Problem Set 3

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2. Exercises

1. Sequence problem:

The firm decides on the sequence of replacements for the machines to maximize this expected present value.

The firm's sequence problem can be formulated as follows:

$$\max_{\{i_t\}_{t=0}^{\infty}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\mu a_t \cdot (1 - i_t) + \varepsilon_{0t} \cdot (1 - i_t) + R \cdot i_t + \varepsilon_{1t} \cdot i_t\right)\right]$$

Subject to:

$$a_{t+1} = \begin{cases} 1 & \text{if } i_t = 1 \\ \min(a_t + 1, 5) & \text{if } i_t = 0 \end{cases}$$

2. Bellman's equation for the value function of the firm.

The conditional value function when the firm chooses not to replace the machine ($i_t = 0$):

$$\bar{V}_0(a; \theta) = \mu a + \mathbb{E}[\varepsilon_{0t}] + \beta \mathbb{E}[V(a + 1; \theta)]$$

- If $a < 5$: $a_{t+1} = a_t + 1$ - If $a = 5$: $a_{t+1} = 5$

The conditional value function when the firm chooses to replace the machine ($i_t = 1$).:

$$\bar{V}_1(a; \theta) = R + \mathbb{E}[\varepsilon_{1t}] + \beta \mathbb{E}[V(1; \theta)]$$

The Bellman equation then becomes:

$$V(a; \theta) = \mathbb{E}\left[\max\{\bar{V}_0(a; \theta), \bar{V}_1(a; \theta)\}\right]$$

3. Contraction mapping results from Julia:Converged after 140 iterations.

Optimal Value Function: [-9.333394245953798, -10.185013690891463, -10.571714214669761, -10.727726142541009, -10.786615069637898]

Epsilon difference for indifference at age 2: 0.11448797184436699

Probability of replacement at age 2: 0.5285907703313703

Value at state ($a_t = 4$, $_{0t} = 1$, $_{1t} = 1.5$): -9.263973102108519

4. Simulate data results from Julia

Data written to "machine_data.csv"

E. NFP

Estimated $\theta_{NFP} = [-0.5778940771939708, -1.7752135619178595]$

F. CCP

Estimated replacement probabilities: [0.3037483843171047, 0.5148514851485149, 0.7008928571428571, 0.8251599147121536, 0.9213483146067416]

Estimated$\theta_{CCP} = [2.973133573684728, 5.083174522621682]$