Supplementary Material of

"Linguistic granular computing: Interval-based granulation and probability-sampling-based degranulation"

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I. Introduction

This supplementary material provides (1) collected linguistic expressions used for analysis (Section II); (2) consistency calculation results under different power-law indexes (Section III); and (3) intermediate calculation results (Section IV).

II. Linguistic expressions

Using the linguistic term set $L = \{l_1$: "Much low", l_2: "Low", l_3: "Slightly low", l_4: "Medium", l_5: "Slightly high", l_6: "High", l_7: "Much high"$\}$, a customer on an e-commerce platform posted evaluations for some of products with the following linguistic expressions.

(1) F_1

"Overall, the quality of *Tesla Model S* is **higher** than that of *Renault Zoe*, and **much higher** than that of *Geely Emgrand* and *Nissan Leaf*; the quality of *Geely Emgrand* is **slightly higher** than that of *Nissan Leaf*, and **slightly lower** than that of *Renault Zoe*; the quality of *Nissan Leaf* is **much lower** than that of *Renault Zoe*."

Given $X = \{x_1 = \textit{Tesla Model S}, x_2 = \textit{Geely Emgrand}, x_3 = \textit{Nissan Leaf}, x_4 = \textit{Renault Zoe}\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_1 = \begin{pmatrix} l_4 & l_7 & l_7 & l_6 \\ Neg(l_7) & l_4 & l_5 & l_3 \\ Neg(l_7) & Neg(l_5) & l_4 & l_1 \\ Neg(l_6) & Neg(l_3) & Neg(l_1) & l_4 \end{pmatrix}$$

(2) F_2

"The design quality of *iPhone 14* is **significantly higher** than that of *iPhone 13*, and **higher** than that of *iPhone 12*; the design quality of *iPhone 13* is **slightly higher** than that of *iPhone 12*."

Given $X = \{x_1 = \textit{iPhone 14}, x_2 = \textit{iPhone 13}, x_3 = \textit{iPhone 12}\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_2 = \begin{pmatrix} l_4 & l_7 & l_6 \\ Neg(l_7) & l_4 & l_5 \\ Neg(l_6) & Neg(l_5) & l_4 \end{pmatrix}$$

(3) F_3

"The price-performance ratio of *APPLE* is **higher** than that of *OPPO*, **slightly higher** than that of *HUWEI* and *MI*, and **quite higher** than that of *SAMSUNG*; the price-performance ratio of *OPPO* is **lower** than that of *HUWEI*, **slightly lower** than that of *MI*, and **slightly higher** than that of *SAMSUNG*; the price-performance ration of *HUWEI* is **slightly higher** than that of *MI*, and **higher** than that of *SAMSUNG*; the price-performance ratio of *MI* is **higher** than that of *SAMSUNG*."

Given $X = \{x_1 = APPLE, x_2 = OPPO, x_3 = HUWEI, x_4 = MI, x_5 = SAMSUNG\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_3 = \begin{pmatrix} l_4 & l_6 & l_5 & l_5 & l_7 \\ Neg(l_6) & l_4 & l_2 & l_3 & l_5 \\ Neg(l_5) & Neg(l_2) & l_4 & l_5 & l_6 \\ Neg(l_5) & Neg(l_3) & Neg(l_5) & l_4 & l_6 \\ Neg(l_7) & Neg(l_5) & Neg(l_6) & Neg(l_6) & l_4 \end{pmatrix}$$

(4) F_4

"The quality of *EsteeLauder* is **slightly higher** than that of *L'Oreal*, **higher** than that of *LANCOME*, and **quite higher** than that of *OLAY* and *SK-II*; the quality of *L'Oreal* is **slightly higher** than that of *LANCOME*, **higher** than that of *OLAY*, and **quite higher** than that of *SK-II*; the quality of *LANCOME* is **slightly higher** than that of *OLAY*, and **higher** than that of *SK-II*; the quality of *OLAY* is **slightly higher** than that of *SK-II*."

Given $X = \{x_1 = EsteeLauder, x_2 = L'Oreal, x_3 = LANCOME, x_4 = OLAY, x_5 = SK\text{-}II\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_4 = \begin{pmatrix} l_4 & l_5 & l_6 & l_7 & l_7 \\ Neg(l_5) & l_4 & l_5 & l_6 & l_7 \\ Neg(l_6) & Neg(l_5) & l_4 & l_5 & l_6 \\ Neg(l_7) & Neg(l_6) & Neg(l_5) & l_4 & l_5 \\ Neg(l_7) & Neg(l_7) & Neg(l_6) & Neg(l_5) & l_4 \end{pmatrix}$$

(5) F_5

"The quality of *ANTA* is **equal to** that of *LINING*, **slightly lower** than that of *NewBalance*, and **much lower** than that of *Adadas* and *Nike*; the quality of *LINING* is **slightly lower** than that of *NewBalance*, **lower** than that of *Adadas*, and **quite lower** than that of *Nike*; the quality of *NewBalance* is **slightly lower** than that of *Adadas*, and **lower** than that of *Nike*; the quality of *Adadas* is **slightly higher** than that of *Nike*."

Given $X = \{x_1 = ANTA, x_2 = LINING, x_3 = NewBalance, x_4 = Adadas, x_5 = Nike\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_5 = \begin{pmatrix} l_4 & l_4 & l_3 & l_1 & l_1 \\ Neg(l_4) & l_4 & l_3 & l_2 & l_1 \\ Neg(l_3) & Neg(l_3) & l_4 & l_3 & l_2 \\ Neg(l_1) & Neg(l_2) & Neg(l_3) & l_4 & l_3 \\ Neg(l_1) & Neg(l_1) & Neg(l_2) & Neg(l_3) & l_4 \end{pmatrix}$$

(6) F_6

"The quality of *Brioni* is **equal to** that of *ARMANI*, **slightly higher** than that of *ERMENEGILDO ZEGNA* and *Ferragamo*, **higher** than that of *CANALI*, and **much higher** than that of *PRADA*; the quality of *ARMANI* is **equal to** than that of *ERMENEGILDO ZEGNA*, **slightly higher** than that of *Ferragamo*, and **quite higher** than that of *CANALI* and *PRADA*; the quality of *ERMENEGILDO ZEGNA* is **slightly higher** than that of *Ferragamo*, **higher** than that of *CANALI*, and **quite higher** than that of *PRADA*; the quality of *Ferragamo* is **slightly higher** than that of *CANALI*, and **higher** than that of *PRADA*; the quality of *CANALI* is **slightly higher** than that of *PRADA*."

Given $X = \{x_1 = \text{Brioni}, x_2 = ARMANI, x_3 = ERMENEGILDO\ ZEGNA, x_4 = \text{Ferragamo}, x_5 = CANALI, x_6 = PRADA\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_6 = \begin{pmatrix} l_4 & l_4 & l_5 & l_5 & l_6 & l_7 \\ Neg(l_4) & l_4 & l_4 & l_5 & l_7 & l_7 \\ Neg(l_5) & Neg(l_4) & l_4 & l_5 & l_6 & l_7 \\ Neg(l_5) & Neg(l_5) & Neg(l_5) & l_4 & l_5 & l_6 \\ Neg(l_6) & Neg(l_7) & Neg(l_6) & Neg(l_5) & l_4 & l_5 \\ Neg(l_7) & Neg(l_7) & Neg(l_7) & Neg(l_6) & Neg(l_5) & l_4 \end{pmatrix}$$

(7) F_7

"The quality of *SYMINGTON* is **significantly lower** than that of *CATENA*, and **lower** than that of *CONCHA Y TORO*; the quality of *CATENA* is **slightly lower** than that of *CONCHA Y TORO*."

Given $X = \{x_1 = SYMINGTON, x_2 = CATENA, x_3 = CONCHA\ Y\ TORO\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_7 = \begin{pmatrix} l_4 & l_1 & l_2 \\ Neg(l_1) & l_4 & l_3 \\ Neg(l_2) & Neg(l_3) & l_4 \end{pmatrix}$$

(8) F_8

"The brand value of *Red Bull* is **slightly lower** than that of *Pepsi*, **slightly higher** than that of *Nescafe*, and **much lower** than that of *Coca-Cola*; the brand value of *Pepsi* is **higher** than that of *Nescafe*, and **slightly lower** than that of *Coca-Cola*; the brand value of *Nescafe* is **much lower** than that of *Coca-Cola*."

Given $X = \{x_1 = Red\ Bull, x_2 = Pepsi, x_3 = Nescafe, x_4 = Coca\text{-}Cola\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_8 = \begin{pmatrix} l_4 & l_3 & l_5 & l_1 \\ Neg(l_3) & l_4 & l_6 & l_3 \\ Neg(l_5) & Neg(l_6) & l_4 & l_1 \\ Neg(l_1) & Neg(l_3) & Neg(l_1) & l_4 \end{pmatrix}$$

(9) F_9

"The number of *Lenovo* sold is **higher** than that of *Apple* and *ThinkPad*, **slightly higher** than that of *Dell*, and **much higher** than that of *ASUS* and *HP*; the number of *Apple* sold is **lower** than that of *Dell*, **slightly lower** than that of *ThinkPad*, **slightly higher** than that of *ASUS*, and **higher** than that of *HP*; the number of *Dell* sold is **slightly higher** than that of *ThinkPad*, **higher** than that of *ASUS*, and **much higher** than that of *HP*; the number of *ThinkPad* sold is **higher** than that of *ASUS*, and **much higher** than that of *HP*; the number of *ASUS* sold is **slightly higher** than that of *HP*."

Given $X = \{x_1 = Lenovo, x_2 = Apple, x_3 = Dell, x_4 = ThinkPad, x_5 = ASUS, x_6 = HP\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_9 = \begin{pmatrix} l_4 & l_6 & l_5 & l_6 & l_7 & l_7 \\ Neg(l_6) & l_4 & l_2 & l_3 & l_5 & l_6 \\ Neg(l_5) & Neg(l_2) & l_4 & l_5 & l_6 & l_7 \\ Neg(l_6) & Neg(l_3) & Neg(l_5) & l_4 & l_6 & l_7 \\ Neg(l_7) & Neg(l_5) & Neg(l_6) & Neg(l_6) & l_4 & l_5 \\ Neg(l_7) & Neg(l_6) & Neg(l_7) & Neg(l_7) & Neg(l_5) & l_4 \end{pmatrix}$$

(10) F_{10}

"The quality of *Mengniu* is **slightly lower** that of *Yili*, **lower** than that of *Jindian*, **slightly higher** than that of *Telunsu*, **much higher** than that of *Guangming*, **higher** than that of *Sanyuan*, and **much lower** than that of *Yantang*; the quality of *Yili* is **slightly lower** than that of *Jindian*, **slightly higher** than that of *Telunsu*, **much higher** than that of *Guangming* and *Sanyuan*, and **lower** that of *Yantang*; the quality of *Jindian* is **equal to** that of *Telunsu*, **much higher** than that of *Guangming* and *Sanyuan*, and **slightly lower** that of *Yantang*; the quality of *Telunsu* is **higher** than that of *Guangming*, **slightly higher** than that of *Sanyuan*, and **much lower** than that of *Yantang*; the quality of *Guangming* is **slightly lower** than that of *Sanyuan*, and **much lower** than that of *Yantang*; the quality of *Sanyuan* is **much lower** than that of *Yantang*."

Given $X = \{x_1 = Mengniu, x_2 = Yili, x_3 = Jindian, x_4 = Telunsu, x_5 = Guangming, x_6 = Sanyuan, x_7 = Yantang\}$, the following linguistic preference relation that is equivalent to the linguistic expression can be obtained.

$$F_{10} = \begin{pmatrix} l_4 & l_3 & l_2 & l_5 & l_7 & l_6 & l_1 \\ Neg(l_3) & l_4 & l_3 & l_5 & l_7 & l_7 & l_2 \\ Neg(l_2) & Neg(l_3) & l_4 & l_4 & l_7 & l_7 & l_3 \\ Neg(l_5) & Neg(l_5) & Neg(l_4) & l_4 & l_6 & l_5 & l_1 \\ Neg(l_7) & Neg(l_7) & Neg(l_7) & Neg(l_6) & l_4 & l_3 & l_1 \\ Neg(l_6) & Neg(l_7) & Neg(l_7) & Neg(l_5) & Neg(l_3) & l_4 & l_1 \\ Neg(l_1) & Neg(l_2) & Neg(l_3) & Neg(l_1) & Neg(l_1) & Neg(l_1) & l_4 \end{pmatrix}$$

III. Consistency calculation results under different power-law indexes

See Figs. S1-S10.

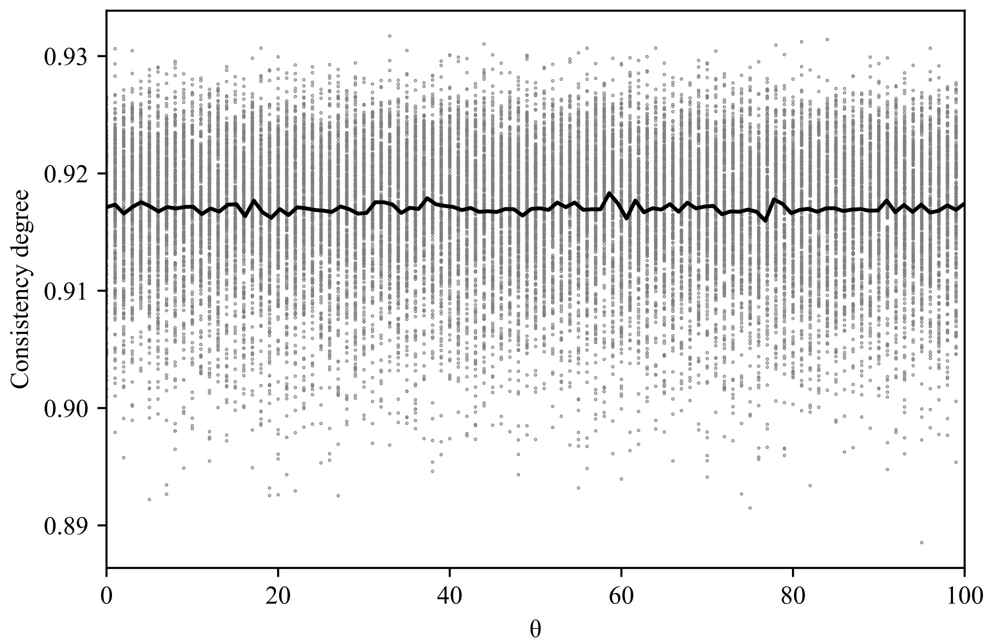

Fig. S1. Consistency calculation results of F_1 under different power-law indexes

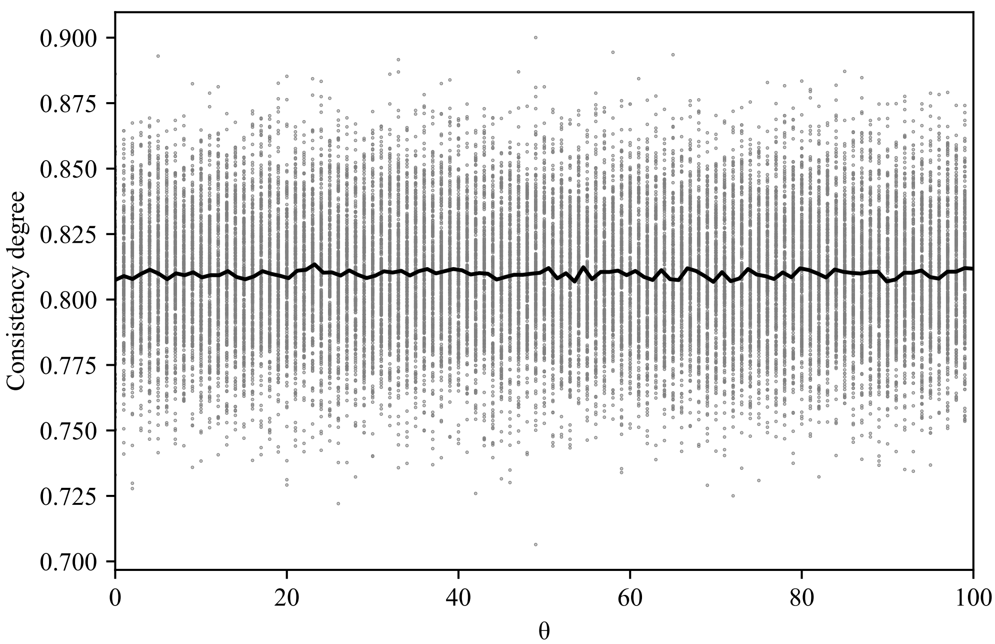

Fig. S2. Consistency calculation results of F_2 under different power-law indexes

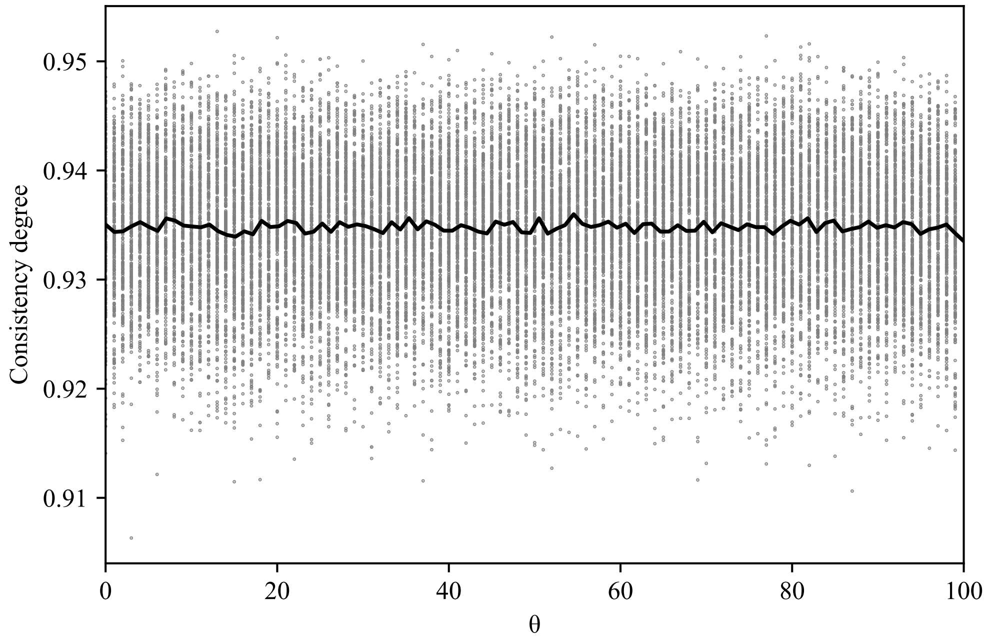

Fig. S3. Consistency calculation results of F_3 under different power-law indexes

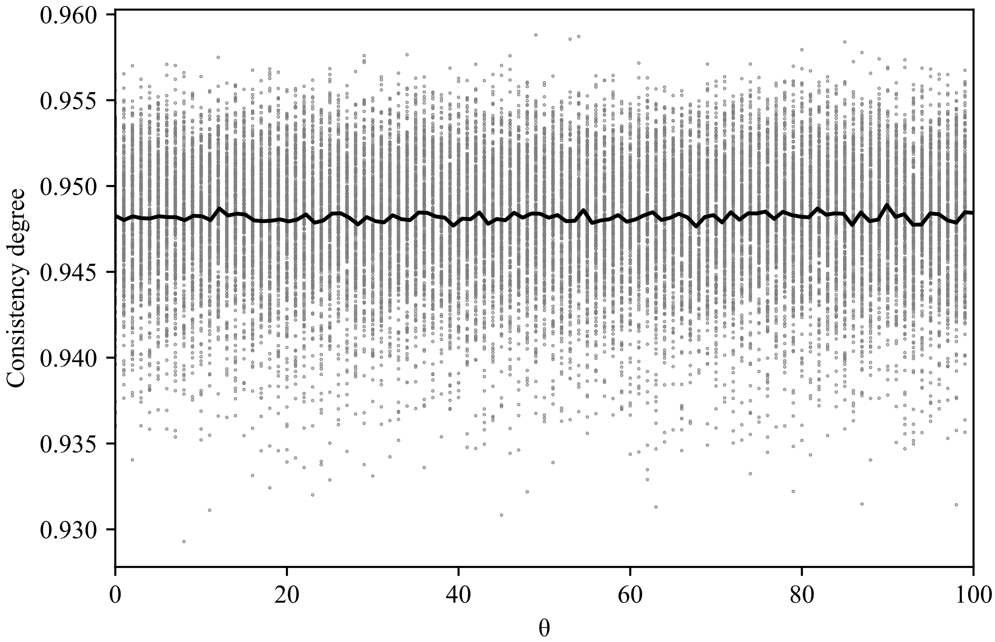

Fig. S4. Consistency calculation results of F_4 under different power-law indexes

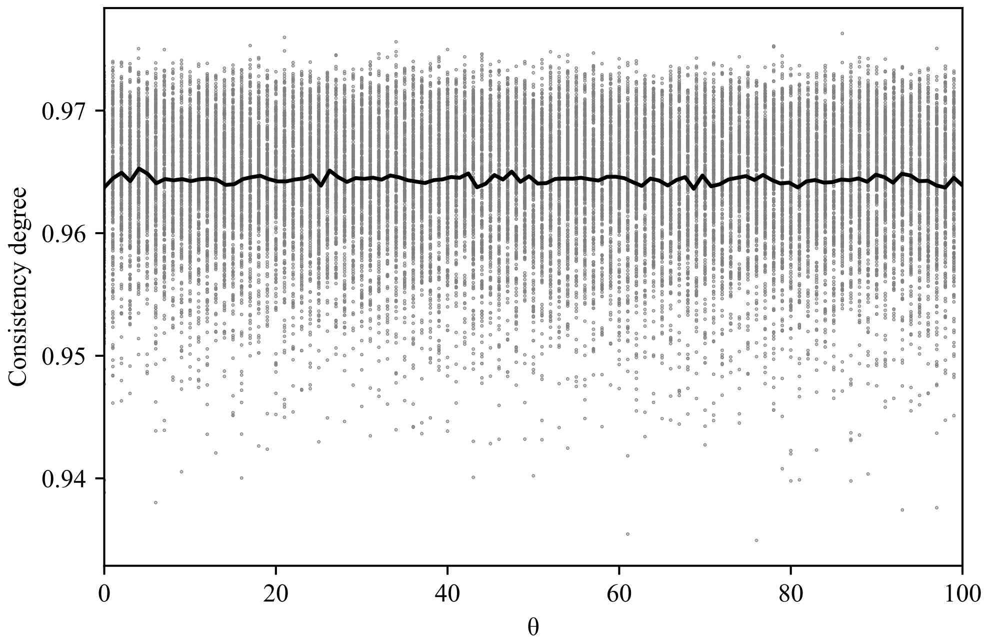

Fig. S5. Consistency calculation results of F_5 under different power-law indexes

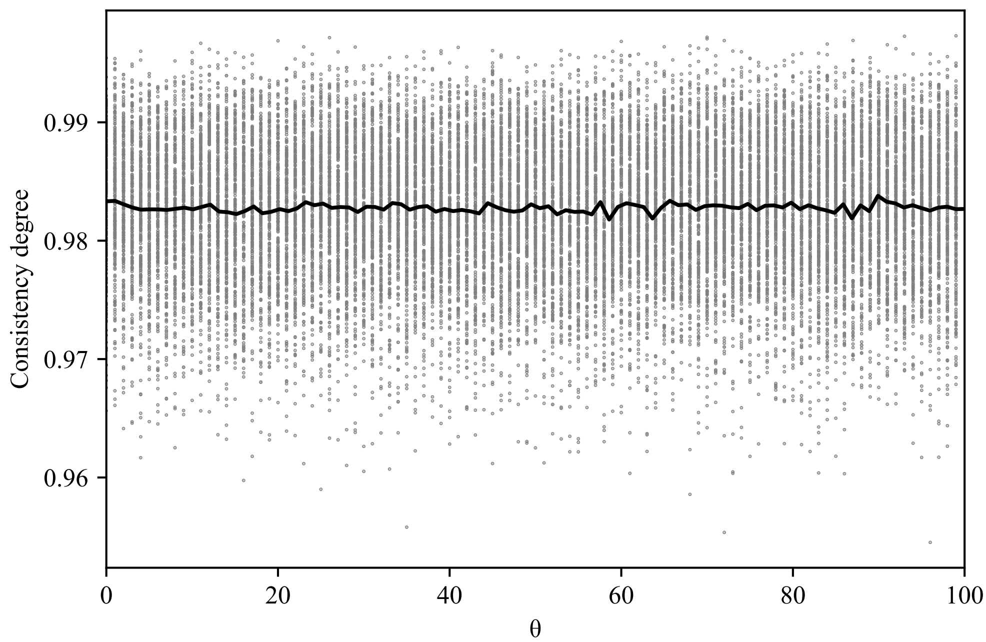

Fig. S6. Consistency calculation results of F_6 under different power-law indexes

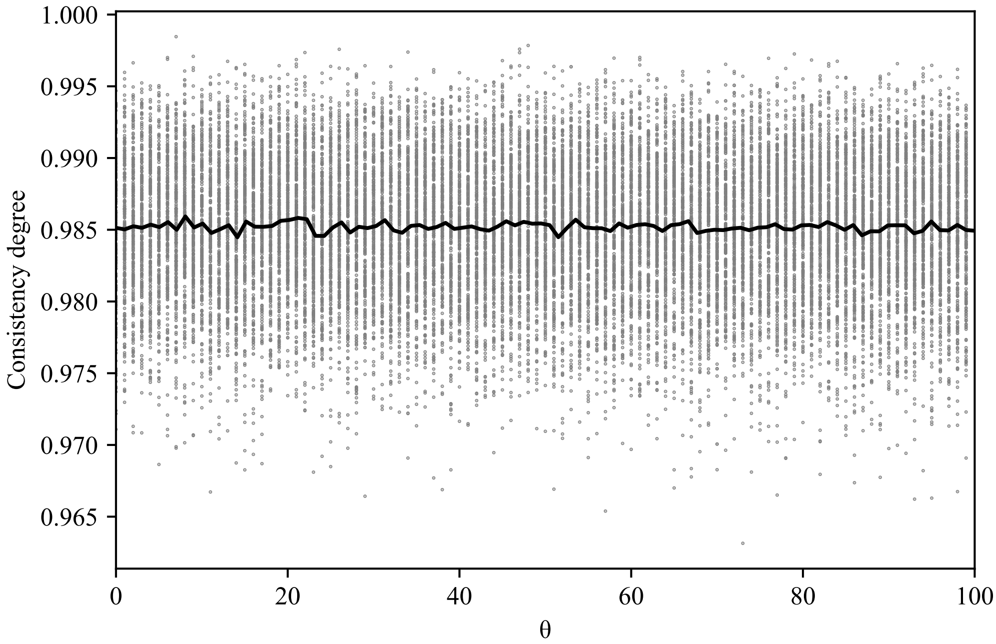

Fig. S7. Consistency calculation results of F_7 under different power-law indexes

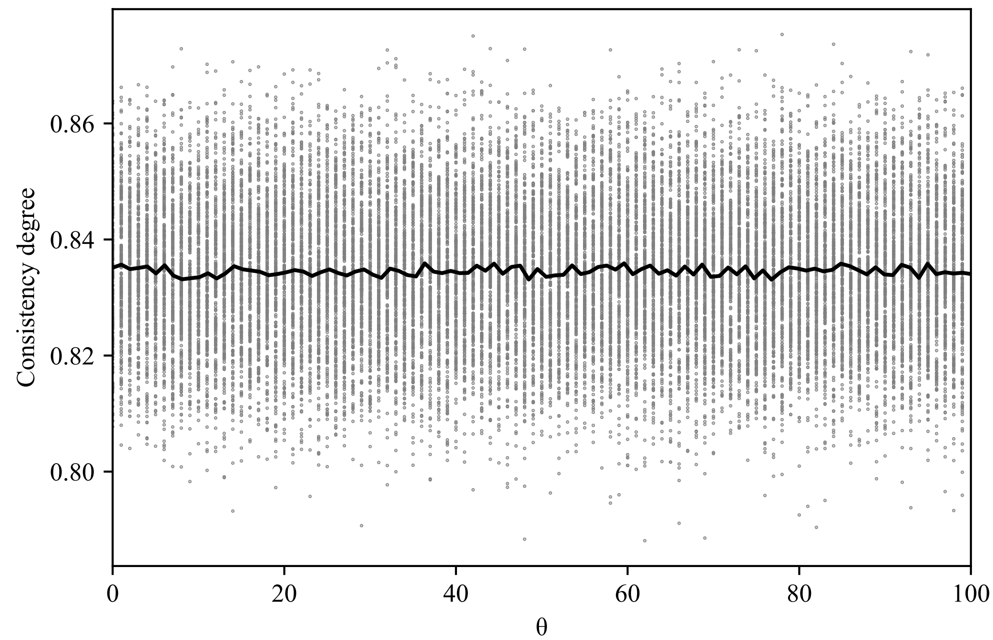

Fig. S8. Consistency calculation results of F_8 under different power-law indexes

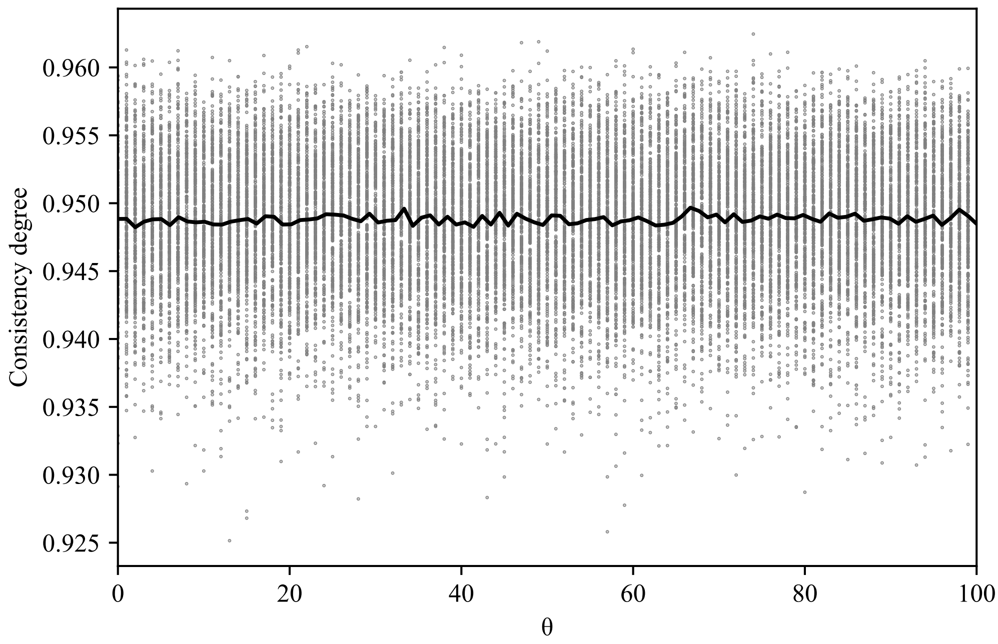

Fig. S9. Consistency calculation results of F_9 under different power-law indexes

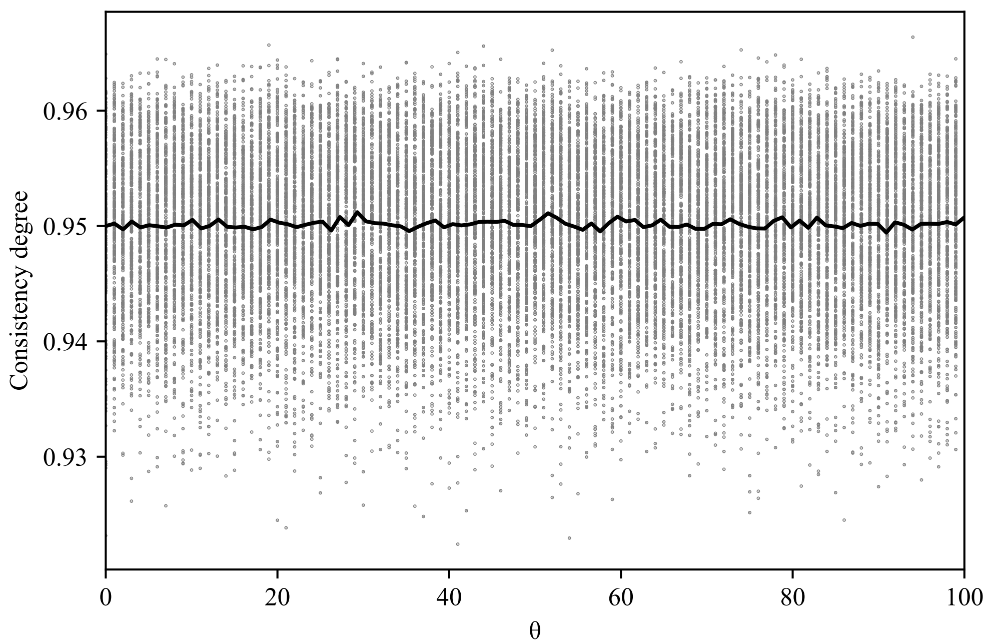

Fig. S10. Consistency calculation results of F_{10} under different power-law indexes

IV. Intermediate calculation results

This subsection presents the intermediate calculation results, that is, the obtained numerical preference relations, the comprehensive scores, and the final ranking results.

(1) F_1

$$\tilde{F}_1 = \begin{pmatrix} 0.5052 & 0.8616 & 0.8616 & 0.6963 \\ 0.1369 & 0.5052 & 0.6084 & 0.3744 \\ 0.1369 & 0.3744 & 0.5052 & 0.1369 \\ 0.2858 & 0.6084 & 0.8616 & 0.5052 \end{pmatrix}$$

$score(x_1) = 0.8065$, $score(x_2) = 0.3732$, $score(x_3) = 0.2161$, $score(x_4) = 0.5853$, and thus $x_1 \succ x_4 \succ x_2 \succ x_3$.

(2) F_2

$$\tilde{F}_2 = \begin{pmatrix} 0.5025 & 0.8666 & 0.6955 \\ 0.1326 & 0.5025 & 0.6087 \\ 0.2858 & 0.3736 & 0.5025 \end{pmatrix}$$

$score(x_1) = 0.7811$, $score(x_2) = 0.3706$, $score(x_3) = 0.3298$, and thus $x_1 \succ x_2 \succ x_3$.

(3) F_3

$$\tilde{F}_3 = \begin{pmatrix} 0.5016 & 0.6969 & 0.6108 & 0.6108 & 0.8642 \\ 0.2860 & 0.5016 & 0.2860 & 0.3726 & 0.6108 \\ 0.3726 & 0.6969 & 0.5016 & 0.6108 & 0.6969 \\ 0.3726 & 0.6108 & 0.3726 & 0.5016 & 0.6969 \\ 0.1397 & 0.6108 & 0.2860 & 0.2860 & 0.5016 \end{pmatrix}$$

$score(x_1) = 0.6957$, $score(x_2) = 0.3888$, $score(x_3) = 0.5843$, $score(x_4) = 0.5132$, $score(x_5) = 0.3306$, and thus $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5$.

(4) F_4

$$\tilde{F}_4 = \begin{pmatrix} 0.5021 & 0.6095 & 0.6971 & 0.8590 & 0.8590 \\ 0.3738 & 0.5021 & 0.6095 & 0.6971 & 0.8590 \\ 0.2857 & 0.3738 & 0.5021 & 0.6095 & 0.6971 \\ 0.1435 & 0.2857 & 0.3738 & 0.5021 & 0.6095 \\ 0.1435 & 0.1435 & 0.2857 & 0.3738 & 0.5021 \end{pmatrix}$$

$score(x_1) = 0.7561$, $score(x_2) = 0.6348$, $score(x_3) = 0.4915$, $score(x_4) = 0.3531$, $score(x_5) = 0.2366$, and thus $x_1 \succ x_2 \succ x_3 \succ x_4 \succ x_5$.

(5) F_5

$$\tilde{F}_5 = \begin{pmatrix} 0.5038 & 0.5038 & 0.3718 & 0.1351 & 0.1351 \\ 0.5038 & 0.5038 & 0.3718 & 0.2858 & 0.1351 \\ 0.6074 & 0.6074 & 0.5038 & 0.3718 & 0.2858 \\ 0.8636 & 0.6957 & 0.6074 & 0.5038 & 0.3718 \\ 0.8636 & 0.8636 & 0.6957 & 0.6074 & 0.5038 \end{pmatrix}$$

$score(x_1) = 0.2865$, $score(x_2) = 0.3241$, $score(x_3) = 0.4681$, $score(x_4) = 0.6346$, $score(x_5) = 0.7576$, and thus $x_5 \succ x_4 \succ x_3 \succ x_2 \succ x_1$.

(6) F_6

$$\tilde{F}_6 = \begin{pmatrix} 0.5019 & 0.5019 & 0.6071 & 0.6071 & 0.6968 & 0.8607 \\ 0.5019 & 0.5019 & 0.5019 & 0.6071 & 0.8607 & 0.8607 \\ 0.5019 & 0.5019 & 0.5019 & 0.6071 & 0.6968 & 0.8607 \\ 0.3743 & 0.3743 & 0.3743 & 0.5019 & 0.6071 & 0.6968 \\ 0.2857 & 0.2857 & 0.2857 & 0.3743 & 0.5019 & 0.6071 \\ 0.1306 & 0.1306 & 0.1306 & 0.2857 & 0.3743 & 0.5019 \end{pmatrix}$$

$score(x_1) = 0.6547$, $score(x_2) = 0.6664$, $score(x_3) = 0.6337$, $score(x_4) = 0.4854$, $score(x_5) = 0.3677$, $score(x_6) = 0.2104$, and thus $x_2 \succ x_1 \succ x_3 \succ x_4 \succ x_5 \succ x_6$.

(7) F_7

$$\tilde{F}_7 = \begin{pmatrix} 0.5018 & 0.1391 & 0.2860 \\ 0.1391 & 0.5018 & 0.3729 \\ 0.6966 & 0.6095 & 0.5018 \end{pmatrix}$$

$score(x_1) = 0.2125$, $score(x_2) = 0.2560$, $score(x_3) = 0.6530$, and thus $x_3 \succ x_2 \succ x_1$.

(8) F_8

$$\tilde{F}_8 = \begin{pmatrix} 0.5021 & 0.3721 & 0.6111 & 0.1439 \\ 0.6111 & 0.5021 & 0.6965 & 0.3721 \\ 0.3721 & 0.2859 & 0.5021 & 0.1439 \\ 0.8658 & 0.6111 & 0.8658 & 0.5021 \end{pmatrix}$$

$score(x_1) = 0.3757$, $score(x_2) = 0.5599$, $score(x_3) = 0.2673$, $score(x_4) = 0.7809$, and thus $x_4 \succ x_2 \succ x_1 \succ x_3$.

(9) F_9

$$\tilde{F}_6 = \begin{pmatrix} 0.5025 & 0.6965 & 0.6092 & 0.6965 & 0.8650 & 0.8650 \\ 0.2857 & 0.5025 & 0.2857 & 0.3742 & 0.6092 & 0.6965 \\ 0.3742 & 0.6965 & 0.5025 & 0.6092 & 0.6965 & 0.8650 \\ 0.2857 & 0.6092 & 0.3742 & 0.5025 & 0.6965 & 0.8650 \\ 0.1434 & 0.3742 & 0.2857 & 0.2857 & 0.5025 & 0.6092 \\ 0.1434 & 0.2857 & 0.1434 & 0.1434 & 0.3742 & 0.5025 \end{pmatrix}$$

$score(x_1) = 0.7464$, $score(x_2) = 0.4503$, $score(x_3) = 0.6483$, $score(x_4) = 0.5661$, $score(x_5) = 0.3396$, $score(x_6) = 0.2180$, and thus $x_1 \succ x_3 \succ x_4 \succ x_2 \succ x_5 \succ x_6$.

(10) F_{10}

$$\tilde{F}_{10} = \begin{pmatrix} 0.5029 & 0.3707 & 0.2860 & 0.6092 & 0.8659 & 0.6960 & 0.1391 \\ 0.6092 & 0.5029 & 0.3707 & 0.6960 & 0.8659 & 0.8659 & 0.2860 \\ 0.6960 & 0.6092 & 0.5029 & 0.8659 & 0.8659 & 0.8659 & 0.3707 \\ 0.3707 & 0.2860 & 0.1391 & 0.5029 & 0.6960 & 0.6092 & 0.1391 \\ 0.1391 & 0.1391 & 0.1391 & 0.2860 & 0.5029 & 0.3707 & 0.1391 \\ 0.2860 & 0.1391 & 0.1391 & 0.3707 & 0.6092 & 0.5029 & 0.1391 \\ 0.8659 & 0.6960 & 0.6092 & 0.8659 & 0.8659 & 0.8659 & 0.5029 \end{pmatrix}$$

$score(x_1) = 0.4945$, $score(x_2) = 0.6156$, $score(x_3) = 0.7123$, $score(x_4) = 0.3734$, $score(x_5) = 0.2022$, $score(x_6) = 0.2805$, $score(x_7) = 0.7948$, and thus $x_7 \succ x_3 \succ x_2 \succ x_1 \succ x_4 \succ x_6 \succ x_5$.