Appendix C notes

Fourier Transform

Signal

$$y(t) = \sum_{j=1}^{5} V_j \sin(2\pi f_j t + \phi_j)$$

V_j: amp. f_j: freq

integral of frequency

$$y(t) = \int_{-\infty}^{\infty} Y(f) e^{-2\pi i f t} df = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(\omega/2\pi) e^{-i\omega t} d\omega$$

inverse

$$Y(f) = \int_{-\infty}^{\infty} y(t) e^{2\pi i f t} dt = \int_{-\infty}^{\infty} y(t) e^{i\omega t} dt$$

Discrete

$$y_m = \frac{1}{N} \sum_{n=0}^{N-1} Y_n e^{-2\pi i m n/N}$$

$$Y_n = \sum_{m=0}^{N-1} y_m e^{2\pi i m n/N}$$

inverse

$$\sum_{n=0}^{N-1} e^{2\pi i n(m-m')/N} = N\delta_{m,m'}$$

FFT algorithm

- read in data y_m ($m = 0, 1, \ldots, N-1$)
- find power: p, where $N = 2^p$
- reverse the indices of input data array
- outer loop through levels $i = 1, 2, \ldots, p$ w/ final decomp. of y_m to first decomp. Y_n^e & Y_n^o
- nested middle loop where k in $e^{2\pi i k/N}$ is in each sum.
- nested inner loop where groups split into individual sums

Power

autocorrelation

$$Corr[Y](f) = \int_{-\infty}^{\infty} Y(t)^* Y(t+\tau) d\tau$$

Power spectrum

$$PS[Y](f) = \int_{-\infty}^{\infty} Y(t)^* Y(t+\tau) e^{2\pi i f \tau} d\tau = [Y(f)]^2$$

Power Spectrum