1. Task 1

We want to know if vector a and vector b are parallel.

To to that, they need to satisfy $a = k \cdot b$

① $(-2,1,3)$ and $(4,6,1)$

$\because -2(-2,1,3) = (4,-2,-6)$

$\therefore$ not satisfy $a = kb$

$\Rightarrow (-2,1,3)$ and $(4,6,1)$ two vectors not parallel.

② $(1,2)$ and $(-3,-6)$

$\because -3(1,2) = (-3,-6)$

$\therefore$ satisfy $a = kb$

$\Rightarrow (1,2)$ and $(-3,-6)$ two vectors parallel.

③ $(1,-2,0,1)$ and $(3,0,2,-5)$

$\because 3(1,-2,0,1) = (3,-6,0,3) \neq (3,0,2,-5)$

$\therefore$ not satisfy $a = kb$

$\Rightarrow (1,-2,0,1)$ and $(3,0,2,-5)$ two vectors not parallel.

④ $(10, 0, 2, -4, -8)$ and $(5, 0, 1, -2, -4)$

$\because \frac{1}{2}(10, 0, 2, -4, -8) = (5, 0, 1, -2, -4)$

$\therefore$ satisfy $a = kb$

$\Rightarrow (10, 0, 2, -4, -8)$ and $(5, 0, 1, -2, -4)$ two vectors parallel.

Task 2

When the three points are not on the same line, they can define a single plane. But if the three points are collinear, there are many planes that can pass through the line, so we can't define a unique plane.

Thus, we can use $\boxed{X = A + su + tv}$, A is a point in the planes, u and v are two vectors on the plane that aren't parallel, and s and t can be any scalars.

① $(2, -5, -1)$, $(0, 4, 6)$, $(-3, 7, 1)$

We choose $A = (2, -5, -1)$ as the starting point.

$B = (0, 4, 6)$ to second point.

$u = B - A = (0-2, 4-(-5), 6-(-1)) = (-2, 9, 7)$

$C = (-3, 7, 1)$ to third point.

$V = C - A = (-3-2, 7-(-5), 1-(-1)) = (-5, 12, 2)$

We can find a plane as : $\boxed{X = (2, -5, -1) + s(-2, 9, 7) + t(-5, 12, 2)}$ #

② $(1,2,1)$, $(2,4,2)$ and $(-3,-6,-3)$

We choose $A=(1,2,1)$ as the starting point,

$\overrightarrow{AB} = (2-1, 4-2, 2-1) = (1,2,1)$

$\overrightarrow{AC} = (-3-1, -6-2, -3-1) = (-4,-8,-4) = -4(1,2,1)$

$\because \overrightarrow{AB} = -4\overrightarrow{AC}$

$\therefore$ We can know P_1, P_2, P_3 are collinear, so there is no unique plane. #

③ $(1,1,1), (2,5,2), (0,0,0)$

We choose $A=(1,1,1)$ as the starting point,

$B=(2,5,2)$ to second point,

$u = B-A = (2-1, 5-1, 2-1) = (1,4,1)$

$C=(0,0,0)$ to third point,

$v = C-A = (-1,-1,-1)$

We can find a plane as: $x = (1,1,1) + s(1,4,1) + t(-1,-1,-1)$ #

Task 3:

① In $M_{2\times5}(\mathbb{R})$ zero vector is $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

② $\because F = N$

$\therefore f(x) \in P(N) \Rightarrow$ It have to satisfy that every coefficient of the polynomial is in N,

But 3.2 and -0.5 not N, so $f(x)$ not belong $P(N)$ #

Task 4

① Fasle,

When $f(x) = x^2+1$, $g(x) = -x^2+3$

$f(x) + g(x) = (x^2+1) + (-x^2+3) = 4$

The degreen of the polynomial become 0 , not 2 .

② True

③ Fasle,

If $a \neq 0$, so $ax = ay$ is True.

When $a=0$, we get $0x = 0y$, and at this point , they can have many solutions.