Derivation of $\frac{\delta F}{\delta E^*_{ij}}$ in terms of E_{ij} and its derivatives – New version using $\frac{\delta E_{ij}}{\delta E_{ab}} = \delta_{ai}\delta_{bj} + \delta_{aj}\delta_{bi}$, assuming normality doesn't enter

1 Initial setup

$$
\begin{aligned}
F &= \int f_{\text{bulk}} + f_{\text{comp}} + f_{\text{curv}} \,\mathrm{d}V = F_{\text{bulk}} + F_{\text{comp}} + F_{\text{curv}} && (1)\\
f_{\text{bulk}} &= \frac{A}{2} E_{ij} E^*_{ij} + \frac{C}{4} (E_{ij} E^*_{ij})^2 && (2)\\
f_{\text{comp}} &= b_1^{\|} \Pi_{kl} E_{ij,k} E^*_{ij,l} + b_1^{\perp} T_{kl} E_{ij,k} E^*_{ij,l} \quad \text{maybe try adding} \quad b_1^d E_{ij,j} E^*_{ik,k} \quad \text{later too} && (3)\\
f_{\text{curv}} &= \quad \ldots \quad \text{for later} \quad \ldots && (4)\\
& && (5)
\end{aligned}
$$

where

$$\underline{\underline{\Pi}} = \underline{N}\,\underline{N} \qquad \text{and} \qquad \underline{\underline{T}} = \underline{\underline{\delta}} - \underline{\underline{\Pi}} \tag{6}$$

are the projection operators. We need to express these using $\underline{\underline{E}}$ as well, there are 2 options which I quote here

$$
\begin{aligned}
\underline{\underline{\Pi}} &= \frac{d-1}{d-2} \left(\frac{\underline{\underline{E}} \cdot \underline{\underline{E}}^*}{\underline{\underline{E}} : \underline{\underline{E}}^*} - \frac{\underline{\underline{\delta}}}{d(d-1)} \right) \quad \text{or} && (7)\\
\underline{\underline{\Pi}} &= \sqrt{\frac{d-1}{d \underline{\underline{E}} : \underline{\underline{E}}}}\, \underline{\underline{E}} - \frac{\underline{\underline{\delta}}}{d} \quad \text{which has a complex square root} && (8)
\end{aligned}
$$

$\underline{\underline{T}}$ just being calculated from $\underline{\underline{\Pi}}$.