An Online Bayesian Comparison of Volatility Models: Does anything beat a GARCH(1,1)?

Nicolas Koch, Chris Stanton

June 2023

1 Project Abstract

The problem of forecasting the conditional volatility of the returns of financial assets has applications in derivative pricing, evaluating risk measures and quantitative trading. Two competing modelling approaches have emerged for the forecasting of volatility from daily data: GARCH models (Engle, 2001), (Bollerslev, 1986) and Stochastic Volatility Models (Kim et al., 1998), (Shephard and Andersen, 2009). Within each of these 2 base classes of models, numerous extensions have emerged in the literature, involving student-t distributed innovations, leverage effects, state-switching, introduction of jumps and extension to higher dimensions among others. Existing comparisons of variants of volatility forecasting models look directly at the forecasting performance of the models. Hansen and Lunde (2005) evaluate the one-day ahead forecasts of 330 different ARCH models. Hansen et al. (2003) evaluate the forecast performance of 55 different volatility forecasting models: predominantly focusing on GARCH models and their variants, but also including the base stochastic volatility model, using the model confidence set approach (Hansen et al., 2011). Model evaluation based solely on forecasting performance does suffer from the limitation of 'test dataset overfitting'. This is the phenomenon whereby evaluating models out of the sample on the same dataset does not necessarily generalise to out of sample performance on other datasets. One can also evaluate a model through considering how well a particular model represents the process that generated the data. The principled Bayesian approach to this type of problem is through evaluation of the marginal likelihood, and comparison between models with Bayes Factors (Kass and Raftery, 1995). In Nakajima (2009), Bayesian inference is conducted in various GARCH and Stochastic volatility models using MCMC methods. Estimates of the marginal likelihood for various GARCH and Stochastic Volatility models are obtained from the MCMC output using the methods of (Chib, 1995), (Chib and Jeliazkov, 2001), (Chib and Jeliazkov, 2005). However, the bias and variance properties of these estimators are not well understood. Further, the model selection is conducted offline, thus it is unclear how the results are affected by the length of the time series, and no attempt is made to determine whether the best fitting models according to the Bayes factor correspond to models with superior volatility forecasts.

The aim of this work is to conduct online Bayesian inference and model selection on various GARCH and Stochastic Volatility models along with their extensions, in order to determine both which models are most representative of the financial data, and to determine whether evaluating models using principled Bayesian model selection results in selecting models that generate high quality predictions of volatility. For the GARCH models, as the likelihood of the model given the parameters is analytically tractable, we conduct online inference by using the IBIS algorithm for SMC sampling (Chopin, 2002), (Del Moral et al., 2006). For stochastic volatility models, due to conditional volatility being treated as a latent variable, the likelihood is intractable. To conduct online inference in stochastic volatility models, we use the SMC^2 (Chopin et al., 2013) algorithm. Both the IBIS and SMC^2 methods generate online unbiased estimators of the marginal likelihood, and approximate confidence intervals for these estimators can be evaluated by running the SMC samplers multiple times. If the project is extended to higher dimensions, one could also evaluate whether the best fitting models according to the Bayesian paradigm result in more profitable portfolios, with the simplest case being the $d = 2$ pairs trade. Another possible extension is to consider the extensions of the GARCH and Stochastic Volatility Models that incorporate realised measures.

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