Derivation of $\frac{\delta F}{\delta E^*_{ij}}$ in terms of E_{ij} and its derivatives – New version using $\frac{\delta E_{ij}}{\delta E_{ab}} = \delta_{ai}\delta_{bj} + \delta_{aj}\delta_{bi}$, assuming normality doesn't enter

1 Initial setup

$$F = \int f_{\text{bulk}} + f_{\text{comp}} + f_{\text{curv}}\, \mathrm{d}V = F_{\text{bulk}} + F_{\text{comp}} + F_{\text{curv}} \tag{1}$$

$$f_{\text{bulk}} = \frac{A}{2} E_{ij} E^*_{ij} + \frac{C}{4}(E_{ij}E^*_{ij})^2 \tag{2}$$

$$f_{\text{comp}} = b_1^{\|}\Pi_{kl} E_{ij,k} E^*_{ij,l} + b_1^{\perp} T_{kl} E_{ij,k} E^*_{ij,l} \quad \text{maybe try adding} \quad b_1^d E_{ij,j} E^*_{ik,k} \quad \text{later too} \tag{3}$$

$$f_{\text{curv}} = \dots \quad \text{for later} \quad \dots \tag{4}$$

$$\tag{5}$$

where

$$\underline{\underline{\Pi}} = \underline{N}\,\underline{N} \qquad \text{and} \qquad \underline{\underline{T}} = \underline{\underline{\delta}} - \underline{\underline{\Pi}} \tag{6}$$

are the projection operators. We need to express these using $\underline{\underline{E}}$ as well, there are 2 options which I quote here

$$\underline{\underline{\Pi}} = \frac{d-1}{d-2}\left(\frac{\underline{\underline{E}} \cdot \underline{\underline{E}}^*}{\underline{\underline{E}} : \underline{\underline{E}}^*} - \frac{\underline{\underline{\delta}}}{d(d-1)}\right) \quad \text{or} \tag{7}$$

$$\underline{\underline{\Pi}} = \sqrt{\frac{d-1}{d\underline{\underline{E}} : \underline{\underline{E}}}}\,\underline{\underline{E}} - \frac{\underline{\underline{\delta}}}{d} \quad \text{which has a complex square root} \tag{8}$$

$\underline{\underline{T}}$ just being calculated from $\underline{\underline{\Pi}}$.