**#Homework\_3**

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Chap1.

1. Let p, q, and r be the propositions

p : You get an A on the final exam.

q : You do every exercise in this book.

r : You get an A in this class.

Write these propositions using p, q, and r and logical connectives (including negations).

1. You get an A in this class, but you do not do every exercise in this book.

(r ∧­­­￢q)

1. You get an A in the final, you do every exercise in this book, and you get an A in this class.

(p ∧­­­ q ∧­­­ r)

c) To get an A in this class, it is necessary for you to get an A on the final.

(p → r)

2. Determine whether these biconditionals are true or false.

a) 2+2=4 if and only if 1+1=2. → true

b) 1+1=2 if and only if 2+3=4 → false

c) 1+1=3 if and only if monkeys can fly. → true

d) 0>1 if and only if 2>1. → false

3. Use De Morgan’s laws to find the negation of each of the following statements.

a) Jan is rich and happy.

p : Jan is rich, q: Jan is happy

￢(p ∧ q) = ￢p ∨￢q

= Jan isn’t rich or happy.

b) Carlos will bicycle or run tomorrow.

p : Carlos will bicycle tomorrow, q : Carlos will run tomorrow.

￢(p ∨ q ) = ￢p ∧ ￢q

= Carlos will not bicycle and run tomorrow.

c) Mei walks or takes the bus to class.

p : Mei walks to class, q : Mei takes the bus to class.

￢(p ∨ q) = ￢p ∧￢q

= Carlos don’t walk and take the bus to class.

d) Ibrahim is smart and hard working.

p : Ibrahim is smart, q : Ibrahim is hard working.

￢(p ∧ q) = ￢p ∨￢q

= Ibrahim isn’t smart or hard working.

4. Show that each of these conditional statements is a tautology by using truth tables.

a) [¬p∧(p∨q)] → q

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | ￢p | (p∨q) | [¬p∧(p∨q)] | [¬p∧(p∨q)] → q |
| T | T | F | T | F | T |
| T | F | F | T | F | T |
| F | T | T | T | T | T |
| F | F | T | F | F | T |

b) [(p→q)∧(q→r)]→(p→r)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | q | r | (p→q) | (q→r) | [(p→q)∧(q→r)] | (p→r) | [(p→q)∧(q→r)]→(p→r) |
| T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | T |
| T | F | T | F | T | F | T | T |
| T | F | F | F | T | F | F | T |
| F | T | T | T | T | T | T | T |
| F | T | F | T | F | F | T | T |
| F | F | T | T | T | T | T | T |
| F | F | F | T | T | T | T | T |

c) [p∧(p→q)]→q

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| P | q | (p→q) | [p∧(p→q)] | [p∧(p→q)]→q |
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

d) [(p∨q)∧(p→r)∧(q→r)]→r

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P | q | r | (p∨q) | (p→r) | (q→r) | [(p∨q)∧(p→r) | [(p∨q)∧(p→r)∧(q→r)] | [(p∨q)∧(p→r)∧(q→r)]→r |
| T | T | T | T | T | T | T | T | T |
| T | T | F | T | F | F | F | F | T |
| T | F | T | T | T | T | T | T | T |
| T | F | F | T | F | T | F | F | T |
| F | T | T | T | T | T | T | T | T |
| F | T | F | T | T | F | T | F | T |
| F | F | T | F | T | T | F | F | T |
| F | F | F | F | T | T | F | F | T |

5. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

a) Something is not in the correct place.

P(t) = t is in the correct place.

￢∃t P(t)

b) All tools are in the correct place and are in excellent condition.

P(t) = t is in the correct place.

Q(t) = t is in excellent condition.

∀t(P(t) ∧ Q(t))

c) Everything is in the correct place and in excellent condition.

P(t) = t is in the correct place.

Q(t) = t is in excellent condition.

∀t(P(t) ∧ Q(t))

d) Nothing is in the correct place and is in excellent condition.

P(t) = t is in the correct place.

Q(t) = t is in excellent condition.

￢∃t(P(t) ∧ Q(t))

e) One of your tools is not in the correct place, but it is in excellent condition.

P(t) = t is in the correct place.

Q(t) = t is in excellent condition.

∃t(￢P(t) ∧ Q(t))

6. Let P(x), Q(x), and R(x) be the statements “x is a clear explanation,” “x is satisfactory,” and “x is an excuse,” respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and P(x), Q(x), and R(x).

a) All clear explanations are satisfactory.

∀x(P(x) → Q(x))

b) Some excuses are unsatisfactory

∃x(R(x) ∧￢Q(x))

c) Some excuses are not clear explanations.

∃x(R(x) ∧￢P(x))

d) Does (c) follow from (a) and (b)?

1. ∃x(R(x) ∧￢Q(x))

2. R(a) ∧ ￢Q(a) – E.I. from (1)

3. R(a) - Simplification from (1)

4. ∀x(P(x) → Q(x))

5. P(a) → Q(a) -U.I. from (4)

6. ￢Q(a) -E.I. from (1)

7. ￢P(a) -Modus tollens from (5)(6)

8. R(a) ∧￢P(a) -Conjunction from (3)(7)

9. ∃x(R(x) ∧￢P(x)) -E.G. from (8)

7. Express each of these statements using mathematical and logical operators, predicates, and quantifiers, where the domain consists of all integers.

a) The sum of two negative integers is negative.

∀x ∀y((x<0)∧(y<0) → (x + y <0))

b) The difference of two positive integers is not necessarily positive.

∃x∃y((x>0)∧(y>0)→(x – y < 0)

c) The sum of the squares of two integers is greater than or equal to the square of their sum.

∀x ∀y((x^2 + y^2) ≥ (x + y)^2)

d) The absolute value of the product of two integers is the product of their absolute values.

∀x ∀y(|xy|=|x||y|)

8. Show that the two statements ¬∃x∀yP(x, y) and ∀x∃y¬P(x, y), where both quantifiers over the first variable in P(x, y) have the same domain, and both quantifiers over the second variable in P(x, y) have the same domain, are logically equivalent.

¬∃x∀y P(x, y)

=∀x¬∀y P(x, y)

=∀x∃y￢P(x, y)

9. What rule of inference is used in each of these arguments?

a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.

- Addition

b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.

-Simplification

c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

-Modus ponens

d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

-Modus tollens

e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

-hypothetical syllogism

10. Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

a) If n is a real number such that n>1, then >1. Suppose that >1 Then n>1.

-it’s invaild because if >1, then n > 1 or n > -1

b) If n is a real number with n>3, then >9. Suppose that ≤9. Then n≤3.

-it’s invalid because if n < -3, then

c) If n is a real number with n>2, then >4. Suppose that n≤2. Then ≤4.

-it’s invalid because if n < -2, then > 4

11. Show that the square of an even number is an even number using a direct proof.

Assume that n = 2k ( k is integer). Squaring both side of equation, = 4k2 = 4r = 2 \* 2r( r = k2 is integer). So = 4k2 is even.

12. Show that if n is an integer and +5 is odd, then n is even using

a) A proof by contraposition.

Assume that n is odd. So n = 2k + 1(k is integer).

+5 = (2k+1)3 + 5

=8k3+12k2+6k+1+5

=2(4k3+6k2+3k) + 6

=2r+6( r = 4k3+6k2+3k is integer)

So, if n is odd, +5 is even.

b) A proof by contradiction.

Assume that n is odd such that +5 is odd

If n is odd, then n3 is also odd. So +5 is even.

This contradicts that +5 is odd.

13. Prove that there are no positive perfect cubes less than 1000 that are the sum of the cubes of two positive integers.

Proof by cases.

Integer p, q

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P  q | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1 | 9 | 28 | 65 | 126 | 217 | 344 | 513 | 730 | 1001 |
| 2 | 9 | 16 | 35 | 72 | 133 | 224 | 351 | 520 | 737 | 1008 |
| 3 | 28 | 35 | 54 | 91 | 152 | 243 | 370 | 539 | 756 | 1027 |
| 4 | 65 | 72 | 91 | 128 | 189 | 280 | 407 | 576 | 793 | 1064 |
| 5 | 126 | 133 | 152 | 189 | 250 | 341 | 468 | 637 | 854 | 1125 |
| 6 | 217 | 224 | 243 | 280 | 341 | 432 | 559 | 728 | 945 | 1216 |
| 7 | 344 | 351 | 370 | 407 | 468 | 559 | 686 | 855 | 1072 | 1343 |
| 8 | 513 | 520 | 539 | 576 | 637 | 728 | 855 | 1024 | 1241 | 1512 |
| 9 | 730 | 737 | 756 | 793 | 854 | 945 | 1072 | 1241 | 1458 | 1729 |
| 10 | 1001 | 1008 | 1027 | 1064 | 1125 | 1216 | 1343 | 1512 | 1729 | 2000 |

There are no positive perfect cubes less than 1000 that are the sum of the cubes of two positive integers.

14. The quadratic mean of two real numbers x and y equals. By computing the arithmetic and quadratic means of different pairs of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.

Show that ≥ (x + y)/ 2

Proof.

(x - y)2 ≥ 0

x2 + y2- 2xy ≥ 0

2x2 + 2y2 ≥ x2 + y2+ 2xy

2x2 + 2y2 ≥ (x + y)2

x2/2 + y2/2 ≥ (x + y)2/4

≥ (x + y)/ 2

Chap2

1. What is the cardinality of each of these sets?

a) {a} = 1

b) {{a}} = 1

c) {a, {a}} = 2

d) {a, {a}, {a, {a}}} = 3

2. Let A = {a, b, c}, B={x, y}, and C = {0, 1}. Find

a) A×B×C = {(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)}

b) C×B×A = {(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)}

c) C×A×B = {(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x), (0, c, y), (1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y)}

d) B×B×B = {(x, x, x), (x, x, y), (x, y, x), (x, y, y), (y, x, x), (y, x, y), (y, y, x), (y, y, y)}

3. Show that if A, B, and C are sets, then =

a) By showing each side is a subset of the other side.

={x|x ∉}

={x|￢(x ∈))}

={x|￢(x∈A ∧ x∈B ∧ x∈C)}

={x|￢(x∈A)∨￢(x∈B)∨￢(x∈C)}

={x|(x ∉ A) ∨ (x ∉ B) ∨ (x ∉ C) }

={x|(x∈x∈) ∨ (x∈}

={x|x∈}

=

b) Using a membership table.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| A | B | C |  |  |  |  |  |  |  |  |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |

4. Show that if A and B are sets with A⊆B, then

a) A∪B=B.

Show that (A∪B)⊆B,

If x ∈ (A∪B), then x ∈ B (Because A⊆B, A∪B = {x| (x ∈ A) ∨ (x ∈ B)}) and It follows (A∪B)⊆B. (A∪B)⊇B

If x ∈ B, then x ∈ (A∪B) (A∪B = {x| (x ∈ A) ∨ (x ∈ B)}). So (A∪B)⊇B

b) A∩B=A.

Show that (A∩B)⊆A, (A∩B)⊇A

If x ∈ (A∩B), then by definition(A∩B = {x| (x ∈ A) ∧ (x ∈ B)}) x ∈ A and It follows (A∩B)⊆A.

If x ∈ A, then x ∈ B (Because A⊆B), x ∈ (A∩B). So, (A∩B)⊇A

5. Let = {…, -2, -1, 0, 1, …, i}. Find

a) {…, -2, -1, 0, 1, 2, …, n}

b) {A1}

6. Let f be the function from R to R defined by f(x) = . Find

a) ({1}). = 1

b) ({x|0<x<1}). {y| 0<y<1}

c) ({x|x>4}). {y|y>2}

7. What are the terms , , , and of the sequence {}, where equals

(A domain is a natural number.)

a) + 1?

a0 = 2

a1 = 3

a2 = 5

a3 = 9

b) ?

a0 = 1

a1 = 4

a2 = 27

a3 = 256

c) ?

a0 = 0

a1 = 0

a2 = 1

a3 = 1

d) + ?

a0 = 0

a1 = 1

a2 = 2

a3 = 3

8. Show that if A and B are sets with the same cardinality, then |A| ≤ |B| and |B| ≤ |A|.

If A and B are sets with the same cardinality, there is a one-to-one correspondence from A to B.

If there is a one-to-one correspondence from A to B, the cardinality of A is less than or the same as the cardinality of B (|A| ≤ |B|).

Also, If A and B are sets with the same cardinality, there is a one-to-one correspondence from B to A.

If there is a one-to-one correspondence from B to A, the cardinality of B is less than or the same as the cardinality of A (|B| ≤ |A|).

9. Show that matrix addition is commutative; that is, show that if A and B are both m×n matrices, then A+(B+C) = (A+B)+C.

Let A = and B = and C =

Case 1: A+(B+C)

B + C = , A+(B+C) =

Case 2: (A+B)+C

A + B= , (A+B)+C=

Therefore, A+(B+C) = (A+B)+C.

10. Let A = and B = . Find

a) A∨B.

b) A∧B.

c) A⊙B.