HW2 Jin Kweon (3032235207)

Jin Kweon

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Problem 3

Part a

```
summary(soi)
```

```
##      Min.  1st Qu.   Median     Mean  3rd Qu.     Max.
## -1.00000 -0.18000  0.11500  0.08004  0.36600  1.00000
```

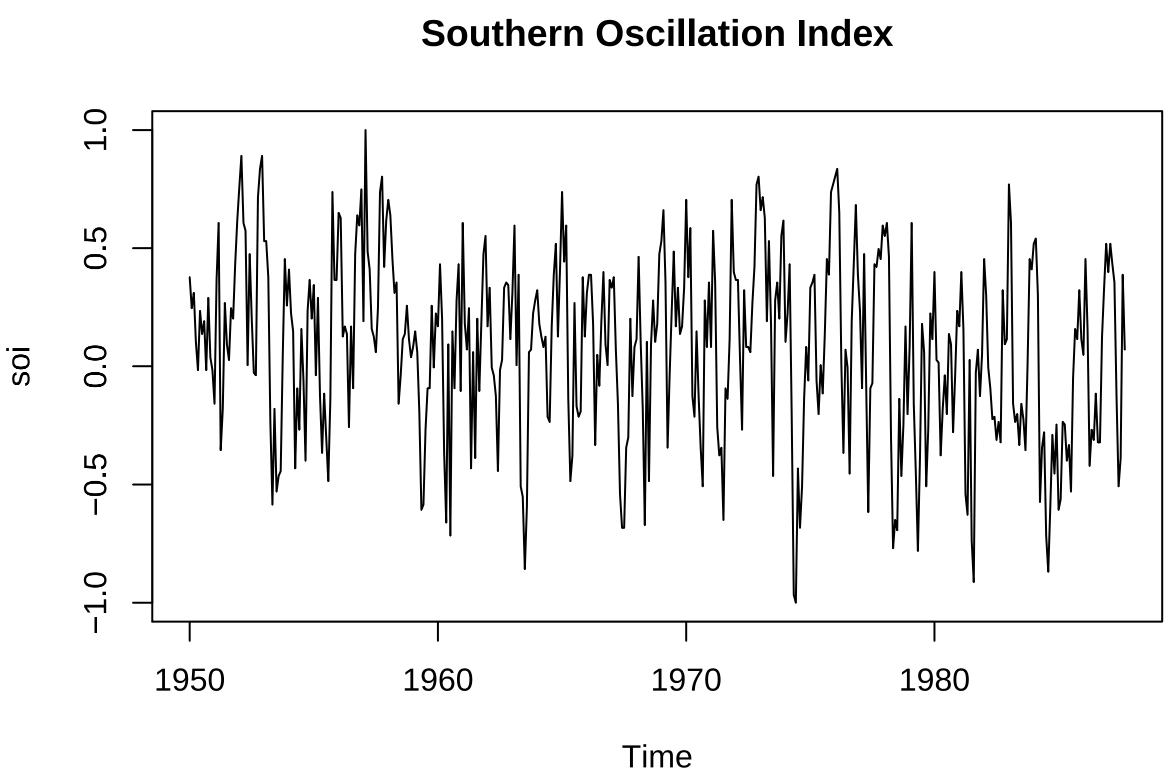
```
str(soi)
```

```
##  Time-Series [1:453] from 1950 to 1988: 0.377 0.246 0.311 0.104 -0.016 0.235 0.137 0.191 -0.016 0.29
```

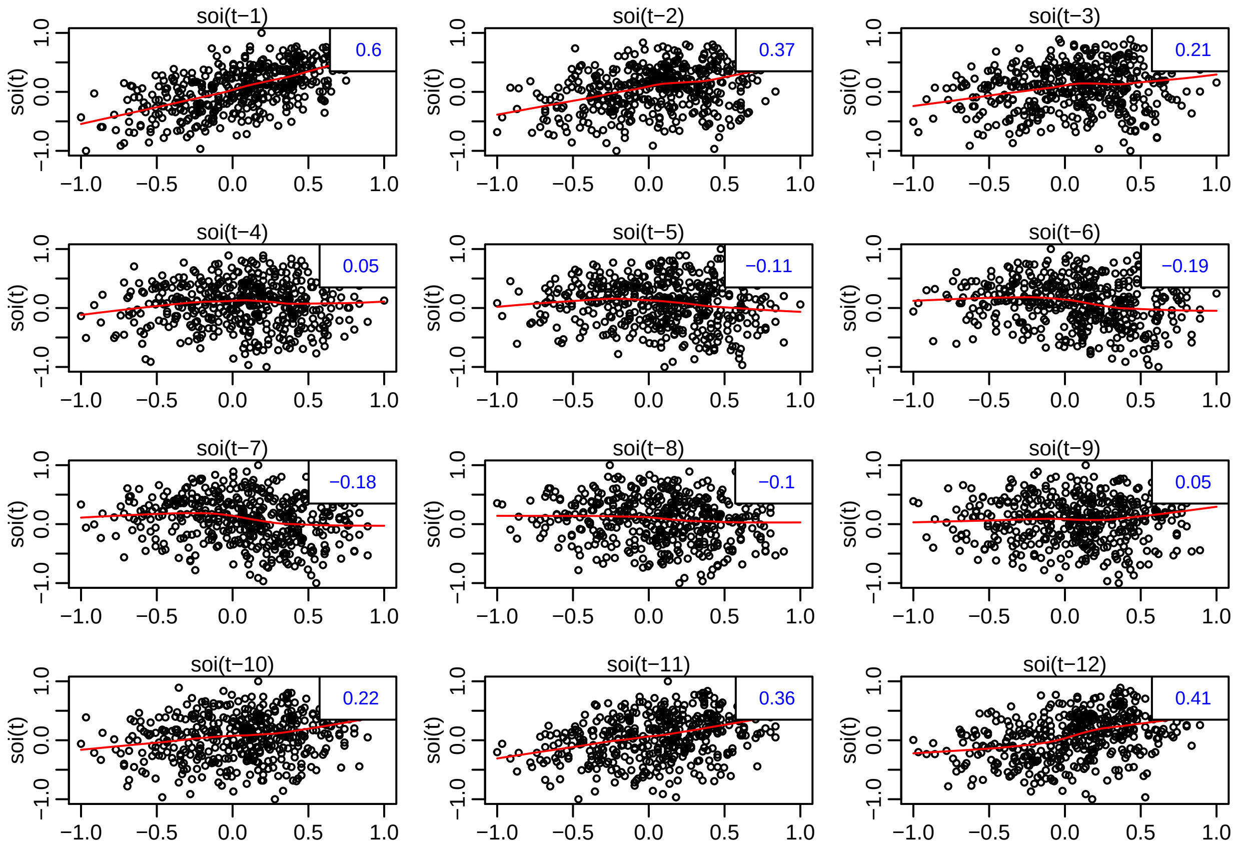
```
class(soi)
```

```
## [1] "ts"
```

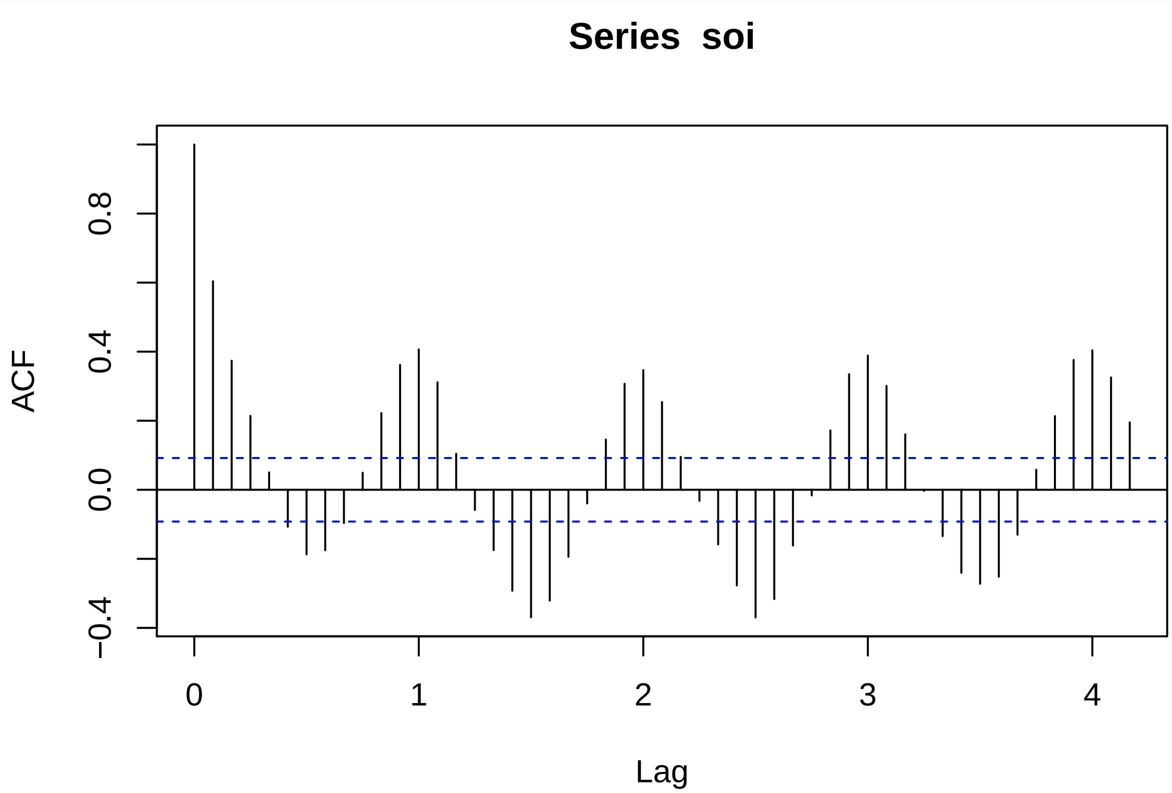
```
plot.ts(soi, main = "Southern Oscillation Index")
```



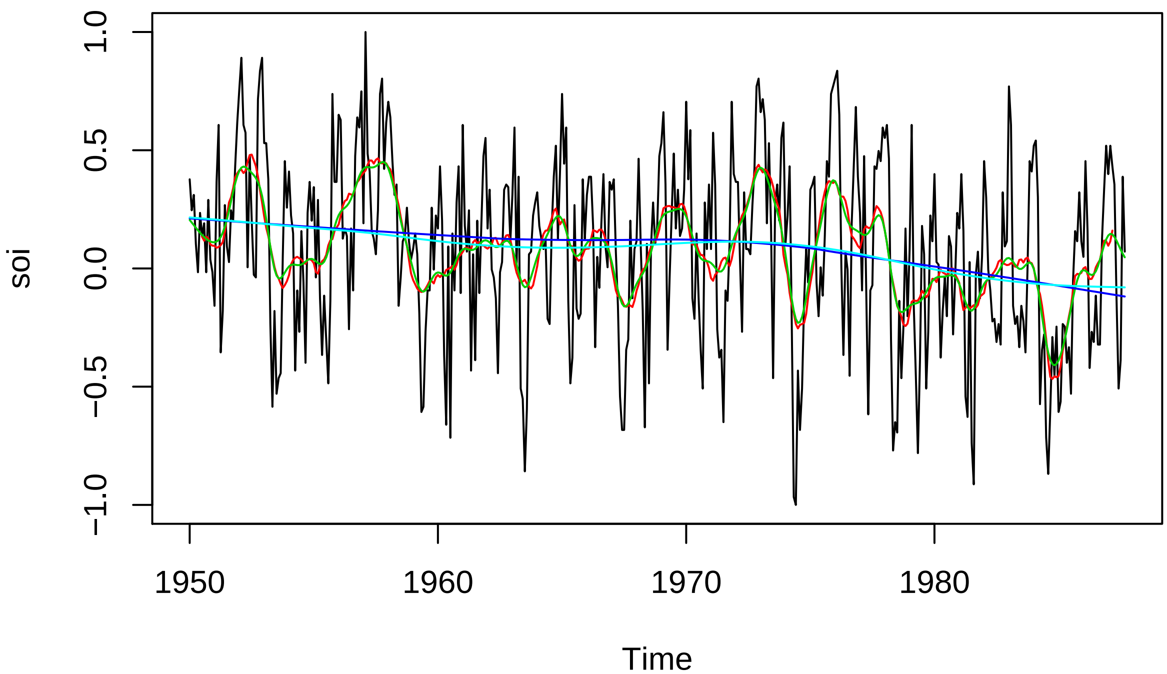

```
#Univariate scatterplot matrix
lag1.plot(soi, 12)
```



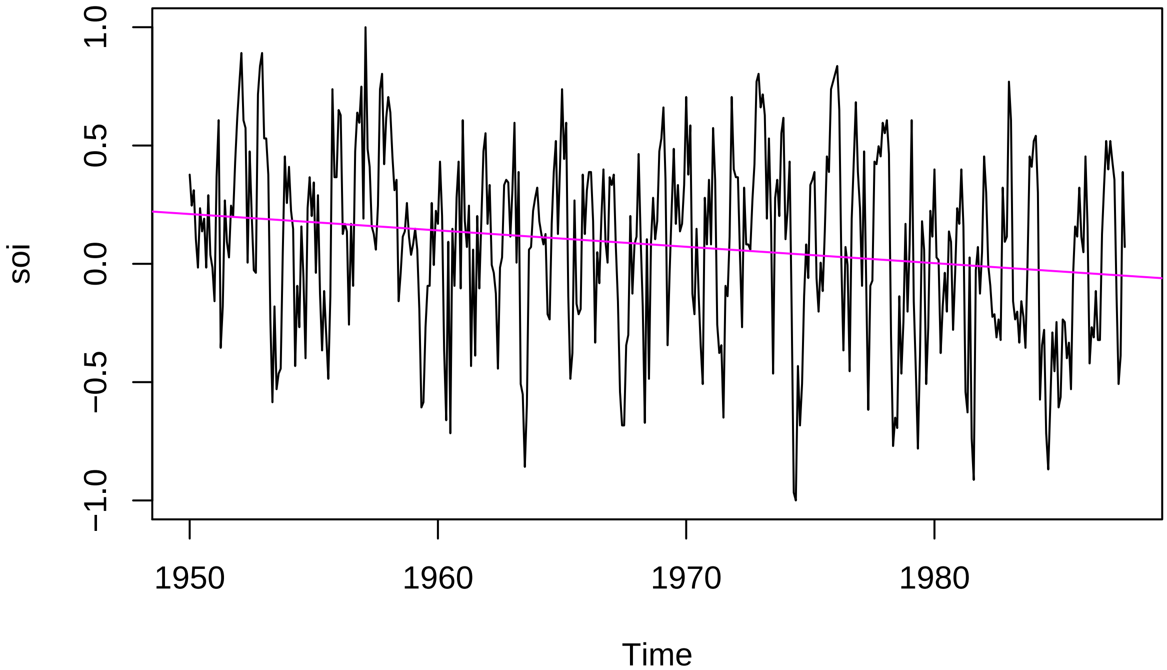

```
#ACF
acf(soi, lag.max = 50)
```



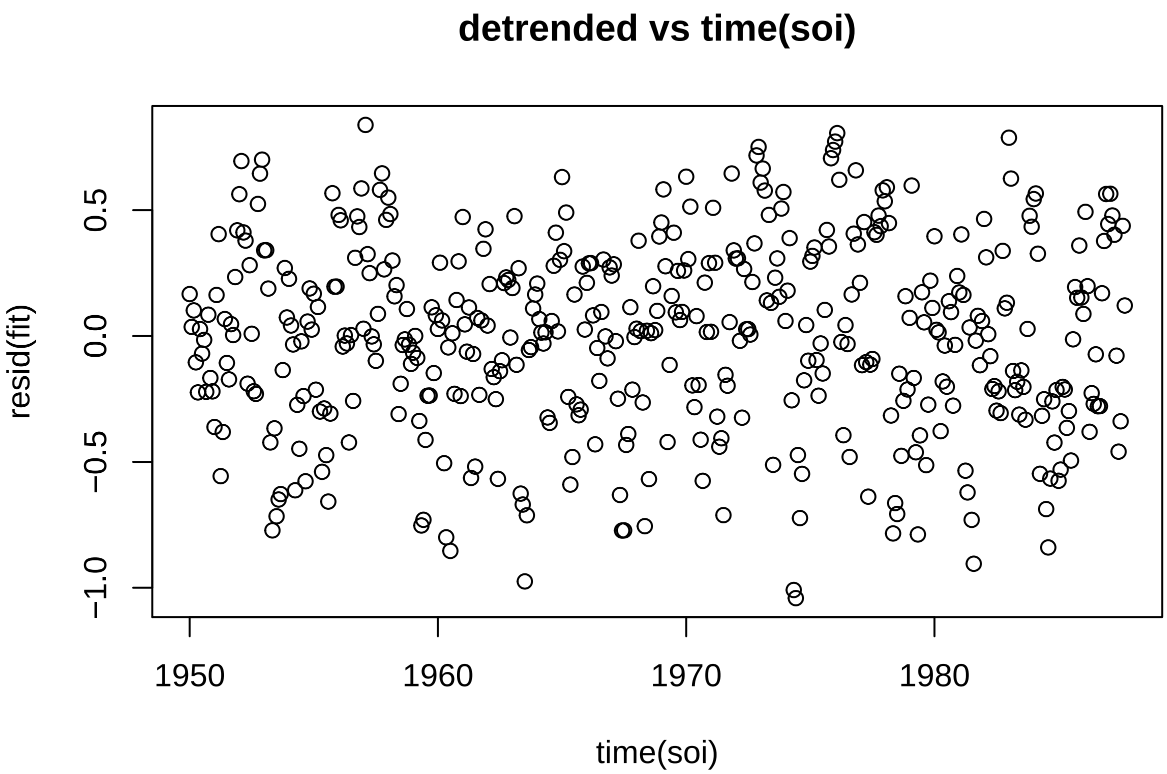
```r
plot(soi)
#moving average smoothings
soif <- filter(soi, sides = 2, filter = c(0.5, rep(1, 11), 0.5)/12)
lines(soif, col = 2)
#lines(ksmooth(time(soi), soi, kernel = "box", bandwidth = 1), col = 3) -> other way
#Kernel smooth
lines(ksmooth(time(soi), soi, kernel = "normal", bandwidth = 1), col = 3)
#Lowess
lines(lowess(soi), col = 4)
#smooth splines
lines(smooth.spline(time(soi), soi, spar = 1), col = 5)
```



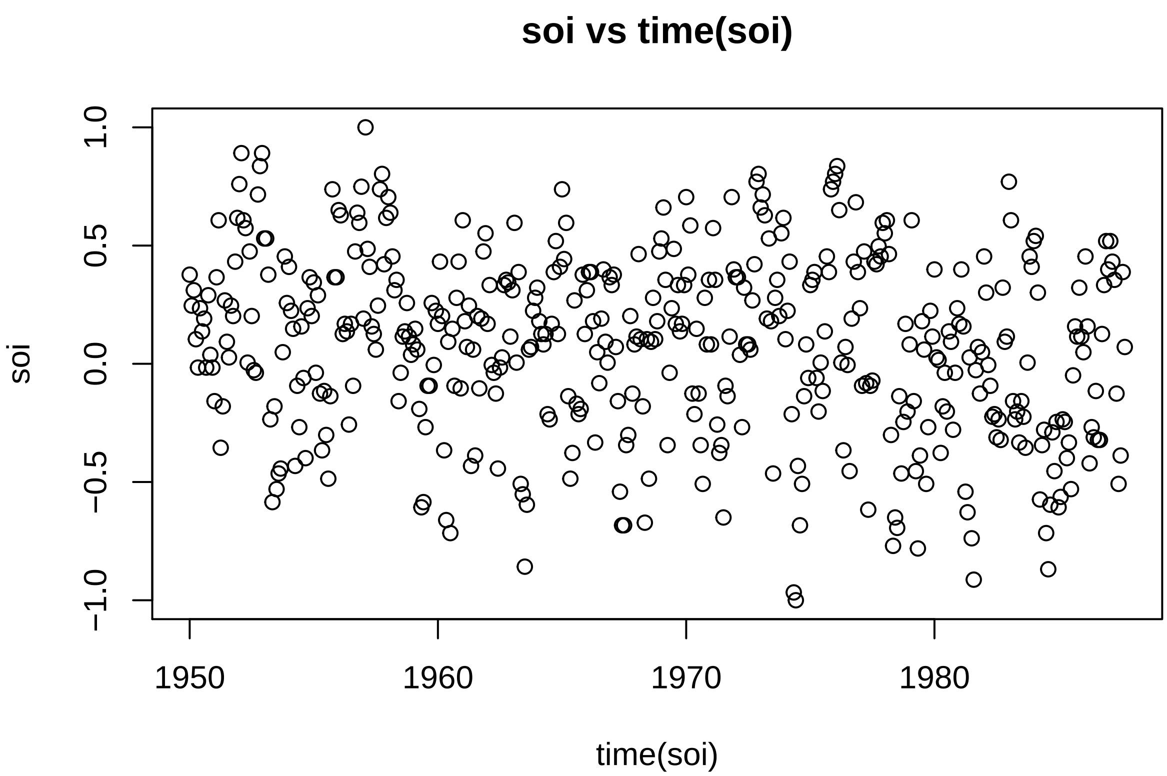

```r
plot(soi)
#Polynomial - linear
fit <- lm(soi ~ time(soi))
abline(fit, col = 6)
```



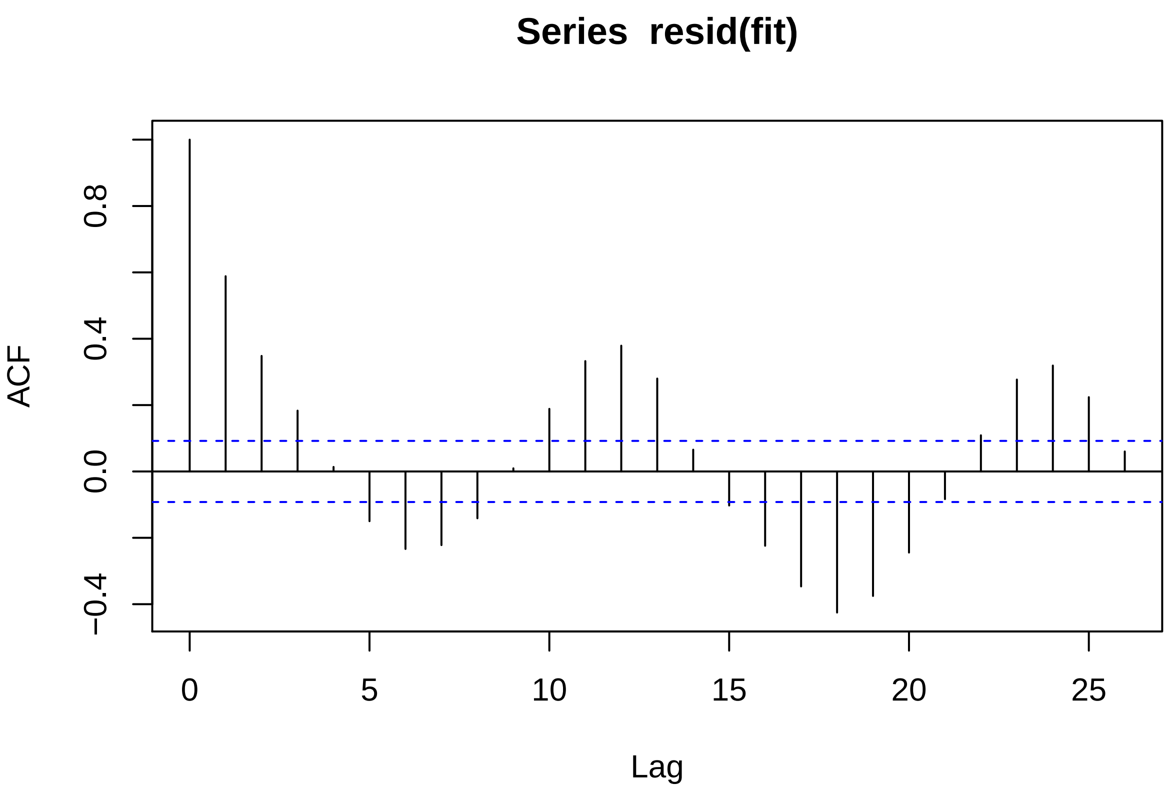

```r
plot.ts(time(soi), resid(fit), main = "detrended vs time(soi)")
```




```r
plot.ts(time(soi), soi, main = "soi vs time(soi)")
```

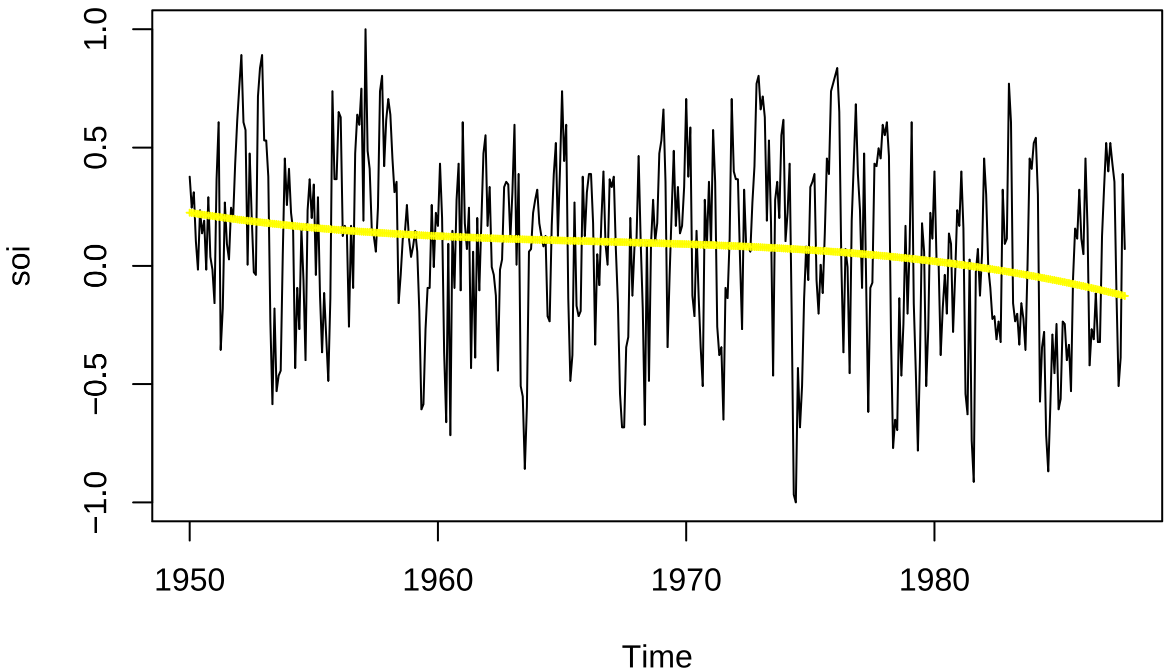



```r
acf(resid(fit))
```

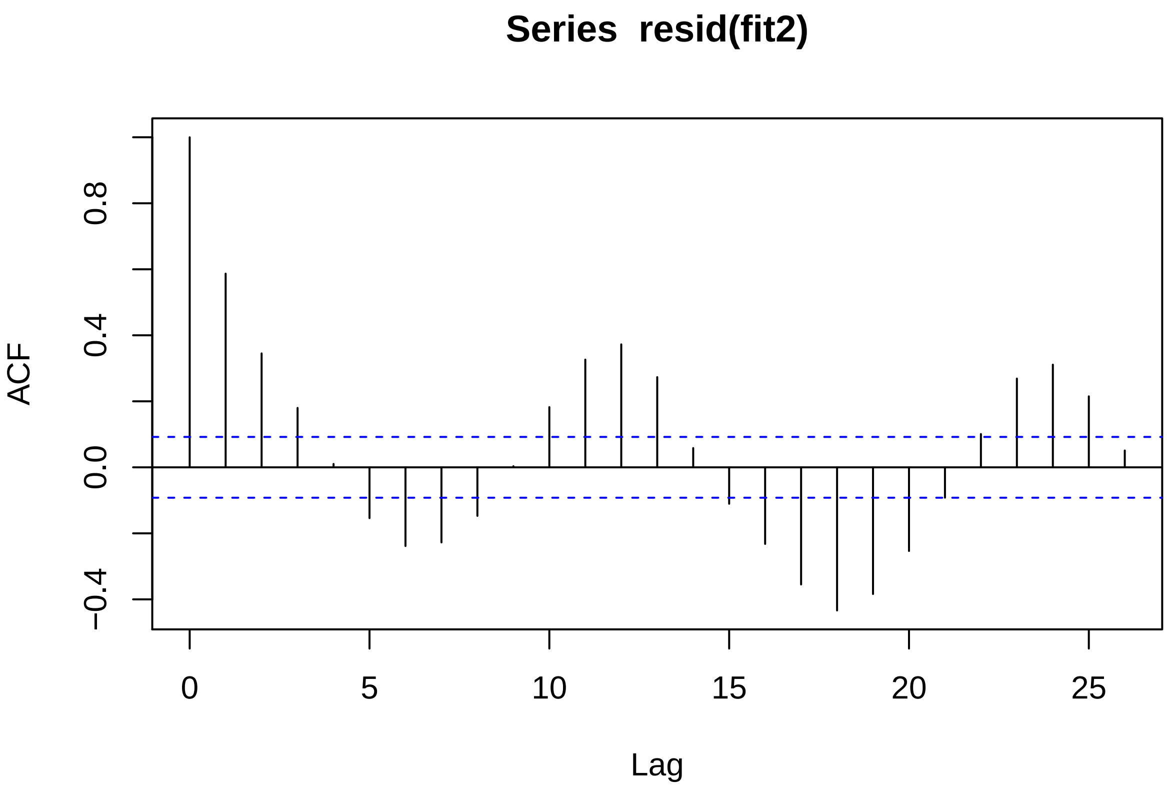



```r
#Polynomial - cubic
plot(soi)
fit2 <- lm(soi ~ time(soi) + I(time(soi)^2) + I(time(soi)^3))
```

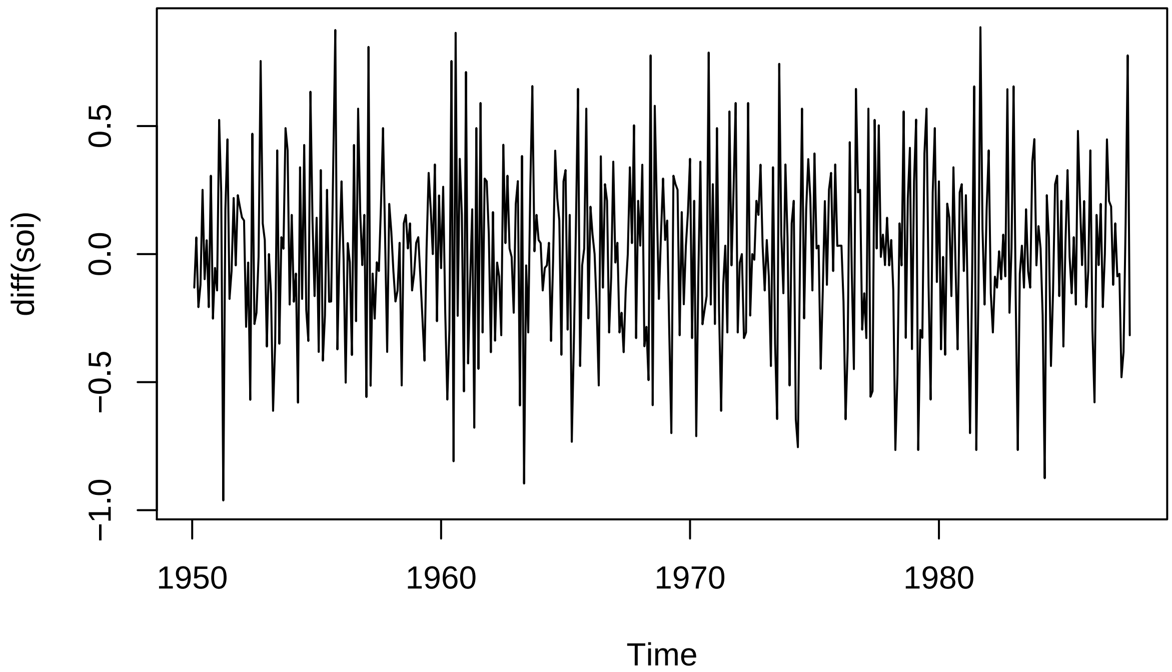
```r
points(time(soi), fitted(fit2), col = 7, pch = 3, cex = 0.3)
```



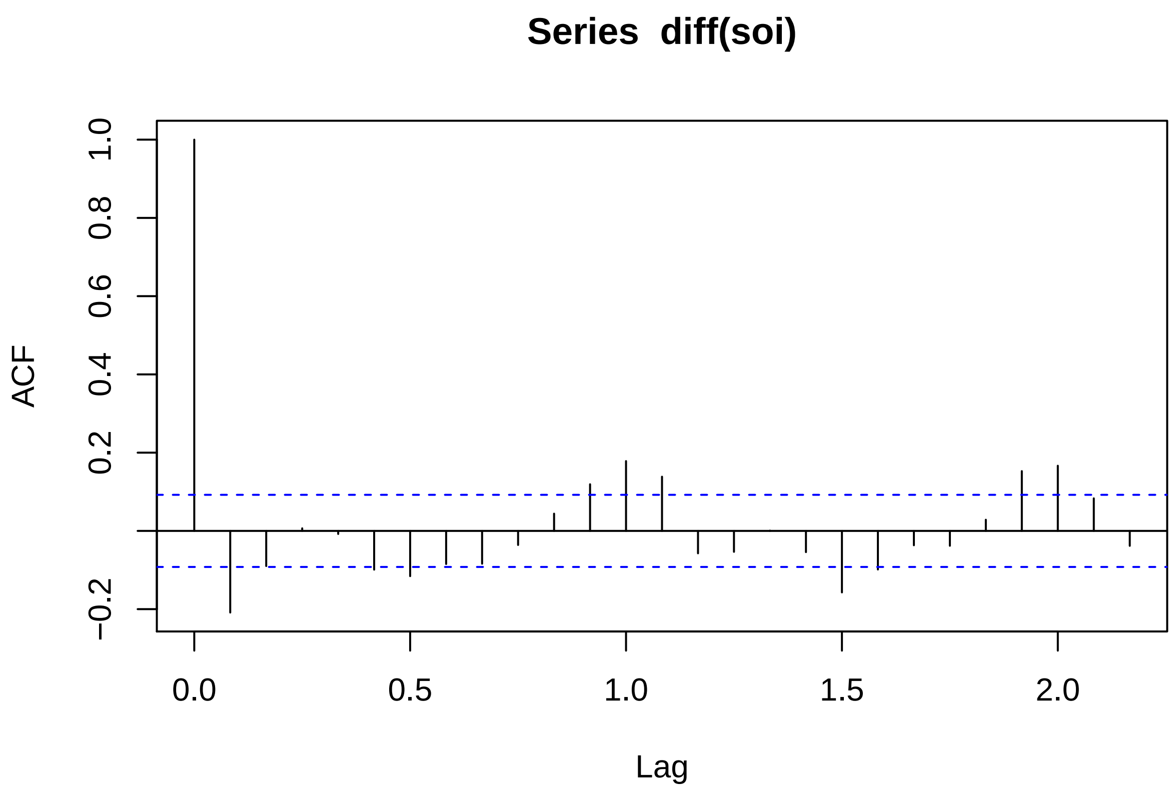

```r
acf(resid(fit2))
```



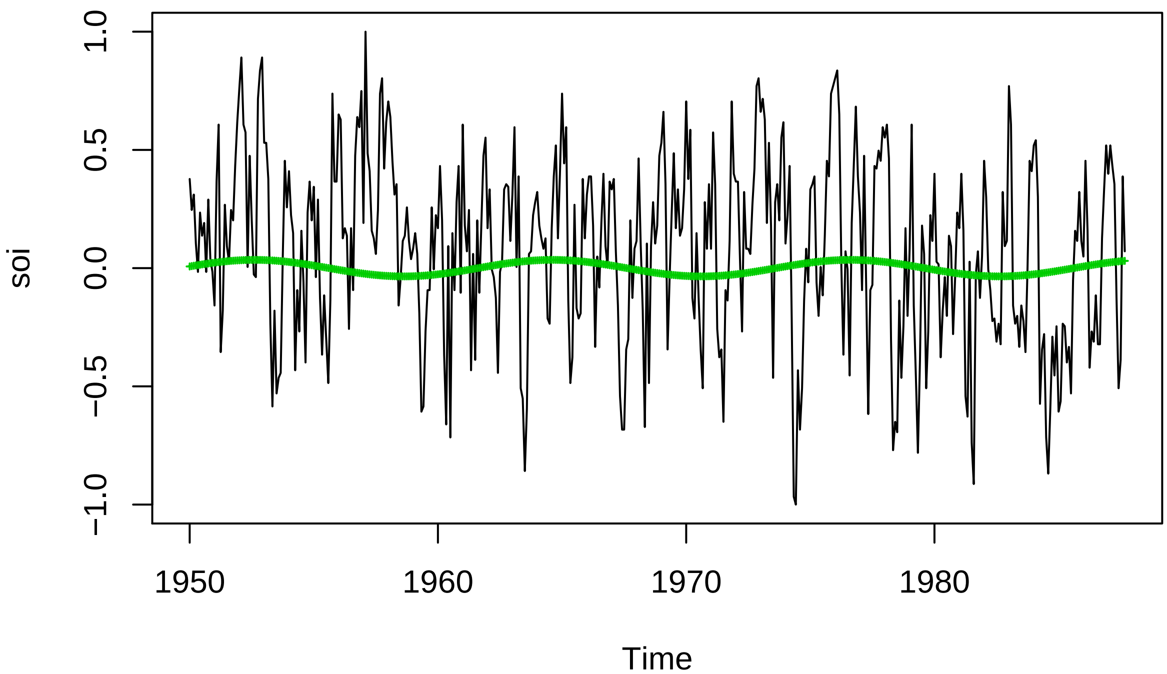

```r
#Differencing
plot(diff(soi))
```



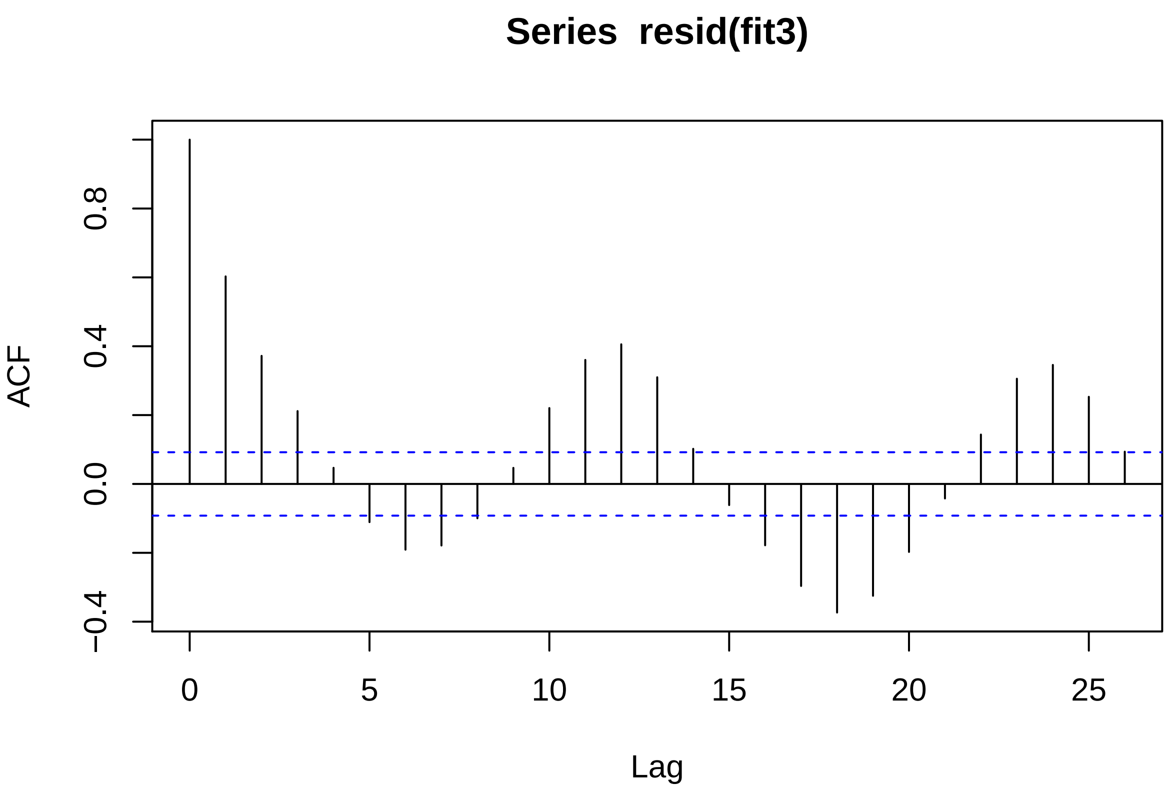

```
acf(diff(soi))
```




```
#Periodic Signal
z1 <- cos(2*pi*(as.vector(time(soi)))/12)
z2 <- sin(2*pi*(as.vector(time(soi)))/12)
fit3 <- lm(soi ~ 0 + z1 + z2)
plot(soi)
points(time(soi), fitted(fit3), col = 3, pch = 3, cex = 0.3)
```




```
acf(resid(fit3))
```




```
#Compare with other Time Series
#summary(fit <- dynlm(rec ~ L(soi, 6)))
```

Comment: As the textbook says, the plot of SOI shows repetitive behavior, with regularly repeating cycles. These periodic behaviors are useful to find the regular pattern and rate/frequency of oscillation. These cycles might happen based on obvious annual cycles and non-obvoious slower frequency to repeat every 4-year. And, those are clear, by looking at ACF plots.

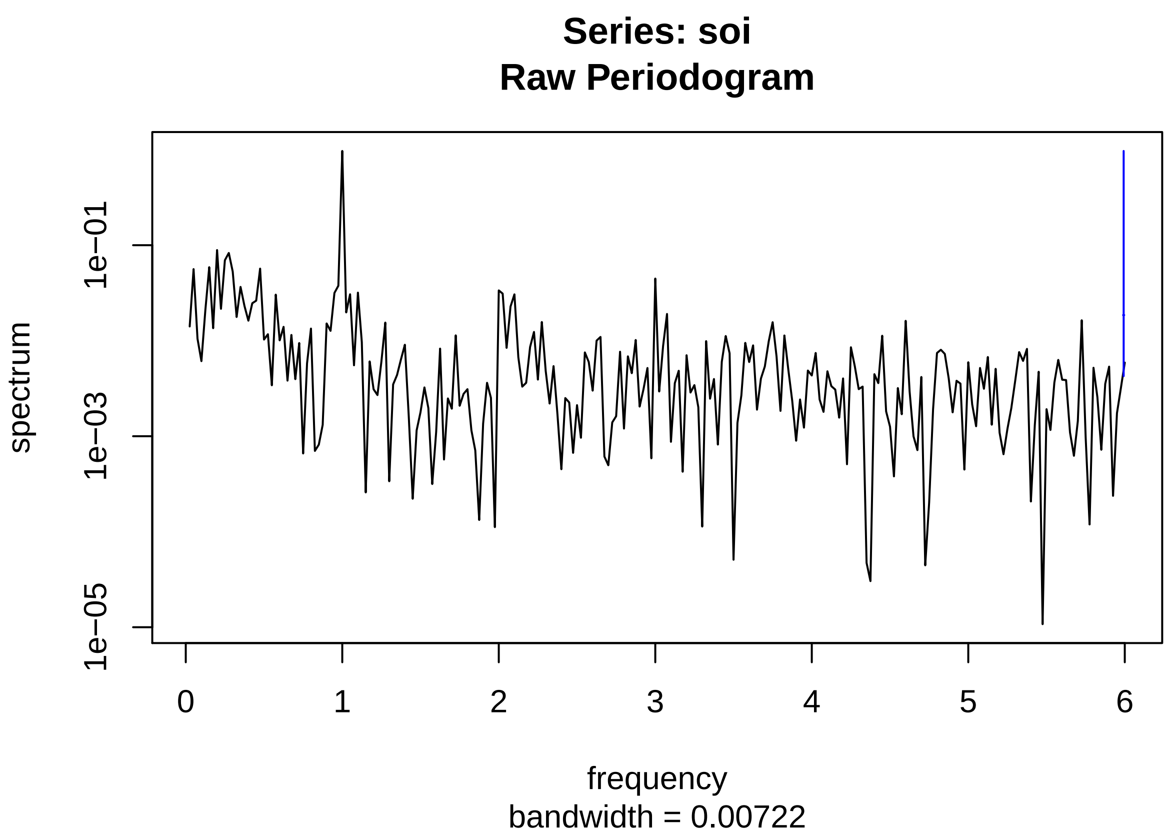
To chase the stationarity, I am mainly going to focus on detrending and differencing (skipping Box-Cox).

Since smoothing and splinings are pretty popular detrending methods, I used both. Linear regression detrending seems inappropriate here. However, first differencing did better job that linear regressoin detrending.

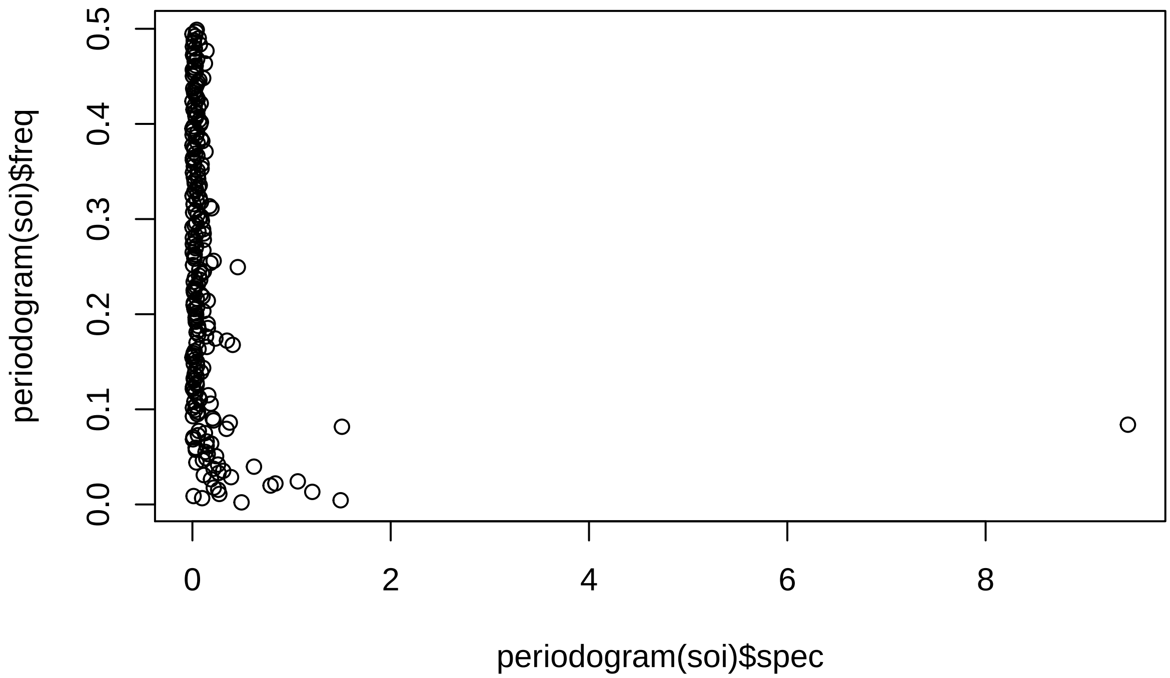
I found a trend of decreasing sea surface temperature over 30-year. And, as I already read in the textbook, there was up-and-down patterns for every 4-5 year. After 1980, it seems like the sea temeperature goes down more rapidly than the past 30 years.

Part b

```r
spec.pgram(soi)
```



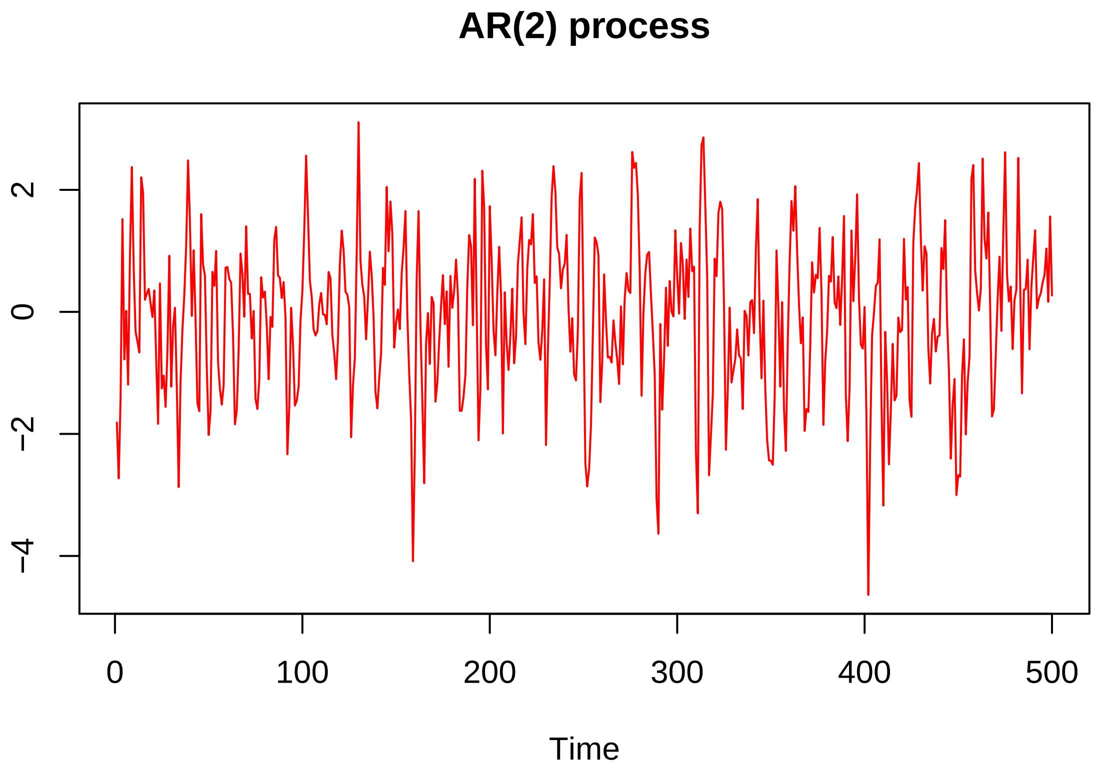

```r
plot.ts(periodogram(soi)$spec,
        periodogram(soi)$freq)
```



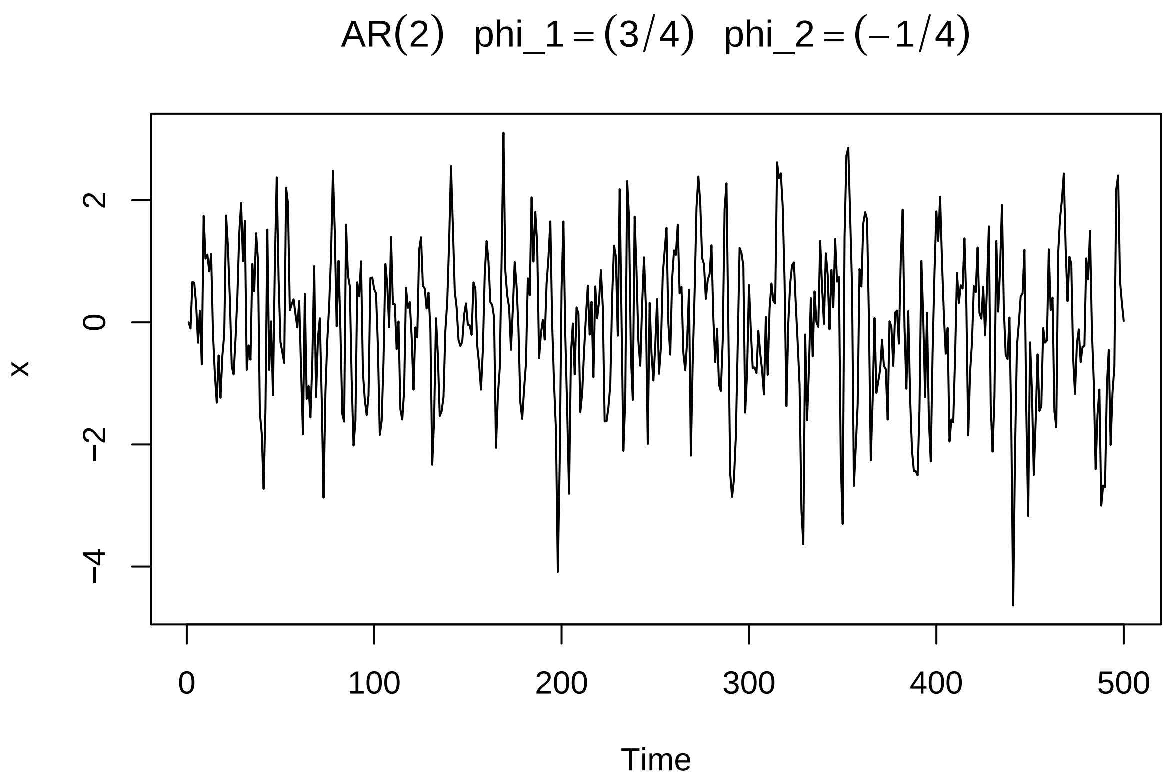
Problem 5

Part a

```
#Generate AR process
set.seed(100)
w <- rnorm(550, 0, 1)
x <- filter(w, filter = c((3/4), (-1/4)), method = "recursive", init = rnorm(2, 0, 1))[-(1:50)]

plot.ts(x, main = "AR(2) process", ylab = "", col = "red")
```




```r
set.seed(100)
#Other way using arima.sim
plot.ts(arima.sim(list(order = c(2, 0, 0), ar = c((3/4), (-1/4))), n = 500), ylab = "x",
     main = (expression(AR(2)~~~phi_1==(3/4)~~~phi_2==(-1/4))))
```

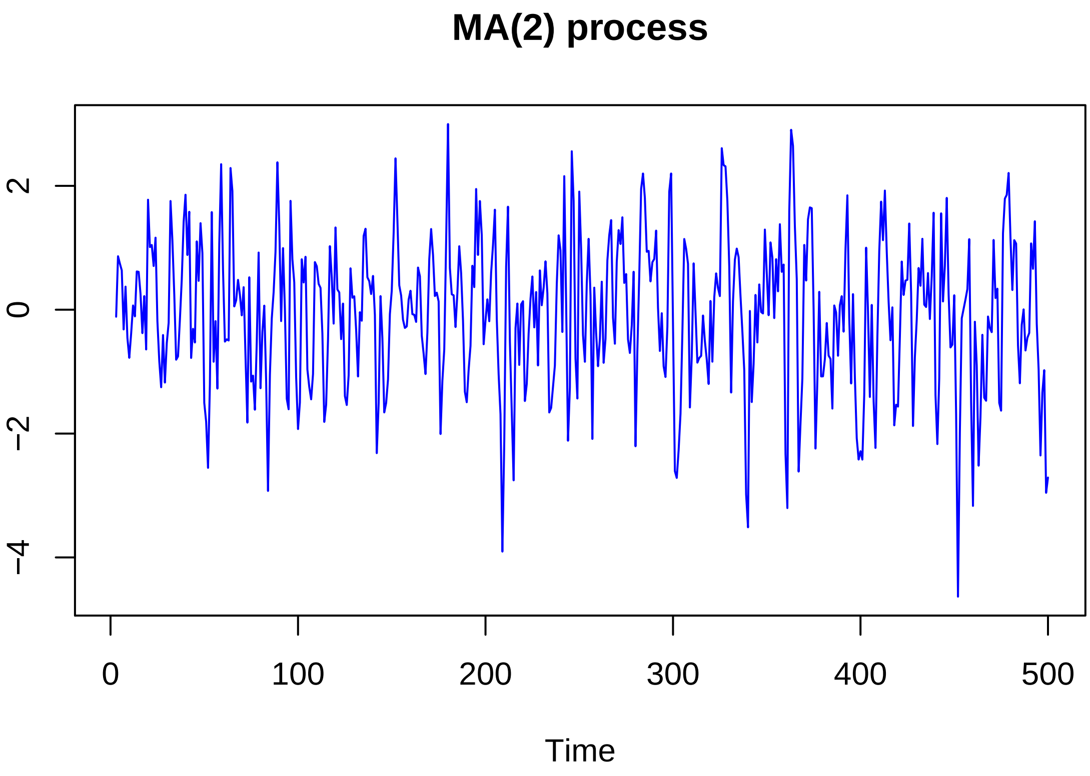


Comment: By the definition, an AR model of order p will be $x_t = \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} + w_t$, where x_t is stationary and w_t is a white noise given (zero mean and unit variance). And, AR(2) process is equivalent with ARMA(2,0). So, I would say my AR(2) process will be $x_t = \frac{3}{4}x_{t-1} + \frac{-1}{4}x_{t-2} + w_t$. And, this process is both invertible (can be express w_t with x in one-sided) and causal (as the absolute value of two roots are both 2).

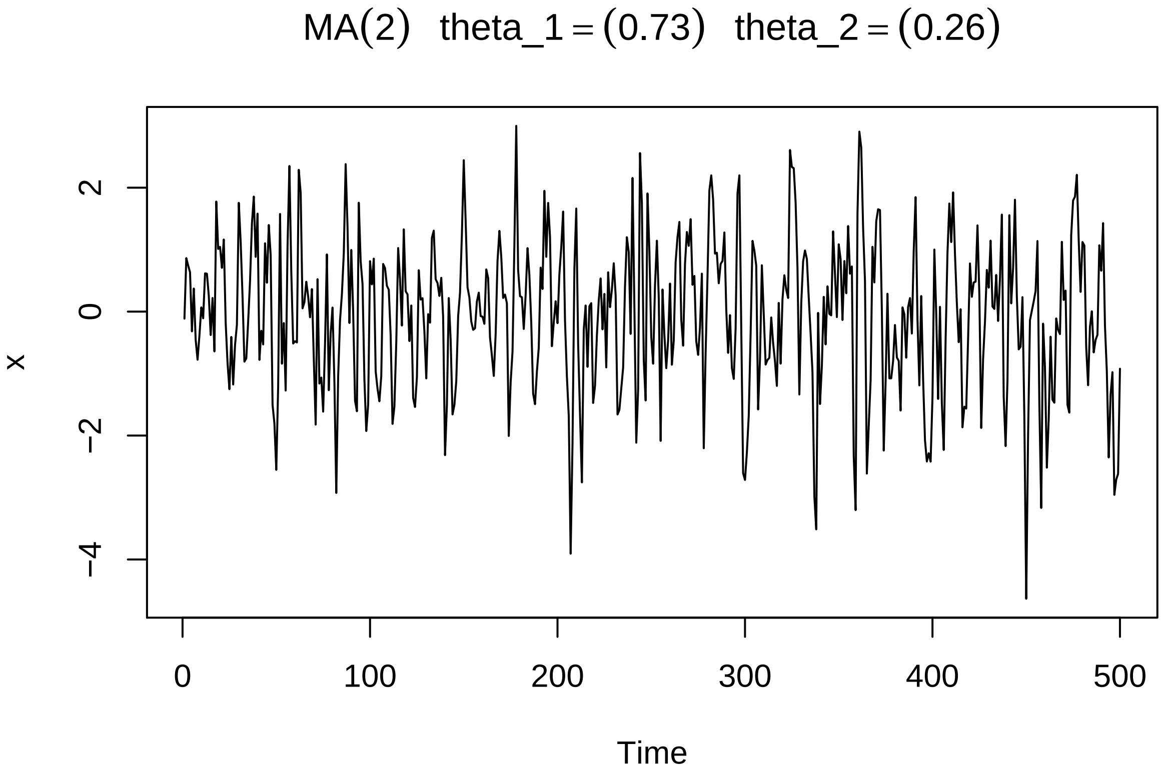
By the way, the two plots look slightly different, because when I used the filter funciton, I generated extra 50, and removed the first 50 to get better simulations.

Part b

```r
#Generate MA process
set.seed(100)
w <- rnorm(500, 0, 1)
x2 <- filter(w, filter = c(1, 0.73, 0.26), method = "convolution", sides = 1)

plot.ts(x2, main = "MA(2) process", ylab = "", col = "blue")
```



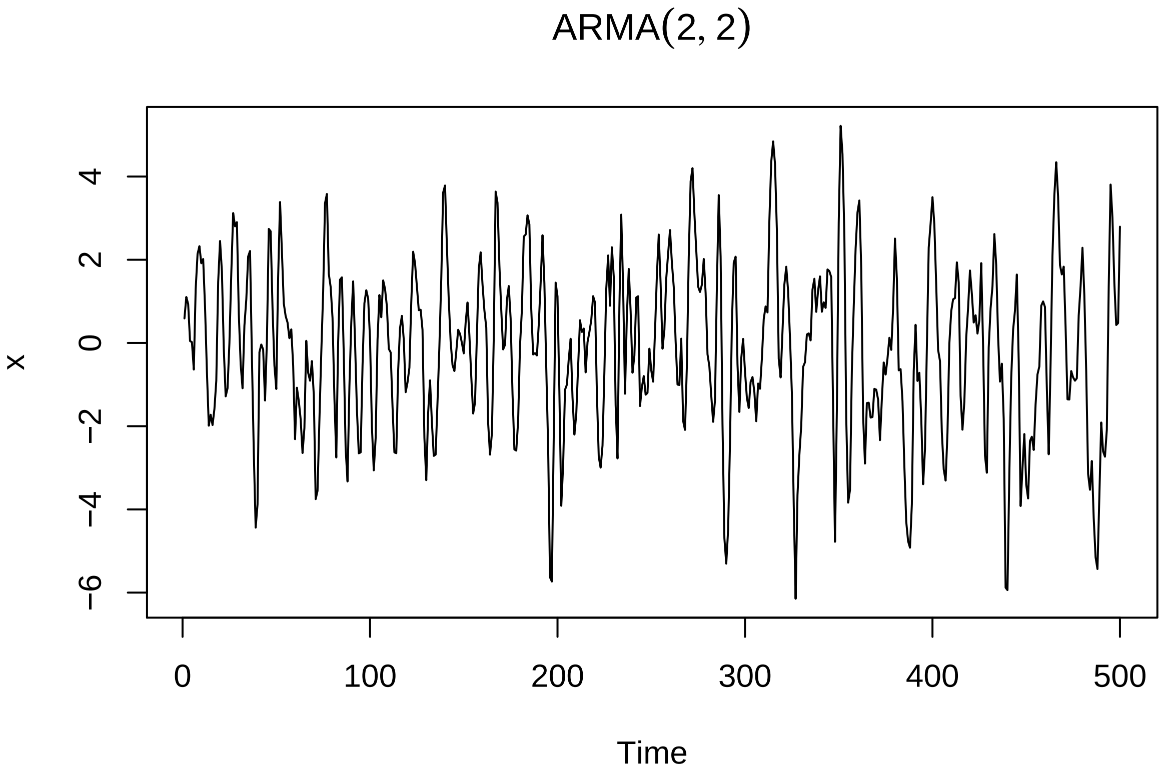

```r
set.seed(100)
#Other way using arima.sim
plot.ts(arima.sim(list(order = c(0, 0, 2), ma = c(0.73, 0.26)), n = 500), ylab = "x",
     main = (expression(MA(2)~~~theta_1==(0.73)~~~theta_2==(0.26))))
```



Comment: By the definition, an MA model of order q will be $x_t = \theta_1 w_{t-1} + \dots + \theta_q w_{t-q} + w_t$, where w_t is a white noise given (zero mean and unit variance). And, MA(2) process is equivalent with ARMA(0,2). So, I would say my MA(2) process will be $x_t = 0.73w_{t-1} + 0.26x_{t-2} + w_t$. And, this process is both invertible (as the absolute value of two roots are both around 1.9612) and causal (can be express x_t with w in one-sided).

Part c

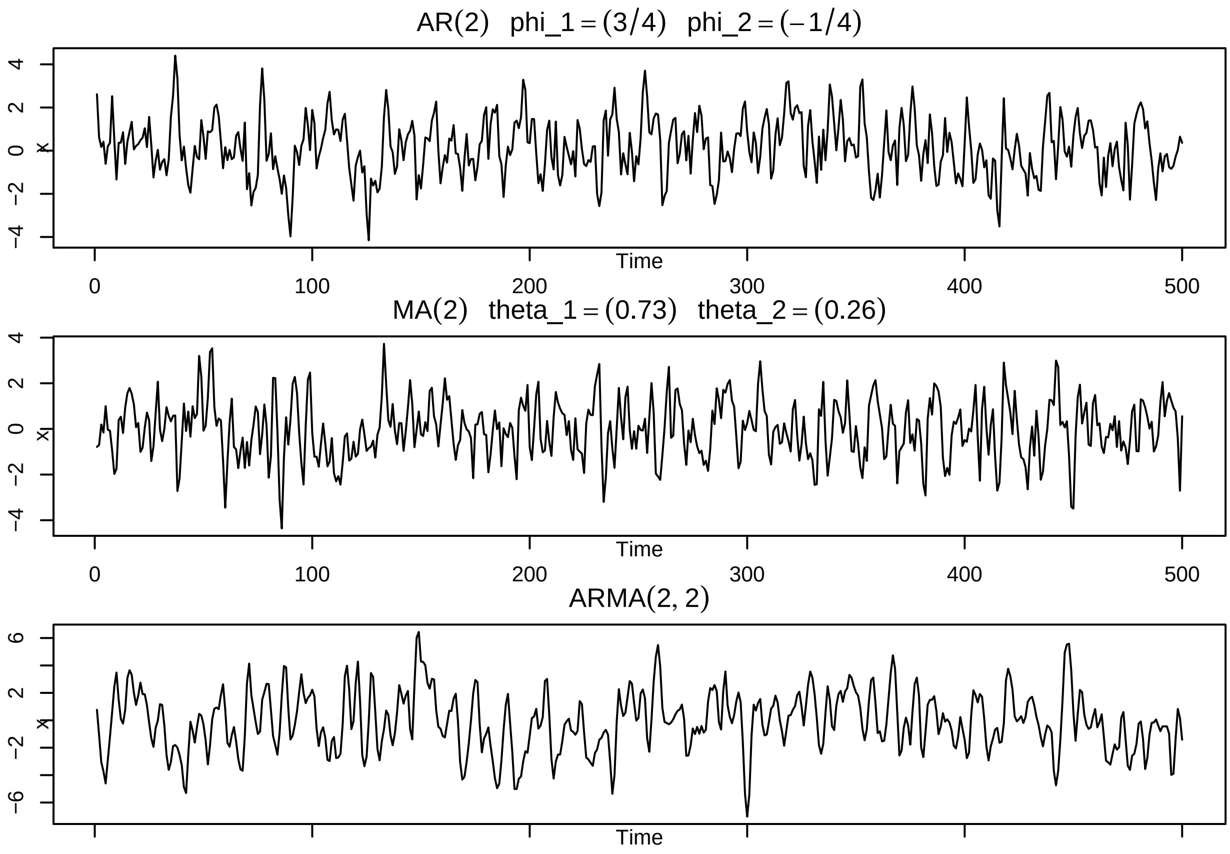
```
set.seed(100)
#Generate ARMA process
plot.ts(arima.sim(list(order = c(2, 0, 2), ar = c((3/4), (-1/4)), ma = c(0.73, 0.26)),
     n = 500), ylab = "x", main = (expression(ARMA(2,2))))
```



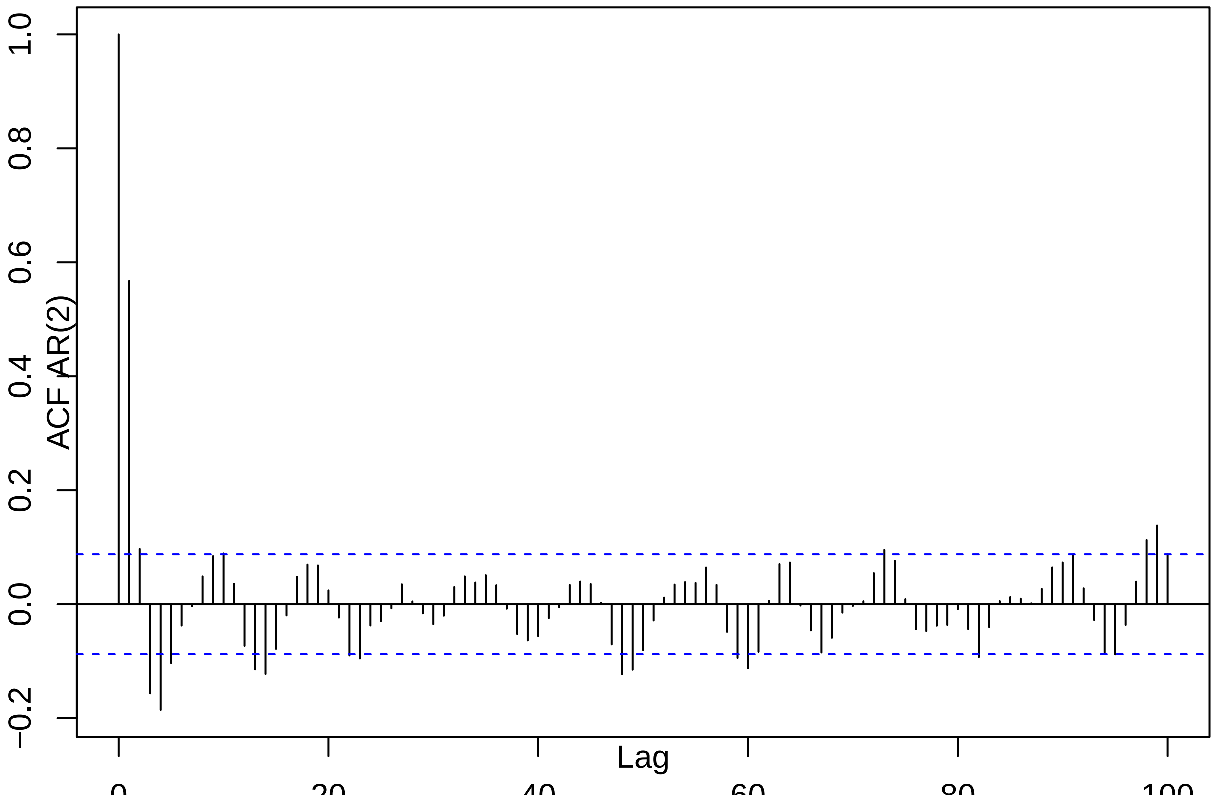
Comment: By the definition, an ARMA model of order p and q will be $x_t = \phi_1 x_{t-1} + \ldots + \phi_p x_{t-p} + w_t + \theta_1 w_{t-1} + \ldots + \theta_q w_{t-q} + \alpha$, where w_t is a white noise given (zero mean and unit variance) and α is an intercept. So, I would say my ARMA(2,2) process will be $x_t = \frac{3}{4}x_{t-1} + \frac{-1}{4}x_{t-2} + 0.73w_{t-1} + 0.26x_{t-2} + w_t$. And, this process is both invertible and causal.

Part d

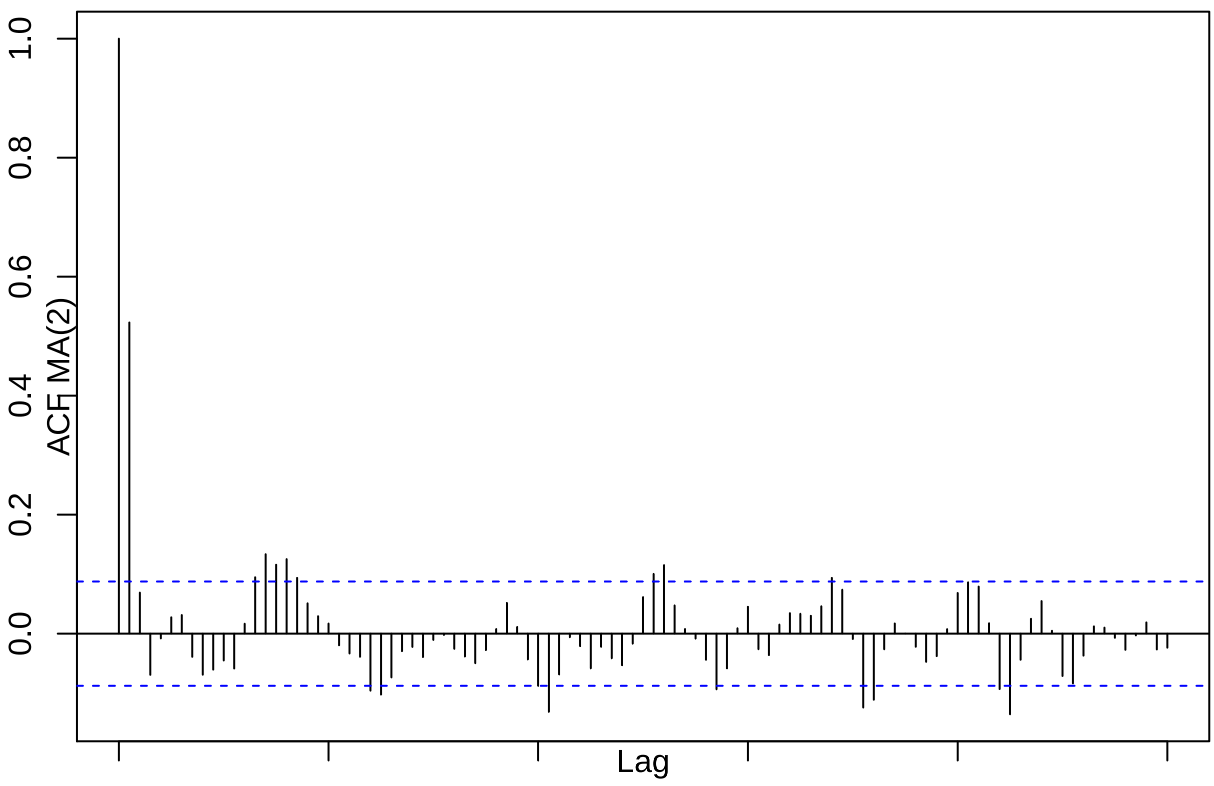
```
par(mfrow = c(3,1), mar = c(1.5,2,2,1), mgp = c(0, 1, 0))
plot.ts(arima.sim(list(order = c(2, 0, 0), ar = c((3/4), (-1/4))), n = 500), ylab = "x",
     main = (expression(AR(2)~~~phi_1==(3/4)~~~phi_2==(-1/4))))

plot.ts(arima.sim(list(order = c(0, 0, 2), ma = c(0.73, 0.26)), n = 500), ylab = "x",
     main = (expression(MA(2)~~~theta_1==(0.73)~~~theta_2==(0.26))))

plot.ts(arima.sim(list(order = c(2, 0, 2), ar = c((3/4), (-1/4)), ma = c(0.73, 0.26)),
     n = 500), ylab = "x", main = (expression(ARMA(2,2))))
```



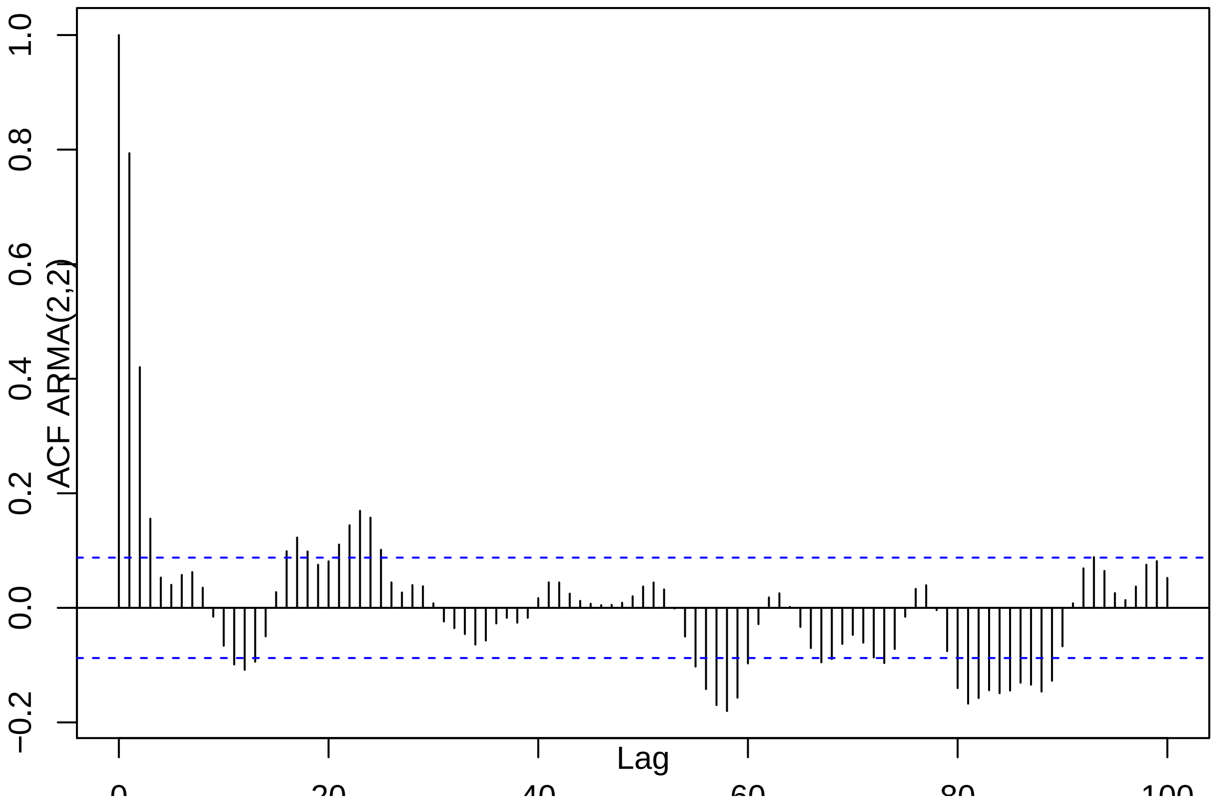

```
par(mfrow = c(1, 1))
acf(arima.sim(list(order = c(2, 0, 0), ar = c((3/4), (-1/4))), n = 500), lag.max = 100,
    ylab = "ACF AR(2)", main = "")
```



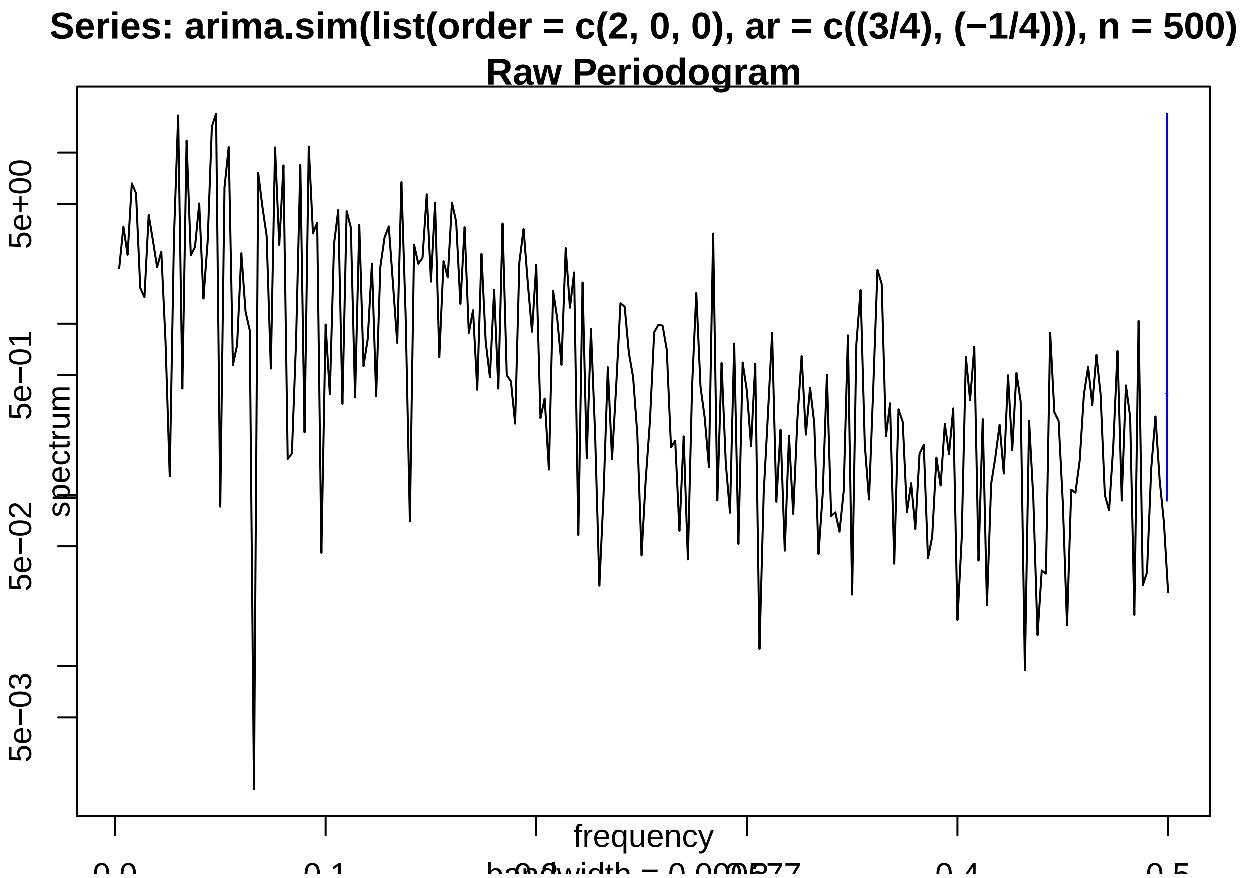

```r
acf(arima.sim(list(order = c(0, 0, 2), ma = c(0.73, 0.26)), n = 500), lag.max = 100,
    ylab = "ACF MA(2)", main = "")
```



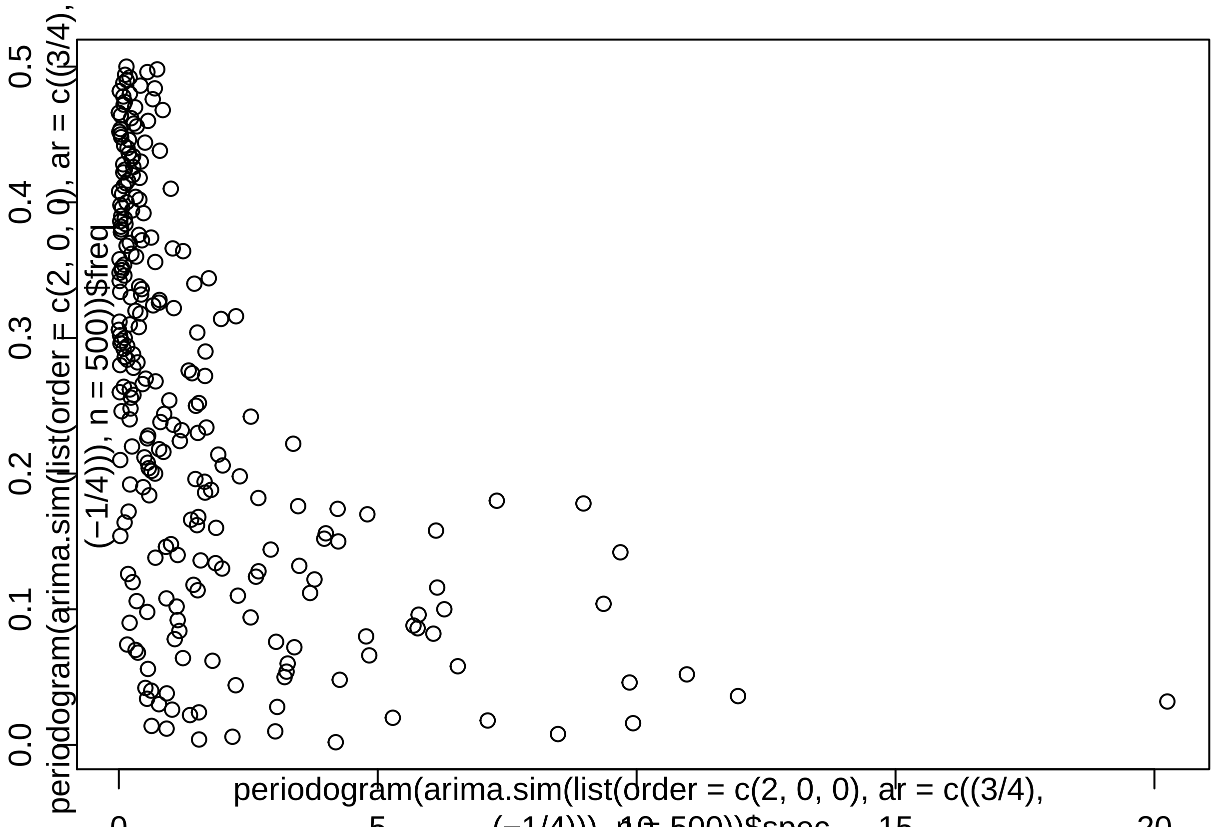
```r
acf(arima.sim(list(order = c(2, 0, 2), ar = c((3/4), (-1/4)), ma = c(0.73, 0.26)),
    n = 500), lag.max = 100, ylab = "ACF ARMA(2,2)", main = "")
```




```r
spec.pgram(arima.sim(list(order = c(2, 0, 0), ar = c((3/4), (-1/4))), n = 500))
```




```
plot.ts(periodogram(arima.sim(list(order = c(2, 0, 0), ar = c((3/4), (-1/4))), n = 500))$spec,
        periodogram(arima.sim(list(order = c(2, 0, 0), ar = c((3/4), (-1/4))), n = 500))$freq)
```



Comment:

It seems like all the processes look stationary, and that pretty makes sense, as I computed the roots and turned out that all of them are both causal and invertible. If I was asked to pick one process that might not look to be stationary, I would pick ARMA(2,2), as it seems like there are some peaks.

Also, when I looked at the ACFs, I can tell AR(2) and MA(2) are mostly having unsignificant ACFs; however, many of ARMA(2,2) have the significant ACFs.

Parte

```r
#AR(2)
polyroot(c(1, (-3/4), (1/4)))
```

```
## [1] 1.5+1.322876i 1.5-1.322876i
```

```r
abs(polyroot(c(1, (-3/4), (1/4)))[1])
```

```
## [1] 2
```

```r
abs(polyroot(c(1, (-3/4), (1/4)))[2])
```

```
## [1] 2
```

```r
#MA(2)
polyroot(c(1, 0.73, 0.26))
```

```
## [1] -1.403846+1.369441i -1.403846-1.369441i
```

```r
abs(polyroot(c(1, 0.73, 0.26))[1])
```

```
## [1] 1.961161
```

```r
abs(polyroot(c(1, 0.73, 0.26))[2])
```

```
## [1] 1.961161
```

Comment:

For AR(2), the roots of the AR polynomial are 1.5 ± 1.323i (absolute value of 2). And, there is no root for MA polynomial.

For MA(2), there is no root for the AR polynomial. And, the roots of the MA polynomial are -1.4038 ± 1.369i (absolute value of 1.96).