Assignment Solution

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1 Assignment 01

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1.1 Question 01

```
In [33]: from math import *
         import cmath
         from IPython.display import Markdown

         freq = 25124390 # Hz
         A = 25.0

         mu_0 = 4*pi*10**(-7) # Vaccum permeability
         eps_0 = 8.854*10**(-12) # Vacuum permittivity

         omega = 2*pi*freq

         c = 1/sqrt(mu_0*eps_0) # Speed of Light in a Vacuum

         eps_ground = 20
         mu_ground = 1

         eps_air = 1.000589
         mu_air = 1
```

1.1.1 Brewster Angle

```
In [2]: n1 = sqrt(eps_air*mu_air)
        n2  = sqrt(eps_ground*mu_ground)
        theta_b = atan(n2/n1)

        print("The brewster angle is {:f} degrees".format(degrees(theta_b)))
```

```
The brewster angle is 77.392025 degrees
```

1.1.2 Phase Constant

In [3]:
```
B_air = 2*pi*freq/c

v_ground = 1/sqrt(eps_ground*eps_0*mu_ground*mu_0)
B_ground = 2*pi*freq/v_ground

print("Phase constant for air: {:f} rad/s".format(B_air))
print("Phase constant for ground: {:f} rad/s".format(B_ground))
```

```
Phase constant for air: 0.526563
Phase constant for ground: 2.354860
```

1.1.3 Intrinsic Wave Impedances

In [4]:
```
eta1 = sqrt((mu_0*mu_air)/(eps_0*eps_air))
eta2 = sqrt((mu_0*mu_ground)/(eps_0*eps_ground))

print("Intrinsic wave impedance for air: {:f} ohms".format(eta1))
print("Intrinsic wave impedances for ground: {:f} ohms".format(eta2))
```

```
Intrinsic wave impedance for air: 376.623410
Intrinsic wave impedances for ground: 84.240352
```

1.1.4 Angle of Transmission

In [5]:
```
sin_t = n1/n2*sin(theta_b)
theta_t = asin(sin_t)

print("The transmission angle is {:f} degrees".format(degrees(theta_t)))
```

```
The transmission angle is 12.607975 degrees
```

1.1.5 TE Polarised Reflection Coefficient

In [6]:
```
Gamma_te = (eta2*cos(theta_b)-eta1*cos(theta_t))/(eta2*cos(theta_b)+eta1*cos(theta_t))
tau_te = 1 + Gamma_te;
print("The Polarised TE Reflection Coefficient is {:f}".format(Gamma_te))
```

```
The Polarised TE Reflection Coefficient is -0.904708
```

1.1.6 TE Polarised Electric Fields

Incident

$$\vec{E}_s^i(x,z) = E_0^i \cdot \exp(-jB_1 \sin(\theta_i)x) \cdot \exp(-jB_1 \cos(\theta_i)z) V/m$$

$$\vec{E}_s^i(x,z,t) = \Re\{\vec{E}_s^i \exp(jwt)\}$$

In [214]:
```
E_i_0 = A

Markdown("""
$$\\vec{{E^i_s}}(x,z) = {a_y}\\hat{{a_y}}\exp({x:.3}jx)\cdot\exp({z:.3}jz)V/m$$
$$\\vec{{E^i_s}}(x,z,t) = {a_y}\\hat{{a_y}}\cos({omega:.0f}t{x:+.3}x{z:+.3}z)V/m$$
""".format(a_y=E_i_0,
           omega=omega,
           x=-B_air*sin(theta_b),
           z=-B_air*cos(theta_b)
          ))
```

Out[214]:

$$\vec{E^i_s}(x,z) = 25\hat{a_y}\exp(-0.514jx)\cdot\exp(-0.115jz)V/m$$

$$\vec{E^i_s}(x,z,t) = 25\hat{a_y}\cos(157861198t-0.514x-0.115z)V/m$$

Reflected

$$\vec{E^i_s}(x,z) = E^i_0\cdot\exp(-jB_1\sin(\theta_r)x)\cdot\exp(jB_1\cos(\theta_r)z)V/m$$

$$\vec{E^i_s}(x,z,t) = \Re\{\vec{E^i_s}\exp(jwt)\}$$

In [213]:
```
E_r_0 = A*Gamma_te

Markdown("""
$$\\vec{{E^r_s}}(x,z) = {a_y:.3f}\\hat{{a_y}}\exp({x:.3}jx)\cdot\exp({z:.3}jz)V/m$$
$$\\vec{{E^r_s}}(x,z,t) = {a_y:.3f}\\hat{{a_y}}\cos({omega:.0f}t{x:+.3f}x{z:+.3f}z)V/
""".format(
    a_y=E_r_0,
    omega=omega,
    x=-B_air*sin(theta_b),
    z=B_air*cos(theta_b)
))
```

Out[213]:

$$\vec{E^r_s}(x,z) = -22.618\hat{a_y}\exp(-0.514jx)\cdot\exp(0.115jz)V/m$$

$$\vec{E^r_s}(x,z,t) = -22.618\hat{a_y}\cos(157861198t-0.514x+0.115z)V/m$$

Transmitted

$$\vec{E^t_s}(x,z) = E^t_0\cdot\exp(-jB_2\sin(\theta_t)x)\cdot\exp(jB_2\cos(\theta_t)z)V/m$$

$$\vec{E^i_s}(x,z,t) = \Re\{\vec{E^i_s}\exp(jwt)\}$$

In [221]:
```
E_t_0 = A*tau_te

Markdown("""
$$\\vec{{E^t_s}}(x,z) = {a_y:.3f}\\hat{{a_y}}\cdot\exp({x:.3}jx)\cdot\exp({z:.3f}jz)\
```

```
$$\\vec{{E^t_s}}(x,z,t) = {a_y:.3f}\\hat{{a_y}}\cdot\cos({omega:.0f}t{x:+.3f}x{z:+.3
""".format(
    a_y=E_t_0,
    omega=omega,
    x=-B_ground*sin(theta_t),
    z=-B_ground*cos(theta_t)
))
```

Out[221]:

$$\vec{E^t_s}(x,z) = 2.382\hat{a_y}\cdot\exp(-0.514jx)\cdot\exp(-2.298jz)V/m$$

$$\vec{E^t_s}(x,z,t) = 2.382\hat{a_y}\cdot\cos(157861198t-0.514x-2.298z)V/m$$

1.1.7 TM Polarised Reflection Coefficient

```
In [168]: Gamma_tm = (eta2*cos(theta_t)-eta1*cos(theta_b))/(eta2*cos(theta_t)+eta1*cos(theta_b
          tau_tm = (1 + Gamma_tm)*cos(theta_b)/cos(theta_t);
          print("The Polarised TM Reflection Coefficient is {:f}".format(Gamma_tm))
```

```
The Polarised TM Reflection Coefficient is 0.000000
```

1.1.8 TM Polarized Electric Fields

Incident

$$\vec{E^i_s}(x,z) = E^i_0\cdot\exp(-jB_1\sin(\theta_i)x)\cdot\exp(-jB_1\cos(\theta_i)z)(\cos(\theta_i)\hat{a_x}-\sin(\theta_i)\hat{a_z})V/m$$

$$\vec{E^i_s}(x,z,t) = \Re\{\vec{E^i_s}\exp(jwt)\}$$

```
In [205]: E_i_0 = A

          Markdown("""
          $$\\vec{{E^i_s}}(x,z) = \exp({x:.3f}jx)\cdot\exp({z:.3}jz)({a_x:.3}\hat{{a_x}}{a_z:+
          $$\\vec{{E^i_s}}(x,z,t) = \cos({omega:.0f}t{x:+.3}x{z:+.3}z)({a_x:0.3f}\hat{{a_x}}{a_
          """.format(
              omega=omega,
              x=-B_air*sin(theta_b),
              z=-B_air*cos(theta_b),
              a_x=cos(theta_b)*E_i_0,
              a_z=-sin(theta_b)*E_i_0
          ))
```

Out[205]:

$$\vec{E^i_s}(x,z) = \exp(-0.514jx)\cdot\exp(-0.115jz)(5.46\hat{a_x}-24.4\hat{a_z})V/m$$

$$\vec{E^i_s}(x,z,t) = \cdot\cos(157861198t-0.514x-0.115z)(5.457\hat{a_x}-24.397\hat{a_z})V/m$$

Reflected

$$\vec{E^i_s}(x,z) = E^i_0 \cdot \exp(-jB_1\sin(\theta_r)x) \cdot \exp(-jB_1\cos(\theta_r)z)(\cos(\theta_r)\hat{a_x} + \sin(\theta_r)\hat{a_z})V/m$$

$$\vec{E^i_s}(x,z,t) = \Re\{\vec{E^i_s}\exp(jwt)\}$$

```
In [217]: E_r_0 = A*Gamma_tm

          Markdown("""
          $$\\vec{{E^r_s}}(x,z) = \exp({x:.3}jx)\cdot\exp({z:.3}jz)({a_x:.3f}\hat{{a_x}}+{a_z:
          $$\\vec{{E^r_s}}(x,z,t) = \cos({omega:.0f}t{x:+.4}x{z:+.4}z)({a_x:0.2}\hat{{a_x}}+{a_
          """.format(
              omega=omega,
              x=-B_air*sin(theta_b),
              z=B_air*cos(theta_b),
              a_x=cos(theta_b)*E_r_0,
              a_z=sin(theta_b)*E_r_0
          ))
```

Out[217]:

$$\vec{E^r_s}(x,z) = \exp(-0.514jx) \cdot \exp(0.115jz)(0.000\hat{a_x} + 0.000\hat{a_z})V/m = 0V/m$$

$$\vec{E^r_s}(x,z,t) = \cos(157861198t - 0.5139x + 0.1149z)(0.0\hat{a_x} + 0.0\hat{a_z})V/m$$

Transmitted

$$\vec{E^i_s}(x,z) = E^i_0 \cdot \exp(-jB_2\sin(\theta_r)x) \cdot \exp(-jB_2\cos(\theta_r)z)(\cos(\theta_r)\hat{a_x} - \cos(\theta_r)\hat{a_z})V/m$$

$$\vec{E^i_s}(x,z,t) = \Re\{\vec{E^i_s}\exp(jwt)\}$$

```
In [220]: E_t_0 = A*tau_tm

          Markdown("""
          $$\\vec{{E^t_s}}(x,z) = \exp({x:.4}jx)\cdot\exp({z:.4}jz)({a_x:0.2}\hat{{a_x}}{a_z:+
          $$\\vec{{E^t_s}}(x,z,t) = \cos({omega:.0f}t{x:+.3f}x{z:+.3f}z)({a_x:0.2}\hat{{a_x}}{a
          """.format(
              omega=omega,
              x=-B_ground*sin(theta_t),
              z=-B_ground*cos(theta_t),
              a_x=cos(theta_t)*E_t_0,
              a_z=-sin(theta_t)*E_t_0
          ))
```

Out[220]:

$$\vec{E^t_s}(x,z) = \exp(-0.514jx) \cdot \exp(-2.298jz)(5.5\hat{a_x} - 1.2\hat{a_z})V/m$$

$$\vec{E^t_s}(x,z,t) = \cos(157861198t - 0.514x - 2.298z)(5.5\hat{a_x} - 1.2\hat{a_z})V/m$$

1.1.9 Randomly Polarized Electric Fields (Uniform Plane Waves)

Incident

In [222]:
```
E_i_0 = A

Markdown("""
$$\\vec{{E^i_s}}(x,z) = \exp({x:.3f}jx)\cdot\exp({z:.3}jz)({a_x:.3}\hat{{a_x}}{a_z:+
$$\\vec{{E^i_s}}(x,z,t) = \cos({omega:.0f}t{x:+.3}x{z:+.3}z)({a_x:0.3f}\hat{{a_x}}{a
""".format(
    omega=omega,
    x=-B_air*sin(theta_b),
    z=-B_air*cos(theta_b),
    a_x=cos(theta_b)*E_i_0,
    a_z=-sin(theta_b)*E_i_0,
    a_y=E_i_0
))
```

Out[222]:

$$\vec{E^i_s}(x,z) = \exp(-0.514jx)\cdot\exp(-0.115jz)(5.46\hat{a_x} - 24.4\hat{a_z} + 25.000\hat{a_y})V/m$$

$$\vec{E^i_s}(x,z,t) = \cos(157861198t - 0.514x - 0.115z)(5.457\hat{a_x} - 24.397\hat{a_z} + 25.000\hat{a_y})V/m$$

Reflected

In [224]:
```
Markdown("""
$$\\vec{{E^r_s}}(x,z) = \exp({x:.3}jx)\cdot\exp({z:.3}jz)({a_x:.3f}\hat{{a_x}}+{a_z:
$$\\vec{{E^r_s}}(x,z,t) = \cos({omega:.0f}t{x:+.4}x{z:+.4}z)({a_x:0.2}\hat{{a_x}}+{a
""".format(
    omega=omega,
    x=-B_air*sin(theta_b),
    z=B_air*cos(theta_b),
    a_x=cos(theta_b)*A*Gamma_tm,
    a_z=sin(theta_b)*A*Gamma_tm,
    a_y=A*Gamma_te
))
```

Out[224]:

$$\vec{E^r_s}(x,z) = \exp(-0.514jx)\cdot\exp(0.115jz)(0.000\hat{a_x} + 0.000\hat{a_z} - 22.618\hat{a_y})V/m = 0V/m$$

$$\vec{E^r_s}(x,z,t) = \cos(157861198t - 0.5139x + 0.1149z)(0.0\hat{a_x} + 0.0\hat{a_z} - 22.618\hat{a_y})V/m$$

Transmitted

In [226]:
```
E_t_0 = A*tau_tm

Markdown("""
```

```
$$\\vec{{E^t_s}}(x,z) = \exp({x:.4}jx)\cdot\exp({z:.4}jz)({a_x:0.2}\hat{{a_x}}{a_z:+
$$\\vec{{E^t_s}}(x,z,t) = \cos({omega:.0f}t{x:+.3f}x{z:+.3f}z)({a_x:0.2}\hat{{a_x}}{a
""".format(
    omega=omega,
    x=-B_ground*sin(theta_t),
    z=-B_ground*cos(theta_t),
    a_x=cos(theta_t)*E_t_0,
    a_z=-sin(theta_t)*E_t_0,
    a_y=A*tau_te
))
```

Out[226]:

$$\vec{E^t_s}(x,z) = \exp(-0.514jx) \cdot \exp(-2.298jz)(5.5\hat{a_x} - 1.2\hat{a_z} + 2.382\hat{a_y})V/m$$

$$\vec{E^t_s}(x,z,t) = \cos(157861198t - 0.514x - 2.298z)(5.5\hat{a_x} - 1.2\hat{a_z} + 2.382\hat{a_y})V/m$$

1.2 Question 02

In [231]:
```
R_a = 215/2
X_a = 251/1.5
Z_a = R_a+1j*X_a

Z_0 = 50

freq = 2.2*10**(9)

eps_r = 4.2
```

```
complex reflection coefficient of the antenna is: (0.7017423611589649+0.31687901523428474j)
```

1.2.1 Reflection Coefficient

In [232]:
```
Gamma = (Z_a-Z_0)/(Z_a+Z_0)
print("complex reflection coefficient of the antenna is: {0}".format(Gamma))
```


```
complex reflection coefficient of the antenna is: (0.7017423611589649+0.31687901523428474j)
```


1.2.2 Stub Matching

In [235]:
```
y_l = Z_0/Z_a

d = 0.5-0.472+0.188
l = 0.313-0.25
print("Through line length is :{0:f} meters".format(d*c/freq))
print("Open circuit stub length:{0:f} meters".format(l*c/freq))
```

```
Through line length is :0.029434 meters
Open circuit stub length:0.008585 meters
```