Języki Formalne i Techniki Translacji

Lista 1, Zadanie 3

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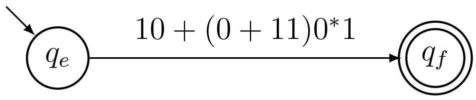
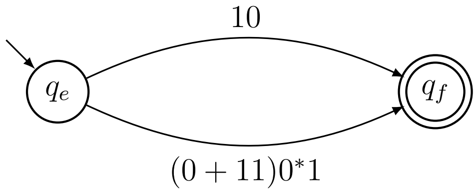
1 Regular Expression $10 + (0 + 11)0^*1$

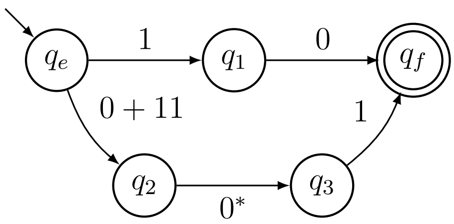
The regular expression $10 + (0 + 11)0^*1$ may have its corresponding FA constructed in a few steps by iteratively lowering its subexpressions. Firstly, we start with an automaton that has only one transition matching the whole expression.



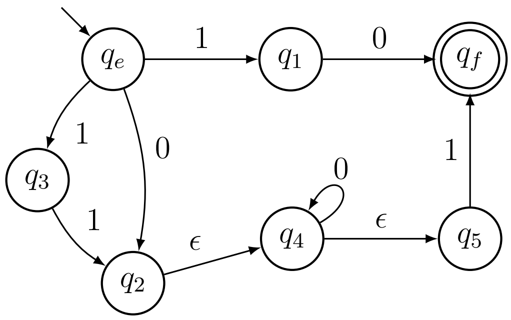
We separate the expressions at the alternative and create two transitions from q_e to q_f.



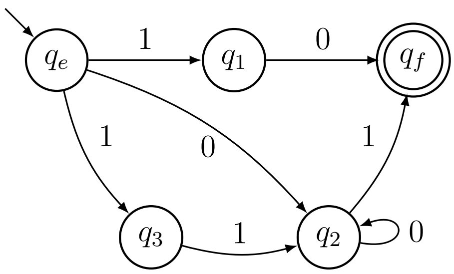
Concatenations are sequence events, hence we create sequences of nodes with transitions inbetween corresponding to the atomic expressions of the concatenation.



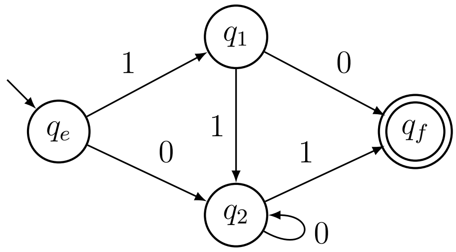
The tricky part in this step is the transition with Kleene star. One possible solution is to create a state that loops to itself on the expression that Kleene star operates on and provide ϵ transtions to and from that state.



At this point we have transformed the entire RE into states and transitions on the elements of the alphabet, hence what remains is simplification and elimination of ϵ transitions. We note that we have a chain of ϵ transitions between q_2 and q_5 with no transitions apart from the loop in q_4. We may simplify that by merging both q_4 and q_5 into q_2.



The resulting FA is non-deterministic because of the two transitions on 1 from q_e. We may turn it into a DFA by merging q_1 and q_3.



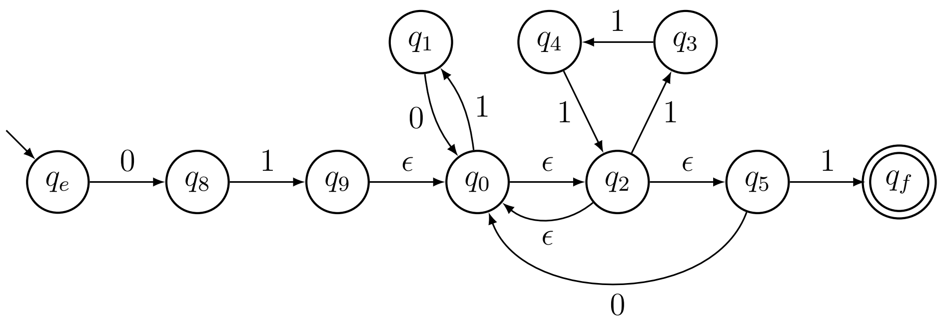
From the above we may conclude that the DFA for $10 + (0 + 11)0^*1$ is the quintuple $(\{0, 1\}, \{q_e, q_f, q_1, q_2\}, \{q_e\}, \delta, \{q_f\})$ where δ is the transition function shown in Table 1.

	0	1
q_e	q_2	q_1
q_1	q_f	q_2
q_2	q_2	q_f

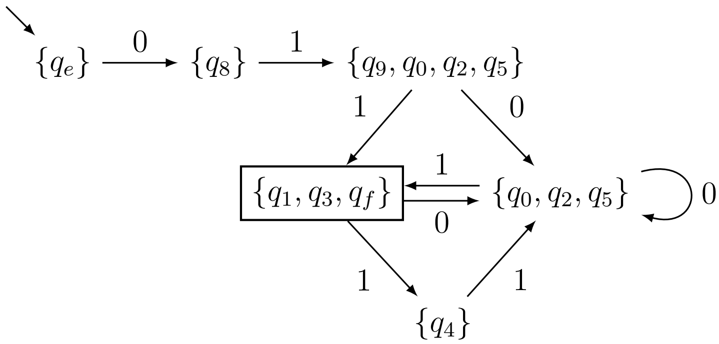
Table 1: Transition function δ.

2 Regular Expression $01[((10)^* + 111)^* + 0]^*1$

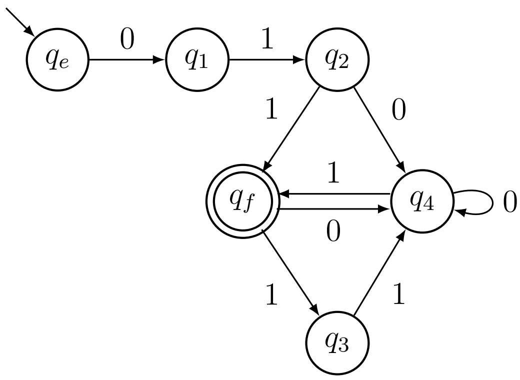
We begin by constructing an NFA for the expression $01[((10)^* + 111)^* + 0]^*1$ following similar procedure as above.



We then transform this to a DFA by repeatedly applying the procedure of finding the set of states for a given transition and computing the ϵ closure of that set. Missing transitions are implicitly assumed to be going into the trap state $\varnothing$.



The DFA has a single accpeting state marked with a rectangular outline. Renaming the states we obtain



Hence the DFA for $01[((10)^* + 111)^* + 0]^*1$ is the quintuple

$$(\{0, 1\}, \{q_e, q_f, q_1, q_2, q_3, q_4\}, \{q_e\}, \delta, \{q_f\})$$

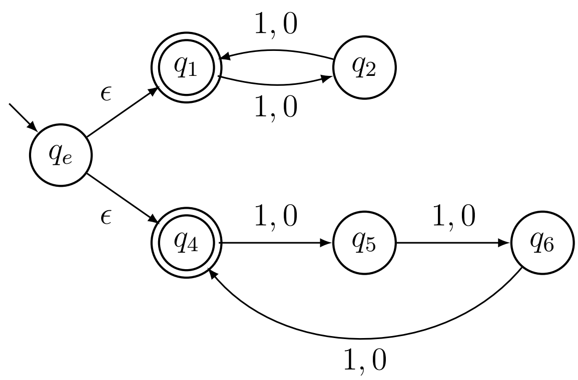
where δ is the transition function shown in Table 2.

	0	1
q_e	q_1	$\varnothing$
q_1	$\varnothing$	q_2
q_2	q_5	q_f
q_3	$\varnothing$	q_4
q_4	q_4	q_f
q_f	q_4	q_3

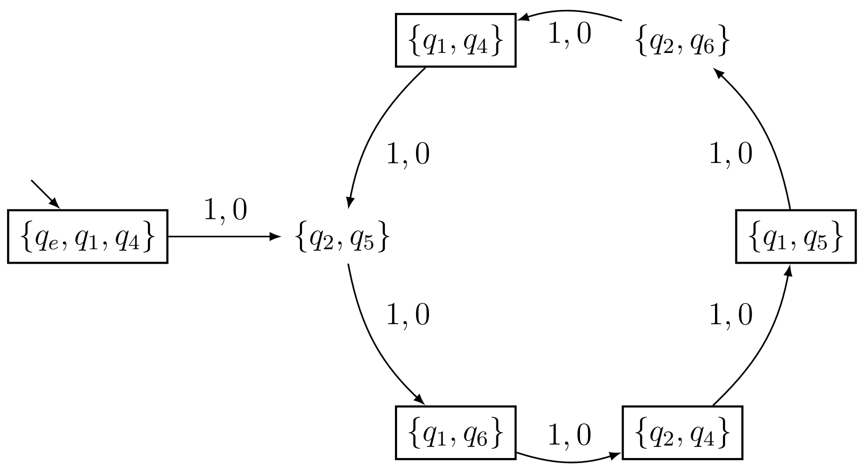
Table 2: Transition function δ.

3 Regular Expression $((0+1)(0+1))^* + ((0+1)(0+1)(0+1))^*$

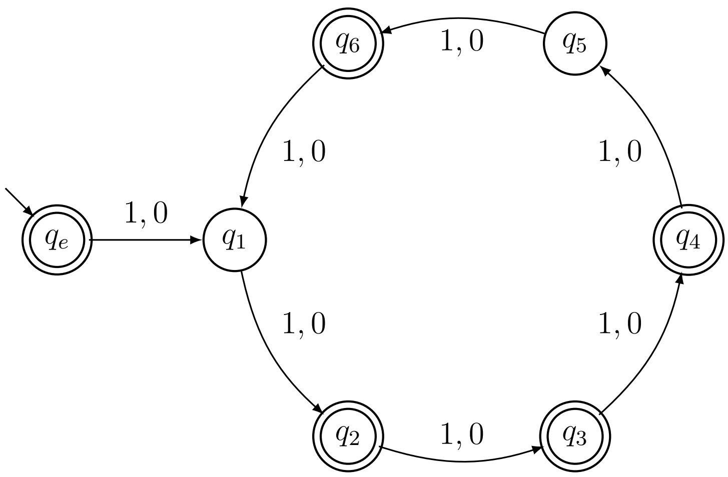
We begin by constructing an NFA for the expression $((0+1)(0+1))^* + ((0+1)(0+1)(0+1))^*$ identically as before.



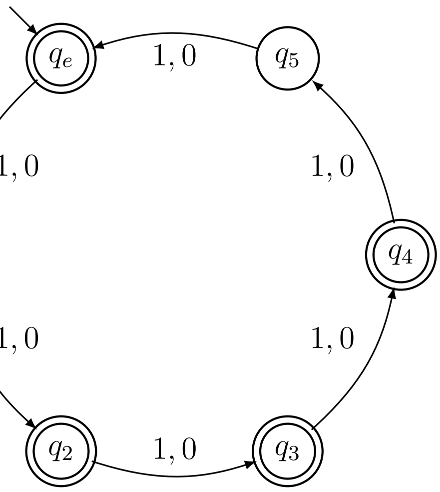
We then transform this to a DFA. Missing transitions are implicitly assumed to be going into the trap state $\varnothing$.



The resulting DFA has multiple accepting states marked with rectangular outlines. Renaming the states we obtain



We note that the state q_e is reduntant as the state q_6 is equivalent to it (accepting state, identical transition function). Thus we may remove q_e and make q_6 the entry state (and also rename it to q_e).



The final DFA for $((0+1)(0+1))^* + ((0+1)(0+1)(0+1))^*$ is the quintuple

$$(\{0,1\}, \{q_e, q_f, q_1, q_2, q_3, q_4, q_5\}, \{q_e\}, \delta, \{q_e, q_2, q_3, q_4\})$$

where δ is the transition function shown in Table 3.

	0	1
q_1	q_2	q_2
q_2	q_3	q_3
q_3	q_4	q_4
q_4	q_5	q_5
q_5	q_6	q_6
q_e	q_1	q_1

Table 3: Transition function δ.