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# TODO

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# Contest (1)

template.cpp 38 lines

```
#pragma GCC optimize("O3")
#include "bits/stdc++.h"
using namespace std;

#define rep(i, b, e) for(int i = (b); i <= (e); i++)
#define per(i, b, e) for(int i = (e); i >= (b); i--)
#define FOR(i, b, e) rep(i, b, (e) - 1)
#define SZ(x) int(x.size())
#define all(x) x.begin(), x.end()
#define pb push_back
#define mp make_pair
#define st first
#define nd second
using ll = long long;
using vi = vector<int>;
using pii = pair<int, int>;

auto &operator<<(auto &o, pair<auto, auto> p) {
  return o << "(" << p.st << ", " << p.nd << ")"; }
auto operator<<(auto &o, auto x)->decltype(end(x), o) {
  o << "{"; int i=0; for(auto e: x) o << ", " + 2*!i++ << e;
  return o << "}"; }
#ifdef LOCAL
#define deb(x...) cerr << "[" #x "]: ", [](auto...$) { \
  ((cerr << $ << "; "),...) << endl; }(x)
#else
#define deb(...)
#endif

void solve() { }

int main() {
  cin.tie(0)->sync_with_stdio(0);
  int tt = 1;
  // cin >> tt;
  FOR(te, 0, tt) solve();
  return 0;
}
```

Makefile 2 lines

```
CXXFLAGS=-std=c++20 -O3 -g -DLOCAL -Wall -Wextra -Wshadow
-fsanitize=address,undefined
```

hash.sh 3 lines

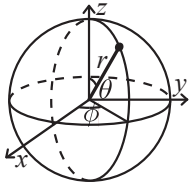
```
# Hashes a file, ignoring all whitespace and comments.
# Use for verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

test.sh 9 lines

```
# Example usage: "./test.sh prog"
for i in {1..1000}
do
  ./gen > $1.in
  ./$1 < $1.in > $1.out
  ./brute < $1.in > $1.brute
  diff $1.out $1.brute || break
  echo $i
done
```

# Mathematics (2)

## 2.1 Spherical coordinates




$$x = r\sin\theta\cos\phi \qquad r = \sqrt{x^2+y^2+z^2}$$
$$y = r\sin\theta\sin\phi \qquad \theta = \operatorname{acos}(z/\sqrt{x^2+y^2+z^2})$$
$$z = r\cos\theta \qquad \phi = \operatorname{atan2}(y,x)$$

# Data structures (3)

OrderStatisticTree.h

**Description:** A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
**Time:** $\mathcal{O}(\log N)$, join probably $\mathcal{O}(N)$ 023ea1, 17 lines

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).st;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // O(N), assuming T<T2 or T>T2, merge t2 into t
}
```

HashMap.h

**Description:** Hash map with mostly the same API as unordered_map, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided). 3e474b, 11 lines

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
// For places where hacking might be a problem:
const int R = chrono::high_resolution_clock::now().
    time_since_epoch().count();
struct chash { // To use most bits rather than just the lowest
  const uint64_t C = ll(4e18 * acos(0)) | 71;
  ll operator()(ll x) const {
    return __builtin_bswap64((x^R)*C); }
};
// 1<<16 is initial size, don't put it if you want small map
gp_hash_table<ll, int, chash> h({},{},{},{}, {1 << 16});
```

UnionFindRollback.h

**Description:** Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().
**Usage:** `int t = uf.time(); ...; uf.rollback(t);`
**Time:** $\mathcal{O}(\log(N))$ 200ca0, 19 lines

```
struct RollbackUF {
  vi e; vector<pii> st;
  RollbackUF(int n) : e(n, -1) {}
  int size(int x) { return -e[find(x)]; }
  int find(int x) { return e[x] < 0 ? x : find(e[x]); }
  int time() { return SZ(st); }
  void rollback(int t) {
    for(int i = time(); i --> t;) e[st[i].st] = st[i].nd;
    st.resize(t);
  }
  bool join(int a, int b) {
    a = find(a), b = find(b);
    if (a == b) return false;
    if (e[a] > e[b]) swap(a, b);
    st.pb({a, e[a]}); st.pb({b, e[b]});
    e[a] += e[b]; e[b] = a;
    return true;
  }
};
```

LineContainer.h

**Description:** Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
**Time:** $\mathcal{O}(\log N)$ 8ec1c7, 29 lines

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
```

```
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty()); auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

## Treap.h

**Description:** A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.
**Time:** $\mathcal{O}(\log N)$, increase works in $\mathcal{O}(\log N \log U)$ amortized. 72a78a, 126 lines

```
struct Treap {
  struct Node {
    int ch[2] = {0, 0};
    int rank = rand(), size = 0;
    int val = 0, mn = 1e9, sum = 0; // Subtree aggregates
    bool flip = 0; int add = 0;   // Lazy tags
    Node() {}
    Node(int v) : size(1), val(v), mn(v), sum(v) {}
  };
  vector<Node> t;

  Treap() : t(1) {}

  void pull(int v) {
    auto [l, r] = t[v].ch;
    t[v].size = t[l].size + 1 + t[r].size;
    t[v].mn = min({t[l].val, t[v].val, t[r].val});
    t[v].sum = t[l].sum + t[v].val + t[r].sum;
  }

  int apply(int v, bool flip, int add) {
    if(!v) return 0;
    // t.pb(t[v]); v = SZ(t) - 1;        // <- persistency
    if(flip) {
      t[v].flip ^= 1;
      swap(t[v].ch[0], t[v].ch[1]);
    }
    t[v].val += add; t[v].mn += add;
    t[v].sum += add * t[v].size;
    t[v].add += add;
    return v;
  }

  void push(int v) {
    FOR(i, 0, 2) {
      t[v].ch[i] = apply(t[v].ch[i], t[v].flip, t[v].add);
    }
    t[v].flip = 0; t[v].add = 0;
  }

  pii split(int v, int k) {
    if(!v) return {0, 0};
    push(v);
    auto [l, r] = t[v].ch;
    if(k <= t[l].size) {
    // if(k <= t[v].val) {          // <- by values
      auto [p, q] = split(l, k);
      t[v].ch[0] = q, pull(v);
      return {p, v};
    }
    else {
      auto [p, q] = split(r, k - t[l].size - 1);
      // auto [p, q] = split(r, k);   // <- by values
      t[v].ch[1] = p, pull(v);
      return {v, q};
    }
  }

  int merge(int v, int u) {
    if(!v || !u) return v ^ u;
    push(v), push(u);
    if(t[v].rank > t[u].rank) {
      t[v].ch[1] = merge(t[v].ch[1], u);
      return pull(v), v;
    }
    else {
      t[u].ch[0] = merge(v, t[u].ch[0]);
      return pull(u), u;
    }
  }

  void insert(int &v, int pos, int val) {
    // if(v) t.pb(t[v]), v = SZ(t) - 1;   // <- persistency
    auto [p, q] = split(v, pos);
    t.pb(Node(val)); int u = SZ(t) - 1;
    // t.pb(Node(pos)); int u = SZ(t) - 1;  // <- by values
    v = merge(merge(p, u), q);
  }

  void erase(int &v, int l, int r) {
    // if(v) t.pb(t[v]); v = SZ(t) - 1;   // <- persistency
    auto [p, q] = split(v, l);
    auto [u, s] = split(q, r - l + 1);
    // auto [u, s] = split(q, r + 1);   // <- by values
    v = merge(p, s);
  }

  void modify(int &v, int l, int r, bool flip, int add) {
    // if(v) t.pb(t[v]); v = SZ(t) - 1;   // <- persistency
    auto [p, q] = split(v, l);
    auto [u, s] = split(q, r - l + 1);
    // auto [u, s] = split(q, r + 1);   // <- by values
    u = apply(u, flip, add);
    v = merge(merge(p, u), s);
  }

  pii get(int &v, int l, int r) {
    // if(v) t.pb(t[v]); v = SZ(t) - 1;   // <- persistency
    auto [p, q] = split(v, l);
    auto [u, s] = split(q, r - l + 1);
    // auto [u, s] = split(q, r + 1);   // <- by values
    int mn = t[u].mn, sum = t[u].sum;
    v = merge(merge(p, u), s);
    return {mn, sum};
  }

  // only when by values
  int join(int v, int u) {
    if(!v || !u) return v ^ u;
    if(t[v].rank < t[u].rank) swap(v, u);
    auto [p, q] = split(u, t[v].val);
    push(v);
    t[v].ch[0] = join(t[v].ch[0], p);
    t[v].ch[1] = join(t[v].ch[1], q);
    return pull(v), v;
  }

  // only when by values, persistency destroys complexity
  void increase(int &v, int l, int r, int increase) {
    // if(v) t.pb(t[v]); v = SZ(t) - 1; // <- persistency
    auto [p, q] = split(v, l);
    auto [u, s] = split(q, r + 1);
    u = apply(u, 0, increase);
    v = join(merge(p, s), u);
  }
};
```

## FenwickTree.h

**Description:** Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
**Time:** Both operations are $\mathcal{O}(\log N)$. b06af0, 22 lines

```
struct FT {
  vector<ll> s;
  FT(int n) : s(n) {}
  void update(int pos, ll dif) { // a[pos] += dif
    for (; pos < SZ(s); pos |= pos + 1) s[pos] += dif;
  }
  ll query(int pos) { // sum of values in [0, pos)
    ll res = 0;
    for (; pos > 0; pos &= pos - 1) res += s[pos-1];
    return res;
  }
  int lower_bound(ll sum) {// min pos st sum of [0, pos] >= sum
    // Returns n if no sum is >= sum, or -1 if empty sum is.
    if (sum <= 0) return -1;
    int pos = 0;
    for (int pw = 1 << 25; pw; pw >>= 1) {
      if (pos + pw <= SZ(s) && s[pos + pw-1] < sum)
        pos += pw, sum -= s[pos-1];
    }
    return pos;
  }
};
```

## FenwickTree2D.h

**Description:** Computes sums a[i,j] for all i<I, j<J, and increases single elements a[i,j]. Requires that the elements to be updated are known in advance (call fakeUpdate() before init()).
**Time:** $\mathcal{O}(\log^2 N)$. (Use persistent segment trees for $\mathcal{O}(\log N)$.)
"FenwickTree.h" cc92f9, 21 lines

```
struct FT2 {
  vector<vi> ys; vector<FT> ft;
  FT2(int limx) : ys(limx) {}
  void fakeUpdate(int x, int y) {
    for (; x < SZ(ys); x |= x + 1) ys[x].pb(y);
  }
  void init() {
    for (vi& v : ys) sort(all(v)), ft.emplace_back(SZ(v));
  }
  int ind(int x, int y) {
    return int(lower_bound(all(ys[x]), y) - ys[x].begin()); }
  void update(int x, int y, ll dif) {
    for (; x < SZ(ys); x |= x + 1)
      ft[x].update(ind(x, y), dif);
  }
  ll query(int x, int y) {
    ll sum = 0;
    for (; x; x &= x - 1) sum += ft[x-1].query(ind(x-1, y));
    return sum;
  }
};
```

## RMQ.h

**Description:** Range Minimum Queries on an array. Returns min(V[a], V[a + 1], ... V[b - 1]) in constant time.
**Usage:** `RMQ rmq(values);`
`rmq.query(inclusive, exclusive);`
**Time:** $\mathcal{O}(|V| \log |V| + Q)$ 1e70c8, 16 lines

```
template<class T>
```

```
struct RMQ {
  vector<vector<T>> jmp;
  RMQ(const vector<T>& V) : jmp(1, V) {
    for (int pw = 1, k = 1; pw * 2 <= SZ(V); pw *= 2, ++k) {
      jmp.emplace_back(sz(V) - pw * 2 + 1);
      FOR(j, 0, SZ(jmp[k]))
        jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
    }
  }
  T query(int a, int b) {
    assert(a < b); // or return inf if a == b
    int dep = 31 - __builtin_clz(b - a);
    return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
  }
};
```

## MoQueries.h

**Description:** Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change `step` to add/remove the edge $(a, c)$ and remove the initial `add` call (but keep `in`).
**Time:** $\mathcal{O}\left(N\sqrt{Q}\right)$ 24a16d, 49 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
  vi s(SZ(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) {
    pii q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();
  }
  return res;
}

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0){
  int N = SZ(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
  vi s(SZ(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;
  };
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) FOR(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  }
  return res;
}
```

## ExtendedLiChao.h

**Description:** Supports Range Line Insertion, Range Line Addition, Point Query. Can be modified to support Range Maximum Query, if Line Addition is replaced by Sum Addition.
**Time:** $\mathcal{O}\left(\log^2 N\right)$ 06b342, 97 lines

```
template<typename data_t>
struct Line {
  data_t a, b;
  Line() : a(0), b(-inf) {}
  Line(data_t a, data_t b) : a(a), b(b) {}

  data_t get(data_t x) { return a * x + b; }

  void add(Line x) { a += x.a; b += x.b; }
};

struct Node {
  Line<data_t> line = Line<data_t>();
  Line<data_t> lazy = Line<data_t>(0, 0);
  Node *lc = nullptr, *rc = nullptr;

  void apply(data_t l, data_t r, Line<data_t> v) {
    line.add(v); lazy.add(v);
  }
};

void PushLazy(Node* &n, data_t tl, data_t tr) {
  if (n == nullptr) return;
  if (n->lc == nullptr) n->lc = new Node();
  if (n->rc == nullptr) n->rc = new Node();
  data_t mid = (tl + tr) / 2;
  n->lc->apply(tl, mid, n->lazy);
  n->rc->apply(mid + 1, tr, n->lazy);
  n->lazy = Line<data_t>(0, 0);
}

void PushLine(Node* &n, data_t tl, data_t tr) {
  if (n == nullptr) return;
  data_t mid = (tl + tr) / 2;
  InsertLineKnowingly(n->lc, tl, mid, n->line);
  InsertLineKnowingly(n->rc, mid + 1, tr, n->line);
  n->line = Line<data_t>();
}

void InsertLineKnowingly(Node* &n, data_t tl, data_t tr, Line<
    data_t> x) {
  if (n == nullptr) n = new Node();
  if (n->line.get(tl) < x.get(tl)) swap(n->line, x);
  if (n->line.get(tr) >= x.get(tr)) return;
  if (tl == tr) return;
  data_t mid = (tl + tr) / 2;
  PushLazy(n, tl, tr);
  if (n->line.get(mid) > x.get(mid)) {
    InsertLineKnowingly(n->rc, mid + 1, tr, x);
  } else {
    swap(n->line, x);
    InsertLineKnowingly(n->lc, tl, mid, x);
  }
}

void InsertLine(Node* &n, data_t tl, data_t tr, data_t l,
    data_t r, Line<data_t> x) {
  if (tr < l || r < tl || tl > tr || l > r) return;
  if (n == nullptr) n = new Node();
  if (l <= tl && tr <= r) return InsertLineKnowingly(n, tl, tr,
      x);
  data_t mid = (tl + tr) / 2;
  PushLazy(n, tl, tr);
  InsertLine(n->lc, tl, mid, l, r, x);
  InsertLine(n->rc, mid + 1, tr, l, r, x);
}

void AddLine(Node* &n, data_t tl, data_t tr, data_t l, data_t r
    , Line<data_t> x) {
  if (tr < l || r < tl || tl > tr || l > r) return;
  if (n == nullptr) n = new Node();
  if (l <= tl && tr <= r) return n->apply(tl, tr, x);
  data_t mid = (tl + tr) / 2;
  PushLazy(n, tl, tr);
  PushLine(n, tl, tr);
  AddLine(n->lc, tl, mid, l, r, x);
  AddLine(n->rc, mid + 1, tr, l, r, x);
}

data_t Query(Node* &n, data_t tl, data_t tr, data_t x) {
  if (n == nullptr) return -inf;
  if (tl == tr) return n->line.get(x);
  data_t res = n->line.get(x);
  data_t mid = (tl + tr) / 2;
  PushLazy(n, tl, tr);
  if (x <= mid) res = max(res, Query(n->lc, tl, mid, x));
  else res = max(res, Query(n->rc, mid + 1, tr, x));
  return res;
}

void InsertLine(data_t l, data_t r, Line<data_t> x) {
  return InsertLine(root, 0, sz - 1, l, r, x);
}

void AddLine(data_t l, data_t r, Line<data_t> x) {
  return AddLine(root, 0, sz - 1, l, r, x);
}

data_t Query(data_t x) {
  return Query(root, 0, sz - 1, x);
}
```

# Numerical (4)

## 4.1 Polynomials and recurrences

### Polynomial.h

81a226, 17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for (int i = SZ(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    FOR(i,1,SZ(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=SZ(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
    a.pop_back();
  }
};
```

### PolyRoots.h

**Description:** Finds the real roots to a polynomial.
**Usage:** `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
**Time:** $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" aa01df, 21 lines

```
vector<double> polyRoots(Poly p, double xmin, double xmax) {
  if (SZ(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p; der.diff();
  auto dr = polyRoots(der, xmin, xmax);
  dr.pb(xmin-1); dr.pb(xmax+1);
  sort(all(dr));
  FOR(i,0,SZ(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      FOR(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.pb((l + h) / 2);
    }
  }
  return ret;
}
```

### PolyInterpolate.h

**Description:** Given $n$ points (x[i], y[i]), computes an n-1-degree polynomial $p$ that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
**Time:** $\mathcal{O}\left(n^2\right)$

b9cd08, 13 lines

```
using vd = vector<double>;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  FOR(k,0,n-1) FOR(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  FOR(k,0,n) FOR(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
}
```

### BerlekampMassey.h

**Description:** Recovers any $n$-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
**Usage:** `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
**Time:** $\mathcal{O}\left(N^2\right)$

"../number-theory/ModPow.h" 3249f4, 18 lines

```
vector<ll> berlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;
  ll b = 1;
  FOR(i,0,n) { ++m;
    ll d = s[i] % mod;
    FOR(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    FOR(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }
  C.resize(L + 1); C.erase(C.begin());
  for (ll& x : C) x = (mod - x) % mod;
  return C;
}
```

### LinearRecurrence.h

**Description:** Generates the $k$'th term of an $n$-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots \geq n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
**Usage:** `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
**Time:** $\mathcal{O}\left(n^2 \log k\right)$

fd833a, 22 lines

```
using Poly = vector<ll>;
ll linearRec(Poly S, Poly tr, ll k) {
  int n = SZ(tr);
  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    FOR(i,0,n+1) FOR(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) FOR(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
  };
  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;
  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }
  ll res = 0;
  FOR(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
}
```

## 4.2 Optimization

### Integrate.h

**Description:** Simple integration of a function over an interval using Simpson's rule. The error should be proportional to $h^4$, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

26598f, 6 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
  double h = (b - a) / 2 / n, v = f(a) + f(b);
  FOR(i,1,n*2) v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
}
```

### IntegrateAdaptive.h

**Description:** Fast integration using an adaptive Simpson's rule.
**Usage:** `double sphereVolume = quad(-1, 1, [](double x) {`
`return quad(-1, 1, [&](double y) {`
`return quad(-1, 1, [&](double z) {`
`return x*x + y*y + z*z < 1; });});});`

f57493, 15 lines

```
using d = double;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
  d c = (a + b) / 2;
  d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
  if (abs(T - S) <= 15 * eps || b - a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
  return rec(f, a, b, eps, S(a, b));
}
```

### Simplex.h

**Description:** Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal $x$ (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
**Usage:** `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`
**Time:** $\mathcal{O}\left(NM * \#pivots\right)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^n\right)$ in the general case.

0904c4, 67 lines

```
using T = double; // long double, Rational, double + mod<P>...
using vd = vector<T>;
using vvd = vector<vd>;

const T eps = 1e-8, inf = 1/.0;
#define ltj(X) if(s == -1 || mp(X[j],N[j]) < mp(X[s],N[s])) s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(SZ(b)), n(SZ(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      FOR(i,0,m) FOR(j,0,n) D[i][j] = A[i][j];
      FOR(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];}
      FOR(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }

  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    FOR(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      FOR(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    FOR(j,0,n+2) if (j != s) D[r][j] *= inv;
    FOR(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }

  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      FOR(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      FOR(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || mp(D[i][n+1] / D[i][s], B[i])
                     < mp(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }

  T solve(vd &x) {
    int r = 0;
```

```
    FOR(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      FOR(i,0,m) if (B[i] == -1) {
        int s = 0;
        FOR(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    FOR(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

## 4.3 Matrices

### Determinant.h

**Description:** Calculates determinant of a matrix. Destroys the matrix.
**Time:** $\mathcal{O}\left(N^3\right)$

1de5db, 15 lines

```
double det(vector<vector<double>>& a) {
  int n = SZ(a); double res = 1;
  FOR(i,0,n) {
    int b = i;
    FOR(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    FOR(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) FOR(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
}
```

### IntDeterminant.h

**Description:** Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
**Time:** $\mathcal{O}\left(N^3\right)$

dfd9dc, 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = SZ(a); ll ans = 1;
  FOR(i,0,n) {
    FOR(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) FOR(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
}
```

### SolveLinear.h

**Description:** Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in $A$ and $b$ is lost.
**Time:** $\mathcal{O}\left(n^2m\right)$

6218ee, 38 lines

```
using vd = vector<double>;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = SZ(A), m = SZ(x), rank = 0, br, bc;
  if (n) assert(SZ(A[0]) == m);
  vi col(m); iota(all(col), 0);

  FOR(i,0,n) {
    double v, bv = 0;
    FOR(r,i,n) FOR(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      FOR(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    FOR(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    FOR(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      FOR(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    FOR(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

### SolveLinear2.h

**Description:** To get all uniquely determined values of $x$ back from SolveLinear, make the following changes:

"SolveLinear.h" a561f5, 7 lines

```
FOR(j,0,n) if (j != i) // instead of FOR(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
FOR(i,0,rank) {
  FOR(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
```

### SolveLinearBinary.h

**Description:** Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys $A$ and $b$.
**Time:** $\mathcal{O}\left(n^2m\right)$

331913, 34 lines

```
using bs = bitset<1000>;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = SZ(A), rank = 0, br;
  assert(m <= SZ(x));
  vi col(m); iota(all(col), 0);
  FOR(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      FOR(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    FOR(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    FOR(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    FOR(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

### MatrixInverse.h

**Description:** Invert matrix $A$. Returns rank; result is stored in $A$ unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where $A^{-1}$ starts as the inverse of A mod p, and k is doubled in each step.
**Time:** $\mathcal{O}\left(n^3\right)$

0d0b59, 35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = SZ(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  FOR(i,0,n) tmp[i][i] = 1, col[i] = i;

  FOR(i,0,n) {
    int r = i, c = i;
    FOR(j,i,n) FOR(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    FOR(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    FOR(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      FOR(k,i+1,n) A[j][k] -= f*A[i][k];
      FOR(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    FOR(j,i+1,n) A[i][j] /= v;
    FOR(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) FOR(j,0,i) {
    double v = A[j][i];
    FOR(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  FOR(i,0,n) FOR(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
}
```

### MatrixInverse-mod.h

**Description:** Invert matrix $A$ modulo a prime. Returns rank; result is stored in $A$ unless singular (rank < n). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where $A^{-1}$ starts as the inverse of A mod p, and k is doubled in each step.
**Time:** $\mathcal{O}\left(n^3\right)$

"../number-theory/ModPow.h" ebf290, 36 lines

```
int matInv(vector<vector<ll>>& A) {
  int n = SZ(A); vi col(n);
  vector<vector<ll>> tmp(n, vector<ll>(n));
  FOR(i,0,n) tmp[i][i] = 1, col[i] = i;

  FOR(i,0,n) {
    int r = i, c = i;
    FOR(j,i,n) FOR(k,i,n) if (A[j][k]) {
      r = j; c = k; goto found;
    }
    return i;
found:
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    FOR(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c
        ]);
    swap(col[i], col[c]);
    ll v = modpow(A[i][i], mod - 2);
    FOR(j,i+1,n) {
      ll f = A[j][i] * v % mod;
      A[j][i] = 0;
      FOR(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod;
      FOR(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
    }
    FOR(j,i+1,n) A[i][j] = A[i][j] * v % mod;
    FOR(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) FOR(j,0,i) {
    ll v = A[j][i];
    FOR(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
  }

  FOR(i,0,n) FOR(j,0,n)
    A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod
        : 0);
  return n;
}
```

## Tridiagonal.h

**Description:** $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where $a_0$, $a_{n+1}$, $b_i$, $c_i$ and $d_i$ are known. $a$ can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all $i$, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
**Time:** $\mathcal{O}(N)$

736a43, 26 lines

```
using T = double;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
  int n = SZ(b); vi tr(n);
  FOR(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

# 4.4 Fourier transforms

## FastFourierTransform.h

**Description:** fft(a) computes $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$ for all $k$. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$ (in practice $10^{16}$; higher for random inputs). Otherwise, use NTT/FFTMod.
**Time:** $\mathcal{O}(N \log N)$ with $N = |A| + |B|$ ($\sim 1s$ for $N = 2^{22}$)

87d73c, 35 lines

```
using C = complex<double>;
using vd = vector<double>;
void fft(vector<C>& a) {
  int n = SZ(a), L = 31 - __builtin_clz(n);
  static vector<complex<long double>> R(2, 1);
  static vector<C> rt(2, 1);  // (^ 10% faster if double)
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n); rt.resize(n);
    auto x = polar(1.0L, acos(-1.0L) / k);
    FOR(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
  }
  vi rev(n);
  FOR(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  FOR(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) FOR(j,0,k) {
      C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
      a[i + j + k] = a[i + j] - z;
      a[i + j] += z;
    }
}
vd conv(const vd& a, const vd& b) {
  if (a.empty() || b.empty()) return {};
  vd res(SZ(a) + SZ(b) - 1);
  int L = 32 - __builtin_clz(SZ(res)), n = 1 << L;
  vector<C> in(n), out(n);
  copy(all(a), begin(in));
  FOR(i,0,SZ(b)) in[i].imag(b[i]);
  fft(in);
  for (C& x : in) x *= x;
  FOR(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
  fft(out);
  FOR(i,0,SZ(res)) res[i] = imag(out[i]) / (4 * n);
  return res;
}
```

## FastFourierTransformMod.h

**Description:** Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice $10^{16}$ or higher). Inputs must be in $[0, \text{mod})$.
**Time:** $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

"FastFourierTransform.h" db1041, 22 lines

```
using vll = vector<ll>;
template<int M> vll convMod(const vll &a, const vll &b) {
  if (a.empty() || b.empty()) return {};
  vll res(SZ(a) + SZ(b) - 1);
  int B=32-__builtin_clz(SZ(res)), n=1<<B, cut=int(sqrt(M));
  vector<C> L(n), R(n), outs(n), outl(n);
  FOR(i,0,SZ(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
  FOR(i,0,SZ(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
  fft(L), fft(R);
  FOR(i,0,n) {
    int j = -i & (n - 1);
    outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
  }
  fft(outl), fft(outs);
  FOR(i,0,SZ(res)) {
    ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
    ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
    res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
  }
  return res;
}
```

## NumberTheoreticTransform.h

**Description:** ntt(a) computes $\hat{f}(k) = \sum_x a[x] g^{xk}$ for all $k$, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most $2^a$. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in $[0, \text{mod})$.
**Time:** $\mathcal{O}(N \log N)$

"../number-theory/ModPow.h" d332d2, 33 lines

```
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
using vll = vector<ll>;
void ntt(vll &a) {
  int n = sz(a), L = 31 - __builtin_clz(n);
  static vll rt(2, 1);
  for (static int k = 2, s = 2; k < n; k *= 2, s++) {
    rt.resize(n);
    ll z[] = {1, modpow(root, mod >> s)};
    FOR(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
  }
  vi rev(n);
  FOR(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
  FOR(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) FOR(j,0,k) {
      ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
      a[i + j + k] = ai - z + (z > ai ? mod : 0);
      ai += (ai + z >= mod ? z - mod : z);
    }
}
vll conv(const vll &a, const vll &b) {
  if (a.empty() || b.empty()) return {};
  int s = sz(a) + sz(b) - 1, B = 32 - __builtin_clz(s), n = 1
      << B;
  int inv = modpow(n, mod - 2);
  vll L(a), R(b), out(n);
  L.resize(n), R.resize(n);
```

```
  ntt(L), ntt(R);
  FOR(i,0,n) out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv %
       mod;
  ntt(out);
  return {out.begin(), out.begin() + s};
}
```

### FastSubsetTransform.h

**Description:** Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of $a$ must be a power of two.
**Time:** $\mathcal{O}(N \log N)$

846b5e, 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = SZ(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) FOR(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) for (int& x : a) x /= SZ(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  FOR(i,0,SZ(a)) a[i] *= b[i];
  FST(a, 1); return a;
}
```

### PolynomialDivision.h

**Description:** Pseudocode for polynomial division.

1066f2, 20 lines

```
vi polydiv(vi a, vi b) {
  int n = SZ(a), m = SZ(b);
  if(m > n) return {0};
  reverse(all(a)), reverse(all(b));
  int s = moddiv(1, b[0], MOD);
  for(int &x : b) x = modmul(x, s, MOD);
  int k = n - m + 1;
  vi r, v, g = {1};
  for (int w = 1; (1 << (w - 1)) < k; w++){
    r = polymul(g, g); r.resize(1 << w);
    v = b; v.resize(1 << w);
    r = polymul(r, v);
    r.resize(1 << w); g.resize(1 << w);
    polyadd(g, g); polysub(g, r);
  }
  g = polymul(g, a); g.resize(k);
  reverse(all(g));
  for(int &x : g) x = modmul(x, s, MOD);
  return g;
}
```

# Number theory (5)

## 5.1 Modular arithmetic

### ModMulLL.h

**Description:** Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \le a, b \le c \le 7.2 \cdot 10^{18}$.
**Time:** $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

bbbd8f, 11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

### ModSum.h

**Description:** Sums of mod'ed arithmetic progressions.
`modsum(to, c, k, m)` $= \sum_{i=0}^{to-1} (ki + c)\%m$. `divsum` is similar but for floored division.
**Time:** $\log(m)$, with a large constant.

aba7f7, 16 lines

```
using ull = unsigned long long;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

### ModSqrt.h

**Description:** Tonelli-Shanks algorithm for modular square roots. Finds $x$ s.t. $x^2 = a \pmod p$ ($-x$ gives the other solution).
**Time:** $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}(\log p)$ for most $p$

"ModPow.h" 19a793, 23 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m) t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

## 5.2 Primality

### MillerRabin.h

**Description:** Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
**Time:** 7 times the complexity of $a^b \mod c$.

"ModMulLL.h" 60dcd1, 12 lines

```
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
       s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) {   // ^ count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

### Factor.h

**Description:** Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
**Time:** $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

"ModMulLL.h", "MillerRabin.h" c5522b, 18 lines

```
ull pollard(ull n) {
  auto f = [n](ull x) { return modmul(x, x, n) + 1; };
  ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
  while (t++ % 40 || gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return gcd(prd, n);
}
vector<ull> factor(ull n) {
  if (n == 1) return {};
  if (isPrime(n)) return {n};
  ull x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), all(r));
  return l;
}
```

## 5.3 Divisibility

### euclid.h

**Description:** Finds two integers $x$ and $y$, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in `gcd` instead. If $a$ and $b$ are coprime, then $x$ is the inverse of $a \pmod b$.

33ba8f, 5 lines

```
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (!b) return x = 1, y = 0, a;
  ll d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

### CRT.h

**Description:** Chinese Remainder Theorem.
`crt(a, m, b, n)` computes $x$ such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, $x$ will obey $0 \le x < \text{lcm}(m, n)$. Assumes $mn < 2^{62}$.
**Time:** $\log(n)$

"euclid.h" 04d93a, 7 lines

```
ll crt(ll a, ll m, ll b, ll n) {
  if (n > m) swap(a, b), swap(m, n);
  ll x, y, g = euclid(m, n, x, y);
  assert((a - b) % g == 0); // else no solution
  x = (b - a) % n * x % n / g * m + a;
  return x < 0 ? x + m*n/g : x;
}
```

## 5.4 Other

### FracBinarySearch.h

**Description:** Given $f$ and $N$, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \le N$. You may want to throw an exception from $f$ if it finds an exact solution, in which case $N$ can be removed.
**Usage:** `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`

**Time:** $\mathcal{O}(\log(N))$ 27ab3e, 24 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv; hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

PrefixSumMultiplicative.h

**Description:** Prefix sum of multiplicative functions : p_f : the prefix sum of f (x) (1 <= x <= th). p_g : the prefix sum of g (x) (0 <= x <= N). p_c : the prefix sum of f * g (x) (0 <= x <= N). th : the thereshold, generally should be $N^{\frac{2}{3}}$.

**Time:** $\mathcal{O}\left(N^{\frac{2}{3}}\right)$ 3ffa99, 28 lines

```
struct prefix_mul {
  typedef ll (*func) (ll);
  func p_f, p_g, p_c;
  ll n, th;
  unordered_map<ll, ll> mem;

  prefix_mul(func p_f, func p_g, func p_c) : p_f(p_f), p_g(p_g)
       , p_c(p_c) {}

  ll calc (ll x) {
    if (x <= th) return p_f (x);
    auto d = mem.find (x);
    if (d != mem.end ()) return d -> second;
    ll ans = 0;
    for (ll i = 2, la; i <= x; i = la + 1) {
      la = x / (x / i);
      ans = ans + (p_g(la) - p_g(i - 1) + mod) * calc(x / i);
    }
    ans = p_c(x) - ans; ans = ans / inv;
    return mem[x] = ans;
  }

  ll solve (ll n, ll th) {
    if (n <= 0) return 0;
    prefix_mul::n = n; prefix_mul::th = th;
    inv = p_g(1);
    return calc(n);
  }
};
```

## 5.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2),\ b = k \cdot (2mn),\ c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either $m$ or $n$ even.

## 5.6 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \le m \le n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \le m \le n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

# Combinatorial (6)

## 6.1 Permutations

### 6.1.1 Burnside's lemma

Given a group $G$ of symmetries and a set $X$, the number of elements of $X$ *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where $X^g$ are the elements fixed by $g$ ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length $n$, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k) \phi(n/k).$$

## 6.2 General purpose numbers

### 6.2.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able).
$B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$
$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

### 6.2.2 Stirling numbers of the first kind

Number of permutations on $n$ items with $k$ cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k) x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

### 6.2.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly $k$ elements are greater than the previous element. $k$ $j$:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ $j$:s s.t. $\pi(j) \ge j$, $k$ $j$:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

### 6.2.4 Stirling numbers of the second kind

Partitions of $n$ distinct elements into exactly $k$ groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!} \sum_{j=0}^{k} (-1)^{k-j} \binom{k}{j} j^n$$

### 6.2.5 Bell numbers

Total number of partitions of $n$ distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For $p$ prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

### 6.2.6 Labeled unrooted trees

# on $n$ vertices: $n^{n-2}$
# on $k$ existing trees of size $n_i$: $n_1 n_2 \cdots n_k n^{k-2}$
# with degrees $d_i$: $(n-2)!/((d_1-1)! \cdots (d_n-1)!)$

## 6.3 Matroids

MatroidIntersection.h

**Description:** Computes a set of maximum size which is independent in both matroids.

**Time:** $\mathcal{O}\left(GI^{3/2}\right)$ calls to oracles

d61fd9, 40 lines

```
template<class M1, class M2> struct MatroidIsect {
  int n; vi iset; M1 m1; M2 m2;
  bool augment() {
    vi pre(n + 1, -1); queue<int> q({n});
    while(SZ(q)) {
      int x = q.front(); q.pop();
      if(iset[x]) {
        m1.clear();
        FOR(i, 0, n) if(iset[i] && i != x) m1.ins(i);
        FOR(i, 0, n) if(!iset[i] && pre[i] == -1 &&
          m1.indep(i)) pre[i] = x, q.push(i);
      }
      else {
        auto backE = [&]() {
          m2.clear();
          FOR(c, 0, 2) FOR(i, 0, n) if((x == i || iset[i]) &&
          (pre[i] == -1) == c) {
            if(!m2.indep(i)) return c ? pre[i] = x, q.push(i),
                 i:-1;
            m2.ins(i);
          }
          return n;
        };
        for(int y; (y = backE()) != -1;) if(y == n) {
          for(; x != n; x = pre[x]) iset[x] = !iset[x];
          return 1;
        }
      }
    }
    return 0;
  }
  MatroidIsect(int _n, M1 _m1, M2 _m2) :
  n(_n), m1(_m1), m2(_m2) {
    iset.resize(n + 1), iset[n] = 1;
    m1.clear(); m2.clear();
    for(int i=n-1; i>=0; i--) if(m1.indep(i) && m2.indep(i)) {
      iset[i] = 1, m1.ins(i), m2.ins(i);
    }
    while (augment());
  }
};
```

MatroidOracle.h

**Description:** Example of simple matroid oracle.

459ccb, 8 lines

```
struct ColorfulMatroid {
  int C = 0; vi col, used;
  ColorfulMatroid(vi _col) : col(_col) {
    for(auto &c: col) C = max(C, c + 1); }
  void clear() { used = vi(C, 0); }
  void ins(int i) { used[col[i]] = 1; }
  bool indep(int i) { return !used[col[i]]; }
};
```

# Graph (7)

## 7.1 Network flow

MinCostMaxFlow.h

**Description:** Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.

**Time:** Approximately $\mathcal{O}\left(E^2\right)$

f90ffa, 81 lines

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;
using vll = vector<ll>;

struct MCMF {
  int N;
  vector<vi> ed, red;
  vector<vll> cap, flow, cost;
  vi seen;
  vll dist, pi;
  vector<pii> par;

  MCMF(int N) :
    N(N), ed(N), red(N), cap(N, vll(N)), flow(cap), cost(cap),
    seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
    ed[from].pb(to);
    red[to].pb(from);
  }

  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});

    auto relax = [&](int i, ll cap, ll cost, int dir) {
      ll val = di - pi[i] + cost;
      if (cap && val < dist[i]) {
        dist[i] = val;
        par[i] = {s, dir};
        if (its[i] == q.end()) its[i] = q.push({-dist[i], i});
        else q.modify(its[i], {-dist[i], i});
      }
    };

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      for (int i : ed[s]) if (!seen[i])
        relax(i, cap[s][i] - flow[s][i], cost[s][i], 1);
      for (int i : red[s]) if (!seen[i])
        relax(i, flow[i][s], -cost[i][s], 0);
    }
    FOR(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        fl = min(fl, r ? cap[p][x] - flow[p][x] : flow[x][p]);
      totflow += fl;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        if (r) flow[p][x] += fl;
        else flow[x][p] -= fl;
    }
    FOR(i,0,N) FOR(j,0,N) totcost += cost[i][j] * flow[i][j];
    return {totflow, totcost};
  }

  // If some costs can be negative, call this before maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      FOR(i,0,N) if (pi[i] != INF)
        for (int to : ed[i]) if (cap[i][to])
          if ((v = pi[i] + cost[i][to]) < pi[to])
            pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

Dinic.h

**Description:** Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching.

286613, 42 lines

```
struct Dinic {
  struct Edge {
    int to, rev;
    ll c, oc;
    ll flow() { return max(oc - c, 0LL); } // if you need flows
  };
  vi lvl, ptr, q;
  vector<vector<Edge>> adj;
  Dinic(int n) : lvl(n), ptr(n), q(n), adj(n) {}
  void addEdge(int a, int b, ll c, ll rcap = 0) {
    adj[a].pb({b, SZ(adj[b]), c, c});
    adj[b].pb({a, SZ(adj[a]) - 1, rcap, rcap});
  }
  ll dfs(int v, int t, ll f) {
    if (v == t || !f) return f;
    for (int& i = ptr[v]; i < SZ(adj[v]); i++) {
      Edge& e = adj[v][i];
      if (lvl[e.to] == lvl[v] + 1)
        if (ll p = dfs(e.to, t, min(f, e.c))) {
          e.c -= p, adj[e.to][e.rev].c += p;
          return p;
        }
    }
    return 0;
  }
  ll calc(int s, int t) {
    ll flow = 0; q[0] = s;
    FOR(L,0,31) do { // 'int L=30' maybe faster for random data
      lvl = ptr = vi(SZ(q));
      int qi = 0, qe = lvl[s] = 1;
      while (qi < qe && !lvl[t]) {
        int v = q[qi++];
        for (Edge e : adj[v])
          if (!lvl[e.to] && e.c >> (30 - L))
            q[qe++] = e.to, lvl[e.to] = lvl[v] + 1;
      }
      while (ll p = dfs(s, t, LLONG_MAX)) flow += p;
    } while (lvl[t]);
    return flow;
  }
  bool leftOfMinCut(int a) { return lvl[a] != 0; }
};
```

### GlobalMinCut.h

**Description:** Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
**Time:** $\mathcal{O}(V^3)$

8d7f68, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
  pair<int, vi> best = {INT_MAX, {}};
  int n = SZ(mat);
  vector<vi> co(n);
  FOR(i,0,n) co[i] = {i};
  FOR(ph,1,n) {
    vi w = mat[0];
    size_t s = 0, t = 0;
    FOR(it,0,n-ph) { // O(V^2) -> O(E log V) with prio. queue
      w[t] = INT_MIN;
      s = t, t = max_element(all(w)) - w.begin();
      FOR(i,0,n) w[i] += mat[t][i];
    }
    best = min(best, {w[t] - mat[t][t], co[t]});
    co[s].insert(co[s].end(), all(co[t]));
    FOR(i,0,n) mat[s][i] += mat[t][i];
    FOR(i,0,n) mat[i][s] = mat[s][i];
    mat[0][t] = INT_MIN;
  }
  return best;
}
```

### GomoryHu.h

**Description:** Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
**Time:** $\mathcal{O}(V)$ Flow Computations

"Dinic.h" ce350b, 13 lines

```
using Edge = array<ll, 3>;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
  vector<Edge> tree;
  vi par(N);
  FOR(i,1,N) {
    Dinic D(N); // Any flow works
    for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
    tree.pb({i, par[i], D.calc(i, par[i])});
    FOR(j,i+1,N)
      if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
  }
  return tree;
}
```

## 7.2 Matching

### hopcroftKarp.h

**Description:** Fast bipartite matching algorithm. Graph $g$ should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex $i$ on the right side, or $-1$ if it's not matched.
**Usage:** `vi btoa(m, -1); hopcroftKarp(g, btoa);`
**Time:** $\mathcal{O}\left(\sqrt{V}E\right)$

a8c363, 42 lines

```
bool dfs(int a, int L, vector<vi>& g, vi& btoa, vi& A, vi& B) {
  if (A[a] != L) return 0;
  A[a] = -1;
  for (int b : g[a]) if (B[b] == L + 1) {
    B[b] = 0;
    if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
      return btoa[b] = a, 1;
  }
  return 0;
}

int hopcroftKarp(vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);
    fill(all(B), 0);
    cur.clear();
    for (int a : btoa) if(a != -1) A[a] = -1;
    FOR(a,0,SZ(g)) if(A[a] == 0) cur.pb(a);
    for (int lay = 1;; lay++) {
      bool islast = 0;
      next.clear();
      for (int a : cur) for (int b : g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        }
        else if (btoa[b] != a && !B[b]) {
          B[b] = lay;
          next.pb(btoa[b]);
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      for (int a : next) A[a] = lay;
      cur.swap(next);
    }
    FOR(a,0,SZ(g))
      res += dfs(a, 0, g, btoa, A, B);
  }
}
```

### MinimumVertexCover.h

**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"hopcroftKarp.h" aa85e2, 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  vi match(m, -1);
  int res = hopcroftKarp(g, match);
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vi q, cover;
  FOR(i,0,n) if (lfound[i]) q.pb(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.pb(match[e]);
    }
  }
  FOR(i,0,n) if (!lfound[i]) cover.pb(i);
  FOR(i,0,m) if (seen[i]) cover.pb(n+i);
  assert(SZ(cover) == res);
  return cover;
}
```

### WeightedMatching.h

**Description:** Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost. Requires $N \leq M$.
**Time:** $\mathcal{O}\left(N^2M\right)$

9603a1, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
  if (a.empty()) return {0, {}};
  int n = SZ(a) + 1, m = SZ(a[0]) + 1;
  vi u(n), v(m), p(m), ans(n - 1);
  FOR(i,1,n) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vi dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      FOR(j,1,m) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      FOR(j,0,m) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  FOR(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

### GeneralMatching.h

**Description:** Matching for general graphs. Fails with probability $N/mod$.
**Time:** $\mathcal{O}\left(N^3\right)$

"../numerical/MatrixInverse-mod.h" 959f0f, 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  for (pii pa : ed) {
    int a = pa.st, b = pa.nd, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  }

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);

  if (M != N) do {
    mat.resize(M, vector<ll>(M));
    FOR(i,0,N) {
      mat[i].resize(M);
      FOR(j,N,M) {
        int r = rand() % mod;
        mat[i][j] = r, mat[j][i] = (mod - r) % mod;
      }
    }
  } while (matInv(A = mat) != M);

  vi has(M, 1); vector<pii> ret;
  FOR(it,0,M/2) {
    FOR(i,0,M) if (has[i])
      FOR(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;
      } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0;
    FOR(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);
      FOR(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        FOR(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
      }
      swap(fi,fj);
    }
```

```
  }
  return ret;
}
```

### GeneralMatchingRandom.h

**Description:** Fast randomized algorithm for maximum general matching.

596196, 24 lines

```
const int N = 510;
vi g[N];
int lnk[N], vis[N], tim;

bool dfs(int x) {
  if(!x) return 1;
  vis[x] = tim;
  random_shuffle(all(g[x]));
  for(int u: g[x]) {
    int v = lnk[u];
    if(vis[v] < tim) {
      lnk[x] = u, lnk[u] = x, lnk[v] = 0;
      if(dfs(v)) return 1;
      lnk[u] = v, lnk[v] = u, lnk[x] = 0;
    }
  }
  return 0;
}

int max_matching(int n) {
  int ans = 0;  // 20 iterations should also work fine
  FOR(_, 0, 500) rep(i, 1, n) if(!lnk[i]) tim++, ans += dfs(i);
  return ans;
}
```

## 7.3 DFS algorithms

### BiconnectedComponents.h

**Description:** Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

**Usage:** `int eid = 0; ed.resize(N);`
`for each edge (a,b) {`
`ed[a].emplace_back(b, eid);`
`ed[b].emplace_back(a, eid++); }`
`bicomps([&](const vi& edgelist) {...});`

**Time:** $\mathcal{O}(E+V)$

3907d3, 32 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
  int me = num[at] = ++Time, e, y, top = me;
  for (auto pa : ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me) st.pb(e);
    } else {
      int si = SZ(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.pb(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.pb(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}

template<class F>
void bicomps(F f) {
  num.assign(SZ(ed), 0);
  FOR(i,0,SZ(ed)) if (!num[i]) dfs(i, -1, f);
}
```

### 2sat.h

**Description:** Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||!b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

**Usage:** `TwoSat ts(number of boolean variables);`
`ts.either(0, ~3); // Var 0 is true or var 3 is false`
`ts.setValue(2); // Var 2 is true`
`ts.atMostOne({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true`
`ts.solve(); // Returns true iff it is solvable`
`ts.values[0..N-1] holds the assigned values to the vars`

**Time:** $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

47a32c, 52 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true
  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int addVar() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f); j = max(2*j, -1-2*j);
    gr[f].pb(j^1); gr[j].pb(f^1);
  }

  void setValue(int x) { either(x, x); }

  void atMostOne(const vi& li) { // (optional)
    if (SZ(li) <= 1) return;
    int cur = ~li[0];
    FOR(i,2,SZ(li)) {
      int next = addVar();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.pb(i);
    for(int e : gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    if (low == val[i]) do {
      x = z.back(); z.pop_back(); comp[x] = low;
      if (values[x>>1] == -1) values[x>>1] = x&1;
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    FOR(i,0,2*N) if (!comp[i]) dfs(i);
    FOR(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
};
```

### EulerWalk.h

**Description:** Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

**Time:** $\mathcal{O}(V+E)$

72d33d, 15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
  int n = SZ(gr);
  vi D(n), its(n), eu(nedges), ret, s = {src};
  D[src]++; // to allow Euler paths, not just cycles
  while (!s.empty()) {
    int x = s.back(), y, e, &it = its[x], end = SZ(gr[x]);
    if (it == end){ ret.pb(x); s.pop_back(); continue; }
    tie(y, e) = gr[x][it++];
    if (!eu[e]) {
      D[x]--, D[y]++;
      eu[e] = 1; s.pb(y);
    }}
  for (int x : D) if (x < 0 || SZ(ret) != nedges+1) return {};
  return {ret.rbegin(), ret.rend()};
}
```

## 7.4 Trees

### HLD.h

**Description:** Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.

**Time:** $\mathcal{O}\left((\log N)^2\right)$

8cd318, 36 lines

```
struct HLD {
  int N, tim = 0, VALS_EDGES = 0; // change to 1 if needed
  vector<vi> adj;
  vi par, siz, depth, rt, pos;
  HLD(vector<vi> adj_) : N(SZ(adj_)), adj(adj_), par(N, -1),
    siz(N, 1), depth(N), rt(N), pos(N) { dfsSz(0); dfsHld(0); }
  void dfsSz(int v) {
    if(par[v] != -1) adj[v].erase(find(all(adj[v]), par[v]));
    for(int &u: adj[v]) {
      par[u] = v, depth[u] = depth[v] + 1;
      dfsSz(u);
      siz[v] += siz[u];
      if(siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
    }
  }
  void dfsHld(int v) {
    pos[v] = tim++;
    for(int u : adj[v]) {
      rt[u] = (u == adj[v][0] ? rt[v] : u);
      dfsHld(u);
    }
  }
  vector<pii> path(int u, int v) {
    vector<pii> paths;
    for (; rt[u] != rt[v]; v = par[rt[v]]) {
      if(depth[rt[u]] > depth[rt[v]]) swap(u, v);
```

```
      paths.pb({pos[rt[v]], pos[v]});
    }
    if(depth[u] > depth[v]) swap(u, v);
    paths.pb({pos[u] + VALS_EDGES, pos[v]});
    return paths;
  }
  pii subtree(int v) {
    return {pos[v] + VALS_EDGES, pos[v] + siz[v] - 1};
  }
};
```

## CentroidTree.h

**Description:** Builds a centroid tree. 0-indexed
**Time:** $\mathcal{O}(N \log N)$

5cd256, 39 lines

```
struct CentroidTree {
  vector<vector<pii>> G;  // {neighbour, distance}
  vector<ll> dist[20];
  vi sz, block, par, dpth;

  CentroidTree(vector<vector<pii>> &_G) : G(_G) {
    int n = SZ(G);
    sz = block = par = dpth = vi(n);
    FOR(i, 0, 20) dist[i].resize(n);
    decomp(0, -1);
  }

  void sz_dfs(int v, int p) {
    sz[v] = 1;
    for(auto &[u, _]: G[v]) if(u != p && !block[u]) {
      sz_dfs(u, v); sz[v] += sz[u];
    }
  }
  int find_centr(int v, int p, int sum) {
    for(auto &[u, _]: G[v])
      if(u != p && !block[u] && sz[u] > sum / 2)
        return find_centr(u, v, sum);
    return v;
  }
  void dist_dfs(int v, int p, int gleb, ll akt) {
    dist[gleb][v] = akt;
    for(auto &[u, d]: G[v]) if(u != p && !block[u])
      dist_dfs(u, v, gleb, akt + d);
  }
  void decomp(int v, int p) {
    sz_dfs(v, -1);
    v = find_centr(v, -1, sz[v]);
    par[v] = p;
    if(p != -1) dpth[v] = dpth[p] + 1;
    dist_dfs(v, -1, dpth[v], 0);
    block[v] = 1;
    for(auto &[u, _]: G[v]) if(!block[u]) decomp(u, v);
  }
};
```

## LinkCutTree.h

**Description:** One-indexed. Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree. Also you can update a value in a vertex, and do standard path and subtree queries. Operation on subtrees must be inversible. For even bigger hammer have a look at TopTree.
**Time:** All operations take amortized $\mathcal{O}(\log N)$.

42b461, 101 lines

```
struct SplayTree {
  struct Node {
    int p = 0, ch[2] = {0, 0};
    ll self = 0, path = 0;  // Path aggregates
    ll sub = 0, vir = 0;  // Subtree aggregates
    bool flip = 0;        // Lazy tags
  };
  vector<Node> t;
  SplayTree(int n) : t(n + 1) {}

  void push(int v) {
    if(!v || !t[v].flip) return;
    auto &[l, r] = t[v].ch;
    t[l].flip ^= 1, t[r].flip ^= 1;
    swap(l, r), t[v].flip = 0;
  }

  void pull(int v) {
    auto [l, r] = t[v].ch; push(l), push(r);
    t[v].path = t[l].path + t[v].self + t[r].path;
    t[v].sub = t[v].vir + t[l].sub + t[v].self + t[r].sub;
  }

  void set(int u, int d, int v) {
    t[u].ch[d] = v, t[v].p = u, pull(u);
  }

  void splay(int v) {
    auto dir = [&](int x) {
      int u = t[x].p;
      return t[u].ch[0] == x ? 0 : t[u].ch[1] == x ? 1 : -1;
    };
    auto rotate = [&](int x) {
      int y = t[x].p, z = t[y].p, dx = dir(x), dy = dir(y);
      set(y, dx, t[x].ch[!dx]), set(x, !dx, y);
      if(~dy) set(z, dy, x);
      t[x].p = z;
    };
    for(push(v); ~dir(v); ) {
      int y = t[v].p, z = t[y].p;
      push(z), push(y), push(v);
      int dv = dir(v), dy = dir(y);
      if(~dy) rotate(dv == dy ? y : v);
      rotate(v);
    }
  }
};

struct LinkCut : SplayTree {  // One-indexed
  LinkCut(int n) : SplayTree(n) {}

  int access(int v) {
    int u = v, x = 0;
    for(; u; x = u, u = t[u].p) {
      splay(u);
      int &ox = t[u].ch[1];
      t[u].vir += t[ox].sub;
      t[u].vir -= t[x].sub;
      ox = x, pull(u);
    }
    return splay(v), x;
  }

  void reroot(int v) {
    access(v), t[v].flip ^= 1, push(v);
  }

  void link(int u, int v) {
    reroot(u), access(v);
    t[v].vir += t[u].sub;
    t[u].p = v, pull(v);
  }

  void cut(int u, int v) {
    reroot(u), access(v);
    t[v].ch[0] = t[u].p = 0, pull(v);
  }

  // Rooted tree LCA. Returns 0 if u and v are not connected.
  int lca(int u, int v) {
    if(u == v) return u;
    access(u); int ret = access(v);
    return t[u].p ? ret : 0;
  }

  // Query subtree of u where v is outside the subtree.
  ll getSub(int u, int v) {
    reroot(v), access(u);
    return t[u].vir + t[u].self;
  }

  ll getPath(int u, int v) {
    reroot(u), access(v);
    return t[v].path;
  }

  // Update vertex u with value val
  void update(int u, ll val) {
    access(u), t[u].self = val, pull(u);
  }
};
```

## DirectedMST.h

**Description:** Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
**Time:** $\mathcal{O}(E \log V)$

"../data-structures/UnionFindRollback.h" 017a63, 60 lines

```
struct Edge { int a, b; ll w; };
struct Node {
  Edge key;
  Node *l, *r;
  ll delta;
  void prop() {
    key.w += delta;
    if (l) l->delta += delta;
    if (r) r->delta += delta;
    delta = 0;
  }
  Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
  if (!a || !b) return a ?: b;
  a->prop(), b->prop();
  if (a->key.w > b->key.w) swap(a, b);
  swap(a->l, (a->r = merge(b, a->r)));
  return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
  RollbackUF uf(n);
  vector<Node*> heap(n);
  for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
  ll res = 0;
  vi seen(n, -1), path(n), par(n);
  seen[r] = r;
  vector<Edge> Q(n), in(n, {-1,-1}), comp;
  deque<tuple<int, int, vector<Edge>>> cycs;
  FOR(s,0,n) {
    int u = s, qi = 0, w;
    while (seen[u] < 0) {
      if (!heap[u]) return {-1,{}};
      Edge e = heap[u]->top();
      heap[u]->delta -= e.w, pop(heap[u]);
```

```
      Q[qi] = e, path[qi++] = u, seen[u] = s;
      res += e.w, u = uf.find(e.a);
      if (seen[u] == s) {
        Node* cyc = 0;
        int end = qi, time = uf.time();
        do cyc = merge(cyc, heap[w = path[--qi]]);
        while (uf.join(u, w));
        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
      }
    }
    FOR(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
  }

  for (auto& [u,t,comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
  }
  FOR(i,0,n) par[i] = in[i].a;
  return {res, par};
}
```

DominatorTree.h

**Description:** Computes Dominator tree. 0-indexed

c099a3, 46 lines

```
struct dominator_tree {
    vector<basic_string<int>> g, rg, bucket;
    basic_string<int> arr, par, rev, sdom, dom, dsu, label;
    int n, t;
    dominator_tree(int n) : g(n), rg(n), bucket(n), arr(n, -1),
          par(n, -1), rev(n, -1), sdom(n, -1), dom(n, -1),
          dsu(n, 0), label(n, 0), n(n), t(0) {}
    void add_edge(int u, int v) { g[u] += v; }
    void dfs(int u) {
        arr[u] = t; rev[t] = u;
        label[t] = sdom[t] = dsu[t] = t; t++;
        for (int w : g[u]) {
            if (arr[w] == -1) {
                dfs(w);
                par[arr[w]] = arr[u];
            }
            rg[arr[w]] += arr[u];
        }
    }
    int find(int u, int x = 0) {
        if(u == dsu[u]) return x ? -1 : u;
        int v = find(dsu[u], x + 1);
        if(v < 0) return u;
        if(sdom[label[dsu[u]]] < sdom[label[u]]) label[u] =
             label[dsu[u]];
        dsu[u] = v;
        return x ? v : label[u];
    } /* returns -1 for unreachable */
    vi run(int root) {
        dfs(root);
        iota(all(dom), 0);
        per(i, 0, t-1) {
            for(int w : rg[i]) sdom[i] = min(sdom[i], sdom[find
                 (w)]);
            if(i) bucket[sdom[i]] += i;
            for (int w : bucket[i]) {
                int v = find(w);
                if (sdom[v] == sdom[w]) dom[w] = sdom[w];
                else dom[w] = v;
            }
            if(i > 1) dsu[i] = par[i];
        }
        FOR(i, 1, t) if(dom[i] != sdom[i]) dom[i] = dom[dom[i
             ]];
        vi outside_dom(n, -1);
        FOR(i, 1, t) outside_dom[rev[i]] = rev[dom[i]];
        return outside_dom;
    }
};
```

## 7.5 Various

EdgeColoring.h

**Description:** Given a simple, undirected graph with max degree $D$, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. ($D$-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

**Time:** $\mathcal{O}(NM)$

0026d7, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
  vi cc(N + 1), ret(SZ(eds)), fan(N), free(N), loc;
  for (pii e : eds) ++cc[e.first], ++cc[e.second];
  int u, v, ncols = *max_element(all(cc)) + 1;
  vector<vi> adj(N, vi(ncols, -1));
  for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
      loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
      swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
      int left = fan[i], right = fan[++i], e = cc[i];
      adj[u][e] = left;
      adj[left][e] = u;
      adj[right][e] = -1;
      free[right] = e;
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
      for (int& z = free[y] = 0; adj[y][z] != -1; z++);
  }
  FOR(i,0,SZ(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
  return ret;
}
```

MaximalCliques.h

**Description:** Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

**Time:** $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

cce8d1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  FOR(i,0,SZ(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

## 7.6 Math

### 7.6.1 Number of Spanning Trees

Create an $N \times N$ matrix `mat`, and for each edge $a \to b \in G$, do `mat[a][b]--, mat[b][b]++` (and `mat[b][a]--, mat[a][a]++` if $G$ is undirected). Remove the $i$th row and column and take the determinant; this yields the number of directed spanning trees rooted at $i$ (if $G$ is undirected, remove any row/column).

### 7.6.2 Erdős–Gallai theorem

A simple graph with node degrees $d_1 \geq \cdots \geq d_n$ exists iff $d_1 + \cdots + d_n$ is even and for every $k = 1 \ldots n$,

$$\sum_{i=1}^{k} d_i \leq k(k-1) + \sum_{i=k+1}^{n} \min(d_i, k).$$

# Geometry (8)

## 8.1 Geometric primitives

Point.h

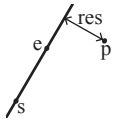
**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

0efdb5, 28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  using P = Point;
  T x, y;
  explicit Point(T _x=0, T _y=0) : x(_x), y(_y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

lineDistance.h

**Description:**
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

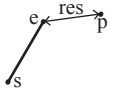



"Point.h" f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

## SegmentDistance.h

**Description:**
Returns the shortest distance between point p and the line segment from point s to e.



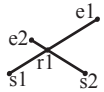

**Usage:** `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

"Point.h" 5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
}
```

## SegmentIntersection.h

**Description:**
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



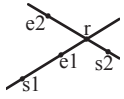

**Usage:** `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

"Point.h", "OnSegment.h" 9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
}
```

## lineIntersection.h

**Description:**
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.




**Usage:** `auto res = lineInter(s1,e1,s2,e2);`
`if (res.first == 1)`
`cout << "intersection point at " << res.second << endl;`

"Point.h" a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

## sideOf.h

**Description:** Returns where $p$ is as seen from $s$ towards $e$. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument $eps$ is given 0 is returned if $p$ is within distance $eps$ from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

**Usage:** `bool left = sideOf(p1,p2,q)==1;`

"Point.h" 3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

## OnSegment.h

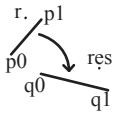
**Description:** Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

"Point.h" c597e8, 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

## linearTransformation.h

**Description:**
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.




"Point.h" 03a306, 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

## LineProjectionReflection.h

**Description:** Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

"Point.h" b5562d, 5 lines

```
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

## Angle.h

**Description:** A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

**Usage:** `vector<Angle> v = {w[0], w[0].t360() ...}; // sorted`
`int j = 0; FOR(i,0,n) { while (v[j] < v[i].t180()) ++j; }`
`// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i`

486a5e, 34 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ? mp(a, b) : mp(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

# 8.2 Circles

## CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 93169e, 11 lines

```
using P = Point<double>;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
  *out = {mid + per, mid - per};
  return true;
}
```

## CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" 31cca4, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.pb({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
}
```

## CircleLine.h

**Description:** Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

"Point.h" e0cfba, 9 lines

```
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
  P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
  double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
  if (h2 < 0) return {};
  if (h2 == 0) return {p};
  P h = ab.unit() * sqrt(h2);
  return {p - h, p + h};
}
```

## CirclePolygonIntersection.h

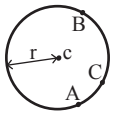
**Description:** Returns the area of the intersection of a circle with a ccw polygon.
**Time:** $\mathcal{O}(n)$

"Point.h" 22d843, 19 lines

```
using P = Point<double>;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  FOR(i,0,SZ(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % SZ(ps)] - c);
  return sum;
}
```

## circumcircle.h

**Description:**




The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h" ccf7fd, 9 lines

```
using P = Point<double>;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

## MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points.
**Time:** expected $\mathcal{O}(n)$

"circumcircle.h" 1aa727, 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(1337));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  FOR(i,0,SZ(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    FOR(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      FOR(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

# 8.3 Polygons

## InsidePolygon.h

**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
**Usage:** vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
bool in = inPolygon(v, P{3, 3}, false);
**Time:** $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" fd40d6, 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  FOR(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

## PolygonArea.h

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" 45e517, 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  FOR(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

## PolygonCenter.h

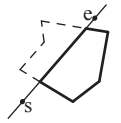
**Description:** Returns the center of mass for a polygon.
**Time:** $\mathcal{O}(n)$

"Point.h" 015ec8, 9 lines

```
using P = Point<double>;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = SZ(v) - 1; i < SZ(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

## PolygonCut.h

**Description:**




Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
**Usage:** vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h" 6c139b, 12 lines

```
using P = Point<double>;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  FOR(i,0,SZ(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.pb(lineInter(s, e, cur, prev).second);
    if (side) res.pb(cur);
  }
  return res;
}
```

## PolygonUnion.h

**Description:** Calculates the area of the union of $n$ polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
**Time:** $\mathcal{O}\left(N^2\right)$, where $N$ is the total number of points

"Point.h", "sideOf.h" c59ead, 33 lines

```
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
  double ret = 0;
  FOR(i,0,SZ(poly)) FOR(v,0,SZ(poly[i])) {
    P A = poly[i][v], B = poly[i][(v + 1) % SZ(poly[i])];
    vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
    FOR(j,0,SZ(poly)) if (i != j) {
      FOR(u,0,SZ(poly[j])) {
        P C = poly[j][u], D = poly[j][(u + 1) % SZ(poly[j])];
        int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
        if (sc != sd) {
          double sa = C.cross(D, A), sb = C.cross(D, B);
          if (min(sc, sd) < 0)
            segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
        } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
          segs.emplace_back(rat(C - A, B - A), 1);
          segs.emplace_back(rat(D - A, B - A), -1);
        }
      }
    }
    sort(all(segs));
```

```
    for (auto& s : segs) s.st = min(max(s.st, 0.0), 1.0);
    double sum = 0;
    int cnt = segs[0].nd;
    FOR(j,1,SZ(segs)) {
      if (!cnt) sum += segs[j].st - segs[j - 1].st;
      cnt += segs[j].nd;
    }
    ret += A.cross(B) * sum;
  }
  return ret / 2;
}
```

## ConvexHull.h

**Description:**
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

**Time:** $\mathcal{O}(n \log n)$

"Point.h" c06f0e, 13 lines

```
using P = Point<ll>;
vector<P> convexHull(vector<P> pts) {
  if (SZ(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(SZ(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    for (P p : pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

## HullDiameter.h

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

**Time:** $\mathcal{O}(n)$

"Point.h" cff1ec, 12 lines

```
using P = Point<ll>;
array<P, 2> hullDiameter(vector<P> S) {
  int n = SZ(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  FOR(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

## PointInsideHull.h

**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

**Time:** $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 7d714c, 13 lines

```
using P = Point<ll>;
bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = SZ(l) - 1, r = !strict;
  if (SZ(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

## LineHullIntersection.h

**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner $i$, • $(i,i)$ if along side $(i,i+1)$, • $(i,j)$ if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner $i$ is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.

**Time:** $\mathcal{O}(\log n)$

"Point.h" f0fd21, 39 lines

```
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0) return {-1, -1};
  array<int, 2> res;
  FOR(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1])) {
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  }
  return res;
}
```

## PolygonTangents.h

**Description:** Polygon tangents from a given point. The polygon must be ccw and have no collinear points. Returns a pair of indices of the given polygon. Should work for a point on border (for a point being polygon vertex returns previous and next one).

**Time:** $\mathcal{O}(\log n)$

"Point.h" 096fab, 21 lines

```
#define pdir(i) (ph ? p - poly[(i)%n] : poly[(i)%n] - p)
#define cmp(i,j) sgn(pdir(i).cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P>
array<int, 2> polygonTangents(vector<P>& poly, P p) {
  auto bs = [&](int ph) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while(lo + 1 < hi) {
      int m = (lo + hi) / 2;
      if (extr(m)) return m;
      int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
      (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi:lo) = m;
    }
    return lo;
  };
  array<int, 2> res = {bs(0), bs(1)};
  if(res[0] == res[1]) res[0] = (res[0] + 1) % sz(poly);
  if(poly[res[0]] == p) res[0] = (res[0] + 1) % sz(poly);
  return res;
}
```

## HalfplaneIntersection.h

**Description:** Checks if half-planes intersection is not empty. Note that the intersection can be non-empty, despite the area being zero.

**Time:** expected $\mathcal{O}(n)$

"sideOf.h", "lineIntersection.h" 7c1c40, 24 lines

```
using P = Point<double>;
pair<int, P> hpInter(vector<pair<P, P>> hps) {
  shuffle(all(hps), mt19937(1337));
  P cand;
  FOR(i, 0, SZ(hps)) {
    auto [s, e] = hps[i];
    if(sideOf(s, e, cand) >= 0) continue;
    P d = (e - s).unit();
    P l = s - d * 1e7, r = s + d * 1e7;
    FOR(j, 0, i) {
      auto [s2, e2] = hps[j];
      auto [res, p] = lineInter(s, e, s2, e2);
      if(res == 1) {
        int cnt = 0;
        if(sideOf(s2, e2, l) == -1) l = p, cnt++;
        if(sideOf(s2, e2, r) == -1) r = p, cnt++;
        if(cnt == 2) return {0, P()};
      }
      else if(sideOf(s2, e2, s) == -1) return {0, P()};
    }
    cand = l;
  }
  return {1, cand};
}
```

## HalfplaneIntersectionDynamic.h

**Description:** Data structure that dynamically keeps track of the intersection of half-planes.

**Time:** amortized $\mathcal{O}(\log n)$

"sideOf.h", "lineIntersection.h" 20c7cf, 69 lines

```
using P = Point<double>;
int half(P a) { return a.y < 0 || (a.y == 0 && a.x < 0); }

struct polarCmp {
  bool operator()(const P &a, const P &b) const {
    if(half(a) == half(b)) return a.cross(b) > 0;
    return half(a) < half(b);
  }
};

struct HalfplaneSet : map<P, P, polarCmp> {
  double INF = 1e6, area = 8 * INF * INF; // two times area

  HalfplaneSet() {
    P p(-INF, -INF);
    for(auto &d: {P(1, 0), P(0, 1), P(-1, 0), P(0, -1)}) {
      insert({d, p});
      p = p + d * 2 * INF;
```

```
    }
  }

  auto fix(auto it) { return it == end() ? begin() : it; }
  auto getNext(auto it) { return fix(next(it)); }
  auto getPrev(auto it) {
    return it == begin() ? prev(end()) : prev(it);
  }

  auto crossNext(auto it) {
    return it->nd.cross(getNext(it)->nd);
  }

  auto del(auto it) {
    area -= crossNext(getPrev(it)) + crossNext(it);
    it = fix(erase(it));
    if(size()) area += crossNext(getPrev(it));
    return it;
  };

  void add(P s, P e) {
    auto eval = [&](auto it) { return sideOf(s, e, it->nd); };
    auto intersect = [&](auto it) {
      return lineInter(s, e, it->nd, it->st + it->nd).nd;
    };

    auto it = fix(lower_bound(e - s));
    if(empty() || eval(it) >= 0) return;
    while(size() && eval(getPrev(it)) < 0) del(getPrev(it));
    while(size() && eval(getNext(it)) < 0) it = del(it);
    if(empty()) return;
    if(eval(getNext(it)) > 0) {
      area -= crossNext(getPrev(it)) + crossNext(it);
      it->nd = intersect(it);
      area += crossNext(getPrev(it)) + crossNext(it);
    }
    else it = del(it);
    it = getPrev(it);
    area -= crossNext(it);
    insert(it, {e - s, intersect(it)});
    area += crossNext(it) + crossNext(getNext(it));
    if(eval(it) == 0) del(it);
  }

  double maxDot(P a) {
    auto it = fix(lower_bound(a.perp()));
    return a.dot(it->nd);
  }

  double getArea() { return area / 2; }
};
```

# 8.4 Misc. Point Set Problems

## ClosestPair.h

**Description:** Finds the closest pair of points.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" 963a68, 17 lines

```
using P = Point<ll>;
pair<P, P> closest(vector<P> v) {
  assert(SZ(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.nd;
}
```

## ManhattanMST.h

**Description:** Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = —p.x - q.x— + —p.y - q.y—. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
**Time:** $\mathcal{O}(N \log N)$

"Point.h" 3dd86e, 23 lines

```
using P = Point<int>;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
  vi id(SZ(ps));
  iota(all(id), 0);
  vector<array<int, 3>> edges;
  FOR(k,0,4) {
    sort(all(id), [&](int i, int j) {
         return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
        int j = it->second;
        P d = ps[i] - ps[j];
        if (d.y > d.x) break;
        edges.pb({d.y + d.x, i, j});
      }
      sweep[-ps[i].y] = i;
    }
    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p.y);
  }
  return edges;
}
```

## kdTree.h

**Description:** KD-tree (2d, can be extended to 3d)

"Point.h" 9e4b16, 60 lines

```
using T = long long;
using P = Point<T>;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;

  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }

  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if width >= height (not ideal...)
      sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = SZ(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};

struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return mp((p - node->pt).dist2(), node->pt);
    }

    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);

    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first) best = min(best, search(s, p));
    return best;
  }

  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) { return search(root, p); }
};
```

## DelaunayTriangulation.h

**Description:** Computes the Delaunay triangulation of a set of points. Each circumcircle contains none of the input points. If any three points are collinear or any four are on the same circle, behavior is undefined.
**Time:** $\mathcal{O}(n^2)$

"Point.h", "3dHull.h" c0e7bc, 10 lines

```
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
  if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0);
    trifun(0,1+d,2-d); }
  vector<P3> p3;
  for (P p : ps) p3.emplace_back(p.x, p.y, p.dist2());
  if (sz(ps) > 3) for(auto t:hull3d(p3)) if ((p3[t.b]-p3[t.a]).
      cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
    trifun(t.a, t.c, t.b);
}
```

## FastDelaunay.h

**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" 1731b6, 88 lines

```
using P = Point<ll>;
using Q = struct Quad*;
using lll = __int128_t; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point

struct Quad {
  Q rot, o; P p = arb; bool mark;
  P& F() { return r()->p; }
  Q& r() { return rot->rot; }
```

```
  Q prev() { return rot->o->rot; }
  Q next() { return r()->prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
  lll p2 = p.dist2(), A = a.dist2()-p2,
      B = b.dist2()-p2, C = c.dist2()-p2;
  return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
  Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
  H = r->o; r->r()->r() = r;
  FOR(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
  r->p = orig; r->F() = dest;
  return r;
}
void splice(Q a, Q b) {
  swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
  Q q = makeEdge(a->F(), b->p);
  splice(q, a->next());
  splice(q->r(), b);
  return q;
}

pair<Q,Q> rec(const vector<P>& s) {
  if (SZ(s) <= 3) {
    Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
    if (SZ(s) == 2) return { a, a->r() };
    splice(a->r(), b);
    auto side = s[0].cross(s[1], s[2]);
    Q c = side ? connect(b, a) : 0;
    return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
  }

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
  Q A, B, ra, rb;
  int half = SZ(s) / 2;
  tie(ra, A) = rec({all(s) - half});
  tie(B, rb) = rec({SZ(s) - half + all(s)});
  while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
         (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
  Q base = connect(B->r(), A);
  if (A->p == ra->p) ra = base->r();
  if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
    while (circ(e->dir->F(), H(base), e->F())) { \
      Q t = e->dir; \
      splice(e, e->prev()); \
      splice(e->r(), e->r()->prev()); \
      e->o = H; H = e; e = t; \
    }
  for (;;) {
    DEL(LC, base->r(), o);  DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
      base = connect(RC, base->r());
    else
      base = connect(base->r(), LC->r());
  }
  return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
  sort(all(pts));  assert(unique(all(pts)) == pts.end());
  if (SZ(pts) < 2) return {};
  Q e = rec(pts).first;
  vector<Q> q = {e};
  int qi = 0;
  while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.pb(c->p); \
  q.pb(c->r()); c = c->next(); } while (c != e); }
  ADD; pts.clear();
  while (qi < SZ(q)) if (!(e = q[qi++])->mark) ADD;
  return pts;
}
```

### manySegmentsIntersection.h

**Description:** Finds one of the segments intersections.
**Time:** $\mathcal{O}(N \log N)$

"SegmentIntersection.h" fe47d2, 42 lines

```
template<class T>
pii allIntersect(vector<pair<Point<T>, Point<T>>> a) {
  using P = Point<T>;
  vector<tuple<P, int, int>> e;
  FOR(i, 0, SZ(a)) {
    if(a[i].nd < a[i].st) swap(a[i].st, a[i].nd);
    e.pb({a[i].st, 0, i}), e.pb({a[i].nd, 1, i});
  }
  sort(all(e));
  auto cmp = [](auto bb, auto cc) {
    auto [bs, be] = bb.st;
    auto [cs, ce] = cc.st;
    P sh(max(bs.x, cs.x), 0);
    auto bv = be - bs, cv = ce - cs;
    T l = bv.cross(bs - sh), r = cv.cross(cs - sh);
    // care! M^3
    return (sgn(cv.x) ? cv.x : 1) * (sgn(bv.x) ? l : bs.y) <
         (sgn(bv.x) ? bv.x : 1) * (sgn(cv.x) ? r : cs.y);
  };
  auto inter = [](auto bb, auto cc) {
    return segInter(bb.st, bb.nd, cc.st, cc.nd);
  };
  set<pair<pair<P, P>, int>, decltype(cmp)> s(cmp);
  for(auto &[_, tp, id]: e) {
    auto akt = a[id];
    if(!tp) {
      auto it = s.lower_bound({akt, id});
      if(it != end(s) && SZ(inter(it->st, akt)))
        return {it->nd, id};
      if(it != begin(s) && SZ(inter((*--it).st, akt)))
        return {it->nd, id};
      s.insert({akt, id});
    }
    else {
      auto it = s.erase(s.find({akt, id}));
      if(it != begin(s) && it != end(s) &&
        SZ(inter(it->st, prev(it)->st)))
        return {it->nd, prev(it)->nd};
    }
  }
  return {-1, -1};
}
```

## 8.5 3D

### PolyhedronVolume.h

**Description:** Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
  double v = 0;
  for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
}
```

### Point3D.h

**Description:** Class to handle points in 3D space. T can be e.g. double or long long.

e0e4cf, 32 lines

```
template<class T> struct Point3D {
  using P = Point3D;
  using R = const P&;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

### 3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
**Time:** $\mathcal{O}(n^2)$

"Point3D.h" 3bb555, 48 lines

```
using P3 = Point3D<double>;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(SZ(A) >= 4);
  vector<vector<PR>> E(SZ(A), vector<PR>(SZ(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i])) q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.pb(f);
  };
  FOR(i,0,4) FOR(j,i+1,4) FOR(k,j+1,4)
```

```
    mf(i, j, k, 6 - i - j - k);

  FOR(i,4,SZ(A)) {
    FOR(j,0,SZ(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = SZ(FS);
    FOR(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
};
```

### sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 ($\phi_1$) and f2 ($\phi_2$) from x axis and zenith angles (latitude) t1 ($\theta_1$) and t2 ($\theta_2$) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

611f07, 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

# Strings (9)

### KMP.h

**Description:** pi[x] computes the length of the longest prefix of s that ends at x, other than s[0...x] itself (abacaba -> 0010123). Can be used to find all occurrences of a string.

**Time:** $\mathcal{O}(n)$

e144f7, 16 lines

```
vi pi(const string& s) {
  vi p(SZ(s));
  FOR(i, 1, SZ(s)) {
    int g = p[i-1];
    while (g && s[i] != s[g]) g = p[g-1];
    p[i] = g + (s[i] == s[g]);
  }
  return p;
}

vi match(const string& s, const string& pat) {
  vi p = pi(pat + '\0' + s), res;
  FOR(i, SZ(p)-SZ(s), SZ(p))
    if(p[i] == SZ(pat)) res.pb(i - 2 * SZ(pat));
  return res;
}
```

### Zfunc.h

**Description:** z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

**Time:** $\mathcal{O}(n)$

19b0b9, 11 lines

```
vi Z(const string& S) {
  vi z(SZ(S));
  int l = -1, r = -1;
  FOR(i, 1, SZ(S)) {
    z[i] = i >= r ? 0 : min(r - i, z[i - l]);
    while(i + z[i] < SZ(S) && S[i + z[i]] == S[z[i]])
      z[i]++;
    if(i + z[i] > r) l = i, r = i + z[i];
  }
  return z;
}
```

### Manacher.h

**Description:** Computes palindromic ranges.

s: a b a a b a a c a a a b b b b a a
r: 0 0 1 0 0 3 0 0 2 0 0 1 0 0 3 0 0 1 0 0 0 1 1 4 1 1 0 0 0 1 0

**Time:** $\mathcal{O}(N)$

73ef68, 10 lines

```
vi Manacher(string s) {
  vi r(SZ(s) * 2 - 1);
  for(int i=0, m=0, k=0, p=0; i < 2*SZ(s) - 1; m = i++ - 1) {
    while(p < k && i / 2 + r[m] != k)
      r[i++] = min(r[m--], (k + 1 - p++) / 2);
      while(k+1 < SZ(s) && p > 0 && s[k+1] == s[p-1]) k++, p--;
    r[i] = (k + 1 - p++) / 2;
  }
  return r;
}
```

### MinRotation.h

**Description:** Finds the lexicographically smallest rotation of a string.

**Usage:** `rotate(v.begin(), v.begin()+minRotation(v), v.end());`

**Time:** $\mathcal{O}(N)$

811762, 8 lines

```
int minRotation(string s) {
  int a = 0, N = SZ(s); s += s;
  FOR(b, 0, N) FOR(k, 0, N) {
    if(a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if(s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
}
```

### SuffixArray.h

**Description:** Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is $i$'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array.

**Time:** $\mathcal{O}(n \log n)$

d987b7, 34 lines

```
int s[N]; /* s[i] > 0, n = len(s), A = sigma(s) */
int n, A, sa[N], lcp[N], cnt[N];
vi x, y;

bool dif(int a, int b, int k) {
    return y[a] != y[b] || (a + k <= n ? y[a + k] : -1) != (b +
          k <= n ? y[b + k] : -1);
}
/* 1-indexed */
void build() {
    x.clear(); x.resize(max(A, n) + 2); y = x; // +2 enough?
    int j = 0;
    rep(i, 1, n) cnt[x[i] = s[i]]++;
    rep(i, 1, A) cnt[i] += cnt[i - 1];
    per(i, 1, n) sa[cnt[x[i]]--] = i;
    for(int k = 1; k < n; k *= 2) {
        int p = 0;
        rep(i, n - k + 1, n) y[++p] = i;
        rep(i, 1, n) if (sa[i] > k) y[++p] = sa[i] - k;
        rep(i, 1, A) cnt[i] = 0;
        rep(i, 1, n) cnt[x[i]]++;
        rep(i, 1, A) cnt[i] += cnt[i - 1];
        per(i, 1, n) sa[cnt[x[y[i]]]--] = y[i];
        swap(x, y);
        A = x[sa[1]] = 1;
        rep(i, 2, n) x[sa[i]] = (A += dif(sa[i - 1], sa[i], k))
            ;
        if(n == A) break;
    }
    rep(i, 1, n) {
        if(x[i] == n) { lcp[x[i]] = 0; continue; }
        int nxt = sa[x[i] + 1];
        while(max(i, nxt) + j <= n && s[i + j] == s[nxt + j]) j
            ++;
        lcp[x[i]] = j; j = max(j - 1, 0);
    }
}
```

### TandemRepeats.h

**Description:** Find all $(i, p)$ such that `s.substr(i,p) == s.substr(i+p,p)`. No two intervals with the same period intersect or touch.

**Usage:** `solve("aaababababa") // {{0, 1, 1}, {2, 5, 2}}`

**Time:** $\mathcal{O}(N \log N)$

3e559b, 13 lines

```
vector<array<int, 3>> solve(string s) {
  int N = SZ(s); SuffixArray A, B;
  A.init(s); reverse(all(s)); B.init(s);
  vector<array<int, 3>> runs;
  for (int p = 1; 2*p <= N; ++p) { // do in O(N/p) for period p
    for (int i = 0, lst = -1; i+p <= N; i += p) {
      int l = i-B.getLCP(N-i-p,N-i), r = i-p+A.getLCP(i,i+p);
      if (l > r || l == lst) continue;
      runs.pb({lst = l,r,p}); // for each i in [l,r],
    } // s.substr(i,p) == s.substr(i+p,p)
  }
  return runs;
}
```

### Hasher.h

**Description:** Self-explanatory methods for string hashing.

19f1f5, 22 lines

```
const __int128 C = 311;
const ll mod = ll(1e18) + 31;

struct Hasher {
  vector<ll> ha, pw;
  Hasher(string &str) : ha(SZ(str)+1), pw(SZ(str)+1, 1) {
    FOR(i, 0, SZ(str)) {
      ha[i+1] = (ha[i] * C + str[i]) % mod;
      pw[i+1] = pw[i] * C % mod;
    }
  }
  ll hashInterval(int a, int b) { // hash [a, b]
    ll res = (ha[b + 1] - __int128(ha[a]) * pw[b + 1 - a]) %
        mod;
    return res < 0 ? res + mod : res;
  }
};

ll hashString(string &s) {
  ll h = 0;
  for(char c: s) h = (h * C + c) % mod;
  return h;
```

```
}
```

### FastHasher.h

**Description:** Self-explanatory methods for string hashing. b4cfda, 17 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "using H = ull;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
using ull = unsigned long long;
struct H {
  ull x; H(ull x=0) : x(x) {}
  H operator+(H o) { return x + o.x + (x + o.x < x); }
  H operator-(H o) { return *this + ~o.x; }
  H operator*(H o) { auto m = (__uint128_t)x * o.x;
    return H((ull)m) + (ull)(m >> 64); }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
  bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)
```

### AhoCorasick.h

**Description:** Famous Bohun's Aho f89aac, 26 lines

```
const int N = 1e6 + 5;
int cnt, fail[N], go[N][26];

void add(string s) {
  int u = 0;
  for(auto ch : s) {
    int c = ch - 'a';
    if(!go[u][c]) go[u][c] = ++cnt;
    u = go[u][c];
  }
}

void build() {
  queue<int> q; q.push(0);
  while(!q.empty()) {
    int u = q.front(); q.pop();
    FOR(c, 0, 26) {
      int &v = go[u][c];
      if(v == 0) v = go[fail[u]][c];
      else {
        fail[v] = u == 0 ? 0 : go[fail[u]][c];
        q.push(v);
      }
    }
  }
}
```

### EertreE.h

**Description:** TODO 7f5814, 25 lines

```
const int N = 1e6 + 5, A = 26;
int nxt[N][A], fail[N], last[N], len[N], cnt, par[N];
char s[N];

void prepare(int n) { // 0 <= i <= n + 1
  rep(i, 0, n + 1) FOR(j, 0, A) nxt[i][j] = 0;
  s[0] = '#'; // CUSTOM
  last[0] = cnt = 1;
  fail[0] = fail[1] = 1;
  len[1] = -1;
}

void add(int n){
  int c = s[n] - 'a'; // CUSTOM
  int v = last[n - 1];
  while(s[n - len[v] - 1] != s[n]) v = fail[v];
  if(!nxt[v][c]){
    int now = ++cnt, k = fail[v];
    len[now] = len[v] + 2;
    while(s[n - len[k] - 1] != s[n]) k = fail[k];
    fail[now] = nxt[k][c]; nxt[v][c] = now;
    par[now] = v;
  }
  last[n] = nxt[v][c];
}
```

### LyndonFactor.h

**Description:** A string is "simple" if it is strictly smaller than any of its own nontrivial suffixes. The Lyndon factorization of the string $s$ is a factorization $s = w_1 w_2 \dots w_k$ where all strings $w_i$ are simple and $w_1 \geq w_2 \geq \cdots \geq w_k$. Min rotation gets min index i such that cyclic shift of s starting at i is minimum.
**Time:** $\mathcal{O}(N)$ 51014a, 20 lines

```
vector<string> duval(string s) {
  int N = SZ(s); vector<string> factors;
  for (int i = 0; i < N; ) {
    int j = i+1, k = i;
    for (; j < N && s[k] <= s[j]; ++j) {
      if (s[k] < s[j]) k = i;
      else ++k;
    }
    for (; i <= k; i += j-k) factors.pb(s.substr(i,j-k));
  }
  return factors;
}

int minRotation(string s) {
  int N = SZ(s); s += s;
  vector<string> d = duval(s); int ind = 0, ans = 0;
  while (ans + SZ(d[ind]) < N) ans += SZ(d[ind++]);
  while (ind && d[ind] == d[ind-1]) ans -= SZ(d[ind--]);
  return ans;
}
```

### CyclicLCS.h

**Description:** Cyclic Longest Common Subsequence. maximum of lcs(any cyclic shift of s, any cyclic shift of t)
**Time:** $\mathcal{O}(NM)$ bd8b93, 43 lines

```
const int N = 2010;

int dp[N * 2][N], from[N * 2][N];
int cyclic_lcs(string s, string t) {
  int n = SZ(s), m = SZ(t);
  auto eq = [&](int a, int b) {
    return s[(a - 1) % n] == t[(b - 1) % m];
  };
  dp[0][0] = from[0][0] = 0;
  rep(i, 0, n * 2) rep(j, 0, m) {
    dp[i][j] = 0;
    if (j && dp[i][j - 1] > dp[i][j]) {
      dp[i][j] = dp[i][j - 1];
      from[i][j] = 0;
    }
    if (i && j && eq(i, j) && dp[i - 1][j - 1] + 1 > dp[i][j])
        {
      dp[i][j] = dp[i - 1][j - 1] + 1;
      from[i][j] = 1;
    }
    if (i && dp[i - 1][j] > dp[i][j]) {
      dp[i][j] = dp[i - 1][j];
      from[i][j] = 2;
    }
  }
  int ret = 0;
  FOR(i, 0, n) {
    ret = max(ret, dp[i + n][m]);
    // re-root
    int x = i + 1, y = 0;
    while (y <= m && from[x][y] == 0) ++y;
    for (; y <= m && x <= n * 2; ++x) {
      from[x][y] = 0, --dp[x][m];
      if (x == n * 2) break;
      for (; y <= m; ++y) {
        if (from[x + 1][y] == 2) break;
        if (y + 1 <= m && from[x + 1][y + 1] == 1) {
          ++y; break;
        }
      }
    }
  }
  return ret;
}
```

### AllSubstringLCS.h

**Description:** Computes LCS of s and t[i, j]
**Time:** $\mathcal{O}(NM)$ 23ce75, 21 lines

```
const int N = 2002;
int f[N][N], g[N][N], ans[N][N];
void all_substring_lcs(string s, string t) {
  int n = SZ(s), m = SZ(t);
  s = "#" + s; t = "#" + t;
  rep(i, 1, m) f[0][i] = i;
  rep(i, 1, n) rep(j, 1, m) {
    if (s[i] == t[j]) {
      f[i][j] = g[i][j - 1];
      g[i][j] = f[i - 1][j];
    }
    else {
      f[i][j] = max(f[i - 1][j], g[i][j - 1]);
      g[i][j] = min(g[i][j - 1], f[i - 1][j]);
    }
  }
  rep(i, 1, m) for(int j = i, a = 0; j <= m; ++j) {
    a += i > f[n][j];
    ans[i][j] = a;
  }
}
```

# Various (10)

## 10.1 Misc. algorithms

### FastKnapsack.h

**Description:** Given N non-negative integer weights w and a non-negative target t, computes the maximum S <= t such that S is the sum of some subset of the weights.
**Time:** $\mathcal{O}(N \max(w_i))$ 32960d, 16 lines

```
int knapsack(vi w, int t) {
  int a = 0, b = 0, x;
  while (b < SZ(w) && a + w[b] <= t) a += w[b++];
  if (b == SZ(w)) return a;
  int m = *max_element(all(w));
  vi u, v(2*m, -1);
  v[a+m-t] = b;
  FOR(i,b,SZ(w)) {
    u = v;
```

```
    FOR(x,0,m) v[x+w[i]] = max(v[x+w[i]], u[x]);
    for (x = 2*m; --x > m;) FOR(j, max(0,u[x]), v[x])
      v[x-w[j]] = max(v[x-w[j]], j);
  }
  for (a = t; v[a+m-t] < 0; a--) ;
  return a;
}
```

KnightMoves.h

**Description:** Finds minimum number of knight moves from (x,y) to (0,0) in non-negative infinite chessboard.

**Time:** $\mathcal{O}(1)$

cc32ee, 8 lines

```
ll knightMoves(ll x, ll y) {
  ll cnt = max({(x + 1) / 2, (y + 1) / 2, (x + y + 2) / 3});
  while((cnt % 2) != (x + y) % 2) cnt++;
  if(x == 1 && !y) return 3;
  if(y == 1 && !x) return 3;
  if(x == y && x == 2) return 4;
  return cnt;
}
```

## 10.2 Optimization tricks

### 10.2.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `FOR(b,0,K) FOR(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.