$d_{TV}(p,q) = \max_{S \subseteq [n]\times[n]} (p(S) - q(S)) \leq \varepsilon$

some i,j $p(i,j) - q(i,j) \leq \varepsilon$.

Algorithm: pick $O(r)$ samples from p

$(x_1,y_1), \dots (x_r, y_r)$

$X = [x_1, \dots x_r]$ randomly pick (x_i, y_j).

$Y = [y_1, \dots y_r]$

This formed a distribution that is independent say (x,y) Assume r is large

it is t distribution

$E[x_t = a] = E[X_q = a] = E[X_p = a]$

$E[y_t = b] = E[y_q = b] = E[y_p = b]$

$d_{TV}(p,t) \leq ?$ $d_{TV}(q,t) \leq ?$

define $d_{i,j}(p,q) = \mathrm{abs}(p(i,j) - q(i,j))$

If i not in $X = \{x_1, \dots x_r\}$ or j not in $Y = \{y_1, \dots y_r\}$

Then $t(i,j) = 0$ $d_{ij}(p,t) = p(i,j)$ $d_{ij}(q,t) = q(i,j)$

$d_{ij}(q,t) = P_r[x=i] \times P_r[y=j]$

$P_r[i \text{ or } j \text{ not picked in } r \text{ picks}] = (1 - P_r[X=i] - P_r[y=j] + \underbrace{P_r[x=i, y=j]}_{p(i,j)})^r$

$d_{TV}(p,t) \geq d_{ij}(p,t)$ $d_{TV}(q,t) \geq d_{ij}(q,t)$

$\therefore P_r[d_{TV}(q,t) \geq d_{ij}(q,t)] \geq [1 - P_r[X=i] - P_r[y=j] + p(i,j)]^r$

$|p(i,j) - q(i,j)| \leq \varepsilon \Rightarrow p(i,j) \in [q(i,j) - \varepsilon, q(i,j) + \varepsilon]$

$\therefore P_r[d_{TV}(q,t) \geq P_r[x=i] \cdot P_r[y=j]] \geq (1 - P_r[x=i] - P_r[y=j] + P_r[x=i] P_r[y=j] - \varepsilon)^r$.

Not right!

$d_{TV}(p,q) = \frac{1}{2} \sum_{x,y} |p(x,y) - q(x,y)| = \frac{1}{2} \sum_{i,j} d_{ij}(p,q)$

If i and j are both picked $(i \in X)(j \in Y)$ same method pick

$d_{ij}(q,t) = q(i,j) - t(i,j) \approx 0$ (in average). since $q(i,j) = P_r[x=i] \times P_r[y=j] \approx t(i,j)$

$P_r[i \in X \text{ and } j \in Y] = 1 - (1 - P_r[x=i] - P_r[y=j] + p(i,j))^r$ $p(i,j) \in [q(i,j) - \varepsilon, q(i,j) + \varepsilon]$

Let $a = P_r[x=i]$ $b = P_r[y=j]$

$P_r[d_{ij}(q,t) \approx 0] \geq 1 - (1 - a - b + ab - \varepsilon)^r$ i or j not picked

$E[d_{ij}(q,t)] = P_r[d_{ij}(q,t) \approx 0] \times 0 + P_r[d_{ij}(q,t) = ab] \times q(i,j)$

$\leq 0 + (1 - a - b + ab - \varepsilon)^r ab$

$d_{TV}(q,t) \leq \frac{1}{2} \sum_{i,j} (1 - a - b + ab - \varepsilon)^r \times ab \leq \frac{1}{2} n^2 (\underbrace{1 - a - b + ab}_{\leq 1} - \varepsilon)^r \times \underbrace{ab}_{\leq 1} \leq \frac{1}{2} n^2 (1 - \varepsilon)^r$

Let $\frac{1}{2} n^2 (1-\varepsilon)^r \leq \underbrace{c}_{\text{constant}} \times \varepsilon \Rightarrow (1-\varepsilon)^r \leq \frac{2c\varepsilon}{n^2} \Rightarrow \cancel{r \ln(1-\varepsilon)} \geq \frac{\ln 2c\varepsilon - 2\ln n}{\ln(1-\varepsilon)}$

$r \approx O\left(\frac{\ln 2c\varepsilon - 2\ln n}{\ln(1-\varepsilon)}\right)$