

Full marks are not necessarily awarded for a correct answer with no working. Answer supported by working and/or explanations. Where an answer is incorrect, some mark for a correct method, provided this is shown by written working. You are therefore advised to show your working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.

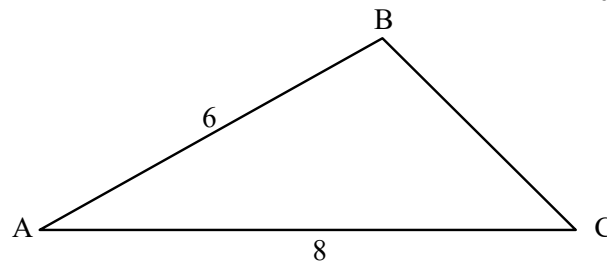


diagram not to scale

(a) Given that $\cos \hat{A} = \frac{5}{6}$, find the value of $\sin \hat{A}$.

[3]

(b) Find the area of triangle ABC.

[2]

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2. [Maximum mark: 5]

Let A and B be events such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.6$.
Find $P(A | B)$.

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3. [Maximum mark: 5]

- (a) Show that $(2n - 1)^2 + (2n + 1)^2 = 8n^2 + 2$, where $n \in \mathbb{Z}$.
- (b) Hence, or otherwise, prove that the sum of the squares of any two consecutive integers is even.

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4. [Maximum mark: 5]

Let $f'(x) = \frac{8x}{\sqrt{2x^2 + 1}}$. Given that $f(0) = 5$, find $f(x)$.



The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x+5$.

- (a) Show that $(g \circ f)(x) = 2x + 11$. [4]
- (b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a . [3]



6. [Maximum mark: 8]

(a) Show that $\log_9 (\cos 2x + 2) = \log_3 \sqrt{\cos 2x + 2}$.

(b) Hence or otherwise solve $\log_3 (2 \sin x) = \log_9 (\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$.

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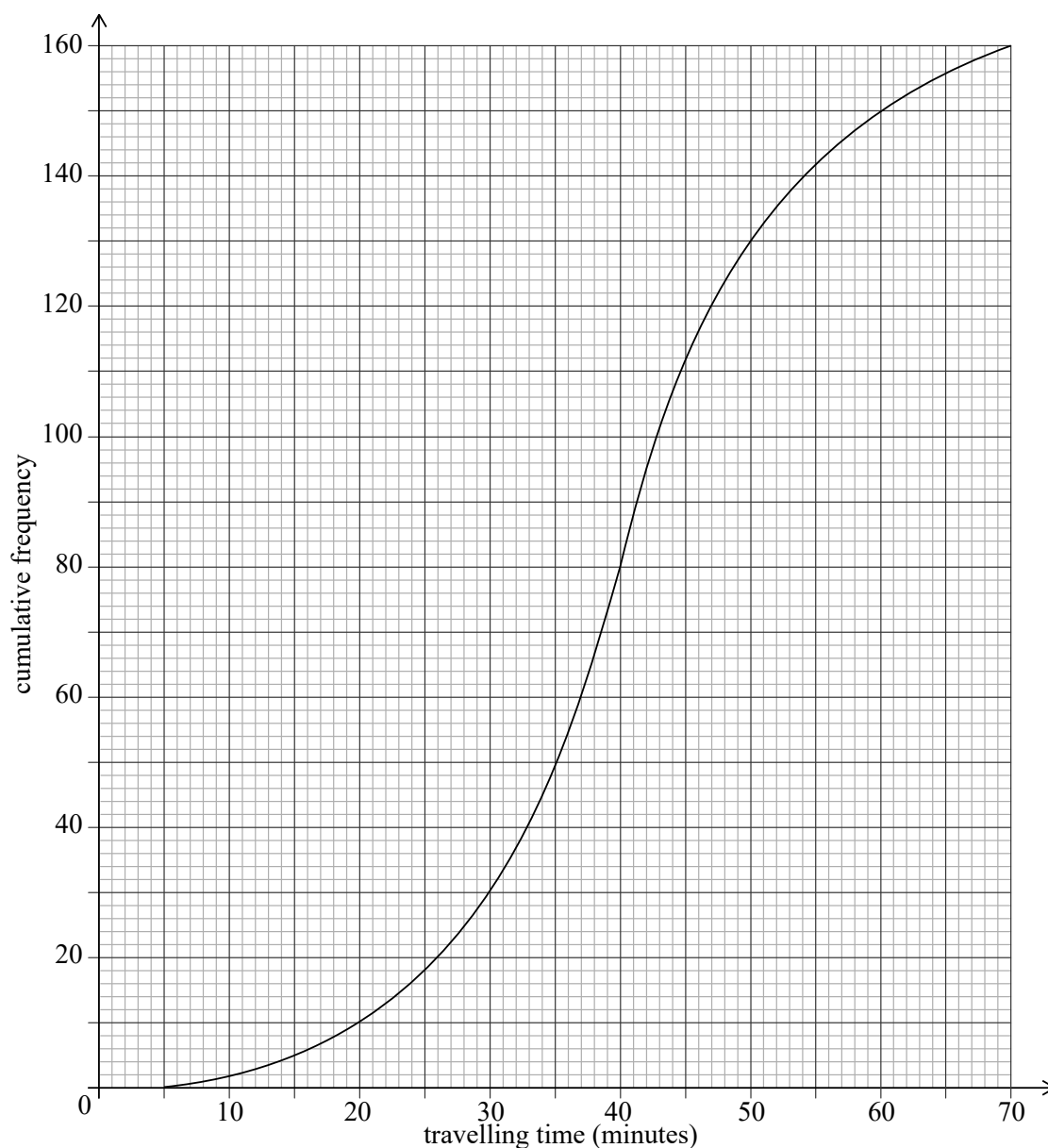
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a

7. [Maximum mark: 15]

A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



(This question continues on the following page)



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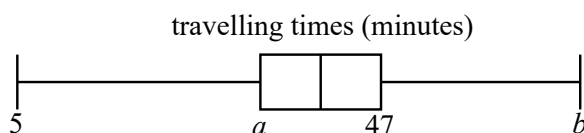
(Question 7 continued)

- (a) Find the median number of minutes spent traveling to work.
- (b) Find the number of employees whose travelling time is within 15 minutes of the median. [3]

Only 10% of the employees spent more than k minutes traveling to work.

- (c) Find the value of k . [3]

The results of the survey can also be displayed on the following box-and-whisker diagram.



- (d) Write down the value of b . [1]
- (e) (i) Find the value of a . [4]
- (ii) Hence, find the interquartile range. [4]

Travelling times of less than p minutes are considered outliers.

- (f) Find the value of p . [2]

8. [Maximum mark: 16]

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$.

- (a) Find $f'(x)$. [2]

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

- (b) Find the value of a and the value of b . [3]
- (c) (i) Sketch the graph of $y = f'(x)$. [2]
- (ii) Hence explain why the graph of f has a local maximum point at $x = a$. [2]
- (d) (i) Find $f''(b)$. [4]
- (ii) Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$.

The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at the point (p, q) .

- (e) Find the value of p and the value of q . [5]



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9. [Maximum mark: 16]

Let $f(x) = \frac{\ln 5x}{kx}$ where $x > 0$, $k \in \mathbb{R}^+$.

(a) Show that $f'(x) = \frac{1 - \ln 5x}{kx^2}$. [3]

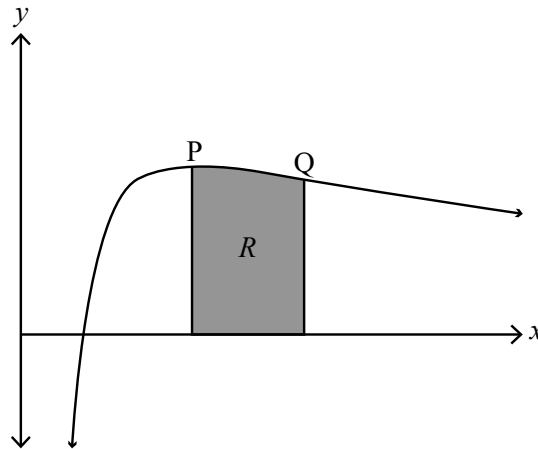
The graph of f has exactly one maximum point P .

(b) Find the x -coordinate of P . [3]

The second derivative of f is given by $f''(x) = \frac{2 \ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q .

(c) Show that the x -coordinate of Q is $\frac{1}{5}e^{\frac{3}{2}}$. [3]

The region R is enclosed by the graph of f , the x -axis, and the vertical lines through the maximum point P and the point of inflexion Q .



(d) Given that the area of R is 3, find the value of k . [7]



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Answers written on this page
will not be marked.

