8. Prove (from the definition of a limit of a sequence) that if the sequence $\{a_n\}_{n=1 \text{ to } \infty}$ tends to limit L as $n \to \infty$, then for any fixed number $M>0$, the sequence $\{Ma_n\}_{n=1 \text{ to } \infty}$ tends to the limit ML.

Claim: Given $\{a_n\}_{n=1 \text{ to } \infty}$ and $L\in\mathbb{R}$

$(\forall\epsilon>0)(\exists N\in\mathbb{N})(\forall n\geq N)[|a_n-L|<\epsilon]\Rightarrow(\forall M\in\mathbb{R}>0)(\forall\epsilon>0)(\exists N\in\mathbb{N})(\forall n\geq N)[|Ma_n-ML|<\epsilon]$

Proof: Suppose the antecedent of the claim. Given an $M\in\mathbb{R}>0$, by the assumption we know that we if we take an arbitrary $\epsilon>0$ there will be a $N\in\mathbb{N}$ such that for any given $n\geq N$ it will be the case that $|a_n-L|<\epsilon$. By algebra and since $M>0$:

$|a_n-L|<\epsilon$

$-\epsilon<a_n-L<\epsilon$

$-M\epsilon<Ma_n-ML<M\epsilon$ (remember $M>0$)

$-M\epsilon<-\epsilon< Ma_n-ML<\epsilon<M\epsilon$, hence,

$-\epsilon< Ma_n-ML<\epsilon$, thus,

$|Ma_n-ML|<\epsilon$

By the numbers given and chosen, and by the assumptions and reasoning, we can deduce the claim and gets proved. □