Theta関数

Ver. 0.09

Chap. 1. 導入

§1 $\vartheta(z,\tau)$

$$\vartheta(z,\tau) = \sum_{n\in\mathbb{Z}} e^{\pi i n^2\tau + 2\pi i n z}$$

$z \in \mathbb{C}$. $\tau \in H$ ($= \mathbb{C}$ の $\mathrm{Im}\,\tau > 0$ 部分)

$|\mathrm{Im}\,z| < c$ かつ $\mathrm{Im}\,\tau > \varepsilon$ で収束する.

$$\vartheta(z+1,\tau) = \vartheta(z,\tau)$$

$$\vartheta(z+\tau,\tau) = \sum e^{\pi i n^2\tau + 2\pi i n(z+\tau)}$$

$$= \sum e^{\pi i (n+1)^2\tau - \pi i\tau + 2\pi i n z}$$

$n+1=m$

$$= \sum e^{\pi i m^2\tau - \pi i\tau + 2\pi i m z - 2\pi i z}$$

$$= e^{-\pi i\tau - 2\pi i z}\,\vartheta(z,\tau)$$

$$\vartheta(z+a\tau+b,\tau) = e^{-\pi i a^2\tau - 2\pi i a z}\,\vartheta(z,\tau)$$

$f(z)$ を正則関数, $a, b \in \mathbb{R}$ とする.

$$(S_a f)(z) = f(z+a)$$
$$(T_a f)(z) = e^{\pi i a^2 \tau + 2\pi i a z} f(z + a\tau)$$
とする.

$$S_{b_1}(S_{b_2} f) = S_{b_1+b_2} f$$
$$T_{a_1}(T_{a_2} f) = T_{a_1+a_2} f$$

$$S_b \circ T_a = e^{2\pi i ab} T_a \circ S_b$$

T_a, S_b で生成される変換群を $\mathcal{G}$ と書く.

$$\mathcal{G} = \mathbb{C}_1^* \times \mathbb{R} \times \mathbb{R} \qquad \text{Heisenberg 群}$$

$\mathbb{C}_1^* = \{z \in \mathbb{C} \mid |z| = 1\}$

$(\lambda, a, b) \in \mathcal{G}$

$$\rightsquigarrow U_{(\lambda, a, b)} f(z) = \lambda (T_a \circ S_b f)(z)$$
$$= \lambda e^{\pi i a^2 \tau + 2\pi i a z} f(z + a\tau + b)$$

$[\mathcal{G}, \mathcal{G}] = \mathbb{C}^*$ よって $\mathcal{G}$ はべき零群

$\Gamma = \{(1, a, b) \in G \mid a, b \in \mathbb{Z}\}$ は

G の部分群.

$l\Gamma$ を $\{(1, la, lb)\} \subset \Gamma$ とおく.

$V_l = \{l\Gamma$ で不変な $f(z)\}$ $\dim V_l = l^2$

μ_m を 1 の m 乗根のなす群とする.

$G_l = \{(\lambda, a, b) \mid \lambda \in \mu_{l^2}, a, b \in \frac{1}{l}\mathbb{Z}\} / l\Gamma$ とし

その積を
$(\lambda, a, b)(\lambda', a', b') = (\lambda\lambda' e^{2\pi i ba'}, a+a', b+b')$

とすると G_l は有限群になる.

$S_{1/l}, T_{1/l} \in G_l$ の V_l への作用を

$S_{1/l}, T_{1/l} \in G$ の作用と同じにできる.

V_l は G_l の作用に関して既約である.

V_ℓ の基底を

$$\vartheta_{a,b} = S_b T_a \vartheta \qquad a, b \in \frac{1}{\ell}\mathbb{Z}$$

とすることができる.

$$\vartheta_{a,b} = \sum_{n\in\mathbb{Z}} e^{\pi i (a+n)^2 \tau + 2\pi i (n+a)(z+b)}$$

$$\vartheta_{0,0} = \vartheta$$

$$S_{b_1}(\vartheta_{a,b}) = \vartheta_{a,b+b_1}$$

$$T_{a_1}(\vartheta_{a,b}) = e^{-2\pi i a_1 b} \vartheta_{a_1+a,b}$$

$$\forall a, a_1, b \in \frac{1}{\ell}\mathbb{Z}$$

$$\vartheta_{a+p,b+q} = e^{2\pi i a q} \vartheta_{a,b}$$

$$\forall p, q \in \mathbb{Z}, \ \forall a, b \in \frac{1}{\ell}\mathbb{Z}$$

§2 $\mathbb{C}/\mathbb{Z}+\mathbb{Z}\tau$

$\ell \geq 2$ とする.

(a_i, b_i) が $i = 0 \sim \ell^2-1$ で

$(\frac{1}{\ell}\mathbb{Z}/\mathbb{Z})^2$ の全代表元になるとし.

$\theta_i = \theta_{a_i, b_i}$ とする.

$\ell = 2$ のとき

$(0, 0)$, $(0, \frac{1}{2})$, $(\frac{1}{2}, 0)$, $(\frac{1}{2}, \frac{1}{2})$

θ_{00} $\theta_{0\frac{1}{2}}$ $\theta_{\frac{1}{2}0}$ $\theta_{\frac{1}{2}\frac{1}{2}}$

0 1 2 3

$\ell = 3$ のとき

$(0, 0)$, $(0, \frac{1}{3})$, $(0, \frac{2}{3})$,

θ_{00} 0 $\theta_{0\frac{1}{3}}$ 1 $\theta_{0\frac{2}{3}}$ 2

$(\frac{1}{3}, 0)$, $(\frac{1}{3}, \frac{1}{3})$, $(\frac{1}{3}, \frac{2}{3})$,

$\theta_{\frac{1}{3}0}$ 3 $\theta_{\frac{1}{3}\frac{1}{3}}$ 4 $\theta_{\frac{1}{3}\frac{2}{3}}$ 5

$(\frac{2}{3}, 0)$ $(\frac{2}{3}, \frac{1}{3})$, $(\frac{2}{3}, \frac{2}{3})$

$\theta_{\frac{2}{3}0}$ 6 $\theta_{\frac{2}{3}\frac{1}{3}}$ 7 $\theta_{\frac{2}{3}\frac{2}{3}}$ 8

$z \in \mathbb{C}$ のとき

$(\vartheta_0(\ell z, \tau), \ldots, \vartheta_{\ell^2-1}(\ell z, \tau))$ を

$\mathbb{P}_{\mathbb{C}}^{\ell^2-1}$ の斉次座標とする.

これらは同時に 0 にならない

$\vartheta_{ab}(z, \tau)$ の零点は

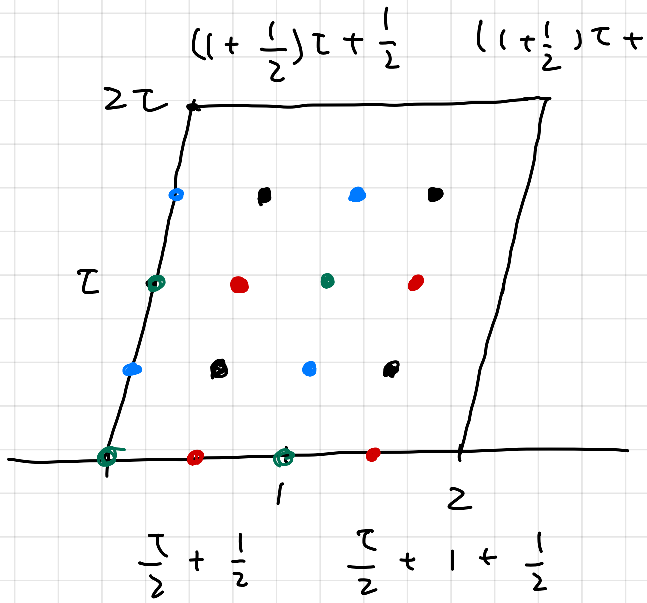
$(a + p + \frac{1}{2})\tau + (b + q + \frac{1}{2})$, $p, q \in \mathbb{Z}$

よって $i \neq j$ のとき ϑ_i と ϑ_j の零点は異なる

$\ell = 2$ のとき

$(a_i, b_i) = (0, 0), (0, \frac{1}{2}), (\frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2})$

ϑ_{00}　$\vartheta_{0\frac{1}{2}}$　$\vartheta_{\frac{1}{2}0}$　$\vartheta_{\frac{1}{2}\frac{1}{2}}$



ϑ_{00} の零点

$\vartheta_{0\frac{1}{2}}$ 〃

$\vartheta_{\frac{1}{2}0}$ 〃

$\vartheta_{\frac{1}{2}\frac{1}{2}}$ 〃

$$(\theta_0(z+l,\tau),\dots,\theta_{l^2-1}(z+l,\tau))$$
$$=(\theta_0(z,\tau),\dots,\theta_{l^2-1}(z,\tau))$$

$$(\theta_0(z+l\tau,\tau),\dots,\theta_{l^2-1}(z+l\tau,\tau))$$
$$=\lambda(\theta_0(z,\tau),\dots,\theta_{l^2-1}(z,\tau))$$

ただし $\lambda = e^{-\pi i l^2\tau - 2\pi i l z}$.

となるから

$\Lambda_\tau = \mathbb{Z}+\mathbb{Z}\tau$

$E_\tau = \mathbb{C}/\Lambda_\tau$

$\varphi_l : E_\tau \to \mathbb{P}^{l^2-1} \quad z \mapsto (\dots \theta_i(lz,\tau) \dots)$

が定義できる.

φ_l は埋め込みである. すなわち.

$\varphi_l(E_\tau)$ は $\mathbb{P}_{\mathbb{C}}^{l^2-1}$ の複素解析部分多様体で

E_τ に同型である.

θ関数の関係式

$$(x+y+u+v)^2+(x+y-u-v)^2$$

$$+(x-y+u-v)^2+(x-y-u+v)^2=4(x^2+y^2+u^2+v^2)$$

$l=2$

$$\theta_{a,b}(z,\tau)=\sum_{n\in\mathbb{Z}} e^{\pi i(a+n)^2\tau+2\pi i(n+a)(z+b)}$$

$$a,b=0,\frac{1}{2}$$

⇓

$\sum\theta\theta\theta\theta=\theta\theta\theta\theta$ $\overbrace{\text{という形の}}^{\text{たくさんの}}$関係式

$$\theta_{11}(-z,\tau)=\sum_{n\in\mathbb{Z}} e^{\pi i(n+\frac{1}{2})^2\tau+2\pi i(n+\frac{1}{2})(-z+\frac{1}{2})}$$

$n+\frac{1}{2}$

$n-\frac{1}{2}$ $\quad z-\frac{1}{2}$

$$=\sum_{n\in\mathbb{Z}} e^{\pi i(n+\frac{1}{2})^2\tau+2\pi i(-n+\frac{1}{2})(-z+\frac{1}{2})}$$

$$=-\theta_{11}(z,\tau)$$

$$\therefore\ \theta_{11}(0,\tau)=0$$

τは固定して書くことを省略

$\vartheta_{00},\ \vartheta_{0\frac{1}{2}},\ \vartheta_{\frac{1}{2}0},\ \vartheta_{\frac{1}{2}\frac{1}{2}} \rightsquigarrow \vartheta_{00},\ \vartheta_{01},\ \vartheta_{10},\ \vartheta_{11}$

$$\vartheta_{00}(x)\vartheta_{00}(y)\vartheta_{00}(u)\vartheta_{00}(v)+\vartheta_{01}(x)\vartheta_{01}(y)\vartheta_{01}(u)\vartheta_{01}(v)$$
$$+\vartheta_{10}(x)\vartheta_{10}(y)\vartheta_{10}(u)\vartheta_{10}(v)+\vartheta_{11}(x)\vartheta_{11}(y)\vartheta_{11}(u)\vartheta_{11}(v)$$
$$=2\vartheta_{00}(x_1)\vartheta_{00}(y_1)\vartheta_{00}(u_1)\vartheta_{00}(v_1)$$

ただし $x_1=\frac{1}{2}(x+y+u+v)$　$y_1=\frac{1}{2}(x+y-u-v)$

$u_1=\frac{1}{2}(x-y+u-v)$　$v_1=\frac{1}{2}(x-y-u+v)$

$$\vartheta_{00}(x)\vartheta_{00}(y)\vartheta_{11}(u)\vartheta_{11}(v)+\vartheta_{01}(x)\vartheta_{01}(y)\vartheta_{10}(u)\vartheta_{10}(v)$$
$$+\vartheta_{10}(x)\vartheta_{10}(y)\vartheta_{01}(u)\vartheta_{01}(v)+\vartheta_{11}(x)\vartheta_{11}(y)\vartheta_{00}(u)\vartheta_{00}(v)$$
$$=2\vartheta_{01}(x_1)\vartheta_{01}(y_1)\vartheta_{10}(u_1)\vartheta_{10}(v_1)$$

など.

$x=y$, $u=v=0$ とおくと.

$x_1=x$, $y_1=x$, $u_1=0$, $v_1=0$

$$\vartheta_{00}(x)^2\vartheta_{00}(0)^2=\vartheta_{01}(x)^2\vartheta_{01}(0)^2+\vartheta_{10}(x)^2\vartheta_{10}(0)^2$$
$$\vartheta_{11}(x)^2\vartheta_{00}(0)^2=\vartheta_{01}(x)^2\vartheta_{10}(0)^2-\vartheta_{10}(x)^2\vartheta_{01}(0)^2$$

さらに $x=0$ とおくと

$$\vartheta_{00}(0)^4=\vartheta_{01}(0)^4+\vartheta_{10}(0)^4$$

$\mathbb{C}/\Lambda_\tau = \mathbb{C}/\mathbb{Z}+\mathbb{Z}\tau$

$$\varphi_2 : E_\tau \longrightarrow \mathbb{P}^3$$

$$z \longmapsto (\underbrace{\theta_{00}(2z)}_{x_0}, \underbrace{\theta_{01}(2z)}_{x_1}, \underbrace{\theta_{10}(2z)}_{x_2}, \underbrace{\theta_{11}(2z)}_{x_3})$$

$$\theta_{00}(0)^2 x_0^2 = \theta_{01}(0)^2 x_1^2 + \theta_{10}(0)^2 x_2^2$$

$$\theta_{00}(0)^2 x_3^2 = \theta_{10}(0)^2 x_1^2 - \theta_{01}(0)^2 x_2^2$$

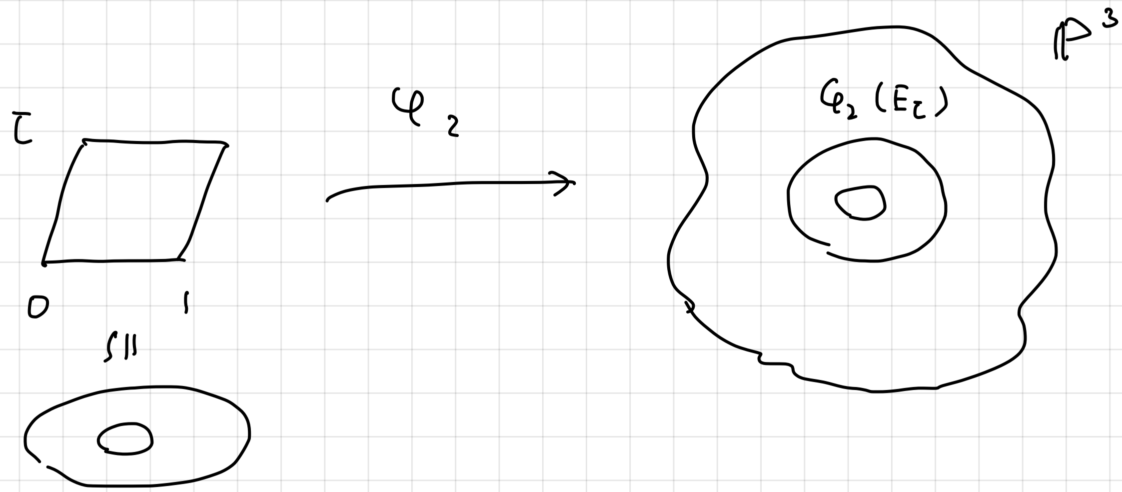
これは $\mathbb{P}^3$ の中の曲線 C

$$\varphi(E_\tau) = C$$

平面 $\sum a_i x_i = 0$ との交点.

$$a_0\theta_{00}(2z) + a_1\theta_{01}(2z) + a_2\theta_{10}(2z) + a_3\theta_{11}(2z) = 0$$

これは $\mathbb{C}/2\Lambda_\tau$ の中に4つの零点をもつ.



§3 2重周期有理型関数

$$z \longrightarrow z+1$$
$$z \longrightarrow z+\tau$$

Method I

$$\frac{\theta_{ab}(2z)}{\theta_{00}(2z)}$$

$$\theta_{ab}(2z+2) = \theta_{ab}(2z)$$

$$\theta_{ab}(2z+2\tau) = e^{-4\pi i\tau - 4\pi i z}\,\theta_{ab}(2z)$$

Method II

$$\prod_{1\le i\le k} \frac{\theta(z-a_i)}{\theta(z-b_i)} \qquad \sum a_i = \sum b_i$$

$$\theta(z-a+1) = \theta(z-a)$$
$$\theta(z-a+\tau) = e^{-\pi i\tau - 2\pi i(z-a)}\,\theta(z-a)$$

Method III.

$$\frac{d^2}{dz^2} \log \vartheta(z)$$

$$\log \vartheta(z+1) = \log \vartheta(z)$$

$$\log \vartheta(z+\tau) = \log \vartheta(z) - \pi i \tau - 2\pi i z$$

Weierstrass のペー関数

$$\wp(z) = \underbrace{- \frac{d^2}{dz^2} \log \vartheta_{11}(z) + \text{const.}}_{\text{全部で定数項を0とする.}}$$

Method IV

$$\sum \lambda_i \frac{d}{dz} \log \vartheta(z - a_i) \qquad \sum \lambda_i = 0$$

$$\vartheta_{11}(z) = \sum_{n \in \mathbb{Z}} e^{\pi i (n+\frac{1}{2})^2 \tau + 2\pi i (n+\frac{1}{2})(z+\frac{1}{2})}$$

$$\vartheta_{11}(0) = 0$$

$$-\frac{d^2}{dz^2} \log \vartheta_{11}(z) = -\frac{d}{dz} \frac{\vartheta_{11}'(z)}{\vartheta_{11}(z)}$$

$$= \frac{\vartheta_{11}'(z)^2 - \vartheta_{11}''(z)\vartheta_{11}(z)}{\vartheta_{11}(z)^2}$$

$$= \frac{1}{z^2} + \text{const.} + a z^2 + \cdots$$

$$\wp(z) = \frac{1}{z^2} + a z^2 + b z^4 + \cdots \qquad z = 0 \text{ 近く.}$$

$$\wp(z) = \frac{1}{z^2} + az^2 + bz^4 + \cdots$$

$$\wp'(z) = -\frac{2}{z^3} + 2az + 4bz^3 + \cdots$$

$$\wp'(z)^2 = \frac{4}{z^6} - \frac{8a}{z^2} - 16b + \cdots$$

$$4\wp(z)^3 = \frac{4}{z^6} + \frac{12a}{z^2} + 12b + \cdots$$

$$\underbrace{\wp'(z)^2 - 4\wp(z)^3 + \overset{-g_2(\tau)}{20a}\,\wp(z)}_{\text{2重周期}} = \underbrace{-28b + \cdots}_{\text{極なし}}$$

↓

zに依らない..

$g_3(\tau)$

$$\wp'(z)^2 = 4\wp(z)^3 + g_2(\tau)\wp(z) + g_3(\tau)$$

$$\wp''\cancel{\wp'} = 6\cancel{\wp'}\wp^2 + \frac{1}{2}g_2\cancel{\wp'}$$

$$\wp''' = 12\wp\wp'$$

§4 θの関数方程式

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$a, b, c, d \in \mathbb{Z}$

$ad - bc = 1$

さらに ab, cd は偶数 とする.

$c \geq 0$

$$\theta\left(\frac{z}{c\tau+d}, \frac{a\tau+b}{c\tau+d}\right) = \zeta (c\tau+d)^{\frac{1}{2}} e^{\pi i \frac{cz^2}{c\tau+d}} \theta(z, \tau)$$

ζ は適当な1の8乗根

導出

$$\left.\begin{array}{l} f(z+1) = f(z) \\ f(z+\tau) = e^{\alpha z+\beta} f(z) \end{array}\right\} \Rightarrow f(z) \sim \theta(z, \tau)$$

$$\psi(y, \tau) = e^{\pi i c(c\tau+d)y^2} \theta((c\tau+d)y, \tau)$$

$y \to y + \frac{a\tau+b}{c\tau+d}$ に対してそうなっている

$$\psi(y, \tau) \sim \theta\left(y, \frac{a\tau+b}{c\tau+d}\right)$$

$$\int_0^1 \theta(y, \tau)\, dy = 1$$

§5 モジュラー形式

$$SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; \begin{array}{l} a, b, c, d \in \mathbb{Z} \\ ad - bc = 1 \end{array} \right\}$$

$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ の $\mathbb{C} \times H$ への作用を

$$(z, \tau) \overset{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}{\longmapsto} \left(\frac{z}{c\tau + d}, \frac{a\tau + b}{c\tau + d} \right)$$

とする.

$$(z, \tau) \overset{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}{\longmapsto} \left(\frac{z}{c\tau + d}, \frac{a\tau + b}{c\tau + d} \right)$$

$$\overset{\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}}{\longmapsto} \left(\frac{\frac{z}{c\tau+d}}{c' \frac{a\tau+b}{c\tau+d} + d'}, \frac{a' \frac{a\tau+b}{c\tau+d} + b'}{c' \frac{a\tau+b}{c\tau+d} + d'} \right)$$

$$\frac{z}{(c'a + d'c)\tau + (c'b + d'd)} \qquad \frac{(a'a + b'c)\tau + (a'b + b'd)}{(c'a + d'c)\tau + (c'b + d'd)}$$

これは

$$\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a'a + b'c & a'b + b'b \\ c'a + d'c & c'b + d'd \end{pmatrix}$$

$$(z, \tau) \xrightarrow{\hspace{6em}} \left(\;〃\;, \;〃\; \right)$$

と同じ.

以下 $\mathbb{H}$ 上でのみ考える.

$$\tau \xmapsto{\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}} \frac{a\tau + b}{c\tau + d}$$

これを $\gamma\tau$ と書く

$$\Gamma_N = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) \;\middle|\; \begin{array}{ll} b, c \equiv 0 & \mathrm{mod}\, N \\ a, d \equiv 1 & \mathrm{mod}\, N \end{array} \right\}$$

これは $SL(2, \mathbb{Z})$ の正規部分群

$$\gamma_N : SL(2, \mathbb{Z}) \longrightarrow SL(2, \mathbb{Z}/N\mathbb{Z})$$

$$\mathrm{Ker}\, \gamma_N = \Gamma_N$$

モジュラー形式

$k \in \mathbb{Z}^+$, $N \in \mathbb{N}$ に対し

次をみたす $\mathbb{H}$ 上の正則関数 $f(\tau)$ を

重さ k. レベル N のモジュラー形式という

(a) $\forall \tau \in \mathbb{H}$, $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_N$

$$f(\gamma\tau) = (c\tau + d)^k f(\tau)$$

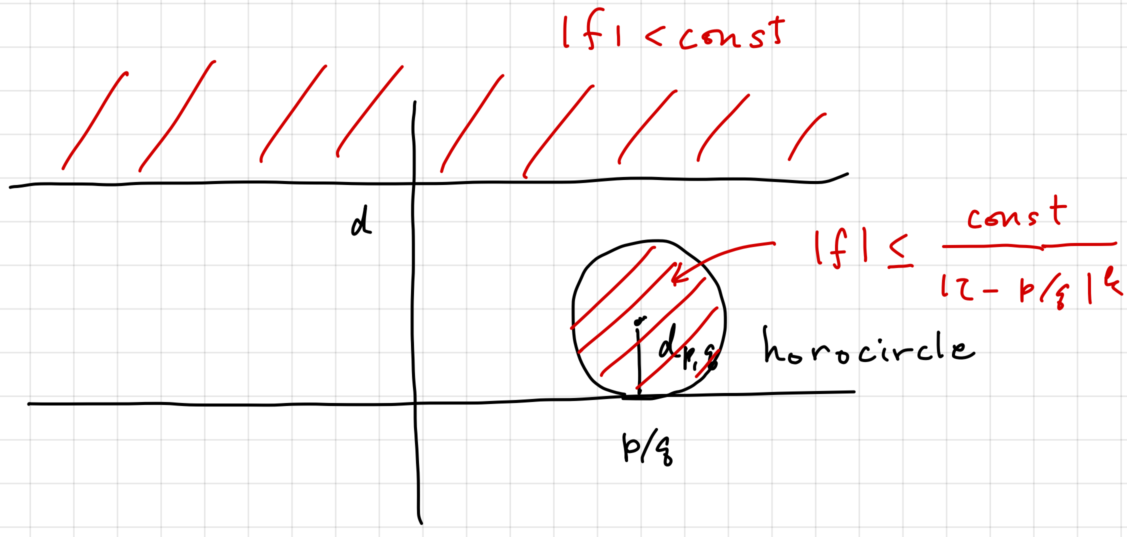
↖ $e_\gamma(\tau)$ と書く.

(b) (i) $\exists c, d$ $\operatorname{Im}\tau > d$ のとき $|f(\tau)| < c$

(ii) $\forall p/q \in \mathbb{Q}$ に対し $\exists c_{p,q}, d_{p,q} > 0$

$|\tau - p/q - i d_{p,q}| < d_{p,q}$ のとき

$$|f(\tau)| \leq c_{p,q} |\tau - (p/q)|^{-k}$$



$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ に対して $e_\gamma(\tau) = (c\tau + d)^k$

$f(\gamma\tau) = e_\gamma(\tau) f(\tau)$

$\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\gamma' = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$ に対して

$e_{\gamma'\gamma}(\tau) = e_{\gamma'}(\gamma\tau) e_\gamma(\tau)$

1-cocycle condition

$e_{\gamma'\gamma}(\tau) = \left((c'a + d'c)\tau + (c'b + d'd)\right)^k$

$e_{\gamma'}(\gamma\tau) e_\gamma(\tau) = \left(c' \frac{a\tau + b}{c\tau + d} + d'\right)^k (c\tau + d)^k$

$f(\gamma'\gamma\tau) = e_{\gamma'\gamma}(\tau) f(\tau)$

$= e_{\gamma'}(\gamma\tau) f(\gamma\tau) = e_{\gamma'}(\gamma\tau) e_\gamma(\tau) f(\tau)$

重さk，レベルNのモジュラー形式たちのなすベクトル空間を$\mathrm{Mod}_k^{(N)}$と書く．

モジュラー形式f，$\gamma \in SL(2,\mathbb{Z})$に対し

$f^\gamma(\tau) = e_\gamma(\tau)^{-1} f(\gamma\tau)$とおく．

$\gamma \in \Gamma_N$のとき$f^\gamma(\tau) = e_\gamma(\tau)^{-1} e_\gamma(\tau) f(\tau) = f(\tau)$で

$f^\gamma \in \mathrm{Mod}_k^{(N)}$ ⊛ だから

$SL(2,\mathbb{Z})/\Gamma_N$の$\mathrm{Mod}_k^{(N)}$への作用

$$(\gamma\Gamma_N)(f) = f^\gamma$$

が与えられたことになる

⊛ $f^\gamma(\gamma'\tau) = e_{\gamma'}(\tau) f^\gamma(\tau) \quad \gamma' \in \Gamma_N$

$$\begin{aligned}
f^\gamma(\gamma'\tau) &= e_\gamma(\gamma'\tau)^{-1} f(\gamma\gamma'\tau) \qquad \Gamma_N \triangleleft SL(2,\mathbb{Z}) \\
&= e_\gamma(\gamma'\tau)^{-1} f(\gamma\gamma'\gamma^{-1}\gamma\tau) \\
&= e_\gamma(\gamma'\tau)^{-1} e_{\gamma\gamma'\gamma^{-1}}(\gamma\tau) f(\gamma\tau) \\
&= e_\gamma(\gamma'\tau)^{-1} e_{\gamma\gamma'}(\tau) e_{\gamma^{-1}}(\gamma\tau) f(\gamma\tau) \\
&= \cancel{e_\gamma(\gamma'\tau)^{-1}} \cancel{e_\gamma(\gamma'\tau)} e_{\gamma'}(\tau) \underbrace{e_{\gamma^{-1}}(\gamma\tau)}_{e_\gamma(\tau)^{-1}} f(\gamma\tau) \\
&= e_{\gamma'}(\tau) f^\gamma(\tau)
\end{aligned}$$

Prop.

$\vartheta_{00}^2(0,\tau)$, $\vartheta_{01}^2(0,\tau)$, $\vartheta_{10}^2(0,\tau)$ は

重さ 1 レベル 4 のモジュラー形式