

# STRUCTURE FORMING PROCESSES IN MESOSCOPIC POLYMER SYSTEMS

by

TOMAS KOCI

(Under the direction of Michael Bachmann)

# ABSTRACT

This is going to be the best abstract ever :)

INDEX WORDS: Index word or phrase, Index word or phrase, Index word or phrase, Index word, Index word, Index word

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MESOSCOPIC POLYMER SYSTEMS

by

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B.A., The Juilliard School, 2008

A Dissertation Submitted to the Graduate Faculty  
of The University of Georgia in Partial Fulfillment  
of the

Requirements for the Degree

DOCTOR OF PHILOSOPHY

ATHENS, GEORGIA

2016

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# Structure Forming Processes in Mesoscopic Polymer Systems

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May 5, 2016

# Acknowledgments

Mention Michael Bachmann, Steven Lewis, Heinz Schuttler, D.P. Landau, Jeff Mike and Shan-Ho, finally all the Links and my family

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# Chapter 1

## Introduction

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# Chapter 2

## Elements of Statistical Mechanics

Statistical mechanics aims at explaining the microscopic origins of macroscopic properties of systems with large numbers of degrees of freedom. The exact solution for a single phase space trajectory of a complex system requires enormous computational efforts and in most cases provides only a limited insight. In contrast to the chaotic nature of most phase space trajectories, collective system properties such as entropy, pressure, or temperature, for the most part exhibit relatively simple behavior. The formalism of statistical mechanics allows us to study these properties by considering the average behavior of a large number of identically prepared systems, i.e. the statistical ensemble. It is well established that in the thermodynamic limit all ensembles become equivalent. However this is emphatically not true in the case of intrinsically finite systems for which the choice of an ensemble is non-trivial. Therefore, I shall briefly discuss several prominent statistical ensembles starting with the most fundamental one, the *microcanonical ensemble*.

## 2.1 The microcanonical ensemble

Let us consider a mechanically and adiabatically isolated system with a constant number of particles ( $N$ ), volume ( $V$ ), and energy ( $E$ ). At any given moment, the system is to be found in a particular microstate  $\mu$ , which is represented by a point in a  $6N$  dimensional phase-space. At a fixed energy  $E$ , the accessible microstates are constrained to the surface of constant energy  $\mathcal{H}(\mu) = E$ , where  $\mathcal{H}$  is the Hamiltonian of the system. The total number of microstates corresponding to a macrostate with a fixed energy  $E$  is obtained by calculating the density of states<sup>1,2</sup>

$$g(E) = \int \mathcal{D}\mathcal{P}\mathcal{D}\mathcal{Q} \delta(E - \mathcal{H}(\mathcal{P}, \mathcal{Q})), \quad (2.1)$$

where

$$\mathcal{D}\mathcal{P}\mathcal{D}\mathcal{X} = \prod_{n=1}^N \frac{d^3 p_n d^3 x_n}{(2\pi\hbar)^3} \quad (2.2)$$

is the Lebesgue measure over phase space. Assuming that no additional quantities are conserved, i.e. the system is ergodic, all accessible microstates have equal a priori probabilities. The microcanonical equilibrium probability distribution is given by

$$p(\mu)_E = \begin{cases} 1/g(E), & \text{if } \mathcal{H}(\mu) = E \\ 0, & \text{if } \mathcal{H}(\mu) \neq E, \end{cases} \quad (2.3)$$

and the expectation value of an observable  $O$  at a fixed energy  $E$  is found by averaging over the surface of constant energy

$$\langle O \rangle_E = \int \mathcal{D}\mathcal{P}\mathcal{D}\mathcal{Q} O(\mathcal{P}, \mathcal{Q}) \delta(E - \mathcal{H}(\mathcal{P}, \mathcal{Q})). \quad (2.4)$$

---

<sup>1</sup>In the context of computer simulations, the energy space becomes by necessity discretized and the density of states is determined by counting the number of microstates within some finite energy range  $[E, E + \Delta E]$ .

<sup>2</sup>Please refer to section 2.3 for detailed discussion of alternative definitions of the density of states.

The density of states of a typical mesoscopic system can easily span several thousands of orders of magnitude. It is therefore convenient to define the microcanonical equilibrium entropy

$$S(E) = k_B \ln g(E), \quad (2.5)$$

as an *extensive* quantity with dimensions of energy over temperature.<sup>3</sup>

### 2.1.1 Microcanonical temperature

Temperature is one of the fundamental concepts of statistical mechanics and has been traditionally defined in terms of the average kinetic energies of particles in a system. Here we motivate a more fundamental definition of temperature as an intrinsic system property, which can be obtained directly from the microcanonical density of states  $g(E)$ . For this purpose, let us consider an adiabatically isolated system composed of two weakly interacting subsystems,  $S_1$  and  $S_2$ . The energy of the combined system is fixed and can be written as the sum of the energies of the two subsystems  $E = E_1 + E_2$ . The probability density for a given pair of subsystem energies  $(E_1, E_2)$  is

$$\rho(E_1, E_2) = \frac{g_1(E_1)g_2(E - E_1)}{g(E)}, \quad (2.6)$$

where the density of states of the combined system is expressed as a convolution

$$g(E) = \int dE_1 g_1(E_1)g_2(E - E_1). \quad (2.7)$$

In systems with many degrees of freedom, the probability density  $\rho(E_1, E_2)$  is a sharply peaked distribution around the equilibrium energies  $(\bar{E}_1, \bar{E}_2)$ <sup>4</sup>. These can be found by setting

---

<sup>3</sup>If temperature is measured in the more natural units of energy, entropy becomes a unitless quantity and the Boltzmann constant equals to unity.

<sup>4</sup>The energy fluctuations per particle around the equilibrium energy  $\bar{E}_1$  scale as  $N^{-1/2}$ .

the energy derivative of the probability density to zero, from which we obtain

$$\frac{1}{g_1} \frac{dg_1}{dE_1} \Big|_{\bar{E}_1} = \frac{1}{g_2} \frac{dg_2}{dE_2} \Big|_{E-\bar{E}_1}, \quad (2.8)$$

or alternatively in terms of the microcanonical entropy

$$\frac{dS_1}{dE_1} \Big|_{\bar{E}_1} = \frac{dS_2}{dE_2} \Big|_{E-\bar{E}_1}. \quad (2.9)$$

Motivated by the familiar observation that interacting systems at thermal equilibrium have equal temperatures, we define the microcanonical temperature as

$$T(E) = \left( \frac{dS(E)}{dE} \right)^{-1}. \quad (2.10)$$

Frequently, it is more convenient to consider the inverse microcanonical temperature defined as

$$\beta(E) = \frac{dS(E)}{dE}. \quad (2.11)$$

In the following section, we discuss the central role of inverse microcanonical temperature and its energy derivatives in the classification of structural phase transitions.

### 2.1.2 Microcanonical analysis of phase transitions

A macrostate of a system is specified by a set of macroscopic variables and possesses the characteristics of the predominant microstates. Macrostates are said to belong to the same thermodynamic phase, if in a given range of some external control parameters<sup>5</sup> all of the system's thermodynamic observables are analytic, i.e. have convergent Taylor expansions. Singularities in the observables signify the presence of phase transitions between distinct

---

<sup>5</sup>Some common examples of external control parameters are the canonical temperature, pressure, or the chemical potential.

phases, typically marked by abrupt changes in macroscopic properties in response to minute variations of external control parameters. Phase transitions can be roughly divided into two categories. *Abrupt* transitions are characterized by the coexistence of two distinct phases and discontinuities in most physical properties. *Continuous* transitions, although less common in nature, have been the object of most intense research. They are marked by diverging correlation lengths, large fluctuations, and scale invariance.

Divergences and singularities in the thermodynamic observables and their derivatives are only found in systems which satisfy the thermodynamic limit. In mesoscopic systems<sup>6</sup>, due to finite size effects, divergences are replaced by peaks and discontinuities are smoothed over. For clarity, we designate the term *pseudophase transition* to represent significant conformational changes in finite systems. Likewise, a thermodynamic phase in a finite system shall be referred to as *pseudophase*. In the following, we present a powerful formalism for the analysis of pseudophase transitions in the microcanonical ensemble; the microcanonical inflection point analysis.

## Microcanonical inflection point analysis

Unlike its canonical counterpart – the heat-bath temperature – the microcanonical inverse temperature is an inherent property of the system, derived directly from the fundamental microcanonical quantities  $S(E)$  and  $E$ . We assert that all essential information about energetically and entropically driven thermodynamic processes, is contained in its curvature. Hence the microcanonical inverse temperature is an ideal starting point for a comprehensive analysis of pseudophase transitions.

In analogy to the principle of minimal sensitivity [?], structural transitions between pseu-

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<sup>6</sup>Typical length scales in a mesoscopic system are of the order of  $\sim 10$  nanometers. In this regime, exact quantum many-body interactions can be replaced by effective classical potentials, and cooperative effects dominate structure formation processes. Mesoscopic systems are distinct from macroscopic systems due to the presence of significant finite size effects which disallow the simplifying assumptions of the thermodynamic limit.



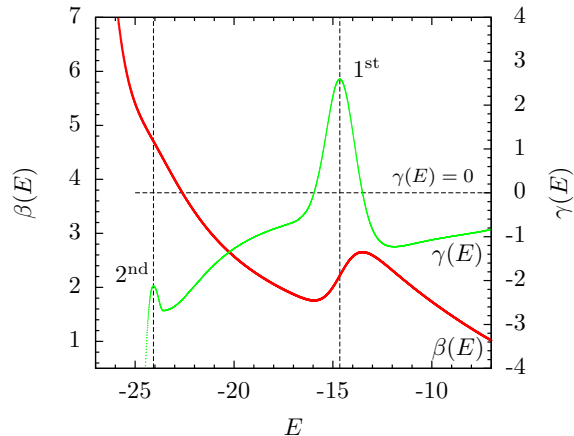


Figure 2.1: Microcanonical inflection-point analysis of the inverse microcanonical temperature  $\beta(E)$ . The prominent back-bending region in  $\beta(E)$ , together with the positive-valued peak in its energy derivative  $\gamma(E)$  at  $E \approx -15$ , indicates a *first-order* transition. The negative-valued peak at  $E \approx -24$  corresponds to a *second-order* transition.

dophases occur if  $\beta(E)$ , or one of its energy derivatives, responds least sensitively to variations in energy. In particular, *first-order* transitions are associated with inflection points in  $\beta(E)$  that have a positive slope. Therefore, it can easily be identified by a positive-valued peak in the energy derivative  $\gamma(E) = d\beta(E)/dE$ .

Similarly, a *second-order* transition occurs if  $\gamma(E)$  attains a negative-valued peak. Examples of microcanonical *first-* and *second-order* transition signals are shown in Fig. 2.1. Based on the principle of minimal sensitivity and Ehrenfest's original idea of characterizing the order of a transition by the free-energy derivative at which a discontinuity occurs, one can likewise introduce a hierarchy of higher-order transitions microcanonically.

## 2.2 The canonical ensemble

## 2.3 Configurational density of states

## 2.4 Generalized ensembles

# Chapter 3

## Computational Methods

### 3.1 Markov chain Monte Carlo

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### 4.2 Interacting homopolymers

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#### 7.2.2 Missing subphases and translational entropy

#### 7.2.3 Density effects on the latent heat

# Chapter 8

## Summary and Outlook

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Figure 8.1: Example of a figure.



Table 8.1: Example of a table.  
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