Task 1

① $\left\{ \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \right\}$ in $M_{2\times2}(\mathbb{R})$

$M_1 \to (1,2,0,0) \quad M_2 \to (0,-1,3,1) \quad M_3 \to (1,1,2,-1)$

We consider $c_1M_1 + c_2M_2 + c_3M_3 = (0,0,0,0)$

$$\begin{cases} c_1 + 0 + c_3 = 0 \\ 2c_1 - c_2 + c_3 = 0 \\ 0 + 3c_2 + 2c_3 = 0 \\ 0 + c_2 - c_3 = 0 \end{cases} \Rightarrow \begin{array}{l} c_2 = c_3 \\ 5c_2 = 0 \to c_2 = 0 \\ c_3 = 0 \end{array}$$

$\because c_2 = c_3 = 0$ back to ①

$\therefore c_1 + 0 + 0 = 0, \; c_1 = 0$

When $c_1 = c_2 = c_3 = 0$

$\therefore \{M_1, M_2, M_3\}$ linearly independent. #

⑦ $\left\{ \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix}, \begin{bmatrix} -2 & 3 \\ 2 & 6 \end{bmatrix} \right\}$ in $M_{2\times 2}(R)$

$M_1 \to (1,0,-1,0)$ $\quad M_2 \to (0,2,0,4)$ $\quad M_3 \to (-2,3,2,6)$

$$c_1M_1 + c_2M_2 + c_3M_3 = (0,0,0,0)$$

$$\begin{cases} c_1 + 0 - 2c_3 = 0 \\ 0 + 2c_2 + 3c_3 = 0 \\ -c_1 + 0 + 2c_3 = 0 \\ 0 + 4c_2 + 6c_3 = 0 \end{cases} \Rightarrow$$

$c_1 = 2c_3$

if $c_3 = 1$

get $c_1 = 2$

$c_2 = -\frac{3}{2}$

$\because c_1 = 2,\ c_2 = -\frac{3}{2},\ c_3 = 1$

We get an answer about this function

So $\{M_1, M_2, M_3\}$ is linearly dependent. #

(3)

$V_1=(1,0,-2,1)$, $V_2=(0,-1,1,1)$, $V_3=(-1,2,1,0)$, $V_4=(2,1,-4,4)$

$c_1V_1+c_2V_2+c_3V_3+c_4V_4=(0,0,0,0)$

$$\begin{cases} c_1+0-c_3+2c_4=0 \\ 0-c_2+2c_3+c_4=0 \\ -2c_1+c_2+c_3-4c_4=0 \\ c_1+c_2+0+4c_4=0 \end{cases}$$

$c_1=c_3-2c_4$

$c_3-2c_4+c_2+4c_4=0$

$c_2=-c_3-2c_4$

$0-(-c_3-2c_4)+2c_3+c_4=0$ $\quad 3c_3+3c_4=0$

$\rightarrow \quad c_3=-c_4$

$c_3=a$, $c_4=-a$

$c_1=3a \quad c_2=a$

$\because (3a,a,a,-a)=(c_1,c_2,c_3,c_4)$

$\therefore$ We can get this four vector linearly dependent. #

④

$V_1 = (1,0,-2,1)$, $V_2 = (0,-1,1,1)$, $V_3 = (-1,2,1,0)$, $V_4 = (2,1,2,-2)$

$$c_1V_1 + c_2V_2 + c_3V_3 + c_4V_4 = (0,0,0,0)$$

$$\begin{cases} c_1 + 0 - c_3 + 2c_4 = 0 \\ 0 - c_2 + 2c_3 + c_4 = 0 \\ -2c_1 + c_2 + c_3 + 2c_4 = 0 \\ c_1 + c_2 + c_3 - 2c_4 = 0 \end{cases} \Rightarrow \begin{array}{l} c_1 = c_3 - 2c_4 \\ c_2 = 2c_3 + c_4 \end{array}$$

$$c_3 - 2c_4 + 2c_3 + c_4 + c_3 - 2c_4 = 0$$

$$\rightarrow 4c_3 - 3c_4 = 0$$

$$-2(c_3 - 2c_4) + 2c_3 + c_4 + c_3 + 2c_4 = 0$$

$$\hookrightarrow -2\cancel{c_3} + 4c_4 + 2\cancel{c_3} + c_4 + c_3 + 2c_4 = 0$$

$$\rightarrow c_3 + 7c_4 = 0$$

$$\begin{cases} 4c_3 - 3c_4 = 0 \\ c_3 + 7c_4 = 0 \end{cases} \Rightarrow -31c_4 = 0 \rightarrow c_4 = 0, c_3 = 0$$

We only can get solution $(0,0,0,0)$

$\therefore$ These four vectors linearly independene. #

Task 2

We can get example

$V_1 = (1, 0, 0) \quad V_2 = (0, 1, 0) \quad V_3 = (1, 1, 0)$

There are linearly dependent, but no two of them are parallel,

Because V_1 & V_2 cannot be sclar multipes of each other.
V_2 & V_3 and V_1 & V_3 the same,

So, we know these vector are not parallel,

In linearly dependent part, we know the z is 0

We can know

$1(1,0,0) + 1(0,1,0) - 1(1,1,0) = (0,0,0)$

$\therefore$ There are linearly dependent,

Task 3

① Fasle , In R^2 we can choose two vector, that there are not in the same line. Like, $\{(1,1),(1,0)\}$ is the example.

② True

③ Fasle, In R^2 $S=\{(1,0),(2,0)\}$

∵ $S=2$, but there are linearly dependent

so it's can't basis of V.

#

Task 4

In bases of P_2, We know the dimension have to under two and vectors have to linearly independent.

① $\{1-x^2, 2+5x+x^2, -4x+3x^2\}$

$c_1(1-x^2) + c_2(2+5x+x^2) + c_3(-4x+3x^2) = 0$

$c_1(1,0,-1) + c_2(2,5,1) + c_3(0,-4,3) = 0$

$\begin{cases} c_1 + 2c_2 + 0 = 0 \\ 0 + 5c_2 - 4c_3 = 0 \\ -c_1 + c_2 + 3c_3 = 0 \end{cases} \Rightarrow$ $c_1 = -2c_2$

$3c_2 = -3c_3$

$c_2 = -c_3$

$0 + 5(-c_3) - 4c_3 = 0$

$-9c_3 = 0$, $c_3 = 0 \Rightarrow c_2 = 0$, $c_1 = 0$

$\therefore$ The sets are bases for $P_2(\mathbb{R})$ #

②

$$\{2-4x+x^2,\ 3x-x^2,\ 6-x^2\}$$

$$C_1(2-4x+x^2)+C_2(3x-x^2)+C_3(6-x^2)=0.$$

$$C_1(2,-4,1)+C_2(0,3,-1)+C_3(6,0,-1)=0$$

$$\begin{cases} 2C_1+0+6C_3=0 \\ -4C_1+3C_2+0=0 \\ C_1-C_2-C_3=0 \end{cases} \Rightarrow \begin{array}{l} C_1=-3C_3 \\ C_2=-4C_3 \end{array}$$

$$-3C_3+4C_3-C_3=0$$

when $C_3 \neq 0$, We have other solution like

$C_3=1$, so we have solution is $(-3,-4,1)$

$\therefore$ The sets are not bases for P_2 #

③ $\{1+2x-x^2,\ 1+2x^2,\ 2+x+x^2\}$

$C_1(1+2x-x^2)+C_2(1+2x^2)+C_3(2+x+x^2)=0$

$$\begin{cases} C_1+C_2+2C_3=0 \\ 2C_1+0+C_3=0 \\ -C_1+2C_2+C_3=0 \end{cases} \Rightarrow \begin{array}{l} C_3=-2C_1 \\ C_2=3C_1 \end{array}$$

$-C_1+6C_1-2C_1=0 \Rightarrow 3C_1=0$

$\therefore C_1=0$

$\because C_1=0, \rightarrow C_2=0,\ C_3=0$

$\therefore$ The sets are buses for P_2 #

Task 5

$W = \{ (a_1, a_2, a_3, a_4, a_5) \in R^5 : a_1 - a_3 - a_4 = 0) \}$

$a_1 = a_3 + a_4$

$(a_1, a_2, a_3, a_4, a_5) = (a_3 + a_4, a_2, a_3, a_4, a_5)$

$\Rightarrow a_2(0,1,0,0,0) + a_3(1,0,1,0,0)$

$+ a_4(1,0,0,1,0) + a_5(0,0,0,0,1)$

$\Rightarrow$ so, we can know these four vector independent.

$\therefore$ The dimension is 4. #