ジョレダン標準形

$$Ax_j = \lambda_{ij} x_i + \lambda_{2j} x_2 + \cdots + k_{ni} x_n$$

$$\chi_{i} = \begin{pmatrix} \chi_{i}, \\ \vdots \\ \chi_{in} \end{pmatrix} \qquad \chi(2)$$

$$P_{ij} = \chi_{ji}$$

$$P = (\lambda_1, \dots, \lambda_n) = \begin{pmatrix} \lambda_{11} & \lambda_{21} & \dots & \lambda_{n1} \\ \vdots & & & \vdots \\ \lambda_{1n} & \lambda_{2n} & \lambda_{nn} \end{pmatrix}$$

$$AP = P\Lambda \qquad P'AP = \Lambda$$

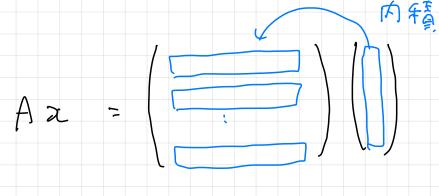
$$Ax_{j} = \sum_{i} \lambda_{ij} x_{i}$$

$$(AP)_{ij} = (Ax, ... Ax_n)_{ij}$$

$$= (A \times j);$$

$$= (\chi_{ij} \chi_i + \chi_{2j} \chi_2 + \cdots + \chi_{nj} \chi_n)_i$$

$$= (P \wedge)_{ij}$$



$$= \chi_1 \alpha_1 + \chi_2 \alpha_2 + \cdots + \chi_n \alpha_n$$

$$= (Ab_1, \dots, Ab_n)$$

一次独立な固有べつトル

$$Ax_i = \lambda_i x_i$$

かいりるのかは

$$P = (\chi_1, \ldots, \chi_n)$$

$$P^{-1}AP = \begin{pmatrix} \lambda_1 & \dots & \lambda_n \end{pmatrix}$$

これから行動の対角化、

しかし、一般にはり個ない、

2. 固有值、広義固有空間

国有值、国有べつにし

 $Ax = \lambda x x \neq 0 9 =$

入はAの固有値、スは入の固有べつfルとら

入の固有べかしいたるのなる空間をVAと書く

 $V_{\lambda} = \text{Ker}(A - \lambda I)$ $A_{\lambda} = \lambda x = \lambda I x$

 $\sim (A - \lambda I) x = 0$

最小约項式

FEC[X] 对項式管

F(A)=0 公司 显示治的好质式之

最高次係数於1,99項式色

An島小夕頃式とよんでやと書く

F(x) = Xn+ axn-1+ ... + b

~> F(A) = A" + a A"-1 + .. + b I

$$\tilde{V}_1 = \ker(A - I)^2$$
 > $V_1 = \ker(A - I)$

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$

$$A x_1 = 3 x_1 = 3 x_1 + 0 x_2$$

 $A x_2 = -x_2 = 0 x_1 - x_2$

$$(A-31)x = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix}x = 0 \qquad x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$(A + I) x = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} x = 0 \qquad x_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$A \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$$

$$X_{1}, X_{2}$$

$$P^{-1}AP = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$(A - I) x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} x = 0 \qquad \lambda_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

国有约项式

ケーリー・ハミルトンの定理.

 $\phi(x) = det(XI - A) z = 3 z$

 $\phi(A) = 0$

中(X)を固有約項式という

Q(x)はめ(x)を書りりた3.

中(x)はや(x)かを割りりたる。

Cは代数的関体でのでは 化炭の物項式は「次式に分割できる、

应義固有空間

$$\varphi = (x - x)^d (y - y - y - y)^d$$

GのFR入の重複度をdとする V_x = Ker (A-XI)^d

を 広義 固有空間でいう、

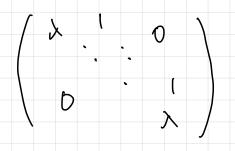
$$C^{n} = \widetilde{V}_{\lambda_{r}} \oplus \cdots \oplus \widetilde{V}_{\lambda_{r}}$$

VX C VX

$$\phi = (X - Y)^{\mathsf{M}} () \cdots$$

固有値入の中が根としての重複度がと入の重複度という。

Jordan 本型 译形



Jordan (73)

Jordan 7739

O Ai Ar

Jordan 产品证书

A は PTAP によって Jordan 本票に無明にできる

A: 部分でけに着目すれば

Q A Z PE (.

A > , = > > 1

 $A \chi_2 = \lambda \chi_2 + \chi_1$

 $A \chi_3 = \lambda \chi_3 + \chi_2$

Axk= Axk+xk-1

 $(A - \lambda I) \chi_{i} = 0$

 $(A - \lambda l) \chi_2 = \chi_1$

 $(A - \lambda 2 | \lambda_3 = \lambda_2$

(A-) 1) Xk = 74-1

3. 13%

$$A = \begin{bmatrix} 4 & -2 & -1 & 0 & 0 & -1 & -2 \\ 1 & 1 & 0 & 1 & 0 & -1 & -2 \\ -2 & 3 & 4 & 0 & 0 & 1 & 3 \\ -2 & -3 & -1 & 3 & 0 & -1 & -3 \\ -4 & 5 & 2 & -1 & 2 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 2 & -2 & -1 & 0 & 0 & -1 & 0 \end{bmatrix}$$

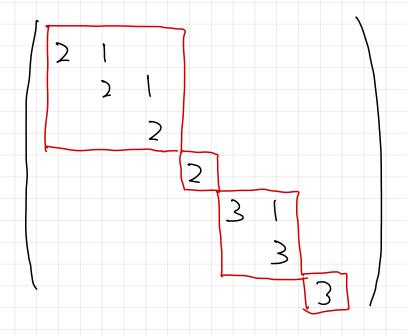
$$\phi = \det(X_{1} - A) = (X - 2)^{4} (X - 3)^{2}$$

$$C(X) = (X - 2)^{3} (X - 3)^{2}$$

$$C^{1} = V_{2} \oplus V_{3}$$

$$V_{2} = \ker(A - 2I)^{3} \dim V_{3} = 4$$

$$V_{2} = \ker(A - 2I)^{3}$$
 din $V_{3} = 4$
 $V_{3} = \ker(A - 3I)^{2}$ din $V_{3} = 3$



$$(A - 2I) \chi_1 = 0$$

 $(A - 2I) \chi_2 = \chi_1$
 $(A - 2I) \chi_3 = \chi_2$
 $(A - 2I) \chi_4 = 0$
 $(A - 3I) \chi_5 = 0$
 $(A - 3I) \chi_6 = \chi_5$
 $(A - 3I) \chi_1 = 0$

$$(A - \lambda 1) \chi_{1} = 0$$

$$(A - \lambda 1) \chi_{2} = \chi_{1}$$

$$(A - \lambda 1) \chi_{3} = \chi_{2}$$

$$\vdots$$

$$(A - \lambda 1) \chi_{k} = \chi_{k-1}$$

$$(A-2I)x=0$$

$$(A - 2I)^3 y = 0$$

$$\mathcal{J} = \left(\begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right) + \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right) + \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right) + \left(\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right)$$

b-2d

$$\left(\begin{array}{c} d - \alpha \\ d - \alpha \end{array} \right)$$

$$\left(\begin{array}{c} A - 2I \end{array} \right)^{2} y = \begin{pmatrix} 2\alpha - 2d \\ 2d - 2\alpha \\ 3\alpha - 3d \\ 0 \\ d - \alpha \end{pmatrix}$$

$$\chi_{2} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\chi_{3} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\chi_{4} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\chi_{5} = \begin{pmatrix} A - 2I \end{pmatrix} \chi_{3} = \begin{pmatrix} -2 \\ -2 \\ -3 \\ 5 \end{pmatrix}$$

$$\chi_{5} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\chi_{1} = (A - 2I)^{2} \chi_{2}^{2} = \begin{pmatrix} 1 \\ -2 \\ 2 \\ -3 \\ 0 \end{pmatrix}$$

$$\chi_{\varphi} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(A - 2I) \chi_1 = 0$$

 $(A - 2I) \chi_2 = \chi_1$
 $(A - 2I) \chi_3 = \chi_2$
 $(A - 2I) \chi_4 = 0$

$$(A-3I)x=0$$

$$\mathcal{L} = \mathcal{V} \begin{pmatrix} 0 \\ 0 \\ -1 \\ -1 \\ 1 \end{pmatrix} + \mathcal{V} \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

$$(A - 3I)^2y = 0$$

$$y = a \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\chi_{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \chi_{s} = (A - 3z) \chi_{b} = \begin{pmatrix} -1 \\ 0 \\ -1 \\ 2 \\ 0 \\ -1 \end{pmatrix}$$

$$\mathcal{C}_{1} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \end{pmatrix}$$

$$(A - 31) \times_5 = 0$$

 $(A - 31) \times_6 = \times_5$
 $(A - 37) \times_1 = 0$

$$P = \begin{pmatrix} 1 & -2 & 0 & 0 & -1 & 0 & 1 \\ 1 & -2 & 0 & 0 & 0 & 0 & 0 \\ -2 & 3 & 0 & 0 & 1 & 1 & 0 \\ -2 & 3 & 0 & 0 & -1 & 0 & 0 \\ 2 & -3 & 0 & 0 & -1 & 0 & 0 \\ -3 & 5 & 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 1 & -2 & 1 & 0 & -1 & 0 & 1 \end{pmatrix}$$

$$x_1, \quad \chi_2, \quad \chi_3, \quad \chi_4, \quad \chi_5, \quad \chi_6, \quad \chi_9$$

$$A = \begin{pmatrix} 4 & 1 & 1 \\ -\gamma & -1 & -3 \\ 3 & 1 & 3 \end{pmatrix}$$

$$\phi = (\chi - 2)^3$$

$$\varphi = (X-2)^2$$

$$(A - 2I) \chi_i = 0$$

$$(A - 2Z) x_i = x_i$$

$$(A - 2T)\chi_3 = \chi_2$$

$$t_{-} \leq \tilde{z} (\tilde{f}) \qquad \chi_{3} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \chi_{2} = \begin{pmatrix} A - 2I \end{pmatrix} \chi_{3} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\chi_1 = (A - 2I) \chi_2 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 1 & 0 \\ -1 & -3 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$C^{3} = V_{2}$$

$$V_{2} = (A-2I)^{3}$$

$$dim V_2 = 3$$

$$A = \begin{bmatrix} 8 & -3 & 3 & -2 \\ 0 & 3 & 0 & 2 \\ -10 & 6 & -3 & 4 \\ 2 & 0 & 1 & 2 \end{bmatrix}$$

$$\phi = (X - 2)^2 (X - 3)^2$$

$$\varphi = (\chi - 2)^2 (\chi - 3)^2$$

$$(A - 2I)x, = 0$$

$$(A - 2I)x_2 = x_1$$

$$(A - 2I)^2 \chi = 0$$

$$\chi_2 = \begin{pmatrix} D \\ -2 \\ 0 \end{pmatrix}$$

$$C^{\xi} = V_{2} \oplus V_{3}$$

$$V_{2} = (er(A-2I)^{2})$$

$$V_{3} = (er(A-3I)^{2})$$

$$V_{3} = (er(A-3I)^{2})$$

$$d:mV_3 = 2$$
 $d:mV_3 = 2$

$$\chi = A \begin{pmatrix} -1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + A \begin{pmatrix} 0 \\ -2 \\ 0 \\ 1 \end{pmatrix}$$

$$\chi_{2} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} \qquad \chi_{1} = \begin{pmatrix} A - 2I \end{pmatrix} \chi_{2} = \begin{pmatrix} 4 \\ 0 \\ -8 \\ 0 \end{pmatrix}$$

$$(A - 3I)x_3 = 0$$

 $(A - 3I)x_4 = x_3$

$$(A - 3 T)^2 \chi = 0$$

$$x = \alpha \begin{pmatrix} 3 \\ 13 \\ 0 \\ 6 \end{pmatrix} + \beta \begin{pmatrix} -3 \\ 1 \\ 6 \\ 0 \end{pmatrix}$$

$$\mathcal{Z}_{4} = \begin{pmatrix} 3 \\ 13 \\ 0 \\ 6 \end{pmatrix}$$

$$\chi_{4} = \begin{pmatrix} 3 \\ 13 \\ 0 \\ L \end{pmatrix} \qquad \chi_{3} = (A - 3I) \chi_{4} = \begin{pmatrix} -36 \\ 12 \\ 12 \\ 0 \end{pmatrix}$$

$$P = \begin{pmatrix} 4 & 0 & -36 & 3 \\ 0 & -2 & 12 & 13 \\ -8 & 0 & 12 & 0 \\ 0 & 1 & 0 & 6 \end{pmatrix}$$

斎藤毅 绿形代数。世界 高杉豊 馬場芬之 演習線形代数 有馬哲、浅枝陽 演習詳解線型代数