7. Prove that for any natural number n, $2+2^2+2^3+...+2^n=2^{n+1}-2$

Claim: $\forall n\in\mathbb{N}[2^1+2^2+...+2^n=2^{n+1}-2]$

Proof: By induction.

Base case: We know that $2^1=2^{1+1}-2$, i.e., $2=4-2$, i.e., $2=2$, which is true.

Induction step: Given an $n\in\mathbb{N}$ suppose $2^1+2^2+...+2^n=2^{n+1}-2$. Adding 2^{n+1} to both sides of the equality we get

$2^1+2^2+...+2^n+2^{n+1}=2^{n+1}+2^{n+1}-2$.

The LHS (left hand side) is already in the form of n+1. Now, by algebra in the RHS (right hand side):

$2^{n+1}+2^{n+1}-2$

$2(2^{n+1})-2$

$2^{(n+1)+1}-2$

making the RHS in the form of n+1. This concludes the induction step and proves the claim. □