HW3 Jin Kweon (3032235207)

Jin Kweon 2/6/2018

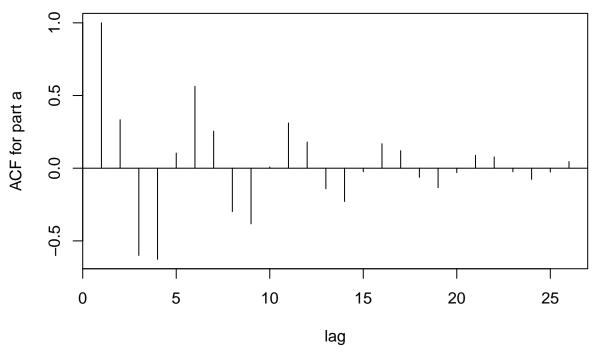
Problem 1

```
#Check answers
ARMAtoMA(ar= -0.5, ma = 1.5, 7)
## [1] 1.000000 -0.500000 0.250000 -0.125000 0.062500 -0.031250 0.015625
```

Problem 4

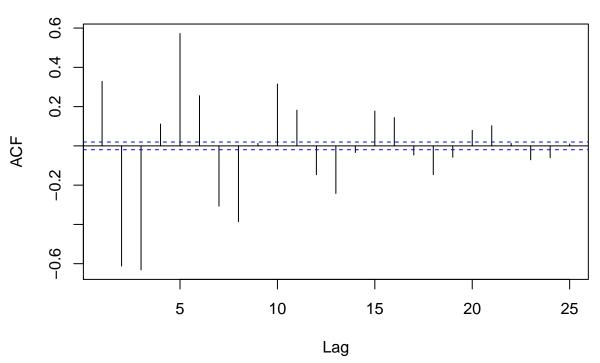
Part a

```
#Check invertibility/causality
abs(polyroot(c(1, (-3/5), (4/5)))[1])
abs(polyroot(c(1, (-3/5), (4/5)))[2])
#Plot Series
# set.seed(100)
\# plot.ts(arima.sim(list(order = c(2, 0, 0), ar = c((3/5), (-4/5))), n = 10000), ylab = "x",
       main = (expression(AR(2) - -phi_1 = (3/5) - -phi_2 = (-4/5))))
# set.seed(100)
\# plot(arima.sim(list(order = c(2, 0, 0), ar = c((3/5), (-4/5))), n = 10000), ylab = "x",
       main = (expression(AR(2) - -phi_1 = (3/5) - -phi_2 = (-4/5))))
# set.seed(100)
# tsplot(arima.sim(list(order = c(2, 0, 0), ar = c((3/5), (-4/5))), n = 10000), ylab = "x",
       main = (expression(AR(2) - -phi_1 = (3/5) - -phi_2 = (-4/5))))
#ACF
set.seed(100)
acfa \leftarrow ARMAacf(ar = c((3/5), (-4/5)), ma = 0, lag.max = 25)
plot(acfa, type = "h", xlab = "lag", ylab = "ACF for part a")
abline(h = 0)
```



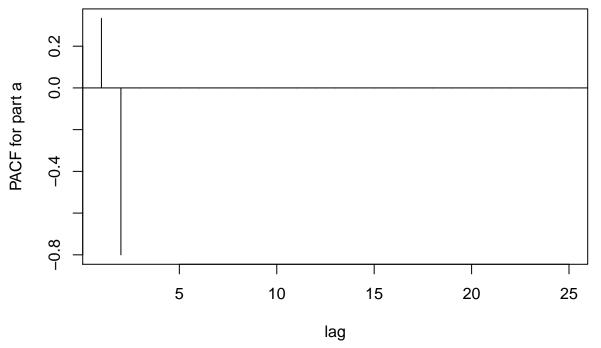
```
#ACF2
set.seed(100)
acfa2 <- arima.sim(list(order = c(2, 0, 0), ar = c((3/5), (-4/5))), n = 10000)
acf(acfa2, lag.max = 25, plot = T)</pre>
```

Series acfa2



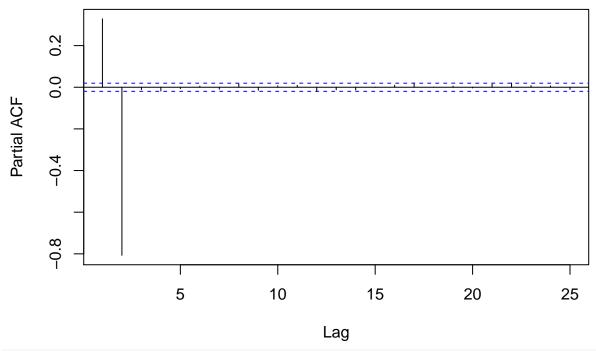
```
#PACF theoretical
set.seed(100)
pacfa <- ARMAacf(ar = c((3/5), (-4/5)), ma = 0, lag.max = 25, pacf = T)</pre>
```

```
pacfa
plot(pacfa, type = "h", xlab = "lag", ylab = "PACF for part a")
abline(h = 0)
```

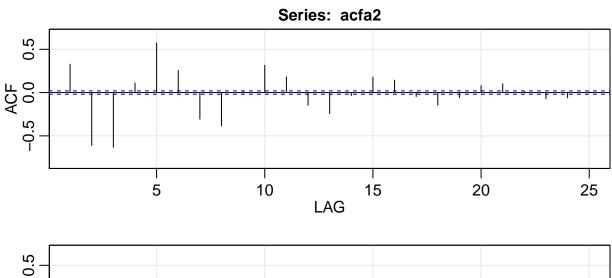


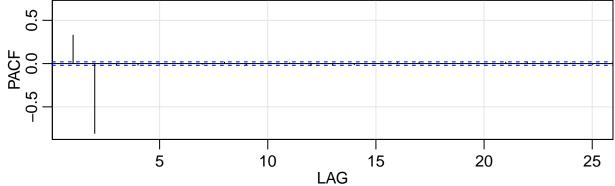
```
#PACF2 simulation
set.seed(100)
pacfa2 <- arima.sim(list(order = c(2, 0, 0), ar = c((3/5), (-4/5))), n = 10000)
pacf(pacfa2, lag.max = 25, plot = T)</pre>
```

Series pacfa2



#Do both acf and pacf
acf2(acfa2, max.lag = 25, plot = T)





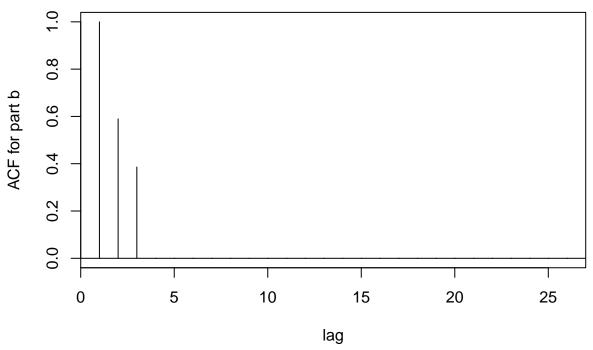
Comment: My time series model is $x_t = \frac{3}{5}x_{t-1} - \frac{4}{5}x_{t-2} + w_t$. So, ϕ_1 will be $\frac{3}{5}$ and ϕ_2 will be $\frac{-4}{5}$. And, this is both causal and invertible. And, by the definition, this is AR(2) process.

The most important thing I can notice from AR(2) process is that ACF is tailing off, and PACF cuts off after lag 2.

PACF starts at lag 1 and ACF starts at lag 0. Since this is AR(2) process, PACF should cut off at h=2.

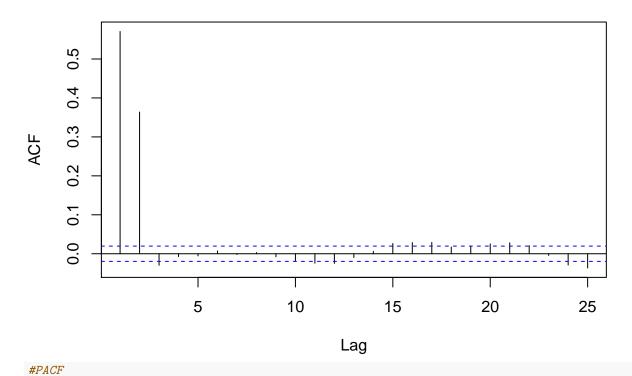
Part b

```
#Check invertibility/causality
abs(polyroot(c(1, 0.8, 1.1))[1])
## [1] 0.9534626
abs(polyroot(c(1, 0.8, 1.1))[2])
## [1] 0.9534626
#Plot Series
# set.seed(100)
\# plot.ts(arima.sim(list(order = c(0, 0, 2), ma = c(0.8, 1.1)), n = 10000), ylab = "x",
     main = (expression(MA(2) \sim theta_1 = (0.8) \sim theta_2 = (1.1))))
# set.seed(100)
\# plot(arima.sim(list(order = c(0, 0, 2), ma = c(0.8, 1.1)), n = 10000), ylab = "x",
     main = (expression(MA(2) --- theta_1 == (0.8) --- theta_2 == (1.1))))
# set.seed(100)
\# tsplot(arima.sim(list(order = c(0, 0, 2), ma = c(0.8, 1.1)), n = 10000), ylab = "x",
     main = (expression(MA(2) \sim theta 1 == (0.8) \sim theta 2 == (1.1))))
#ACF theoretical
set.seed(100)
acfb \leftarrow ARMAacf(ar = 0, ma = c(0.8, 1.1), lag.max = 25)
##
        0
                         2
                                 3
                                         4
                                                 5
                                                          6
                 1
7
                 8
                         9
                                10
                                        11
                                                 12
##
        14
                15
                        16
                                17
                                         18
                                                 19
##
        21
                22
                        23
                                24
                                        25
plot(acfb, type = "h", xlab = "lag", ylab = "ACF for part b")
abline(h = 0)
```



```
#ACF2 simulation
set.seed(100)
acfb2 <- arima.sim(list(order = c(0, 0, 2), ma = c(0.8, 1.1)), n = 10000)
acf(acfb2, lag.max = 25, plot = T)</pre>
```

Series acfb2



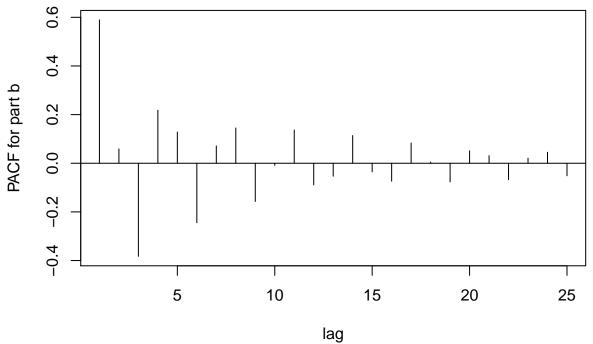
pacfb <- ARMAacf(ar = 0, ma = c(0.8, 1.1), lag.max = 25, pacf = T)

set.seed(100)

```
pacfb
```

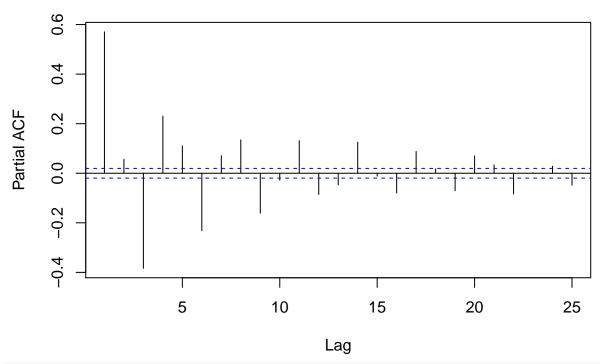
```
## [1] 0.589473684 0.058980019 -0.382720616 0.217835887 0.128330078
## [6] -0.244251517 0.071180617 0.145039704 -0.157116800 -0.008742490
## [11] 0.136624078 -0.089176853 -0.053172237 0.113727343 -0.035367317
## [16] -0.073569931 0.083295264 0.004730005 -0.076611983 0.050895955
## [21] 0.031261115 -0.067609762 0.020970722 0.045192089 -0.051331754

plot(pacfb, type = "h", xlab = "lag", ylab = "PACF for part b")
abline(h = 0)
```

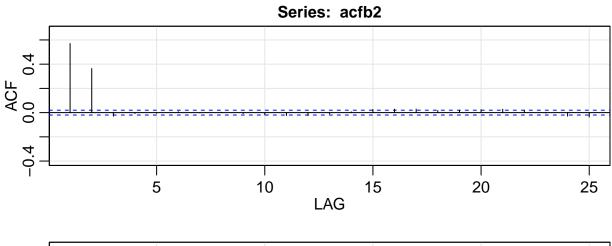


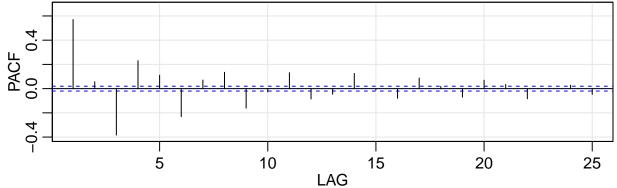
```
#PACF2
set.seed(100)
pacfb2 <- arima.sim(list(order = c(0, 0, 2), ma = c(0.8, 1.1)), n = 10000)
pacf(pacfb2, lag.max = 25, plot = T)</pre>
```

Series pacfb2



#Do both acf and pacf
acf2(acfb2, max.lag = 25, plot = T)





```
##
          ACF PACF
   [1,] 0.57 0.57
##
##
   [2,] 0.36 0.06
   [3,] -0.03 -0.38
##
##
   [4,] -0.01 0.23
   [5,] -0.01 0.11
##
   [6,] 0.01 -0.23
##
   [7,] 0.00 0.07
##
   [8,] 0.00 0.13
##
##
  [9,] -0.01 -0.16
## [10,] -0.02 -0.03
## [11,] -0.02 0.13
## [12,] -0.02 -0.09
## [13,] -0.01 -0.05
## [14,] 0.01 0.13
## [15,]
         0.03 -0.01
## [16,]
         0.03 -0.08
## [17,]
         0.03 0.09
## [18,]
         0.02 0.02
## [19,]
        0.02 - 0.07
## [20,] 0.03 0.07
## [21,] 0.03 0.03
## [22,] 0.02 -0.08
## [23,] 0.00 0.00
## [24,] -0.03 0.03
## [25,] -0.04 -0.05
```

Comment: My time series model is $x_t = w_t - 0.8w_{t-1} + 1.1w_{t-2}$. So, θ_1 will be 0.8 and θ_2 will be 1.1. And, this is causal but not invertible. And, by the definition, this is MA(2) process.

MA(2) process has an ACF of cutting off at lag 2, and has an PACF with tailing off.

Part c

```
#Check invertibility/causality
abs(polyroot(c(1, (-4/5))))

abs(polyroot(c(1, (4/5))))

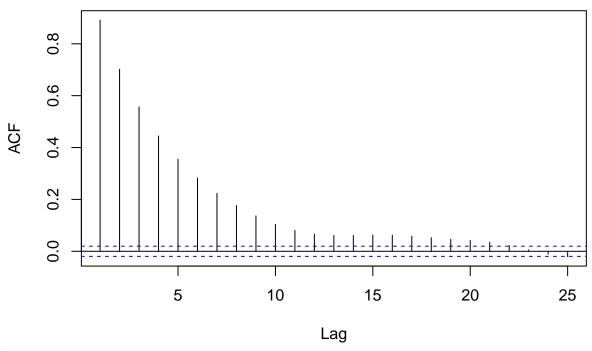
#Plot Series
# set.seed(100)
# plot.ts(arima.sim(list(order = c(1, 0, 1), ar = (4/5), ma = (4/5)), n = 10000), ylab = "x",
# main = (expression(ARMA(1,1)~~~phi_1==(4/5)~~~theta_1==(4/5))))
# set.seed(100)
# plot(arima.sim(list(order = c(1, 0, 1), ar = (4/5), ma = (4/5)), n = 10000), ylab = "x",
# main = (expression(ARMA(1,1)~~~phi_1==(4/5)~~~theta_1==(4/5))))
# set.seed(100)
# tsplot(arima.sim(list(order = c(1, 0, 1), ar = (4/5), ma = (4/5)), n = 10000), ylab = "x",
# main = (expression(ARMA(1,1)~~~phi_1==(4/5)~~~theta_1==(4/5)))
# main = (expression(ARMA(1,1)~~~phi_1==(4/5)~~~theta_1==(4/5)))
```

```
#ACF theoretical
set.seed(100)
acfc \leftarrow ARMAacf(ar = (4/5), ma = (4/5), lag.max = 25)
plot(acfc, type = "h", xlab = "lag", ylab = "ACF for part c")
abline(h = 0)
       0.8
ACF for part c
       9.0
       0.4
       0.2
       0.0
            0
                           5
                                          10
                                                                        20
                                                                                        25
                                                         15
```

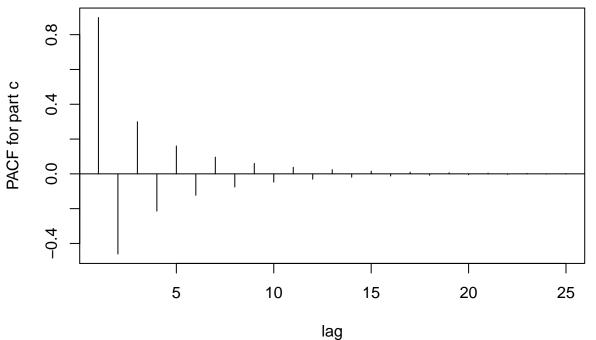
```
#ACF2 simulation
set.seed(100)
acfc2 <- arima.sim(list(order = c(1, 0, 1), ar = (4/5), ma = (4/5)), n = 10000)
acf(acfc2, lag.max = 25, plot = T)</pre>
```

lag

Series acfc2

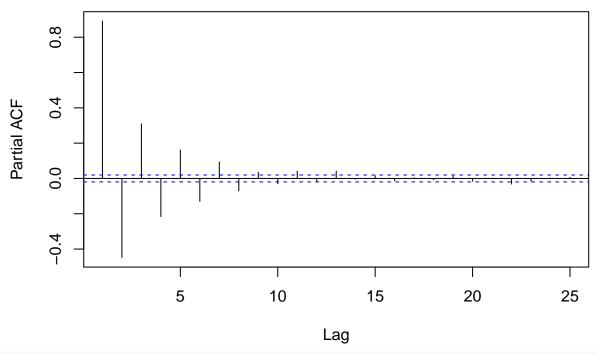


```
#PACF theoretical
set.seed(100)
pacfc <- ARMAacf(ar = (4/5), ma = (4/5), lag.max = 25, pacf = T)
pacfc
plot(pacfc, type = "h", xlab = "lag", ylab = "PACF for part c")
abline(h = 0)</pre>
```

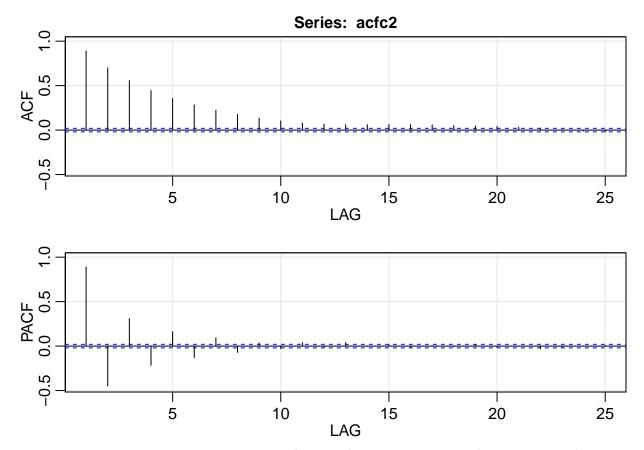


```
#PACF2 simulation
set.seed(100)
pacfc2 <- arima.sim(list(order = c(1, 0, 1), ar = (4/5), ma = (4/5)), n = 10000)
pacf(pacfc2, lag.max = 25, plot = T)</pre>
```

Series pacfc2



```
#Do both acf and pacf
acf2(acfc2, max.lag = 25, plot = T)
```



Comment: My time series model is $x_t = w_t + \frac{4}{5}x_{t-1} + \frac{4}{5}w_{t-1}$. So, ϕ_1 will be $\frac{4}{5}$ and θ_1 will be $\frac{4}{5}$. And, this is both causal and invertible. And, by the definition, this is ARMA(1,1) process.

The ACF and PACF for the $\operatorname{ARMA}(1,1)$ process are both tailing off.