Transsshipment & Risk Minimizing Optimization

Heejae Roh
Northeastern University

ALY 6050: Intro to Enterprise Analytics

Azadeh Mobasher

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1. INTRODUCTION

I perform two optimizations in this module. The first is one of the most common problems as a business problem which is transshipment. The goal of transshipment problem is to develop an optimal route based on the different shipping costs to reach each region. The second is the problem of configuring a portfolio, which is considered the most important in finance. In the process, I use the covariance matrix to find a way to minimize the risk. And based on this, let's find out the relationship between expected return and minimized risk.

2-1. PART 1 ANALYSIS

Understanding of key metrics

1. Transport waste products from its six plants to three waste disposal sites.
2. The cost of transportation from the plant to the waste disposal sites is given. Shipping fee from plant to plant and waste site to site is also given.
3. The amount of waste that plants generate is constant on a weekly basis.
4. The amount that each of the three waste disposal sites can handle is set as the maximum per week.
5. Rockhill does not incur a handling fee. Rockhill's only cost shipping fee.
6. Company wants to know if shipping from the plants to the waste sites is cheaper or if it is cheaper to bring it after transporting it to another plant.

(1) Check the default costs from plants to sites

Cost: Plants to Waste Proposal sites & Capacity

	Waste Proposal Site			
Plant	Orangeburg	Florence	Macon	Waste per Week
Denver	12	15	17	45
Morganton	14	9	10	26
Morrisville	13	20	11	42
Pineville	17	16	19	53
Rockhill	7	14	12	29
Stateville	22	16	18	38
Capacity	65	80	105	

Understanding of data

1. The table above shows plants from Denver to Stateville on the left. The plant located on the left becomes 'From', the starting point of waste, and the column names Orangeburg, Florence and Macon become 'To' as Waste site.
2. Capacity, the last row, represents the capacity of each waste site.
3. The rightmost column, Waste per Week, is the fixed amount of waste generated by each plant per week.

(2) Mathematical formulations of the Problem

X1: Denver, X2: Morganton, X3: Morrisville,
X4: Pinevill, X5: Rockhill, X6: Stateville
W1: Orangeburg, W2: Florence, W3: Macon
Waste quantity Xi to Wj: $XiWj_q$ / Cost of Xi to Wj: $XiWj_c$

Objective: Cost Minimize	$\Sigma WiWj_q * XiWj_c$
Constraints	
Waste of each Plant	$\Sigma X1Wj_q = 45$, $\Sigma X2Wj_q = 26$, $\Sigma X3Wj_q = 42$, $\Sigma X4Wj_q = 53$, $\Sigma X5Wj_q = 29$, $\Sigma X6Wj_q = 38$
Capacity of sites	$\Sigma XiW1_q <= 65$, $\Sigma XiW2_q <= 80$, $\Sigma XiW3_q <= 105$

Explanation

1. The goal is to minimize cost, which is the sum of costs from each plant to each site.
2. Using Σ, I represented each constraint. Each plant has a fixed amount of garbage that it discharges every week. Therefore, the sum of the three waste sites from each plant must match the amount of waste produced by each plant.
3. The capacity of each waste site is set to the maximum. This was also expressed as the sum from each plant using Σ.

(3) Set up the linear programming formulation in an Excel

Plant	Orangeburg	Florence	Macon	Sum	Waste per Week	
Denver				$\Sigma X1Wj_q$	=	45
Morganton				$\Sigma X2Wj_q$	=	26
Morrisville				$\Sigma X3Wj_q$	=	42
Pineville				$\Sigma X4Wj_q$	=	53
Rockhill				$\Sigma X5Wj_q$	=	29
Stateville				$\Sigma X6Wj_q$	=	38
Sum	$\Sigma XiW1_q$	$\Sigma XiW2_q$	$\Sigma XiW3_q$		Total Waste	233
	<=	<=	<=	Total Limit		
Capacity	65	80	105	250		

Explanation

1. The table shows the constraints described in '(2) Mathematical formulations of the Problem'. The yellow tables are decision variables.
2. The amount of waste sent to the waste site for each plant should be equal to Waste per Week.
3. The amount of waste received from 6 plants for each site should not exceed the maximum capacity of each site.

4. The sum of the amount of waste from each plant is 233. The sum of the amount of waste that each waste site can handle is 250, so we can say that we have a margin of 17.

(4) Result of Solver

Plant	Orangeburg	Florence	Macon	Sum	Waste per Week	
Denver	36	9	0	45	=	45
Morganton	0	0	26	26	=	26
Morrisville	0	0	42	42	=	42
Pineville	0	53	0	53	=	53
Rockhill	29	0	0	29	=	29
Stateville	0	18	20	38	=	38
Sum	65	80	88		Total Waste	233
	<=	<=	<=	Total Limit		
Capacity	65	80	105	250	Cost	2988

Interpretation

1. This is the amount of waste going to the waste site from each plant that makes the minimum cost using the excel solver.
2. The minimum cost is 2988, which is obtained through the SUMPRODUCT function. Quantity X quantity per cost from each plant to the waste site.
3. Rows represent 'From' and each plant. The columns represent 'To' and each waste site.
4. By checking Total, I can confirm that the constraint is satisfied once more.

(5) Check costs with relocation

From	To Orangeburg	Florence	Macon	Denver	Morganton	Morrisville	Pineville	Rockhill	Stateville
Orangeburg	…	12	10	…	…	…	…	…	…
Florence	12	…	15	…	…	…	…	…	…
Macon	10	15	…	…	…	…	…	…	…
Denver	12	15	17	…	3	4	9	5	4
Morganton	14	9	10	6	…	7	6	9	4
Morrisville	13	20	11	5	7	…	3	4	9
Pineville	17	16	19	5	4	3	…	3	11
Rockhill	7	14	12	5	9	5	3	…	14
Stateville	22	16	18	4	7	11	12	8	…

Explanation

1. The above table includes the relocation of waste sites. Rows represent From, and Columns represent To.
2. The cost from plant to sites is known, but the cost from sites to plant is not specified.
3. Also, even after relocation, I thought that having to move from sites to plants and send them to sites in the end was something that could not be reviewed when the cost had to be minimized, so I excluded it.
4. At the first calculation, I assumed that the cost from sites to plants is the same as the cost from plant to site, and I solve the problem. After solving the problem, I double-checked to see if the same result is obtained when excluded or not.

(6) Set up the linear programming formulation in an Excel

Plant	W1	W2	W3	X1	X2	X3	X4	X5	X6	Sum From			
W1										C10			
W2										C11	From – To		
W3										C12	(Xi Waste)		
X1										C13	C13-C4	=	45
X2										C14	C14-C5	=	26
X3										C15	C15-C6	=	42
X4										C16	C16-C7	=	53
X5										C17	C17-C8	=	29
X6										C18	C18-C9	=	38
Sum To	C1	C2	C3	C4	C5	C6	C7	C8	C9		Total		233
To - From	C1-C10	C2-C11	C3-C12								Relocation Amount		
	<=	<=	<=		Total						C4+C4+C6		
	65	80	105		250						+C7+C8+C9		

Explanation

1. Row represents ‘from’ and column represents ‘to’. Sum of ‘from’ is the sum of waste sent from each region.
2. Sum of ‘to’ is the sum of the amount received from each region.
3. Finally, the waste obtained by subtracting the amount sent from the amount received by each waste site must be less than or equal to the maximum capacity. These are C1-C10, C2-C11, and C3-C12.
4. And the quantity of waste produced from each plant is the value obtained by subtracting the amount sent ‘To’ from the amount departing from each region ‘From’.
5. In other words, by calculating the amount received and sent through this, you can know the amount produced by each plant. The remaining amount becomes the amount of waste received by each plant for relocation.
6. Therefore, this relocation amount can be found as the sum of C4 through C9. This is the sum of the amount (To) received by each plant.

(7) Result of Solver

Plant	W1	W2	W3	X1	X2	X3	X4	X5	X6	Sum From			
W1	0	0	0	0	0	0	0	0	0	0			
W2	0	0	0	0	0	0	0	0	0	0	From – To		
W3	0	0	0	0	0	0	0	0	0	0	(Xi Waste)		
X1	0	0	0	0	45	0	0	0	0	45	C13-C4	=	45
X2	0	42	29	0	0	0	0	0	0	71	C14-C5	=	26
X3	0	0	59	0	0	0	0	0	0	59	C15-C6	=	42
X4	0	0	0	0	0	17	0	36	0	53	C16-C7	=	53
X5	65	0	0	0	0	0	0	0	0	65	C17-C8	=	29
X6	0	0	0	0	0	0	0	0	0	38	C18-C9	=	38
Sum To	65	80	88	0	45	17	0	36	0		Total		233
To - From	65	80	88								Relocation Amount: C4+C4+C6+C7+C8+C9 = 98		
	<=	<=	<=		Total								
	65	80	105		250								
											Cost 2674		

Interpretation

1. In the end, we used 2 constraints. First, C13- C4 ~ C18 - C9 must be the same as the waste produced in each region.
2. Among columns, W1~W3 have one constraint. One is Sum of 'to' - sum of 'from' must be less than or equal to maximum capacity.
3. For some unknown reason, adding W1-W3 as a Sum of 'to' as Constraint increases the amount of cargo going through X2 by 20. and the cost is the same.
4. In this case, the amount of relocation increases as a result, so minimizing relocation can reduce the amount of work. Therefore, we will make decision

making in a situation where the cost of moving cargo is low and all constraints are satisfied.

5. The minimum cost was 2674, and the amount relocated was 98 barrels, with relocations from X1 to X2, X4 to X3, and X4 to X5.
6. All transportation routes where relocation occurred are cases where the transportation cost between each region is 3 or less. The relocation occurred because the route directly to the waste site averaged 12.94 with a minimum of 7.
7. Of these, Rockhill, which does not impose a handling cost, is included as much as 36 barrels.

2-2. PART 2 ANALYSIS

Understanding of key metrics

1. Bonds, High tech stocks, foreign stocks, Call options, Put options, I would like to compose a portfolio with 6 types of Gold.
2. The expected return is fixed for each type. The highest expected returns are call options and put options, which are derivative products. Next is investing in stocks, with expected returns of 12% for high tech stocks and 11% for foreign stocks. Next is Gold and Bonds, which are considered relatively safe, with expected returns of 9% and 7%.
3. We have a covariance matrix. To put it simply, it represents the relationship between the two types, and if +, one goes up and the other goes down. -If one goes up, the other goes up.
4. We want to invest $10,000, and we will build a portfolio for it. In the process, we will create a portfolio configuration to minimize risk while exceeding the expected return.
5. In addition, we will examine the relationship between 'r', the minimized risk, and 'e', the expected portfolio return, constructed based on portfolio at 0.5% intervals from 10% to 13.5%.

(1) Check the default factors before making portfolio

Expected Returns	
Bonds	7%
High tech stocks	12%
Foreign stocks	11%
Call options	14%
Put options	14%
Gold	9%

Covariance Matrix

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004
High tech stocks	0.0003	0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks	-0.0003	0.0004	0.008	0.0015	-0.0055	-0.0007
Call options	0.00035	0.0016	0.0015	0.012	-0.0005	0.0008
Put options	-0.00035	-0.0016	-0.0055	-0.0005	0.012	-0.0008
Gold	0.0004	0.0006	-0.0007	0.0008	-0.0008	0.005

Understanding

1. We have Expected return data. Multiply this by the allocation to get the expected rate of return.
2. A Covariance Matrix represents each relationship and is symmetric about the diagonal. Based on this, I filled in an empty table.
3. For example, bonds have a negative relationship with foreign stocks and put options. We need to allocate the portfolio with this in mind.
4. What we consider in the process is to minimize risk. We will consider risk based on the covariance matrix.
5. The portfolio risk is also measured by taking the Standard Deviation of variance of actual returns of that portfolio over time. The variability of returns is proportional to the portfolio's risk. This risk can be measured by calculating the Standard Deviation of this variability (DBS, 2022).

(2) Mathematical formulations of the Problem

X1: Bonds, X2: High tech stocks, X3: Foreign stocks, X4: Call options, X5: Put options, X6: Gold
Allocation proportion of Xi: Xi_p / Expected return of Xi: Xi_e

Objective: Risk Minimize	$Xi_p^T \Sigma Xi_p$
Constraints	
Sum of Proportion	$\Sigma Xi_p = 1$
Expected Return	$\Sigma(Xi_p * Xi_e)$ >= Baseline expectation

Explanation

1. We use the covariance matrix as mentioned before to calculate the risk. Our goal is to keep the risk to a minimum.
2. The constraints we have in this process are that the sum of all proportions must first be 1. Next, the total Expected Return, which is the sum of the products of each proportion and the expected return that each investment type has, must be greater than or equal to the baseline expectation.
3. Since humans hate uncertainty the most, we will look for a portfolio that can bring the least risk under conditions that satisfy the expected rate of return.

(3) Set up the linear programming formulation in an Excel

Baseline Expectation 11.0%			
	Allocate Perc.	Allocate Amount	Allocation Total
Bonds	$X1_p$	$=10{,}000 * X2_p$	10,000
High tech stocks	$X2_p$	$=10{,}000 * X3_p$	
Foreign stocks	$X3_p$	$=10{,}000 * X4_p$	
Call options	$X4_p$	$=10{,}000 * X5_p$	
Put options	$X5_p$	$=10{,}000 * X6_p$	
Gold	$X6_p$	$=10{,}000 * X7_p$	
Total	$=\Sigma Xi_p$	=	1
Expected Return	$\Sigma(Xi_p * Xi_e)$	>=	11%
Portfolio Risk	$Xi_p^T \Sigma Xi_p$		

Explanation

1. This is a table including the formula set in (2). The Yellow tables are decision variables.
2. Based on this, the solver is used to derive the result.
3. For how to calculate risk, I refer to how to calculate risk in Excel through the covariance matrix.

(4) Result of Solver

Baseline Expectation 11.0%			
	Allocate Perc.	Allocate Amount	Allocation Total
Bonds	18.98%	1898.1	10000
High tech stocks	10.86%	1086.3	
Foreign stocks	27.08%	2708.3	
Call options	4.79%	479.4	
Put options	25.45%	2544.7	
Gold	12.83%	1283.2	
Total	1.00	=	1
Expected Return	0.11	>=	11%
Portfolio Risk	0.000735635		

Interpretation

1. Each allocation percentage is allocated to each type of investment.
2. The expected return shows the value closest to the baseline return. This is because the purpose of this portfolio is to minimize risk.
3. I set the total 1 to have a number closest to 1 through the solver option.
4. Foreign stocks accounted for the highest share at 27%, followed by put options and bonds at 25% and 19% respectively. This percentage needs to be compared with other baseline expectations.
5. The allocated amount is the product of the total investment amount of 10,000 and the allocated percentage.

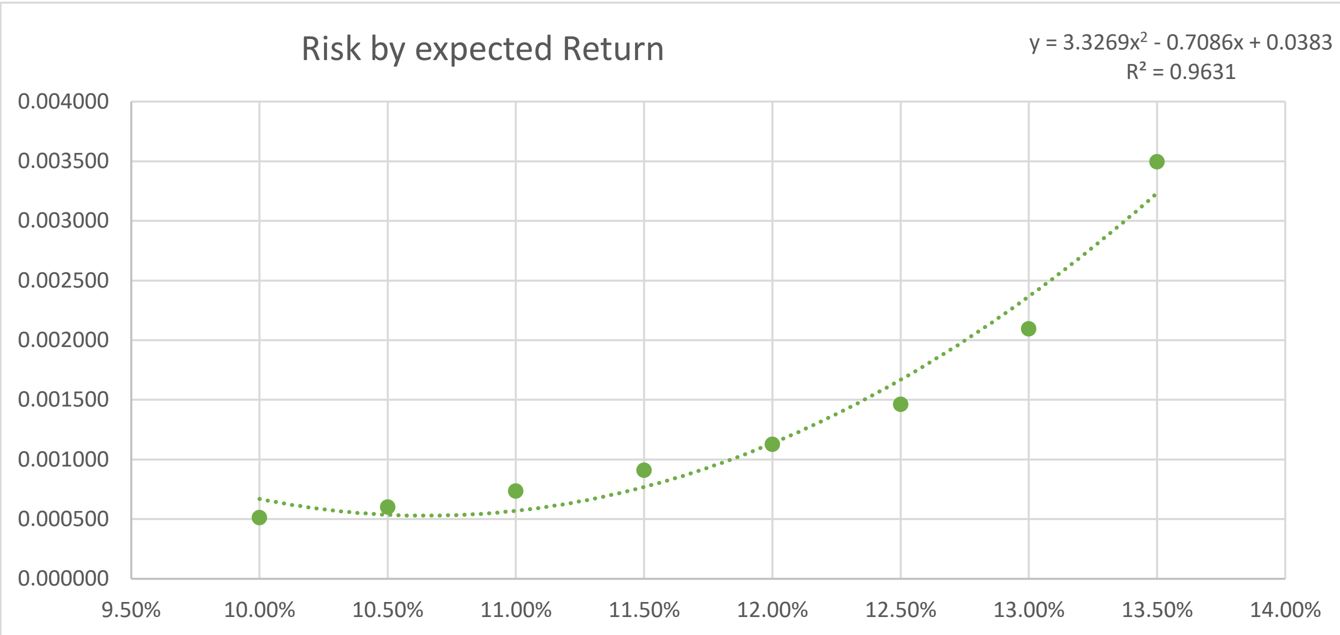
(5) Every minimized risk and expected return from 10.0% to 13.5%

Baseline Return	Portfolio risk	Expected Return
10.00%	0.000514	10.00%
10.50%	0.000603	10.50%
11.00%	0.000736	11.00%
11.50%	0.000911	11.50%
12.00%	0.001129	12.00%
12.50%	0.001463	12.50%
13.00%	0.002098	13.00%
13.50%	0.003496	13.50%

Explanation

1. This is the risk and expected return for each baseline return obtained by repeatedly using the formula for obtaining the baseline expectation of 11% in (4).

(6) Plot of minimized risk & Expected portfolio return



Explanation

1. Basically, as the expected rate of return increases, the risk increases.
2. This is described by the equation y = 3.3269x2 - 0.7086x + 0.0383, where R squared is 0.9631.
3. Specifically, let's calculate Risk/Expected and compare its increase.

Baseline Return	Portfolio risk	Expected Return	Risk / Expected	Difference
10.00%	0.000514	10.00%	0.51%	
10.50%	0.000603	10.50%	0.57%	0.06%
11.00%	0.000736	11.00%	0.67%	0.09%
11.50%	0.000911	11.50%	0.79%	0.12%
12.00%	0.001129	12.00%	0.94%	0.15%
12.50%	0.001463	12.50%	1.17%	0.23%
13.00%	0.002098	13.00%	1.61%	0.44%
13.50%	0.003496	13.50%	2.59%	0.98%

Understanding

1. Difference represents the difference between Risk/Expected. Looking at the change in this, it can be seen that it increased by 0.03 from 0.06 at first, then by 0.08 at 12.5%, and then by 0.21 at 13.0%.
2. Therefore, based on this at the time of decision making, it can be said that when seeking safety, the baseline return can be raised to 12.0%, and if you want a little

aggressive investment, it would be better to choose 12.5%.

3. Answers for Questions

Part 1

1. Solver result without relocation

Plant	Orangeburg	Florence	Macon	Sum	Waste per Week	
Denver	36	9	0	45	=	45
Morganton	0	0	26	26	=	26
Morrisville	0	0	42	42	=	42
Pineville	0	53	0	53	=	53
Rockhill	29	0	0	29	=	29
Stateville	0	18	20	38	=	38
Sum	65	80	88		Total Waste	233
	<=	<=	<=	Total Limit		
Capacity	65	80	105	250	Cost	2988

This uses two of the gray constraints and yellow part is decision variables. The first constraint is that the weekly production per plant is constant. The second is that the capacity for each waste site is less than or equal to the maximum. The amount of waste going to the waste site from each plant that makes the minimum cost using the excel solver. The minimum cost is 2988, which is obtained through the SUMPRODUCT function. Quantity X quantity per cost from each plant to the waste site. Rows represent 'From' and each plant. The columns represent 'To' and each waste site. By checking Total, I can confirm that the constraint is satisfied once more.

2. Solver result with relocation

Plant	W1	W2	W3	X1	X2	X3	X4	X5	X6	Sum From			
W1	0	0	0	0	0	0	0	0	0	0			
W2	0	0	0	0	0	0	0	0	0	0	From – To		
W3	0	0	0	0	0	0	0	0	0	0	(Xi Waste)		
X1	0	0	0	0	45	0	0	0	0	45	C13-C4	=	45
X2	0	42	29	0	0	0	0	0	0	71	C14-C5	=	26
X3	0	0	59	0	0	0	0	0	0	59	C15-C6	=	42
X4	0	0	0	0	0	17	0	36	0	53	C16-C7	=	53
X5	65	0	0	0	0	0	0	0	0	65	C17-C8	=	29
X6	0	0	0	0	0	0	0	0	0	38	C18-C9	=	38
Sum To	65	80	88	0	45	17	0	36	0		Total		233
To - From	65	80	88										
	<=	<=	<=		Total								
	65	80	105		250								

Relocation Amount:
C4+C4+C6+C7+C8+C9 = 98

Cost	2674

This uses two of the gray constraints and yellow part is decision variables. In the end, we used 2 constraints. First, C13- C4 ~ C18 - C9 must be the same as the waste produced in each region. Among columns, W1~W3 (Which is Sum of 'to' - Sum of 'From' of each waste sites) have one constraint. One is Sum of 'to' - sum of 'from' must be less than or equal to maximum capacity. The minimum cost was 2674, and the amount relocated was 98 barrels, with relocations from X1 to X2, X4 to X3, and X4 to X5. All transportation routes where relocation occurred are cases where the transportation cost between each region is 3 or less. The relocation occurred because the route directly to the waste site averaged 12.94 with a minimum of 7. Of these, Rockhill, which does not impose a handling cost, is included as much as 36 barrels.

Part 2

3. <u>Solver Result of Baseline expectation 11.0%</u>

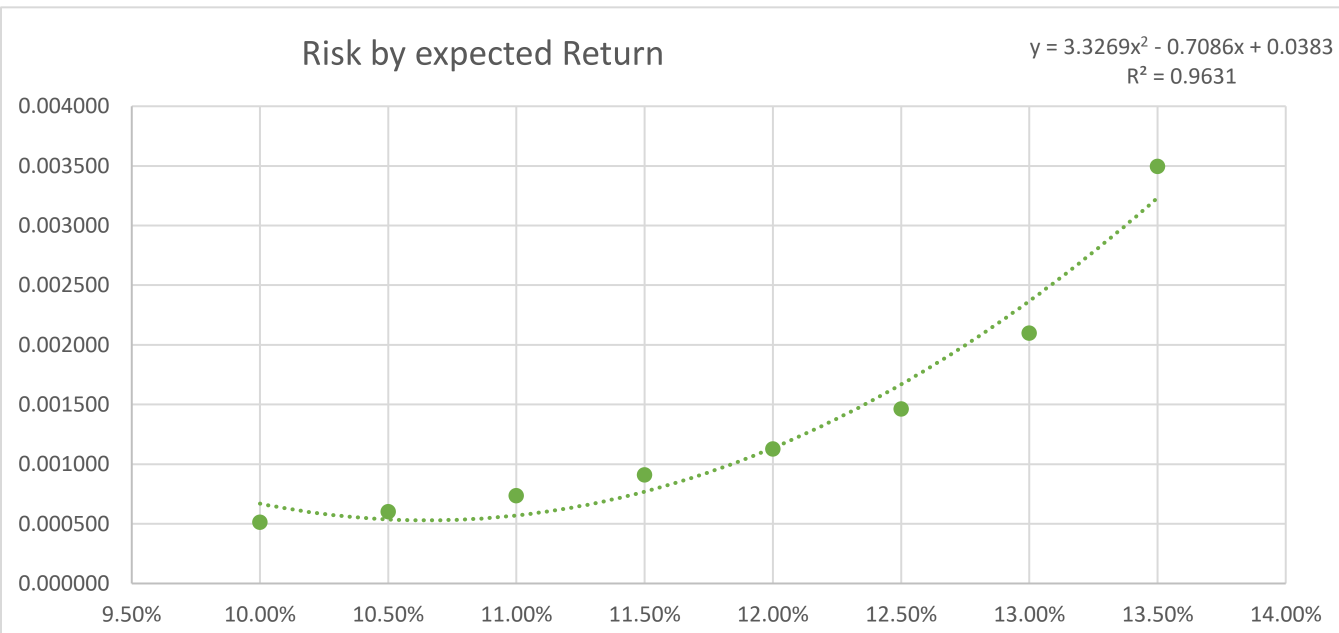
4.

Baseline Expectation 11.0%			
	Allocate Perc.	Allocate Amount	Allocation Total
Bonds	18.98%	1898.1	10000
High tech stocks	10.86%	1086.3	
Foreign stocks	27.08%	2708.3	
Call options	4.79%	479.4	
Put options	25.45%	2544.7	
Gold	12.83%	1283.2	
Total	1.00	=	1
Expected Return	0.11	>=	11%
Portfolio Risk	0.000735635		

Each allocation percentage is allocated to each type of investment. The expected return shows the value closest to the baseline return. This is because the purpose of this portfolio is to minimize risk. I set the total 1 to have a number closest to 1 through the solver option. Foreign stocks accounted for the highest share at 27%, followed by put options and bonds at 25% and 19% respectively. This percentage needs to be compared with other baseline expectations. The allocated amount is the product of the total investment amount of 10,000 and the allocated percentage.

3. <u>Plot Minimized risk and expected return.</u>

Baseline Return	Portfolio risk	Expected Return
10.00%	0.000514	10.00%
10.50%	0.000603	10.50%
11.00%	0.000736	11.00%
11.50%	0.000911	11.50%
12.00%	0.001129	12.00%
12.50%	0.001463	12.50%
13.00%	0.002098	13.00%
13.50%	0.003496	13.50%



The above table is the risk and expected return for each baseline return obtained by repeatedly using the formula for obtaining the baseline expectation of 11% in (4). Basically, as the expected rate of return increases, the risk increases. This is described by the equation y = 3.3269x^2 - 0.7086x + 0.0383, where R squared is 0.9631. In addition to this, let's calculate Risk/Expected and compare its increase.

Baseline Return	Portfolio risk	Expected Return	Risk / Expected	Difference
10.00%	0.000514	10.00%	0.51%	
10.50%	0.000603	10.50%	0.57%	0.06%
11.00%	0.000736	11.00%	0.67%	0.09%
11.50%	0.000911	11.50%	0.79%	0.12%
12.00%	0.001129	12.00%	0.94%	0.15%
12.50%	0.001463	12.50%	1.17%	0.23%
13.00%	0.002098	13.00%	1.61%	0.44%
13.50%	0.003496	13.50%	2.59%	0.98%

Difference represents the difference between Risk/Expected. Looking at the change in this, it can be seen that it increased by 0.03 from 0.06 at first, then by 0.08 at 12.5%, and then by 0.21 at 13.0%.Therefore, based on this at the time of decision making, it can be said that when seeking safety, the baseline return can be raised to 12.0%, and if you want a little aggressive investment, it would be better to choose 12.5%.

4. CONCLUSION

In this module, one of the most frequent business problems is the calculation of transport costs and portfolio construction. There must be many problems in the real world, but I thought that it would be good to solve the problem by simplifying it as much as when the linear regression model was constructed. In the portfolio problem, the risk was different for each expected rate of return. Here, the risk gradually increases up to a certain level of expected return, and then it is identified that it sharply increases. Based on this, we tried to provide a basis for decision making. I will continue to develop not only my analytical skills, but also my ability to make decisions.

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