4. Prove that every odd natural number is of one of the forms 4n+1 or 4n+3, where n is an integer.

Claim: $\forall m \in \mathbb{N}[2 \nmid m \Rightarrow \exists n \in \mathbb{Z}(4n+1=m \lor 4n+3=m)]$

Proof: Given an $m \in \mathbb{N}$ suppose $2 \nmid m$, i.e., $\exists p \in \mathbb{Z}[m=2p+1]$.

If $2|p$ then $\exists q \in \mathbb{Z}[p=2q]$, hence, $m=2(2q)+1=4q+1$, hence, $\exists n \in \mathbb{Z}[4n+1=m]$.

If $2 \nmid p$ then $\exists q \in \mathbb{Z}[p=2q+1]$, hence, $m=2(2q+1)+1=4q+3$, hence, $\exists n \in \mathbb{Z}[4n+3=m]$.

Therefore, since one of both cases must be true, the original claim is true. □