物理授業

2022 後期

すべて「約」です。

重力加速度	$9.8\ m/s^2$
万有引力定数	$6.7 \times 10^{-11}\ m^3/kg\,s^2$
地球の質量	$6.0 \times 10^{24}\ kg$
地球の半径	$6.4 \times 10^{6}\ m$
光速	$3.0 \times 10^{8}\ m/s$
プランク定数	$6.6 \times 10^{-34}\ m^2kg/s$

$$\frac{\text{万有引力定数} \times \text{地球の質量}}{\text{地球の半径}} \fallingdotseq 6.3 \times 10^{7}\ J$$

関数

$x \longmapsto y$

xが与えられるとyが決まる.

$y = 2x + 1$

たとえば　$x = 3$

$y = 2 \times 3 + 1 = 7$

$\underset{x}{3} \longmapsto \underset{y}{7}$

$y = x^2 - 1$

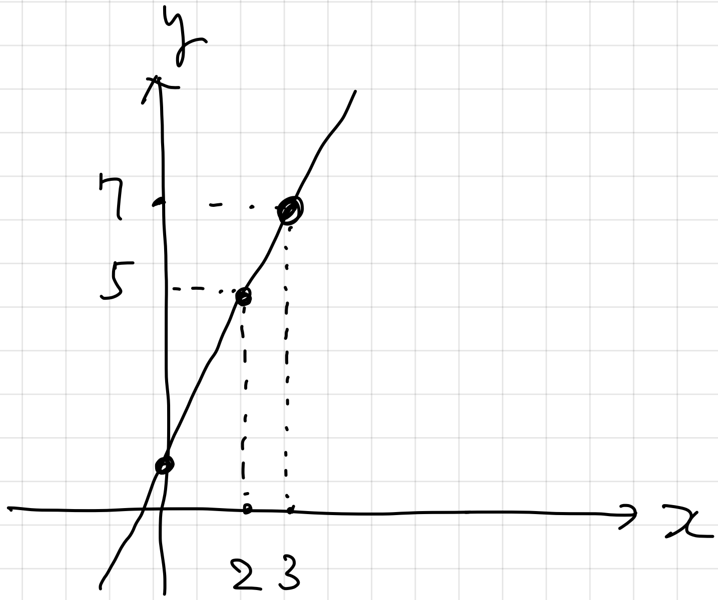
たとえば　$x = 4$

$y = 4^2 - 1 = 16 - 1 = 15$

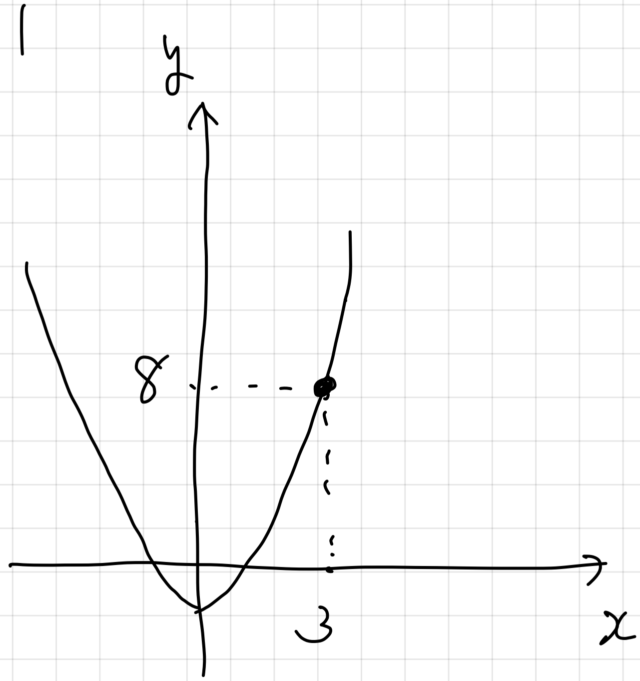
$\underset{x}{4} \longmapsto \underset{y}{15}$

グラフ

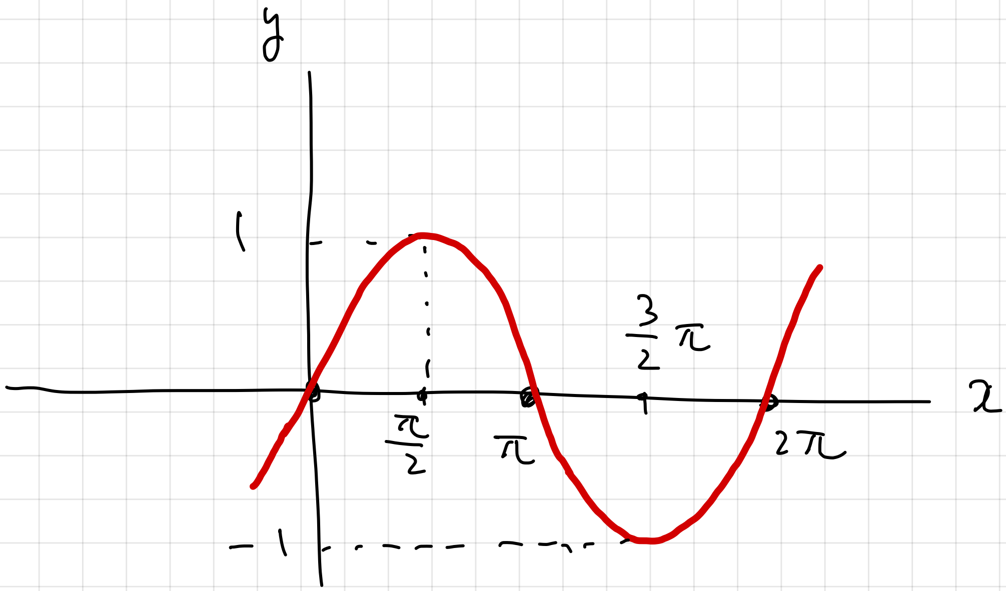
$y = 2x + 1$



$y = x^2 - 1$

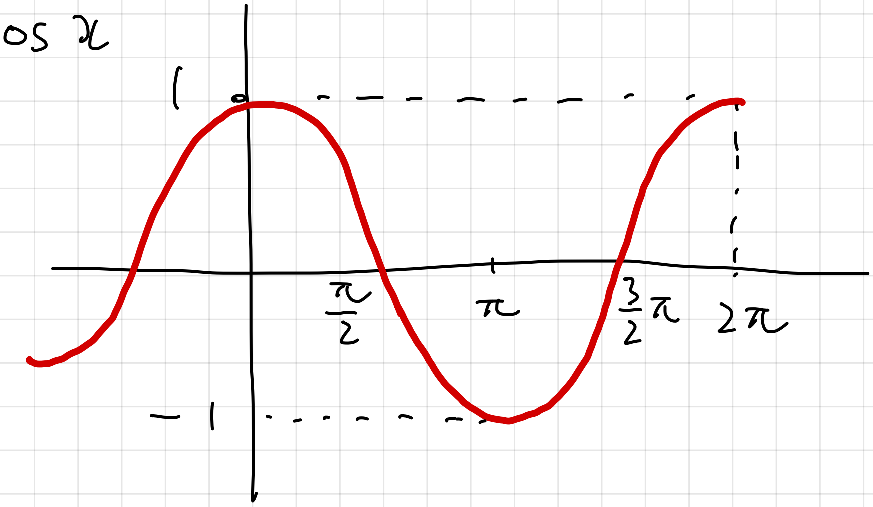


$y = \sin x$



$\pi = 3.14\cdots$

$y = \cos x$



$y = e^x$

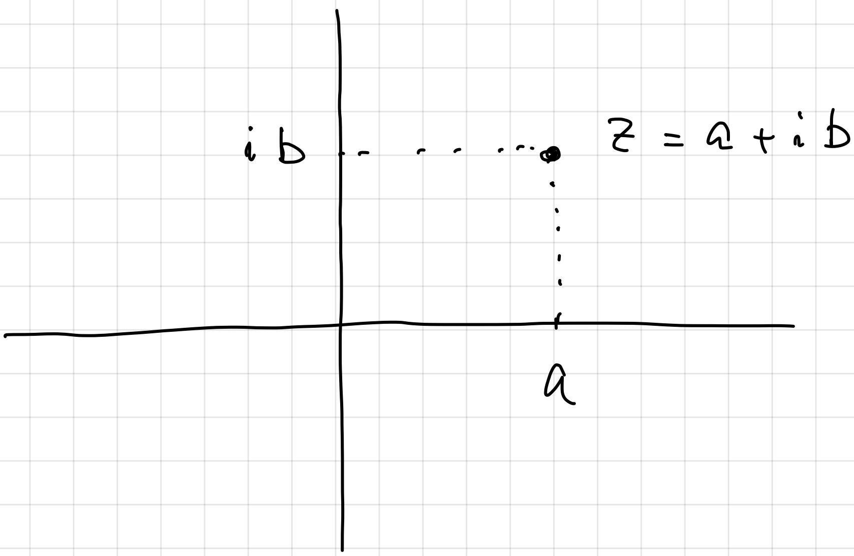
エクスポネンシャル

$e = 2.7\cdots$

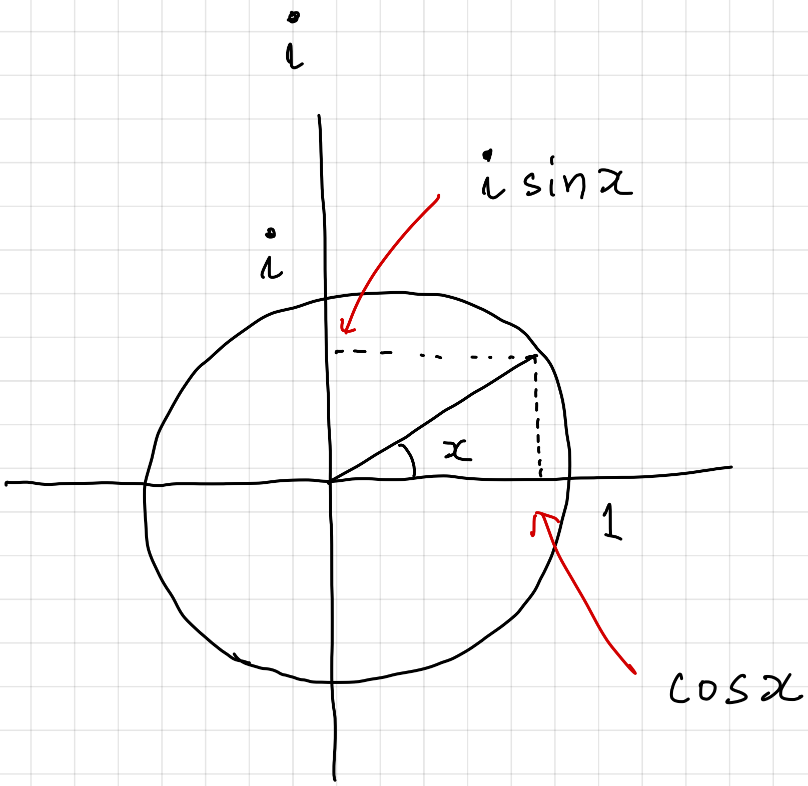
複素数

$i^2 = -1$

$z = a + ib \qquad a, b \in \mathbb{R}$



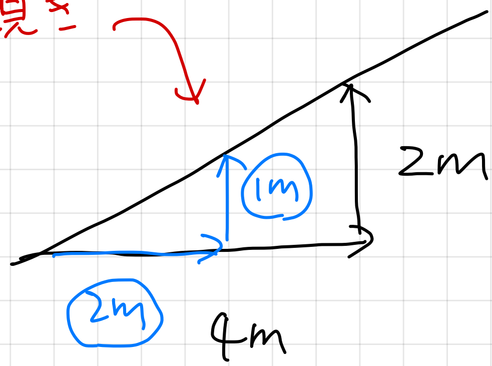
$$e^{ix} = \cos x + i \sin x \qquad x \in \mathbb{R}$$



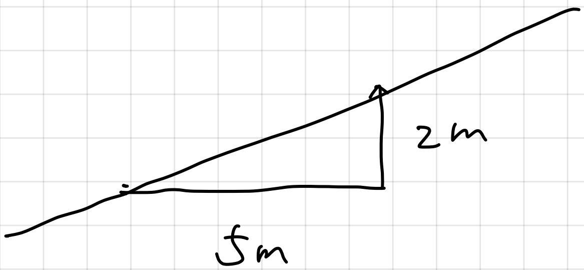
傾き

$$傾き = \frac{y方向の変化}{x方向の変化} = \frac{\overset{デルタ}{\Delta} y}{\Delta x}$$

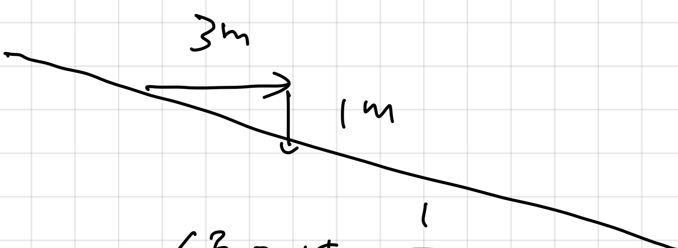
傾き



$$\frac{2}{4} = \frac{1}{2}$$

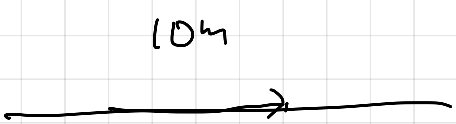


傾きは $\frac{2}{5}$

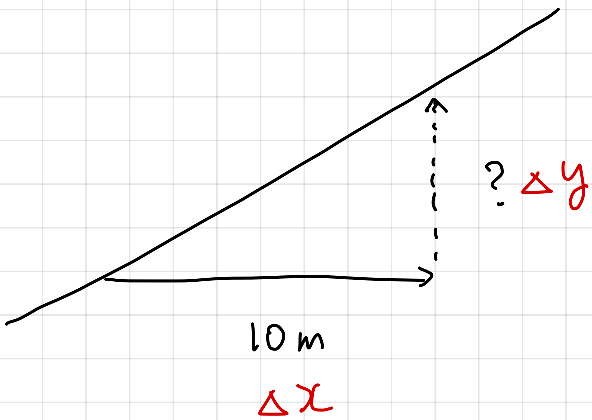


傾きは $-\frac{1}{3}$

下がっている



傾きは 0



傾き a

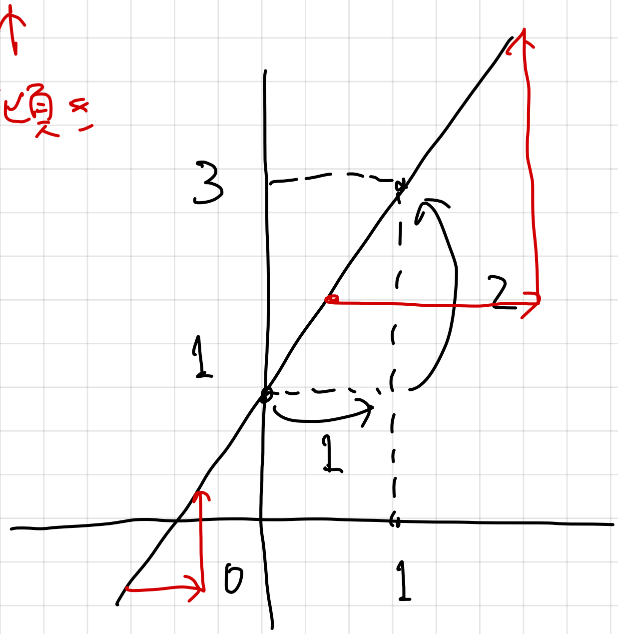
$$a = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{10}$$

$$\Delta y = 10a$$

$$\Delta y = 傾き \times \Delta x$$

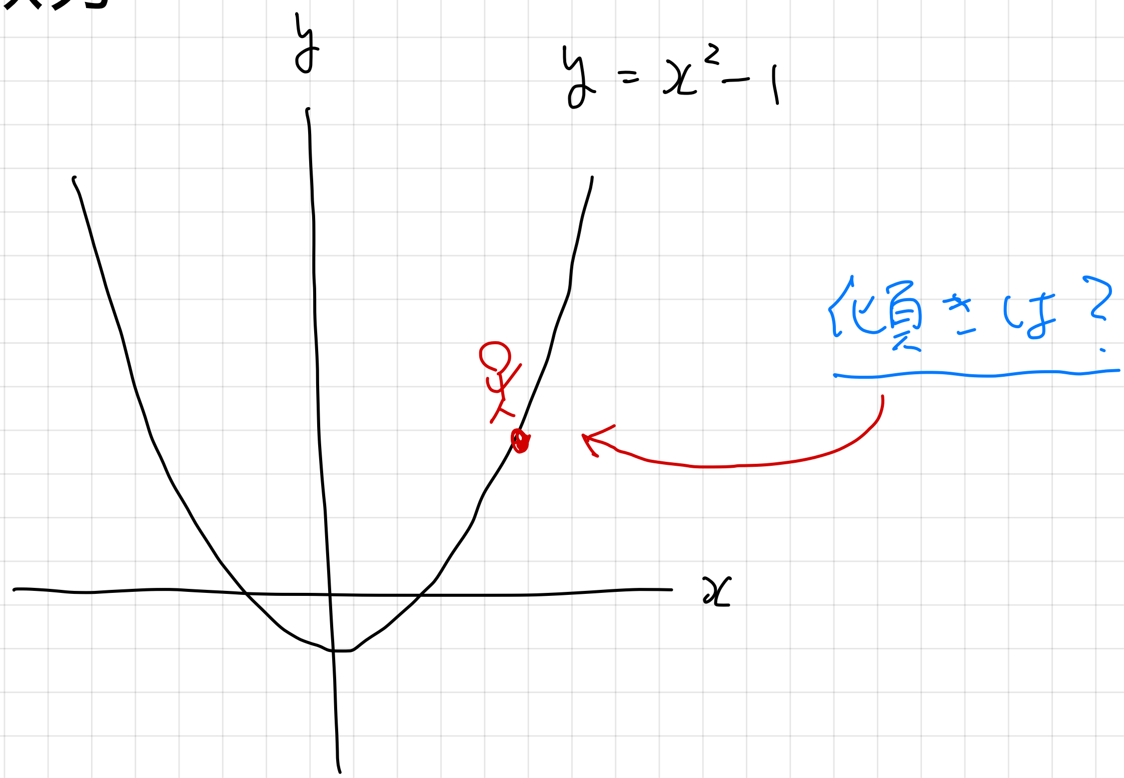
$y = 2x + 1$

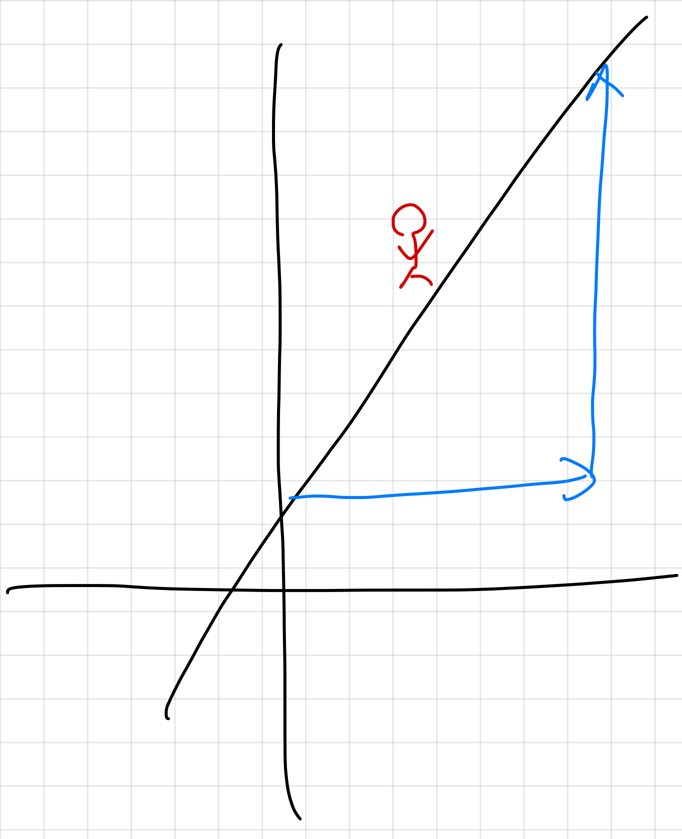
傾き

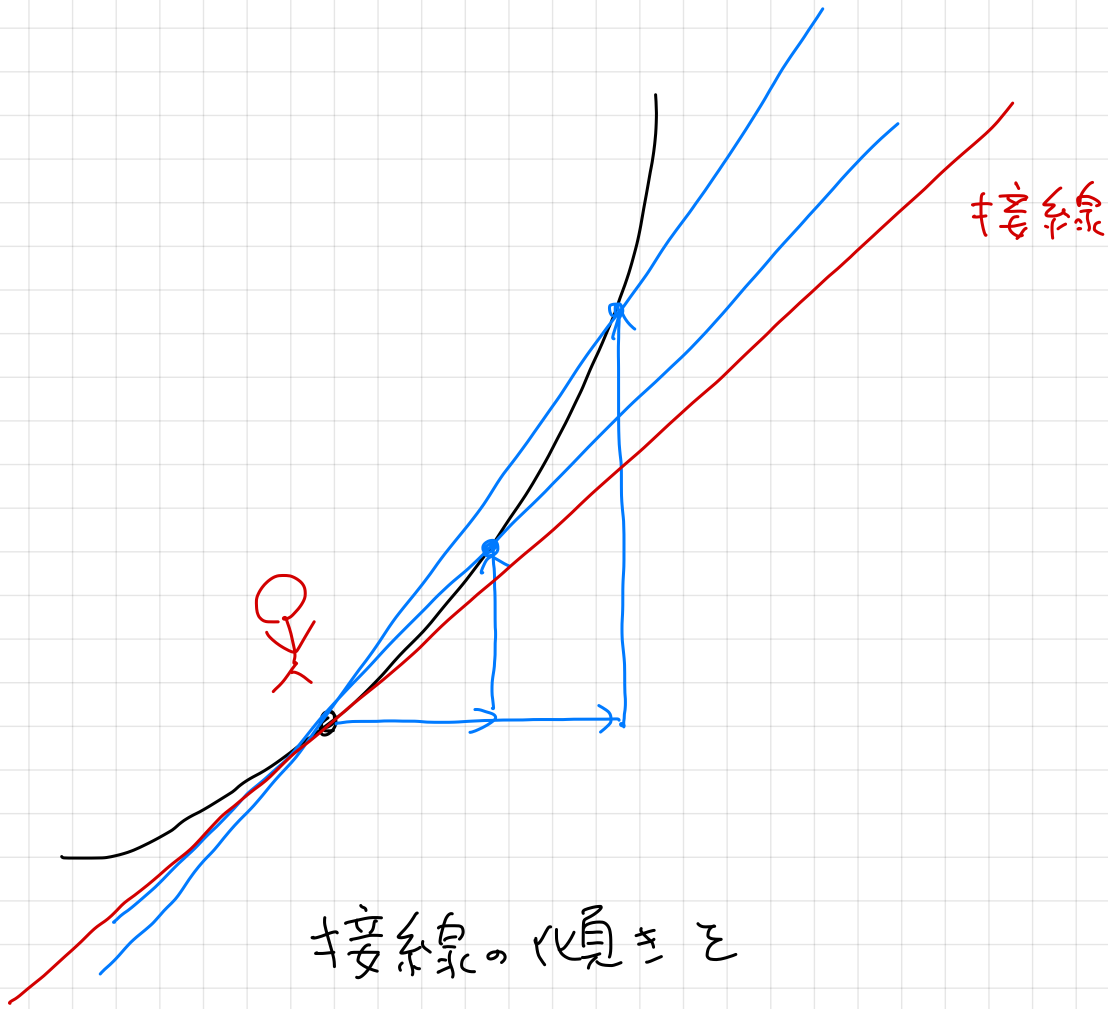


傾きは $\frac{2}{1} = 2$

微分





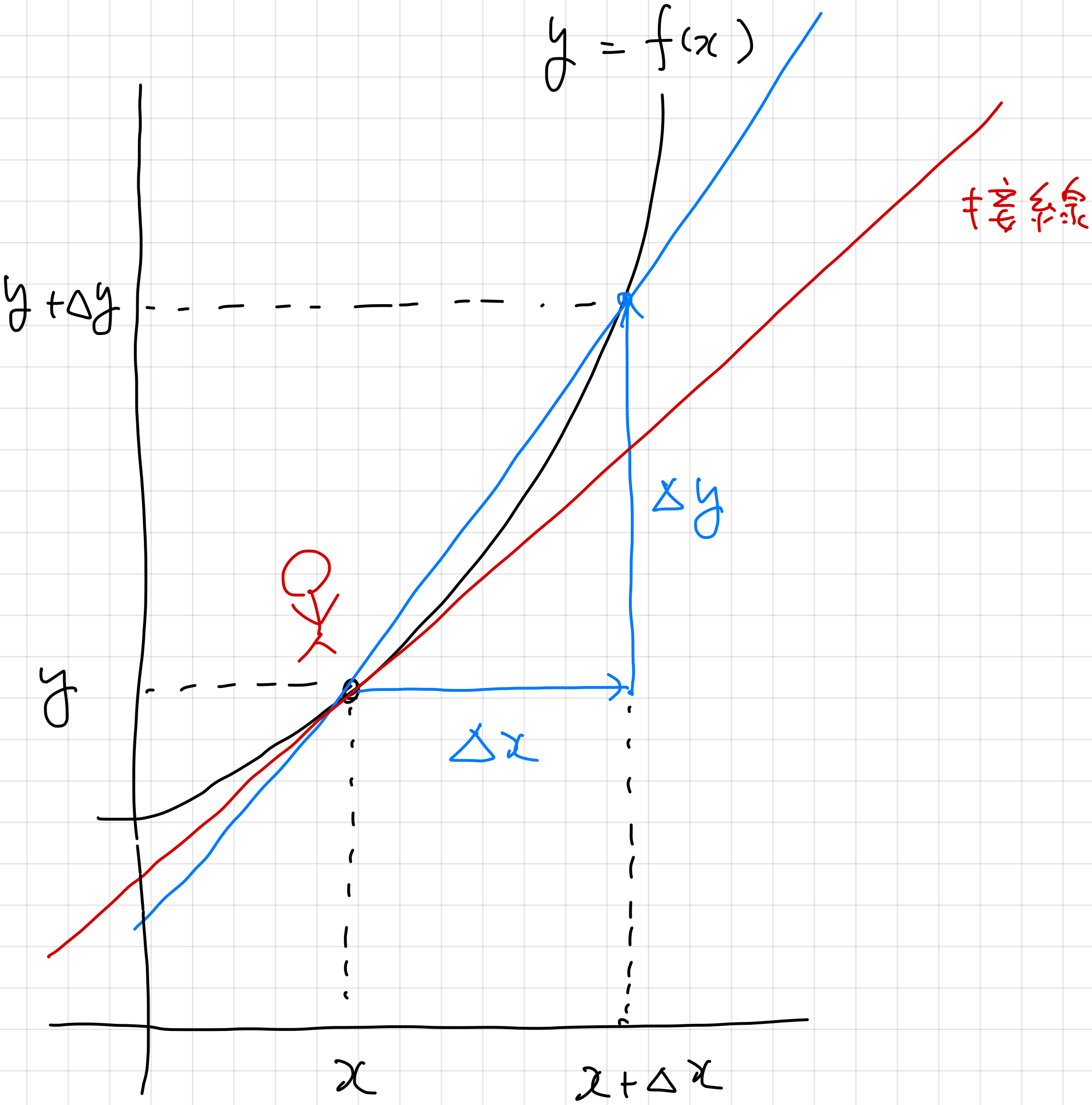


接線の傾きを

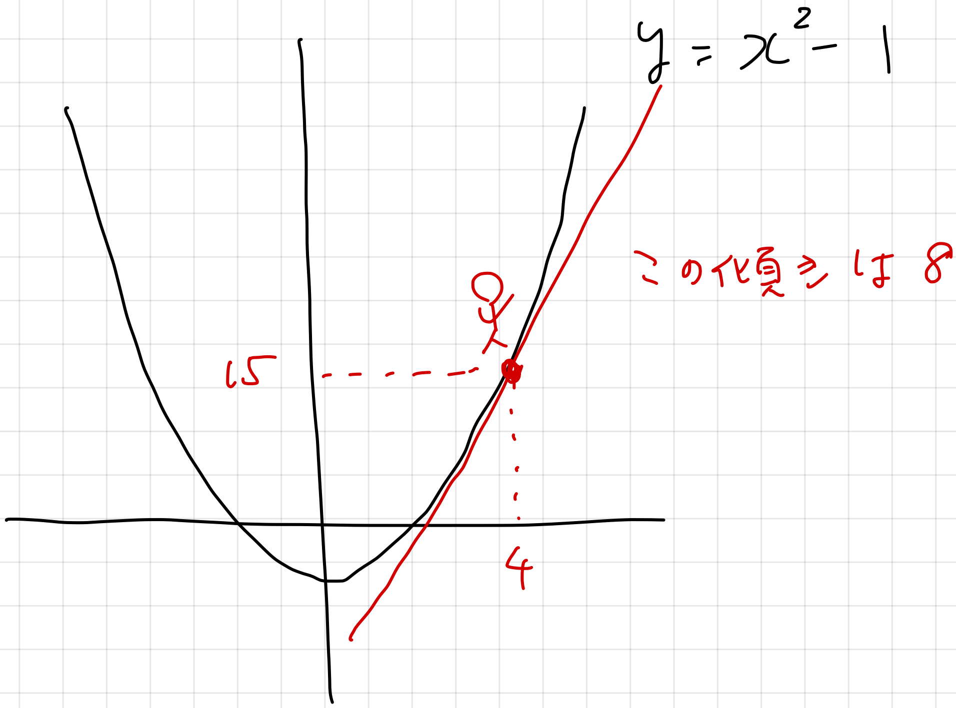
この点での傾きと考える.

⇩

接線の傾きは微分で出せる.



$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$



$y = x^2 - 1$ の微分

$y' = 2x$

$x = 4$ を代入.　　$y' = 2 \times 4 = 8$

微分の公式

$$\boxed{(ax^n)' = nax^{n-1}}$$

ただし x は x^1, 1 は x^0 と考える.

$x' = 1$ 　　 $1' = (x^0)' = 0$

$$y = 2x^3 - 5x^2 + 2x - 3$$

$$y' = 3 \times 2x^{3-1} - 2 \times 5x^{2-1} + 1 \times 2x^{1-1} - 0$$

$$= 6x^2 - 10x + 2$$

$$y = x^5 - 4x^4 + x^3 + 7x^2 + 5x + 1$$

$$y' = 5x^4 - 4 \times 4x^3 + 3x^2 + 2 \times 7x + 5$$

$$= 5x^4 - 16x^3 + 3x^2 + 14x + 5$$

$$\boxed{\begin{aligned}(\sin x)' &= \cos x \\ (\cos x)' &= -\sin x\end{aligned}}$$

$$\boxed{\begin{aligned} y &= e^x \\ y' &= e^x \end{aligned}}$$

積分

微分の逆 〜 不定積分

$y' = 3x^2$

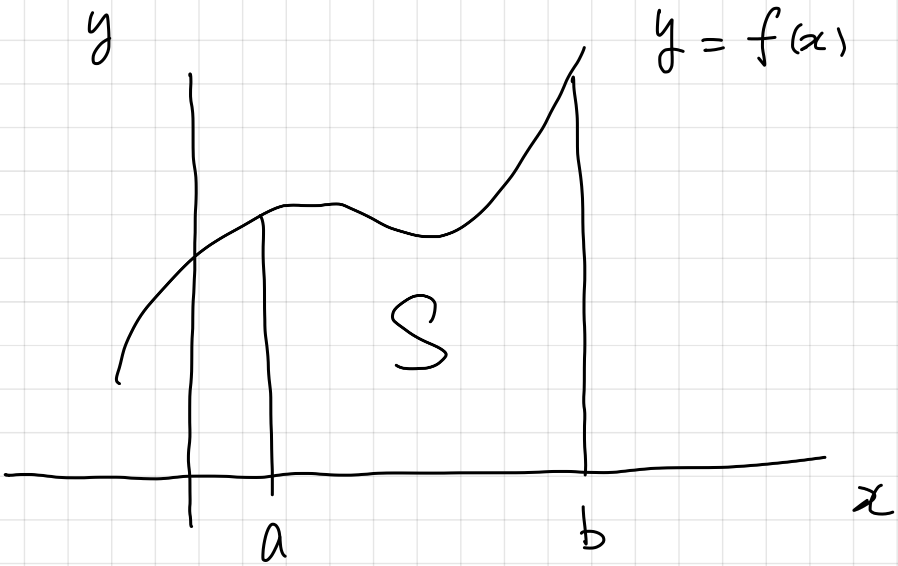
$y = ?$

$y = x^3 + C$

積分定数

$3x^2$の原始関数
$3x^2$の不定積分

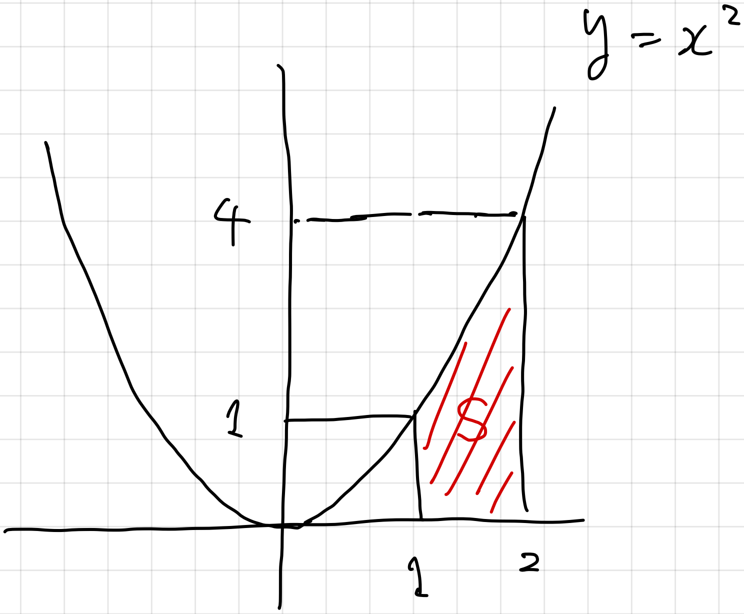
面積や体積を出す 〜 定積分



$$S = \int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

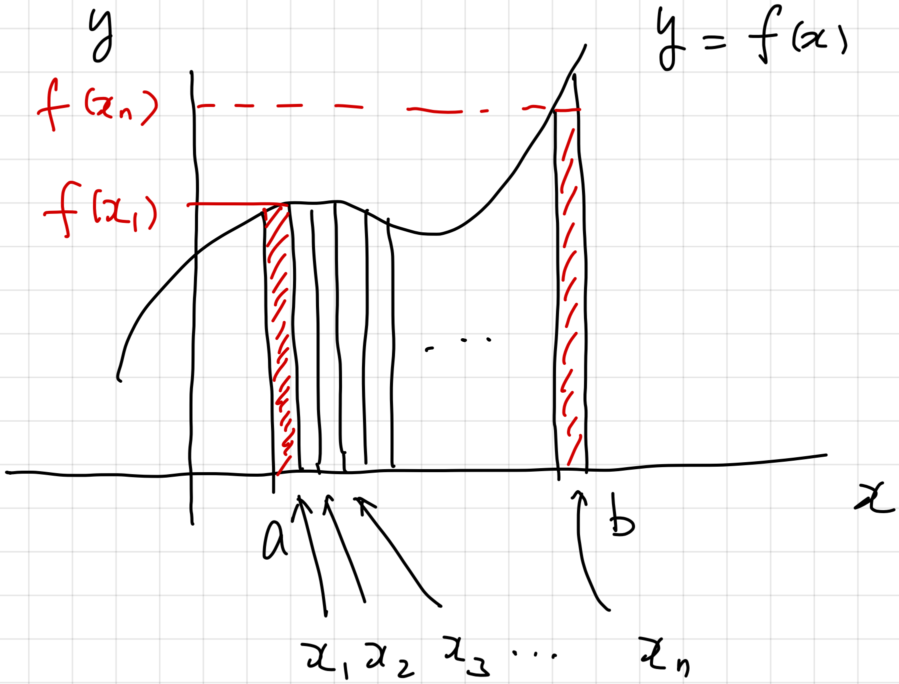
原始関数

例



$$S=\int_1^2 x^2\,dx=\left[\frac{1}{3}x^3\right]_1^2$$

$$=\frac{1}{3}\cdot 2^3-\frac{1}{3}\cdot 1^3 \quad = \frac{7}{3}$$



デルタ

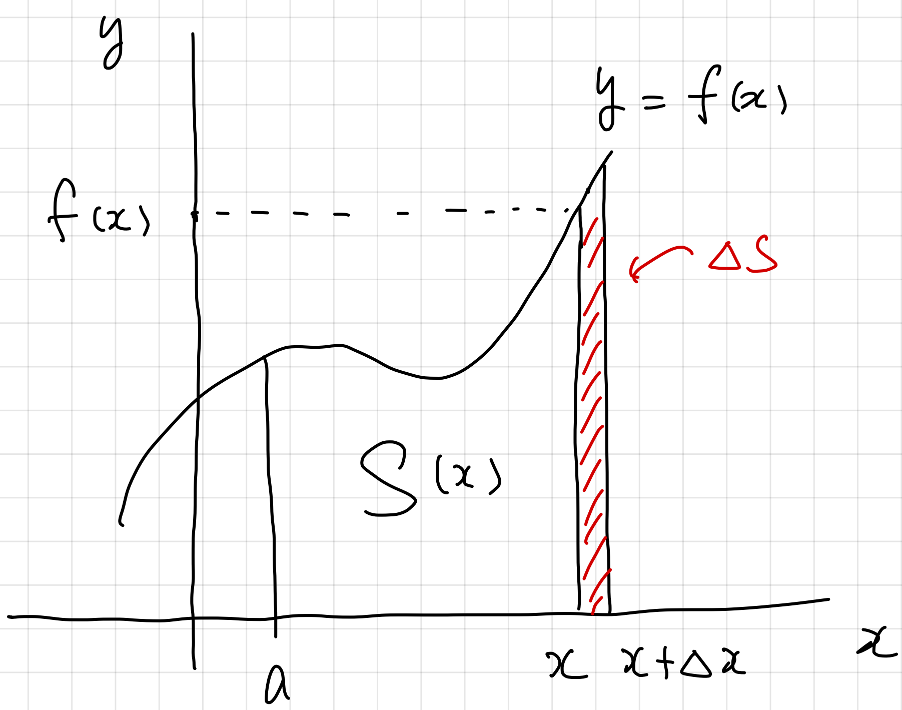
$$S \fallingdotseq f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$$

$$= \sum_{i=1}^{n} f(x_i)\Delta x \rightsquigarrow \int_a^b f(x)\,dx$$

シグマ

$\Delta x \to 0$, $n \to \infty$ のとき 正しい S

タタくの場合



$$S(x) = \int_a^x f(x)\,dx$$

$$S'(x) = \lim_{\Delta x \to 0} \frac{\Delta S}{\Delta x} = f(x)$$

$S(x)$ は $f(x)$ の原始関数 かつ、 $S(a) = 0$.

⤳ $x = a$ で 0 になる $f(x)$ の原始関数を
見つければ それが $S(x)$

$$S(x) = \int_a^x f(x)\,dx = \Big[F(x)\Big]_a^x = F(x) - F(a)$$

↑

原始関数(なんでもよい)