Linear Programming model to Maximize Profits

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1. INTRODUCTION

In this module, I will learn how to do linear programming (LP) with various constraints. Linear programming is a mathematical technique that determines the best way to use available resources. Managers use the process to help make decisions about the most efficient use of limited resources – like money, time, materials, and machinery (MTCT, n.d.). Based on the given dataset about product information such as cost, profit, and square feet, I will run solver in Excel to make decision to make better profit. I will learn how to interpret the sensitivity report and what can be progressed with that.

2. ANALYSIS

Understanding of key metrics

1. This is a problem of maximizing profit by selling 4 products.
2. We have information about each product's price, cost, and warehouse it occupies.
3. Based on this, execute linear programming with our budget and space constraints. For linear programming, I will use continuous numbers rather than integers. And the two marketing constraints are distribution to the company's products pipeline.
4. After making original sensitivity report, I will find out how the increase of each factor affects the profit.

(1) Mathematical formulations of the Problem

P: Pressure washer, K: Go-kart, G: Generator, W: Water Pump
Quantity of each: P_q, K_q, G_q, and W_q

Objective: Profit	P_q*169.99 + K_q*359.99 + G_q*290.99 + W_q*142.99
Constraints	
Cost	P_q*330 + K_q*370 + G_q*410 + W_q*127 ≤ 170,000
Pallet	P_q* 25 + K_q* 40 + G_q* 25 + W_q*1.25 ≤ 12,300
Marketing 1	P_q+K_q ≥ (P+K_q+G_q+W_q)*0.3
Marketing 2	G_q ≥W_q*2

Understanding of data

1. Objective value is profit. Our goal is maximizing the profit. Profit formula uses (Profit – Cost) of each product times product quantity.
2. There are 4 constraints. First factor is cost. The sum cost of whole product should be equal or smaller than 170,000.
3. Second factor is pallet which is related to warehouse available space. The warehouse has 82 shelves, and each shelf is 30 ft long and 5 ft wide. The whole space is 82 * 30 * 5 which is 12,300. The sum of whole pallet should be equal or smaller than 12,300. In this case, water pump's space for each stuff is 5 * 5 / 4 cases * 5 (case of 5 water pumps)
4. Other two constraints are from marketing department. Marketing 1 constraint is

that the quantity of pressure washers and Go karts must account for more than 30% of the total quantity. In other words, the quantity of pressure washers and Go karts is equal or bigger than 30% of the total quantity.

5. Marketing 2 constraint is generators' quantity is equal or bigger than twice of the water pumps quantity.

(2) Set up the linear programming formulation in an Excel

Formulas			
Total Cost	= P_q*330 + K_q*370 + G_q*410 + W_q*127		
Total Length	= P_q*25 + K_q*40 + G_q*25 + W_q*1.25		
Total Profit	= P_q*169.99 + K_q*359.99 + G_q*290.99 + W_q*142.99		
Marketing 1	$P_q+K_q \geq (P+K_q+G_q+W_q)*0.3$	→	$0.7*P_q+0.7*K_q - 0.3* G_q - 0.3* W_q \geq 0$
Marketing 2	$G_q \geq W_q*2$	→	$G_q - W_q*2 \geq 0$

Explanation

1. Using all Mathematical formulations in (1).
2. To show this is linear regression functions, the quantity and every result is displayed in two decimal places.
3. Marketing 1 & 2 have two formula on the table. First one is what I used for solver, and second one is the formula which is normalized.

Maximize Profit Model

Data

	Purchasing & Warehousing					
Item	Pressure washer	go-Kart	Generator	Water Pump	**Total**	Constraint
Cost	330	370	410	127	17000	170000
Price	499.99	729.99	700.99	269.99	312051	
Length of Pallet	**25**	40	25	1.25	12300	12300

	Profit by Quantity Decision				
Item	Pressure washer	go-Kart	Generator	Water Pump	
Quantity	0.00	155.18	237.77	118.88	Decision variables
Profit	0.00	55862.91	69188.48	16999.31	
			Total Profit	142050.70	Objective variables

Marketing Constraints			
	Quantity	>=	Quantity
Pressure washer + go Kart	155.18	Whole	153.55
	Quantity	>=	Quantity
Generator	237.77	Water Pump * 2	237.77

Explanation

1. Decision variables are Quantity of each product.
2. Objective variable is total profit.
3. Decision variables and objective variable are shown in the yellow shadings to stand out.
4. Constraints are shown in the grey shadings.
5. Tried to keep the table as clean as possible so it was easy to read.

(3) Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	0.00	-110.0715237	169.99	110.07	1E+30
D12	Quantity go-Kart	155.18	0	359.99	205.84	76.74
E12	Quantity Generator	237.77	0	290.99	98.20	131.87
F12	Quantity Water Pump	118.88	0	142.99	196.41	89.12

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170000	0.56	170000	428.8	56225
G8	Length of Pallet Total	12300	3.84	12300	6078.38	30.95
K11	Pressure washer + go Kart Quantity	155.18	0	0	1.63	1E+30
K13	Generator Quantity	237.77	-33.68	0	27.92	974.12

Optimal inventory level for four products & monthly report

	Pressure washer	go-Kart	Generator	Water Pump
Quantity	**0**	**155.18**	**237.77**	**118.88**
Total Cost	170,000/ 170,000	Pallet Usage	12,300/ 12,300	
		Monthly Profit	**142050.70**	

Interpretation

1. The optimal inventory quantity for each product is Pressure washer: 0, go-Kart: 155.18, Generator: 237.77, and Water Pump:118.88.
2. Total cost is 170,000 which is running out 100% budget.
3. Pallet usage is 12,300 which is 100% usage of pallet.
4. Total profit by optimal inventory level is 142050.70 which is calculated by (profit – cost) * quantity formula.

(5) Smallest selling price for that item to a non-zero value

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	0.00	-110.0715237	169.99	110.07	1E+30
D12	Quantity go-Kart	155.18	0	359.99	205.84	76.74
E12	Quantity Generator	237.77	0	290.99	98.20	131.87
F12	Quantity Water Pump	118.88	0	142.99	196.41	89.12

Interpretation

1. This is the sensitivity report for quantity variables. Quantity of pressure washer is 0.00. This means that selling pressure washer makes minus or zero profit.
2. Objective formula was maximizing profit. In this case, price 499.99 minus cost 330 is 169.99 which is shown in Objective Coefficient. 169.99 is the profit per selling a one product of pressure washer.
3. The table shows that 169.99 – (-110.07) = 280.06 is the coefficient that we can start selling pressure washer.
4. If we assume that cost is fixed, we have to make the price higher which can affect to revenue.
5. The smallest selling price for pressure washer is 499.99 – (-110.07) = 610.06. After that, the coefficient which is calculated by price (revenue) – cost is 280.06.

(5-1) Result of smallest selling price for pressure washer

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	189.6359743	0	280.062	73.58190244	93.59617391
D12	Quantity go-Kart	0	-115.2415418	359.99	115.2415418	1E+30
E12	Quantity Generator	294.9892934	0	290.99	374.1075224	75.42145
F12	Quantity Water Pump	147.4946467	0	142.99	1E+30	125.32602

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	17000	0	17000	1E+30	0
G8	Length of Pallet Total	12300	13.01130278	12300	1E+30	12300
K11	Pressure washer + go Kart Quantity	189.6359743	-64.6008137	0	344.4	216
K13	Generator Quantity	294.9892934	-53.6728137	0	344.4	2056.119403

Optimal inventory level for four products & monthly report

	Pressure washer	go-Kart	Generator	Water Pump
Quantity	**0 -> 189.64**	155.18 -> 0.00	237.77 -> 294.99	118.88 -> 147.49
Total Cost	170,000/ 170,000	Pallet Usage	12,300/ 12,300	
		Total Profit	**142050.70-> 160039.02**	

Interpretation

1. Based on the sensitivity report, I manually set the price of pressure washer as 610.062 which is rounded up in forth decimal place. This make objective coefficient, which is profit minus cost per unit,
2. It makes the quantity of pressure washer 430.85. At the same time, the quantity of go-Kart get 0, because of the marketing constraint 1 which is the sum of the two products is 30% of the total.
3. You can see that the total profit has risen considerably from 142,050.70 to 160,039.02 as we can buy more products that cost less between pressure washer and go-kart.
4. This is because the number of items that can be sold has increased rapidly within budget and constraint limits.
5. Costs caused by Marketing 1 constraint have been reduced. At the same time, you can see that the number of generators and water pumps has increased significantly.
6. If I had to do decision making at the company, I could argue that based on the first original report, the price of the pressure washer should be lowered. If so, with the rationale that this will bring a lot of profit.

(6) Budget analysis for more profit

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170000	0.56	170000	428.8	56225
G8	Length of Pallet Total	12300	3.84	12300	6078.38	30.95
K11	Pressure washer + go Kart Quantity	155.18	0	0	1.63	1E+30
K13	Generator Quantity	237.77	-33.68	0	27.92	974.12

Explanation

1. The cost total is currently 170,000 which is 100% of budget.
2. The answer to the question of increasing the budget is 'Yes'. Because with 170,000 we already use all budget, if we add more, we can get more profit on this side.
3. The table shows that we can add 428.8 budget with the constraints we have. The total cost with allowable increase is 170,000 + 428.8 which is 170,428.8
4. In the table above, you can see that the additional profit which is the shadow price

when the cost increases by one unit, is marked as 0.56. If the company increase the 428.8 budget, the added profit of the company would be 0.56 * 428.8 which is 240.13.

(6-1) Result of additional budget

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	0	-16.47582609	169.99	16.47582609	1E+30
D12	Quantity go-Kart	154.0173913	0	359.99	205.8402439	20.28608137
E12	Quantity Generator	239.5826087	0	290.99	56.39047619	131.8664063
F12	Quantity Water Pump	119.7913043	0	142.99	112.7809524	102.9368644

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170428.8	0	170428.8	1E+30	2.86218E-11
G8	Length of Pallet Total	12300	11.56827826	12300	2.06566E-12	12300
K11	Pressure washer + go Kart Quantity	154.0173913	-146.7730435	0	215.25	1.08745E-13
K13	Generator Quantity	239.5826087	-42.24886957	0	291.8644068	1.8634E-12

Optimal inventory level for four products & monthly report

	Pressure washer	go-Kart	Generator	Water Pump
Quantity	0 -> 0	**155.18 -> 154.02**	**237.77 -> 239.58**	**118.88 -> 119.79**
Total Cost	170,428.8/ 170,428.8	Pallet Usage	12,300/ 12,300	
		Total Profit	**142050.70 -> 142289.82**	

Interpretation

1. Based on the sensitivity report, I manually set the budget as 170,428.8 which is total budget plus allowable increase. Since there is space left for Marketing constraint 2, we can reduce the number of go-karts and purchase more Generators and Water Pumps.
2. It makes slight increase in quantity of 3 products which were non-zero value in the original report.
5. You can see that the total profit has risen slightly from 142,050.70 to 142,289.82 as the quantity of 3 product slightly increased. It's almost same as 0.56 (Shadow price) * 428.8 (Allowable Increase) which is 240.13.
3. This is because the number of items that can be sold has increased slowly within only small additional budget.
4. If I had to do decision making at the company, this method may not be

recommended because the increase in profit, 239.12, is not greater than the increase in budget, 428.8.

5. However, if the company is in a situation where you need to take market share from another competitor, I can recommend it.

(7) Warehouse size-up analysis for more profit

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170000	0.56	170000	428.8	56225
G8	Length of Pallet Total	12300	3.84	12300	6078.38	30.95
K11	Pressure washer + go Kart Quantity	155.18	0	0	1.63	1E+30
K13	Generator Quantity	237.77	-33.68	0	27.92	974.12

Interpretation

1. The length of pallet total is currently 12,300. which is same as the constraint. And allowable increase is 6078.38 square feet with other constraints fixed. If I have to order in shelves numbers, it would be around 40 ~ 41, because one shelf is 150 square feet each.
2. The answer to the question of larger warehouse is 'Yes', because we already used whole capability of warehouse. In other words, the warehouse is full to maximize profit.
3. In the table above, you can see that the additional profit which is the shadow price when the length of pallet increases by one unit, is marked as 3.84. This means that we can get 3.84 profit by increase the one space of warehouse.
4. Although it doesn't calculate each of the products that can be called integers, it's easy to see how much profit increases when one unit goes up.
5. We have three lengths of four products. That's 1.25 (W) 25 (P & G), and 40 (K) ft. The least common multiple of these three is 200. We can therefore think of increasing the warehouse by 200 units in the real world. Or you can think of increasing it by 40, which is a common multiple of 1.25 and 40.
6. If you increase it by 40 units, we can make an additional profit of 153.6. In other words, if it doesn't cost 153.6 to increase 40 units, then we can increase the warehouse size by 40 units. When making the final decision, we should consider whether other constraints are not violated. Adjust other constraints if necessary. Perhaps if we increase warehouse and sell more products, cost will increase, so we may need to increase budget accordingly. If the company increase the 6078.38 budget, the added profit of the company would be 3.84 * 6078.38 which is 23,340.98.

(7-1) Result of additional budget

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	0	-151.0821622	169.99	151.0821622	1E+30
D12	Quantity go-Kart	459.4594595	0	359.99	1E+30	76.73878564
E12	Quantity Generator	0	0	290.99	98.20490541	1E+30
F12	Quantity Water Pump	0	-196.4098108	142.99	196.4098108	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170000	0.972945946	170000	3.36513E-11	170000
G8	Length of Pallet Total	18378.37838	0	18378.37838	1E+30	3.63798E-12
K11	Pressure washer + go Kart Quantity	459.4594595	0	0	321.6216216	1E+30
K13	Generator Quantity	0	-107.9178378	0	298.9949749	0

Optimal inventory level for four products & monthly report				
	Pressure washer	go-Kart	Generator	Water Pump
Quantity	0 -> 0	**155.18 -> 459.46**	**237.77 -> 0**	**118.88 -> 0**
Total Cost	170,000/ 170,000	Pallet Usage	18,378.38/ 18,378.38	
		Total Profit	**142050.70 -> 165400.81**	

Interpretation

1. Based on the sensitivity report, I manually set the space as 18,378.38 which is total pallet plus allowable increase.
2. It makes only one product out of four available for sale. The product is go-Kart and quantity is 459.46.
3. You can see that the total profit has risen significantly from 142,050.70 to 165400.81 as the quantity of go-Kart increased. It's almost same as 3.84 (Shadow price) * 6078.38 (Allowable Increase) which is 23,340.98.
4. This is because go-kart takes up a lot of pallets and at the same time has the highest profit (objective coefficient) per unit at 359.99.
5. Even though only go-kart is sold, marketing constraints 1 & 2 are satisfied, so only go-kart products are sold.
6. If I had to do decision making at the company, I would recommend this method to companies. Because you can make the best profit. Only if the increase in profit outweighs the cost of increasing the pallet.
7. However, when the company loses market share in other products, the company's stability or long-term disadvantages will be additionally reviewed with this decision.

3. Answers for Questions

1. Write the mathematical formulation of the problem

P: Pressure washer, K: Go-kart, G: Generator, W: Water Pump
Quantity of each: P_q, K_q, G_q, W_q

Objective: Profit	P_q*169.99 + K_q*359.99 + G_q*290.99 + W_q*142.99
Constraints	
Cost	P_q*330 + K_q*370 + G_q*410 + W_q*127 ≤ 170,000
Pallet	P_q* 25 + K_q* 40 + G_q* 25 + W_q*1.25 ≤ 12,300
Marketing 1	P_q+K_q ≥ (P+K_q+G_q+W_q)*0.3
Marketing 2	G_q ≥W_q*2

Objective value is profit. Our goal is maximizing the profit. Profit formula uses (Profit – Cost) of each product times product quantity. There are 4 constraints. First factor is cost. The sum cost of whole product should be equal or smaller than 170,000. Second factor is pallet which is related to warehouse available space. The warehouse has 82 shelves and each shelf is 30 ft long and 5 ft wide. The whole space is 82 * 30 * 5 which is 12,300. The sum of whole pallet should be equal or smaller than 12,300. In this case, water pump's space for each stuff is 5 * 5 / 4 cases * 5 (case of 5 water pumps). Other two constraints are from marketing department. Marketing 1 constraint is that the quantity of pressure washers and Go karts must account for more than 30% of the total quantity. In other words, the quantity of pressure washers and Go karts is equal or bigger than 30% of the total quantity. Marketing 2 constraint is generators' quantity is equal or bigger than twice of the water pumps quantity.

2. Set up the linear programming formulation in an Excel workbook

Formulas			
Total Cost	= P_q*330 + K_q*370 + G_q*410 + W_q*127		
Total Length	= P_q*25 + K_q*40 + G_q*25 + W_q*1.25		
Total Profit	= P_q*169.99 + K_q*359.99 + G_q*290.99 + W_q*142.99		
Marketing 1	$P_q+K_q \geq (P+K_q+G_q+W_q)*0.3$	→	$0.7*P_q+0.7*K_q - 0.3* G_q - 0.3* W_q \geq 0$
Marketing 2	$G_q \geq W_q*2$	→	$G_q - W_q*2 \geq 0$

Maximize Profit Model

Data

	Purchasing & Warehousing					
Item	Pressure washer	go-Kart	Generator	Water Pump	**Total**	Constraint
Cost	330	370	410	127	17000	170000
Price	499.99	729.99	700.99	269.99	312051	
Length of Pallet	**25**	40	25	1.25	12300	12300

	Profit by Quantity Decision				
Item	Pressure washer	go-Kart	Generator	Water Pump	
Quantity	0.00	155.18	237.77	118.88	Decision variables
Profit	0.00	55862.91	69188.48	16999.31	
			Total Profit	142050.70	Objective variables

	Marketing Constraints			
		Quantity	>=	Quantity
	Pressure washer + go Kart	155.18	Whole	153.55
		Quantity	>=	Quantity
	Generator	237.77	Water Pump * 2	237.77

Using all Mathematical formulations in answer number one. To show this is linear regression functions, the quantity and every result is displayed in two decimal places. Marketing 1 & 2 have two formulas on the table. First one is what I used for solver, and second one is the formula which is normalized.

Decision variables are Quantity of each product. Objective variable is total profit. Decision variables and objective variable are shown in the yellow shadings to stand out. Constraints are shown in the grey shadings. Tried to keep the table as clean as possible so it was easy to read.

3. Use the Excel Solver or R to solve the problem and generate a sensitivity report.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	0.00	-110.0715237	169.99	110.07	1E+30
D12	Quantity go-Kart	155.18	0	359.99	205.84	76.74
E12	Quantity Generator	237.77	0	290.99	98.20	131.87
F12	Quantity Water Pump	118.88	0	142.99	196.41	89.12

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170000	0.56	170000	428.8	56225
G8	Length of Pallet Total	12300	3.84	12300	6078.38	30.95
K11	Pressure washer + go Kart Quantity	155.18	0	0	1.63	1E+30
K13	Generator Quantity	237.77	-33.68	0	27.92	974.12

4. Describe the optimal solutions obtained. These will consist of the inventory level for all four products and the optimal monthly profit.

Optimal inventory level for four products & monthly report

	Pressure washer	go-Kart	Generator	Water Pump
Quantity	**0**	**155.18**	**237.77**	**118.88**
Total Cost	170,000/ 170,000	Pallet Usage	12,300/ 12,300	
		Monthly Profit	**142050.70**	

The optimal inventory quantity for each product is Pressure washer: 0, go-Kart: ,155.18, Generator: 237.77, and Water Pump:118.88. Total cost is 170,000 which is running out 100% budget. Pallet usage is 12,300 which is 100% usage of pallet. Total profit by optimal inventory level is 142,050.70 which is calculated by (profit – cost) * quantity formula.

5. One of the decision variables has an optimal value of zero. Use the Solver sensitivity report to determine the smallest selling price for that item so that this optimal zero solution value changes.

This is the sensitivity report for quantity variables. Quantity of pressure washer is 0.00. This means that selling pressure washer makes minus or zero profit. Objective formula was maximizing profit. In this case, price 499.99 minus cost 330

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	0.00	-110.0715237	169.99	110.07	1E+30
D12	Quantity go-Kart	155.18	0	359.99	205.84	76.74
E12	Quantity Generator	237.77	0	290.99	98.20	131.87
F12	Quantity Water Pump	118.88	0	142.99	196.41	89.12

is 169.99 which is shown in Objective Coefficient. 169.99 is the profit per selling a one product of pressure washer. The table shows that 169.99 – (-110.07) = 280.06 is the coefficient that we can start selling pressure washer. If we assume that cost is fixed, we have to make the price higher which can affect to revenue. The smallest selling price for pressure washer is 499.99 – (-110.07) = 610.06. After that, the coefficient which is calculated by price (revenue) – cost is 280.06.

Result of smallest selling price for pressure washer

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	189.6359743	0	280.062	73.58190244	93.59617391
D12	Quantity go-Kart	0	-115.2415418	359.99	115.2415418	1E+30
E12	Quantity Generator	294.9892934	0	290.99	374.1075224	75.42145
F12	Quantity Water Pump	147.4946467	0	142.99	1E+30	125.32602

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	17000	0	17000	1E+30	0
G8	Length of Pallet Total	12300	13.01130278	12300	1E+30	12300
K11	Pressure washer + go Kart Quantity	189.6359743	-64.6008137	0	344.4	216
K13	Generator Quantity	294.9892934	-53.6728137	0	344.4	2056.119403

Optimal inventory level for four products & monthly report

	Pressure washer	go-Kart	Generator	Water Pump
Quantity	**0 -> 189.64**	155.18 -> 0.00	237.77 -> 294.99	118.88 -> 147.49
Total Cost	170,000/ 170,000	Pallet Usage	12,300/ 12,300	
		Total Profit	**142050.70-> 160039.02**	

Based on the sensitivity report, I manually set the price of pressure washer as 610.062 which is rounded up in forth decimal place. It makes the quantity of pressure washer 430.85. In the same time, the quantity of go-Kart get 0, because of the marketing constraint 1 which is the sum of the two products is 30% of the total. You can see that the total profit has risen considerably from 142,050.70 to 160039.02 as we can buy more products that cost less between pressure washer and go-kart. This is because the number of items that can be sold has increased rapidly within budget and constraint limits. Costs caused by Marketing 1 constraint have been reduced. At the same time, you can see that the number of generators and water pumps has increased significantly. If I had to do decision making at the company, I could argue that based on the first original report, the price of the pressure washer should be lowered. If so, with the rationale that this will bring a lot of profit.

6. <u>Should the company allocate additional money? If yes, how much additional investment do you recommend, and how much should the company expect its net monthly profit to increase as a consequence of this increase?</u>

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170000	0.56	170000	428.8	56225
G8	Length of Pallet Total	12300	3.84	12300	6078.38	30.95
K11	Pressure washer + go Kart Quantity	155.18	0	0	1.63	1E+30
K13	Generator Quantity	237.77	-33.68	0	27.92	974.12

The cost total is currently 170,000, which is 100% of budget. The answer to the question of increasing the budget is 'Yes'. Because with 170,000 we already use all budget, if we add more, we can get more profit on this side. The table shows that we can add 428.8 budget with the constraints we have. The total cost with allowable increase is 170,000 + 428.8 which is 170,428.8 In the table above, you can see that the additional profit which is the shadow price when the cost increases by one unit, is marked as 0.56. If the company increase the 428.8 budget, the added profit of the company would be 0.56 * 428.8 which is 240.13.

Result of additional budget

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	0	-16.47582609	169.99	16.47582609	1E+30
D12	Quantity go-Kart	154.0173913	0	359.99	205.8402439	20.28608137
E12	Quantity Generator	239.5826087	0	290.99	56.39047619	131.8664063
F12	Quantity Water Pump	119.7913043	0	142.99	112.7809524	102.9368644

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170428.8	0	170428.8	1E+30	2.86218E-11
G8	Length of Pallet Total	12300	11.56827826	12300	2.06566E-12	12300
K11	Pressure washer + go Kart Quantity	154.0173913	-146.7730435	0	215.25	1.08745E-13
K13	Generator Quantity	239.5826087	-42.24886957	0	291.8644068	1.8634E-12

Optimal inventory level for four products & monthly report				
	Pressure washer	go-Kart	Generator	Water Pump
Quantity	0 -> 0	155.18 -> 154.02	237.77 -> 239.58	118.88 -> 119.79
Total Cost	170,428.8/ 170,428.8	Pallet Usage	12,300/ 12,300	
		Total Profit	142050.70 -> 142289.82	

Based on the sensitivity report, I manually set the budget as 170,428.8 which is total budget plus allowable increase. It makes slight increase in quantity of 3 products which were non-zero value in the original report. You can see that the total profit has risen slightly from 142,050.70 to 142,289.82 as the quantity of 3 product slightly increased. It's almost same as 0.56 (Shadow price) * 428.8 (Allowable Increase) which is 240.13. This is because the number of items that can be sold has increased slowly within only small additional budget. If I had to do decision making at the company, this method may not be recommended because the increase in profit, 239.12, is not greater than the increase in budget, 428.8. However, if the company is in a situation where you need to take market share from another competitor, I can recommend it.

7. Should the company rent a more warehouse? Indicate the ideal size of your recommended warehouse in square feet and indicate how much this change in the size of the warehouse will contribute to the monthly profit.

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170000	0.56	170000	428.8	56225
G8	Length of Pallet Total	12300	3.84	12300	6078.38	30.95
K11	Pressure washer + go Kart Quantity	155.18	0	0	1.63	1E+30
K13	Generator Quantity	237.77	-33.68	0	27.92	974.12

The length of pallet total is currently 12,300. which is same as the constraint. And allowable increase is 6078.38 square feet with other constraints fixed. If I have to order in shelves numbers, it would be around 40 ~ 41, because one shelf is 150 square feet each. The answer to the question of larger warehouse is 'Yes', because we already used whole capability of warehouse. In other words, the warehouse is full to maximize profit. In the table above, you can see that the additional profit which is the shadow price when the length of pallet increases by one unit, is marked as 3.84. This means that we can get 3.84 profit by increase the one space of warehouse. Although it doesn't calculate each of the products that can be called integers, it's easy to see how much profit increases when one unit goes up. We have three lengths of four products. When making the final decision, we should consider whether other constraints are not violated. Adjust other constraints if necessary. Perhaps if we increase warehouse and sell more products, cost will increase, so we may need to increase budget accordingly. If the company increase the 6078.38 budget, the added profit of the company would be 3.84 * 6078.38 which is 23,340.98.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
C12	Quantity Pressure washer	0	-151.0821622	169.99	151.0821622	1E+30
D12	Quantity go-Kart	459.4594595	0	359.99	1E+30	76.73878564
E12	Quantity Generator	0	0	290.99	98.20490541	1E+30
F12	Quantity Water Pump	0	-196.4098108	142.99	196.4098108	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
G6	Cost Total	170000	0.972945946	170000	3.36513E-11	170000
G8	Length of Pallet Total	18378.37838	0	18378.37838	1E+30	3.63798E-12
K11	Pressure washer + go Kart Quantity	459.4594595	0	0	321.6216216	1E+30
K13	Generator Quantity	0	-107.9178378	0	298.9949749	0

Optimal inventory level for four products & monthly report				
	Pressure washer	go-Kart	Generator	Water Pump
Quantity	0 -> 0	155.18 -> 459.46	237.77 -> 0	118.88 -> 0
Total Cost	170,000/ 170,000	Pallet Usage	18,378.38/ 18,378.38	
		Total Profit	142050.70 -> 165400.81	

Based on the sensitivity report, I manually set the space as 18,378.38 which is total pallet plus allowable increase. It makes only one product out of four available for sale. The product is go-Kart and quantity is 459.46. You can see that the total profit has risen significantly from 142,050.70 to 165,400.81 as the quantity of go-Kart increased. It's almost same as 3.84 (Shadow price) * 6078.38 (Allowable Increase) which is 23,340.98. This is because go-kart takes up a lot of pallets and at the same time has the highest profit (objective coefficient) per unit at 359.99. Even though only go-kart is sold, marketing constraints 1 & 2 are satisfied, so only go-kart products are sold. If I had to do decision making at the company, I would recommend this method to companies. Because you can make the best profit. Only if the increase in profit outweighs the cost of increasing the pallet. However, when the company loses market share in other products, the company's stability or long-term disadvantages will be additionally reviewed with this decision.

4. CONCLUSION

This was one of the most exciting challenges. It was a proud and fun thing to be able to increase the company's profit through analysis. I have come to three conclusions. One was to increase the cost of the product. This resulted in a second large profit of 160039.02. In this case, the company need to analyze the factors for demand. The second was to increase the budget. This resulted in the smallest profit of 142,289.82. Of course, if other constraints are different, I think it can create different results. Finally, it was a choice to increase the size of the warehouse. This resulted in the most profit with 165400.81. I thought it might be similar to analysis in real life. I also thought about the problems that could arise from concentrating sales on just one product in this case.

I would like to analyze what will increase the company's profit in any company in the future. And based on that conclusion, I want to contribute to the company, society and country.

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