

Task 1

To determine whether a subset $W \in \mathbb{R}^3$, it must satisfy three conditions :

1. contains the zero vector = $0 \in W$

2. closed under addition = if $u, v \in W$, then $u+v \in W$

3. closed under scalar multiplication = if $v \in W$ and α is any real number, then $\alpha v \in W$.

So, we can following this three conditions to solve these questions.

1.

$$\textcircled{1} \quad W_1 = \{(x, y, z) \in \mathbb{R}^3 : x = 2y\}$$

put $(0, 0, 0)$ to $x = 2y \Rightarrow 0 = 0$, so satisfaction rule 1.

$$\textcircled{2} \quad \because x = 2y \quad u = (2y_u, y_u, z_u) \quad v = (2y_v, y_v, z_v)$$

$$u + v = (2(y_u + y_v), y_u + y_v, z_u + z_v)$$

$$u + v \in \{(x, y, z) : x = 2y\} \in W_1$$

so satisfaction rule 2,



$$\textcircled{2} \quad u = (2y, y, z) \in W_1$$

$$\Rightarrow \alpha u = (2\alpha y, \alpha y, \alpha z)$$

$\therefore x = 2y$, so satisfaction rule 3

$\therefore W_1$ is a subspace.



$$2, \quad W_2 = \{(x, y, z) \in \mathbb{R}^3 : y = 0\}$$

\textcircled{1} put (0, 0, 0) to $y = 0 \Rightarrow 0 = 0$, so satisfaction rule 1.

$$\textcircled{2} \quad u = (x_1, 0, z_1) \quad v = (x_2, 0, z_2)$$

$$u + v = (x_1 + x_2, 0, z_1 + z_2) \Rightarrow y = 0, \text{ so satisfaction rule 2.}$$

$$\textcircled{3} \quad \alpha u = (\alpha x, 0, \alpha z) \Rightarrow y = 0, \text{ so satisfaction rule 3.}$$

$\therefore W_2$ is a subspace.

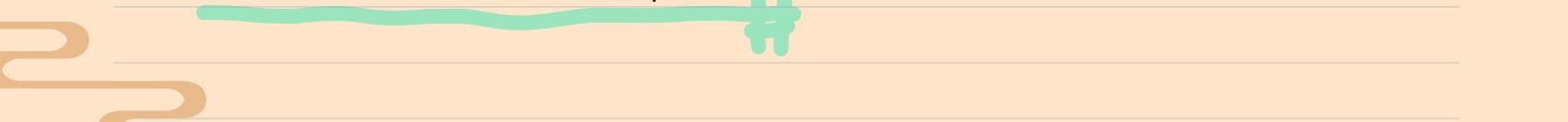



$$3, W_3 = \{(x, y, z) \in \mathbb{R}^3 : x=2y \text{ and } z=2\}$$

① put $(0, 0, 0)$ $\Rightarrow 0=0$ and $z=0$, but not satisfaction $z=2$

\Rightarrow so not satisfaction rule 1,

$\therefore W_3$ is not a subspace.


$$4, W_4 = \{(x, y, z) \in \mathbb{R}^3 : x=y^2\}$$

① put $(0, 0, 0)$ $\Rightarrow 0=0$, satisfaction rule 1.

② $U=(1, 1, 0)$, $V=(4, 2, 0)$

$$U+V = (5, 3, 0) \Rightarrow 5 \neq 3^2$$

\therefore not satisfaction rule 2

$\therefore W_4$ is not a subspace



$$2, \quad B = A + A^t$$

$$B^t = (A + A^t)^t \Rightarrow A + A^t = A^t + (A^t)^t$$

\because transpose twice return original matrix

$$\therefore (A^t)^t = A$$

$$\Rightarrow A + A^t = A^t + A$$

$$\therefore B = B^t$$

$$3, \quad \textcircled{1} -2x^2 + 3, \quad x^2 + 3x, \quad 2x^2 + 4x - 1$$

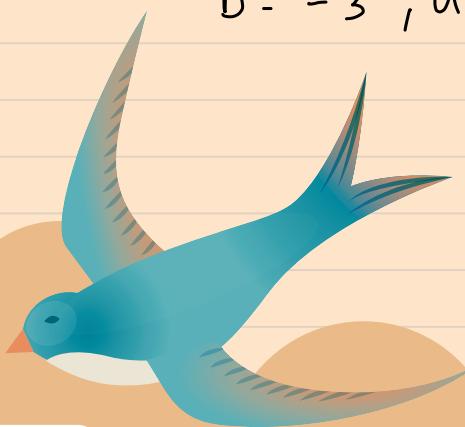
$$-2x^2 + 3 = a(x^2 + 3x) + b(2x^2 + 4x - 1)$$

$$a + 2b = -2$$

$$3a + 4b = 0$$

$$0 - b = 3$$

$b = -3, a = 4 \Rightarrow$ so, we can know the first linearly with last two polynomials.





② $x^2 + 2x - 3$, $-3x^2 + 2x + 1$, $2x^2 - x - 1$

$$x^2 + 2x - 3 = a(-3x^2 + 2x + 1) + b(2x^2 - x - 1)$$

$$\begin{aligned} -3a + 2b &= 1 & a &= 5 \\ 2a - b &= 2 & \Rightarrow & \\ a - b &= -3 & b &= 8 \end{aligned}$$

∴ so, we can know the first linearly with last two polynomials.

③ $3x^2 + 4x + 1$, $x^2 - 2x + 1$, $-2x^2 - x + 1$

$$3x^2 + 4x + 1 = a(x^2 - 2x + 1) + b(-2x^2 - x + 1)$$

$$\left\{ \begin{array}{l} a - 2b = 3 \\ -2a - b = 4 \\ a + b = 1 \end{array} \right. \Rightarrow \text{we can't find the answer in this equation.}$$

so, it's not a linear combination.



Task 4:

$$\textcircled{1} \quad (2, -1, 1), S = \{(1, 0, 2), (-1, 1, 1)\}$$

$$V = aS_1 + bS_2$$

$$(2, -1, 1) = a(1, 0, 2) + b(-1, 1, 1)$$

$$\left\{ \begin{array}{l} a - b = 2 \\ b = -1 \Rightarrow \\ a + b = 1 \end{array} \right. \quad \begin{array}{l} a = 1 \\ b = -1 \end{array} \quad \text{check } 2 + 1 - 1 = 1$$

$\therefore (2, -1, 1)$ in span S

$$\textcircled{2} \quad (-1, 2, 1), S = \{(1, 0, 2), (-1, 1, 1)\}$$

$$(-1, 2, 1) = a(1, 0, 2) + b(-1, 1, 1)$$

$$\left\{ \begin{array}{l} a - b = -1 \\ b = 2 \\ 2a + b = 1 \end{array} \right. \quad \begin{array}{l} a = 1 \\ b = 2 \end{array} \quad \text{check } 2 + 1 + 2 = 4 \neq 1$$

$\therefore (-1, 2, 1)$ not in span S

③

$$(-1, 1, 1, 2), S = \{(1, 0, 1, -1), (0, 1, 1, 1)\}$$

$$(-1, 1, 1, 2) = a(1, 0, 1, -1) + b(0, 1, 1, 1)$$

$$\left\{ \begin{array}{l} a = -1 \\ b = 1 \end{array} \right. \quad \because a = -1 \text{ and } b = 1$$

$$\begin{aligned} a+b &= 1 \\ -a+b &= 2 \end{aligned}$$

$$a+b=0 \neq 1$$

$\therefore (-1, 1, 1, 2)$ not in $\text{Span } S$



Task 5

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, M_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

The 2×2 symmetric is $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

use $M_1, M_2 \notin M_3$, try get symmetric $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$aM_1 + bM_2 + cM_3 = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix} + \begin{bmatrix} 0 & b \\ b & 0 \end{bmatrix}$$

\Rightarrow So we can get symmetric in M_1, M_2 and M_3

and symmetric is $\begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow$ so, we can know the symmetric

have 3 free parameter (a, b, c), thus the dimension of

all 2×2 symmetric matrices is 3.

We know that the $\{M_1, M_2, M_3\}$ is linearly independent

\therefore The span of $\{M_1, M_2, M_3\}$ is the set of all symmetric 2×2 matrices.