Partial Diff. Equations

1. Introduction

1.1. What is a PDE

An equation which relates the partial derivs. of a function of more than variable.

Examples

1. Consider a function $u(x,t)$ which satisfies:

$$\frac{\partial u}{\partial t} + f(x,t) \cdot u = g(x,t).$$

2. The advection equation for a function $u(x,t)$ is:

$$\frac{\partial u}{\partial t} + V \cdot \frac{\partial u}{\partial x} = 0 \quad \text{for all } x + t > 0.$$

where V is a constant. Here x is a spatial variable a t is a temporal variable

3. Consider a function $T(x,t)$ which satisfies.

$$\frac{\partial T}{\partial t} + V \cdot \frac{\partial T}{\partial x} = K \cdot \frac{\partial^2 T}{\partial x^2}. \quad \text{advection - diffusion equation}$$

where V is constant and $K > 0$ is constant. This is an example of a parabolic PDE.

$V \equiv 0 \rightarrow$ heat equation (or diffusion equation).

4. The PDE.

$$\frac{\partial^2 u}{\partial t^2} = a^2 \cdot \frac{\partial^2 u}{\partial x^2} \qquad (a > 0 \text{ constant}).$$

for $u(x,t)$ over $-\infty < x < \infty$ and $t > 0$, is known as the wave equation.

This can be generalized for $u(x,y,z,t)$ to.

$$\frac{\partial^2 u}{\partial t^2} = a^2 \cdot \nabla^2 u.$$

where $\nabla^2 u$ is the Laplacian. These are hyperbolic PDEs.