Task 1

To determine whether a subset $W \in R^3$, it must satisfy three conditons:

1、contains the zero vector: $0 \in W$

2、closed under addition: if $u, v \in W$, then $u+v \in W$

3、closed under sclar multiplication: if $v \in W$ and α is any real number, then $\alpha v \in W$.

So, we can following this three conditons to slove these questions.

1、

① $W_1 = \{(x, y, z) \in R^3 : x = 2y\}$

put $(0,0,0)$ to $x = 2y \Rightarrow 0 = 0$, so satisfation rule 1.

② $\because x = 2y$ $\quad u = (2y_u, y_u, z_u)$ $\quad v = (2y_v, y_v, z_v)$

$u+v = (2(y_u+y_v), y_u+y_v, z_u+z_v)$

$u+v \in \{(x, y, z) : x = 2y\} \in W_1$

so satisfation rule 2,

③ $u=(2y, y, z) \in W_1$

$\Rightarrow \alpha u = (2\alpha y, \alpha y, \alpha z)$

$\because x=2y$, so satisfation rule 3

$\therefore W_1$ is a subspace. #

2. $W_2 = \{(x, y, z) \in R^3 : y=0\}$

① put $(0, 0, 0)$ to $y=0 \Rightarrow 0=0$, so satisfaction rule 1.

② $u=(x_1, 0, z_1)$ $v=(x_2, 0, z_2)$

$u+v=(x_1+x_2, 0, z_1+z_2) \Rightarrow y=0$, so satisfation rule 2.

③ $\alpha u = (\alpha x, 0, \alpha z) \Rightarrow y=0$, so satisfation rule 3.

$\therefore W_2$ is a subspace. #

3. $W_3 = \{(x,y,z) \in \mathbb{R}^3 : x = 2y \text{ and } z = 2\}$

① put $(0,0,0) \Rightarrow 0 = 0$ and $z = 0$, but not satisfation $z = 2$

$\Rightarrow$ so not satisfation rule 1.

$\therefore$ W_3 is not a subspace. #

4. $W_4 = \{(x,y,z) \in \mathbb{R}^3 : x = y^2\}$

① put $(0,0,0) \Rightarrow 0 = 0$, satisfation rule 1.

② $u = (1,1,0)$, $v = (4,2,0)$

$u+v = (5,3,0) \Rightarrow 5 \neq 3^2$

$\therefore$ not satisfation rule 2

$\therefore$ W_4 is not a subspace #

2、 $B = A + A^t$

$B^t = (A+A^t)^t \Rightarrow A + A^t = A^t + (A^t)^t$

$\because$ transpose twice return original matrix

$\therefore (A^t)^t = A$

$\Rightarrow A + A^t = A^t + A$

$\therefore B = B^t$ #

3、① $-2x^2+3$, x^2+3x , $2x^2+4x-1$

$-2x^2+3 = a(x^2+3x) + b(2x^2+4x-1)$

$a+2b=-2$

$3a+4b=0$

$0-b=3$

$b=-3$, $a=4$ $\Rightarrow$ so, we can know the first linearly with last two polynomials.

② x^2+2x-3 , $-3x^2+2x+1$, $2x^2-x-1$

$x^2+2x-3 = a(-3x^2+2x+1)+b(2x^2-x-1)$

$$\begin{aligned} -3a+2b&=1 \\ 2a-b&=2 \\ a-b&=-3 \end{aligned} \Rightarrow \begin{aligned} a&=5 \\ b&=8 \end{aligned}$$

$\Rightarrow$ so, we can know the first linearly with last two polynomials.

③ $3x^2+4x+1$, x^2-2x+1 , $-2x^2-x+1$

$3x^2+4x+1 = a(x^2-2x+1)+b(-2x^2-x+1)$

$$\begin{cases} a-2b=3 \\ -2a-b=4 \\ a+b=1 \end{cases} \Rightarrow$$ we can't find the answer in this equation.

so, it's not a linear combanation.

Task 4:

① $(2,-1,1)$, $S=\{(1,0,2),(-1,1,1)\}$

$$V = aS_1 + bS_2$$

$$(2,-1,1) = a(1,0,2) + b(-1,1,1)$$

$$\begin{cases} a-b=2 \\ b=-1 \\ 2a+b=1 \end{cases} \Rightarrow \begin{matrix} a=1 \\ b=-1 \end{matrix} \quad \text{check } 2\times1-1=1$$

$\therefore (2,-1,1)$ in span S #

② $(-1,2,1)$, $S=\{(1,0,2),(-1,1,1)\}$

$$(-1,2,1) = a(1,0,2) + b(-1,1,1)$$

$$\begin{cases} a-b=-1 \\ b=2 \\ 2a+b=1 \end{cases} \quad \begin{matrix} a=1 \\ b=2 \end{matrix} \quad \text{check } 2\times1+2=4\neq1$$

$\therefore (-1,2,1)$ not in span S #

③ $(-1,1,1,2), S=\{(1,0,1,-1),(0,1,1,1)$

$(-1,1,1,2)=a(1,0,1,-1)+b(0,1,1,1)$

$$\begin{cases} a=-1 \\ b=1 \\ a+b=1 \\ -a+b=2 \end{cases}$$

$\because a=-1$ and $b=1$

$a+b=0\neq 1$

$\therefore (-1,1,1,2)$ not in span S #

Task 5

$M_1=\begin{bmatrix}1&0\\0&0\end{bmatrix}$, $M_2=\begin{bmatrix}0&0\\0&1\end{bmatrix}$, $M_3=\begin{bmatrix}0&1\\1&0\end{bmatrix}$

The 2×2 symmetric is $\begin{bmatrix}a&b\\b&c\end{bmatrix}$

use M_1, M_2 & M_3, try get symmetric $\begin{bmatrix}a&b\\b&c\end{bmatrix}$

$aM_1+cM_2+bM_3=\begin{bmatrix}a&0\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\0&c\end{bmatrix}+\begin{bmatrix}0&b\\b&0\end{bmatrix}$

$\Rightarrow$ So we can get symmetric in M_1, M_2 and M_3

and symmetric is $\begin{bmatrix}a&b\\b&c\end{bmatrix}$ $\Rightarrow$ so, we can know the symmetric have 3 free parameter (a,b,c), thus the dimension of all 2×2 symmetric matrices is 3.

we know that the $\{M_1, M_2, M_3\}$ is linearly independ

$\therefore$ The span of $\{M_1,M_2,M_3\}$ is the set of all symmetric 2×2 matrices.