E theory

d is the number of dimensions, 3 everywhere here

$$\underline{\underline{E}} \sim \psi_1(\underline{N}\,\underline{N} - \frac{\underline{\underline{\delta}}}{d}) + \psi_2(\underline{M}\,\underline{M} - \frac{\underline{\underline{\delta}}}{d}) \quad \text{when biaxial}$$

evolve using

$$\frac{\partial \underline{\underline{E}}}{\partial t} = -\mu \frac{\delta G}{\delta \underline{\underline{E}}^*} \quad \text{where} \quad G = F + \int \lambda_s g_s(\underline{\underline{E}}) + \lambda_t g_t(\underline{\underline{E}}) + \lambda_n g_n(\underline{\underline{E}}) dV$$

and

$$\begin{aligned} g_s &= |E_{ij} - E_{ji}|^2 \\ g_t &= |E_{ii}|^2 \\ g_n &= |E_{ik}E^*_{kj} - E^*_{ik}E_{kj}|^2 \end{aligned}$$

Free energies – 1CA

One constant approximation version

$$
\begin{aligned}
f_{\text{bulk}} &= A|\underline{\underline{E}}|^2 + \frac{C}{2}|\underline{\underline{E}}|^4 \\
f_{\text{comp}} &= b_1|\underline{\nabla}\,\underline{\underline{E}}|^2 \\
f_{\text{curv}} &= b_2|\nabla^2\underline{\underline{E}}|^2
\end{aligned}
$$

$$
\begin{aligned}
\frac{\delta F_{\text{bulk}}}{\delta E^*_{ij}} &= (A + C E_{ab} E^*_{ab}) E_{ij} \\
\frac{\delta F_{\text{comp}}}{\delta E^*_{ij}} &= -b_1 E_{ij,kk} \\
\frac{\delta F_{\text{curv}}}{\delta E^*_{ij}} &= b_2 E_{ij,kkoo}
\end{aligned}
$$

Free energies – Full

The new, more complex version

$$\begin{aligned}
f_{\mathsf{bulk}} &= A|\underline{\underline{E}}|^2 + \frac{C}{2}|\underline{\underline{E}}|^4 \\
f_{\mathsf{comp}} &= b_1^{\parallel}\Pi_{kl}E_{ij,k}E^*_{ij,l} + b_1^{\perp}T_{kl}E_{ij,k}E^*_{ij,l} \\
f_{\mathsf{curv}} &= b_2^{\parallel}\Pi_{kl}E_{ij,lk}\Pi_{mn}E^*_{ij,nm} + b_2^{\perp}T_{kl}E_{ij,lk}T_{mn}E^*_{ij,nm} \\
&\quad + b_2^{\parallel\perp}(\Pi_{kl}E_{ij,lk}T_{mn}E^*_{ij,nm} + T_{kl}E_{ij,lk}\Pi_{mn}E^*_{ij,nm})
\end{aligned}$$

Free energies – Full

$$
\begin{aligned}
\frac{\delta F_{\mathsf{bulk}}}{\delta E^*_{ij}} &= (A + C E_{ab} E^*_{ab}) E_{ij} \\
\frac{\delta F_{\mathsf{comp}}}{\delta E^*_{ij}} &= -(b_1^{\parallel} - b_1^{\perp})(\Pi_{kl,l} E_{ij,k} + \Pi_{kl} E_{ij,kl}) - b_1^{\perp} E_{ij,kk} \\
\frac{\delta F_{\mathsf{curv}}}{\delta E^*_{ij}} &= (b_2^{\parallel} + b_2^{\perp} - 2b_2^{\parallel\perp})\Big((\Pi_{kl}\Pi_{po,po} + 2\Pi_{kl,o}\Pi_{po,p} + \Pi_{kl,po}\Pi_{po})E_{ij,lk} \\
&\qquad + 2(\Pi_{kl,o}\Pi_{po} + \Pi_{kl}\Pi_{po,o})E_{ij,lkp} + \Pi_{kl}\Pi_{po}E_{ij,lkpo}\Big) \\
&\quad + (b_2^{\parallel\perp} - b_2^{\perp})\Big(\Pi_{po,po}E_{ij,kk} + 2\Pi_{po,o}E_{ij,kkp} + \Pi_{po}E_{ij,kkpo} \\
&\qquad + \Pi_{kl,oo}E_{ij,lk} + 2\Pi_{kl,o}E_{ij,lko} + \Pi_{kl}E_{ij,lkoo}\Big) \\
&\quad + b_2^{\perp} E_{ij,kkoo}
\end{aligned}
$$

Gradients of $\underline{\underline{\Pi}}$

- Results using the square root version of $\underline{\underline{\Pi}}$

$$
\begin{aligned}
\Pi_{kl} &= \frac{sE_{kl}}{\sqrt{E_{ab}E_{ab}}} + \frac{\delta_{kl}}{d} \\
\Pi_{kl,m} &= \frac{s}{\sqrt{E_{ab}E_{ab}}}\left(E_{kl,m} - \frac{E_{kl}E_{cd}E_{cd,m}}{E_{ab}E_{ab}}\right) \\
\Pi_{kl,mn} &= \frac{s}{\sqrt{E_{ab}E_{ab}}}\Bigg(E_{kl,mn} \\
&\quad - \frac{E_{kl,n}E_{cd}E_{cd,m} + E_{kl,m}E_{cd}E_{cd,n} + E_{kl}(E_{cd,n}E_{cd,m} + E_{cd}E_{cd,mn})}{E_{ab}E_{ab}} \\
&\quad + 3\frac{E_{kl}E_{cd}E_{cd,m}E_{ef}E_{ef,n}}{(E_{ab}E_{ab})^2}\Bigg)
\end{aligned}
$$