Języki Formalne i Techniki Translacji

Lista 4, Zadanie 7

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We are to prove that the language L over the alphabet $\{1, 2, 3, 4\}$ such that $L = \{w \mid |w|_1 = |w|_2 \land |w|_3 = |w|_4\}$ is either context-free or not. We begin by assuming it is not context-free and use proof by contradiction as this allows us to use the pumping lemma for context-free languages.

Proof. Assume L is context-free. By the pumping lemma let p be the pumping length, then let $s = 1^p3^p2^p4^p$. Then there exists no partition of s into $uvwxy$ such that $|vwx| \leq p$ and $|vw|_1 = |vw|_2 \land |vw|_3 = |vw|_4$. ■

The condition that vw contains the same number of 1,2 and 3,4 symbols is necessary for when we apply the pumping part of the lemma, that is $uv^nwx^ny, n \geq 0$, the lemma produces words that do not satisfy the conditions of the language - if $|vw|_1 \neq |vw|_2$, then while pumping we produce different quantites of 1s and 2s, hence failing the condition.