

5. Prove that for any integer n , at least one of the integers $n, n+2, n+4$ is divisible by 3.

Claim: $\forall n \in \mathbb{Z} [3|n \vee 3|n+2 \vee 3|n+4]$

Proof: Given an $n \in \mathbb{Z}$ we know that it must be true that $3|n$ or $3|n+1$ or $3|n+2$, since one of those 3 consecutive integers must be a multiple of 3. If it is the case that $3|n+1$ then $3|(n+1)+3$, i.e., $3|n+4$. Therefore, the original claim is true. \square