1. Say whether the following is true or false and support your answer by a proof.

$(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m+5n=12)$

Claim: $(\exists m \in \mathbb{N})(\exists n \in \mathbb{N})(3m+5n=12)$ is false

Proof: By cases.

Negating the original claim we get

$(\forall m,n \in \mathbb{N})(3m+5n \neq 12)$

Given $m,n \in \mathbb{N}$

If $m=n=1$ then $3m+5n=8<12$

If $m=2$ and $n=1$ then $3m+5n=11<12$ and

If $m=1$ and $n=2$ then $3m+5n=13>12$.

If $m=n=2$ then $3m+5n=16>12$.

All cases up to here have been not equal than 12, and the last one with $m=n=2$ is greater than 12, so every other combination with $m,n>2$ will be bigger than 12, therefore, for all $m,n \in \mathbb{N}$ $3m+5n \neq 12$, proving the original claim as false. □