3. Say whether the following is true or false and support your answer by a proof: For any integer n, the number n^2+n+1 is odd.

Claim: $\forall n\in\mathbb{Z}[2\nmid n^2+n+1]$

Proof: Given an $n\in\mathbb{Z}$, by the Fundamental Theorem of Arithmetic, we know that $2|n$ iff $2|n^2$. We also know that for all $b,c\in\mathbb{Z}$

if $2|b$ and $2|c$ then $2|b+c$, and,

if $2\nmid b$ and $2\nmid c$ then $2|b+c$ too.

Set $b=n^2$ and $c=n$. Then, no matter if $2|n$ or $2\nmid n$, it is always the case that $2|n^2+n$, hence, $2\nmid n^2+n+1$, proving the original claim. □