Algorytmy Optymalizacji Dyskretnej 2022/2023

Piotr Kocia

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Contents

1 Maximum flow

We construct a graph such that each vertex has directed edges connecting it to vertices whose index has popcount greater by exactly one. We randomly assign capacities of at least 1 to the edges. The total number of vertices in the graph is 2^k where k is a parameter to the program. We gather information for $k \in \{1, .., 16\}$ performing several runs for each value of k.

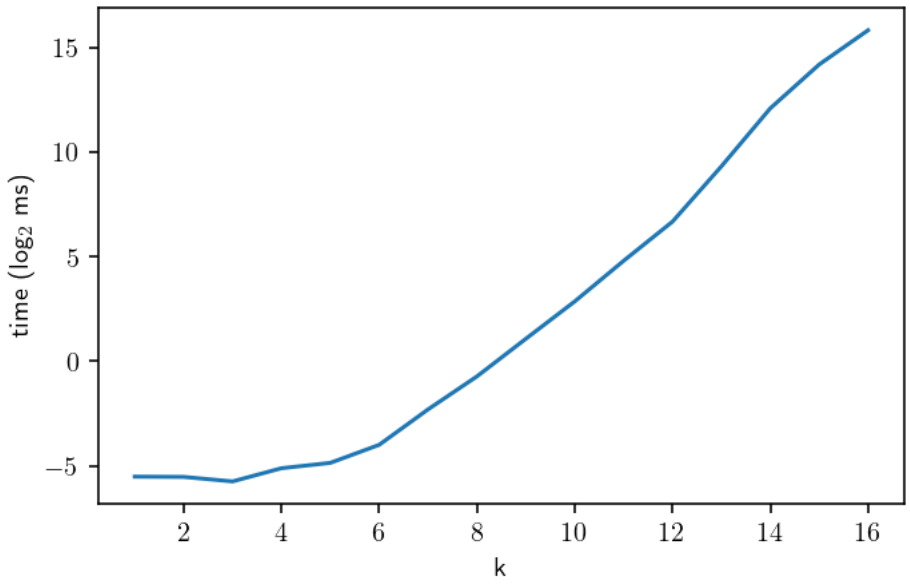

Figure 1: Average computation time across several runs.

The plots are linearised with $\log_2$ to better present the results across the vast range.

It is clearly visible that all metrics (time in Fig. 1, flow in Fig. 2 and augmenting paths in Fig. 3) increase exponentially with k, the function being approximately 2^k, hence proportionally to the number of vertices in the graph.

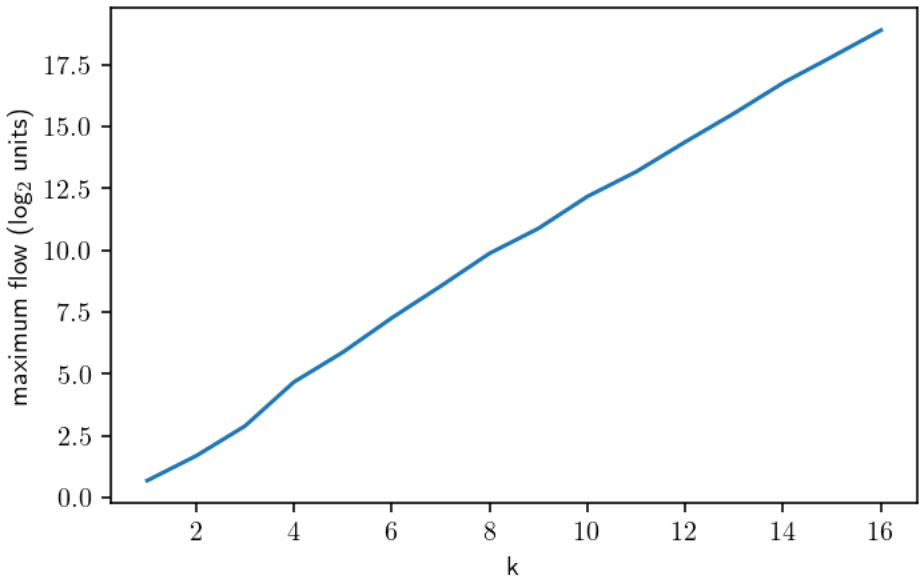

Figure 2: Average maximum flow across several runs.

2 Perfect matchings in bipartite graphs

We use the reduction of the maximal matching problem to the network flow problem in the bipartite graphs and use the Edmonds-Karp algorithm. We construct a bipartite graph consisting of two partitions of size 2^k such that the vertices in the first partition have exactly $d < k$ outgoing edges each. Every edge is assigned the capacity 1. We additionally add a source and a sink which are connected via directed edges to ensure the graph is connected. We then proceed to run Edmonds-Karp algorithm from the source to the sink in order to find the maximal matching.

The degree clearly affects the computation time since it increases the number of edges in the graph, yet its effect appears to diminish with increase in d. However, the degree does not seem to have much of an effect past a certain point in the case of edges in a maximal matching.

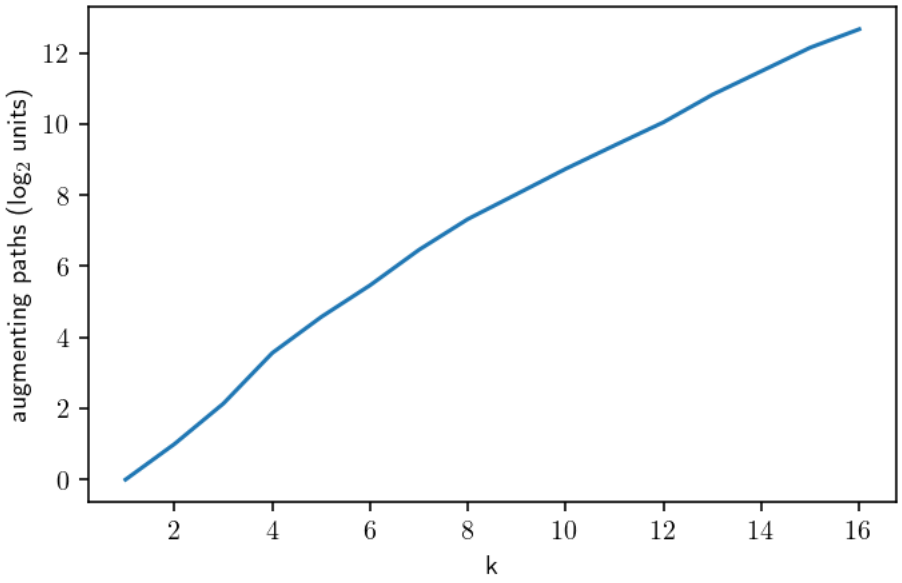

Figure 3: Average number of augmenting paths across several runs.

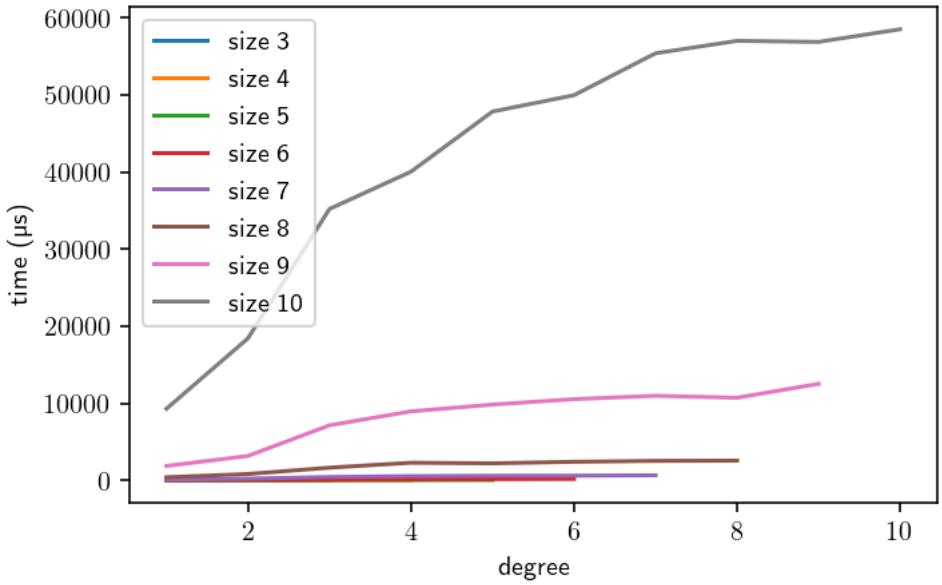

Figure 4: Average computation time with respect to the degree across several runs.

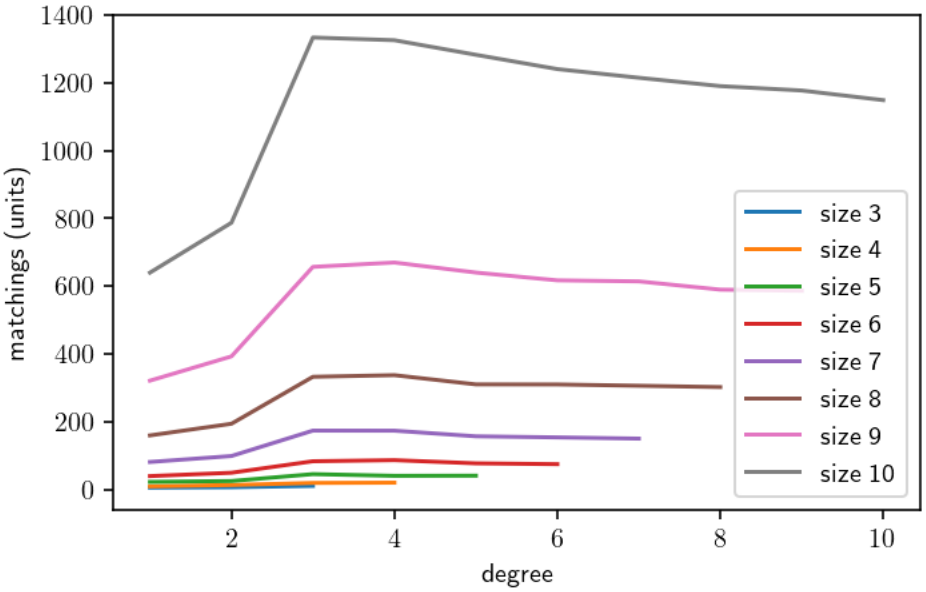

Figure 5: Average number of matchings with respect to the degree across several runs.