julia and linear algebra

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1 Julia & Linear Algebra Tutorial

1.1 Installation Instructions

1.1.1 Install Julia

1. Download Julia from julialang.org/downloads
2. Add Julia to PATH during installation
3. Verify installation in terminal:

```
julia --version
```

1.1.2 Install VS Code

1. Download from code.visualstudio.com
2. Install the **Julia Language Support** extension from VS Code Marketplace

1.1.3 Add some packages

In VS Code's Julia REPL:

```
using Pkg
Pkg.add("LinearAlgebra")
Pkg.add("Plots")
```

Let

$$A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$$

1. Find its eigenvalues/vectors and verify

$$A\mathbf{v} = \lambda\mathbf{v}$$

1.2 QR Decomposition & Gram-Schmidt Connection

2. The *QR decomposition* factorises a matrix $A = QR$, where Q is an orthogonal matrix, and R and upper triangular. This can do the *Gram-Schmidt* decomposition.

For matrix

$$C = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(a) Compute the QR decomposition
(b) Verify orthogonality of Q

1.3 Covariance

3. Compute covariance of

$$X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ 5 & 7 \end{bmatrix}$$

and find its eigenvalues.

1.4 SVD

4. Approximate

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

using a rank-1 SVD.

1.5 Least Squares Approximation

We can solve overdetermined systems using the least squares method, namely minimising the residual $\|Ax - b\|_2$

5. Given points $(1, 2), (2, 3), (3, 5)$ find the best-fit line

$$y = mx + c$$

1.6 Nullspace

The nullspace or kernel of a matrix A is $\{x : Ax = 0\}$.

6. Find the kernel of matrix A where:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -3 & 4 \\ 3 & 5 & -7 \end{bmatrix}$$

and test that a vector generated that way is in the kernel of A.

1.7 Pseudoinverse

The pseudoinverse P of a matrix A is given by

$$P = A(A^*A)^{-1}A^*.$$

We're going to be using this later in the course.

7. Using the pseudoinverse, project vector B onto the column space of matrix A where:

$$B = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$$

and use the `Plots` package to visualize this projection.

1.8 Square-root of a matrix?

8. Find the square root of matrix A where:

$$A = \begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix}$$

1.9 Solving linear equations

9. Solve the following system of linear equations using Julia:

$$\begin{aligned} 2x + 3y - z &= 5 \\ 4x - y + 5z &= 6 \\ -2x + y + 3z &= -4 \end{aligned}$$

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