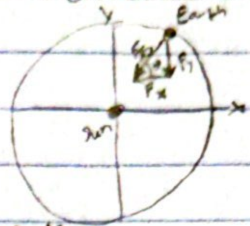
Kolee Stinson CH 4 Sc 1-4

4.1)

Kepler's Law

$$F_G = \frac{G M_S M_E}{r^2}$$ mass of Sun/Earth

Coordinate System



$$\frac{d^2x}{dt^2} = \frac{F_{Gx}}{M_E} \longrightarrow F_{Gx} = -\frac{G M_S M_E}{r^2}\cos\theta = -\frac{G M_S M_E x}{r^3}$$

with a similar result for "y"

Circular Motion

$$G M_S = v^2 r = 4\pi^2 AU^3/yr^2$$

Comp Sol.

$$v_{x\,i+1} = v_{x\,i} - \frac{4\pi^2 x_i}{r_i^3}\Delta t$$

$$x_{i+1} = x_i + v_{x\,i+1}\Delta t$$

$$v_{y\,i+1} = v_{y\,i} - \frac{4\pi^2 y_i}{r_i^3}\Delta t$$

$$y_{i+1} = y_i + v_{y\,i+1}\Delta t$$

4.2) body of reduced mass μ

F_G allows inverse square Law

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{L^2}F(r)$$

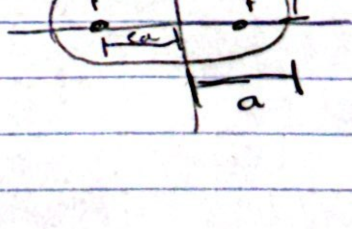
$$\mu \equiv m_1 m_2/(m_1+m_2)$$

$$\vec{r} \equiv \vec{r}_2 - \vec{r}_1$$

$$v_{max} = \sqrt{GM_S}\sqrt{\frac{(1+e)}{a(1-e)}\left(1+\frac{M_P}{M_S}\right)}$$

$$F_G = \frac{G M_S M_E}{r^\beta}$$

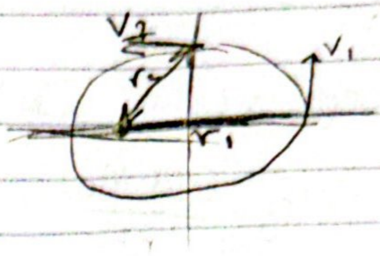
$$v_{min} = \sqrt{GM_S}\sqrt{\frac{(1-e)}{a(1+e)}\left(1+\frac{M_P}{M_S}\right)}$$



4.3)

force law (general relativity)

$$F_G = \frac{GM_s M_m}{r^2}\left(1 + \frac{a}{r^2}\right)$$



Conservation of energy

Total E is the same @ both points

$$-\frac{GM_s M_m}{r_1} + \frac{1}{2} M_m v_1^2 = -\frac{GM_s M_m}{r_2} + \frac{1}{2} M_m v_2^2$$

yields, $r_1 v_1 = b v_2$

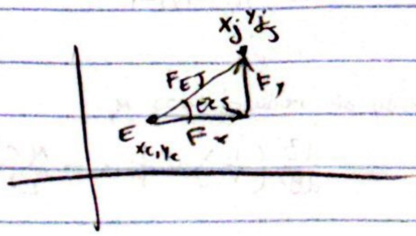
$$v_1 = \sqrt{2GM_s \left[\frac{b^2}{a^2(1+e)^2 - b^2}\right]\left[\frac{1}{\sqrt{e^2a^2+b^2}} - \frac{1}{a+ea}\right]}$$

$$= \sqrt{\frac{GM_s(1-e)}{a(1+e)}}$$

4.4)

Three Body

Earth
Jupiter $\quad F_{EJ} = \frac{GM_J M_E}{r^2_{EJ}}$



$$F_{EJ_x} = \frac{-GM_J M_E (x_e - x_J)}{r^3_{EJ}}$$

$$\frac{dv}{dt} = -\frac{GM_s x_e}{r^3} - \frac{GM_J(x_e - x_J)}{r^3_{EJ}}$$