_{13}, s_0) \rightarrow (q_{13}, s_0, L)), \\ ((q_{13}, s_2) \rightarrow (q_{13}, s_2, L)), \\ ((q_{13}, s_1) \rightarrow (q_{14}, s_2, L)), \\ ((q_{14}, s_0) \rightarrow (q_{15}, s_0, R)), \\ ((q_{14}, s_1) \rightarrow (q_{16}, s_1, L)), \\ ((q_{15}, s_2) \rightarrow (q_{15}, s_1, R)), \\ ((q_{15}, s_3) \rightarrow (q_{15}, s_3, R)), \\ ((q_{15}, s_0) \rightarrow (q_{16}, s_3, L)), \end{array} \begin{array}{l} ((q_{16}, s_1) \rightarrow (q_{16}, s_1, L)), \\ ((q_{16}, s_3) \rightarrow (q_{16}, s_3, L)), \\ ((q_{16}, s_0) \rightarrow (q_{17}, s_0, L)), \\ ((q_{17}, s_1) \rightarrow (q_{17}, s_1, L)), \\ ((q_{17}, s_0) \rightarrow (q_{10}, s_0, R)), \\ ((q_{18}, s_1) \rightarrow (q_{18}, s_0, R)), \\ ((q_{18}, s_3) \rightarrow (q_{18}, s_1, R)), \\ ((q_{18}, s_2) \rightarrow (q_{20}, s_1, R)), \\ ((q_{18}, s_0) \rightarrow (q_{19}, s_0, L)), \\ ((q_{19}, s_1) \rightarrow (q_{19}, s_1, L)), \\ ((q_{19}, s_0) \rightarrow (q_f, s_0, 0)), \\ ((q_{20}, s_2) \rightarrow (q_{20}, s_1, R)), \\ ((q_{20}, s_3) \rightarrow (q_{21}, s_0, R)), \\ ((q_{20}, s_0) \rightarrow (q_{23}, s_0, L)), \\ ((q_{21}, s_3) \rightarrow (q_{21}, s_1, R)), \\ ((q_{21}, s_0) \rightarrow (q_{22}, s_1, L)), \\ ((q_{22}, s_1) \rightarrow (q_{22}, s_1, L)), \\ ((q_{22}, s_0) \rightarrow (q_{23}, s_0, L)), \\ ((q_{23}, s_1) \rightarrow (q_{23}, s_1, L)), \\ ((q_{23}, s_0) \rightarrow (q_f, s_0, R)), \end{array} \}$$

### 3 Input (3, 5)

- (i) The TM starts by checking if the head starts at A (blank) then the divisor is 0 and the ticket is invalidated and the TM halts. Otherwise the divisor is a natural number and goes to the next state.
- (ii) Go to the rightmost of the unary until we reach a blank.

$q_{30}$ :  $\wedge \wedge \wedge 1 @ 11 \wedge 11111 \wedge \wedge \wedge$   
 $q_{30}$ :  $\wedge \wedge \wedge 11 @ 1 \wedge 11111 \wedge \wedge \wedge$   
 $q_{30}$ :  $\wedge \wedge \wedge 111 @ \wedge 11111 \wedge \wedge \wedge$

$q_0$ :  $\wedge \wedge \wedge @ 111 \wedge 11111 \wedge \wedge \wedge$

- (iii) Check if there is a unary and go to (iv).  
 (iv) Once at the rightmost of the unary and check if there is a unary number, move it to the leftmost of the divisor and go back to (ii). Otherwise (if blank,  $\wedge$ ) then go to DIV/MOD part of TM.

$q_1$ :  $\wedge \wedge \wedge 111 \wedge @ 11111 \wedge \wedge \wedge$   
 $q_1$ :  $\wedge \wedge \wedge 111 \wedge 1 @ 1111 \wedge \wedge \wedge$   
 $q_1$ :  $\wedge \wedge \wedge 111 \wedge 11 @ 111 \wedge \wedge \wedge$   
 $q_1$ :  $\wedge \wedge \wedge 111 \wedge 111 @ 11 \wedge \wedge \wedge$   
 $q_1$ :  $\wedge \wedge \wedge 111 \wedge 1111 @ 1 \wedge \wedge \wedge$   
 $q_1$ :  $\wedge \wedge \wedge 111 \wedge 11111 @ \wedge \wedge \wedge$

$q_2$ :  $\wedge \wedge \wedge 111 \wedge 1111 @ 1 \wedge \wedge \wedge$   
 $q_3$ :  $\wedge \wedge \wedge 111 \wedge 111 @ 1 \wedge \wedge \wedge$   
 $q_3$ :  $\wedge \wedge \wedge 111 \wedge 11 @ 11 \wedge \wedge \wedge$   
 $q_3$ :  $\wedge \wedge \wedge 111 \wedge 1 @ 111 \wedge \wedge \wedge$   
 $q_3$ :  $\wedge \wedge \wedge 111 \wedge @ 1111 \wedge \wedge \wedge$   
 $q_3$ :  $\wedge \wedge \wedge 111 @ \wedge 1111 \wedge \wedge \wedge$   
 $q_4$ :  $\wedge \wedge \wedge 11 @ 1 \wedge 1111 \wedge \wedge \wedge$   
 $q_4$ :  $\wedge \wedge \wedge 1 @ 11 \wedge 1111 \wedge \wedge \wedge$   
 $q_4$ :  $\wedge \wedge \wedge @ 111 \wedge 1111 \wedge \wedge \wedge$   
 $q_4$ :  $\wedge \wedge @ \wedge 111 \wedge 1111 \wedge \wedge \wedge$   
 $q_5$ :  $\wedge @ \wedge \wedge 111 \wedge 1111 \wedge \wedge \wedge$   
 $q_6$ :  $\wedge 1 @ \wedge 111 \wedge 1111 \wedge \wedge \wedge$

- (v) Once copying is done(determined by iv.), then go to the leftmost of the divisor and start mod/div operation.
- (vi) Subtract -1 from the leftmost side of the dividend, go to the rightmost side of the divisor and keep track of the subtraction using the character A.

$q_2: \wedge \wedge 11111 \wedge 111 @ \wedge \wedge \wedge$   
 $q_7: \wedge \wedge 11111 \wedge 11 @ 1 \wedge \wedge \wedge$   
 $q_7: \wedge \wedge 11111 \wedge 1 @ 11 \wedge \wedge \wedge$   
 $q_7: \wedge \wedge 11111 \wedge @ 111 \wedge \wedge \wedge$   
 $q_7: \wedge \wedge 11111 @ 1 \wedge 111 \wedge \wedge \wedge$   
 $q_8: \wedge \wedge 1111 @ 1 \wedge 111 \wedge \wedge \wedge$   
 $q_8: \wedge \wedge 111 @ 11 \wedge 111 \wedge \wedge \wedge$   
 $q_8: \wedge \wedge 11 @ 111 \wedge 111 \wedge \wedge \wedge$   
 $q_8: \wedge \wedge 1 @ 1111 \wedge 111 \wedge \wedge \wedge$   
 $q_8: \wedge \wedge @ 11111 \wedge 111 \wedge \wedge \wedge$   
 $q_8: \wedge @ \wedge 11111 \wedge 111 \wedge \wedge \wedge$   
 $q_9: \wedge \wedge @ 11111 \wedge 111 \wedge \wedge \wedge$

$q_{10}: \wedge \wedge @ 11111 \wedge 111 \wedge \wedge$   
 $q_{11}: \wedge \wedge \wedge @ 1111 \wedge 111 \wedge \wedge$   
 $q_{11}: \wedge \wedge \wedge 1 @ 111 \wedge 111 \wedge \wedge$   
 $q_{11}: \wedge \wedge \wedge 11 @ 11 \wedge 111 \wedge \wedge$   
 $q_{11}: \wedge \wedge \wedge 111 @ 1 \wedge 111 \wedge \wedge$   
 $q_{11}: \wedge \wedge \wedge 1111 @ \wedge 111 \wedge \wedge$   
 $q_{12}: \wedge \wedge \wedge 1111 \wedge @ 111 \wedge \wedge$   
 $q_{12}: \wedge \wedge \wedge 1111 \wedge 1 @ 11 \wedge \wedge$   
 $q_{12}: \wedge \wedge \wedge 1111 \wedge 11 @ 1 \wedge \wedge$   
 $q_{12}: \wedge \wedge \wedge 1111 \wedge 111 @ \wedge \wedge$   
 $q_{13}: \wedge \wedge \wedge 1111 \wedge 11 @ 1 \wedge \wedge$   
 $q_{14}: \wedge \wedge \wedge 1111 \wedge 1 @ 1 \wedge \wedge$

- (vii) Go to the leftmost of the dividend and repeat to (vi) until the divisor is only A.
- (viii) Once the divisor is replaced with A's we know it fits once in the dividend, therefore we replace it with 1s again and add AB at the rightmost point of the divisor (indicates our quotient) then we go to the leftmost of the dividend and repeat the process (back to vi) until we can no longer subtract.

$q_{16}: \wedge \wedge \wedge 1111 \wedge @ 11 A \wedge \wedge$   
 $q_{16}: \wedge \wedge \wedge 1111 @ \wedge 11 A \wedge \wedge$   
 $q_{17}: \wedge \wedge \wedge 111 @ 1 \wedge 11 A \wedge \wedge$   
 $q_{17}: \wedge \wedge \wedge 11 @ 11 \wedge 11 A \wedge \wedge$   
 $q_{17}: \wedge \wedge \wedge 1 @ 111 \wedge 11 A \wedge \wedge$   
 $q_{17}: \wedge \wedge \wedge @ 1111 \wedge 11 A \wedge \wedge$   
 $q_{17}: \wedge \wedge @ \wedge 1111 \wedge 11 A \wedge \wedge$

$q_{14}: \wedge \wedge 11 @ \wedge A A A \wedge \wedge$   
 $q_{15}: \wedge \wedge 11 \wedge @ A A A \wedge \wedge$   
 $q_{15}: \wedge \wedge 11 \wedge 1 @ A A \wedge \wedge$   
 $q_{15}: \wedge \wedge 11 \wedge 11 @ A \wedge \wedge$   
 $q_{15}: \wedge \wedge 11 \wedge 111 @ \wedge \wedge$   
 $q_{16}: \wedge \wedge 11 \wedge 11 @ 1 B \wedge \wedge$   
 $q_{16}: \wedge \wedge 11 \wedge 1 @ 11 B \wedge \wedge$   
 $q_{16}: \wedge \wedge 11 \wedge @ 111 B \wedge \wedge$   
 $q_{16}: \wedge \wedge 11 @ \wedge 111 B \wedge \wedge$   
 $q_{17}: \wedge \wedge 1 @ 1 \wedge 111 B \wedge \wedge$   
 $q_{17}: \wedge \wedge @ 11 \wedge 111 B \wedge \wedge$   
 $q_{17}: \wedge @ \wedge 11 \wedge 111 B$

- (ix) We then discard of any left over 1s, and convert any As to ones, then we take the leftmost B, turn it into a blank and move it to the rightmost as a 1, while moving to the rightmost we turn any B's to 1s.
- (x) We then move the leftmost of the reminder.
- End result:
- $q_f: \wedge @ 1 1 \wedge 1$

**4**

(A) hi

(B) hi