

# Some quick notes/documentation of how to actually implement the sqrt in the $\underline{\underline{\Pi}}$ approx

So in general

$$\underline{\underline{E}} = \psi_1(\underline{N}\underline{N} - \frac{\delta}{3}) + \psi_2(\underline{M}\underline{M} - \frac{\delta}{3}) \quad (1)$$

but mostly here **just use the first term** (the uniaxial approximation). Then we want to approximate  $\underline{\underline{\Pi}} = \underline{N}\underline{N}$  using the following method

$$\underline{\underline{\Pi}} = \frac{\underline{\underline{E}}}{\psi_1} + \frac{\delta}{3} \quad (2)$$

where getting  $\psi_1$  is the problem, for that use

$$\frac{3}{2}\underline{\underline{E}} : \underline{\underline{E}} = \psi_1^2 \quad (3)$$

but we need to take a square root – this is the problem, this can give back  $\pm\psi_1$ , if it gives  $-\psi_1$  our approximation yields

$$\underline{\underline{\Pi}}' = -\underline{N}\underline{N} + \frac{2\delta}{3} \quad (4)$$

So we need a method to numerically tell  $\underline{\underline{\Pi}}$  and  $\underline{\underline{\Pi}}'$  apart, for that try to get a matrix square of each

$$\underline{\underline{\Pi}} \cdot \underline{\underline{\Pi}} = \underline{N}\underline{N} = \underline{\underline{\Pi}} \quad (5)$$

$$\underline{\underline{\Pi}}' \cdot \underline{\underline{\Pi}}' = \underline{N}\underline{N} - \frac{4\underline{N}\underline{N}}{3} + \frac{4\delta}{9} = -\frac{1}{3}\underline{N}\underline{N} + \frac{4}{9}\delta \quad (6)$$

so clearly  $\underline{\underline{\Pi}} \cdot \underline{\underline{\Pi}} - \underline{\underline{\Pi}} \sim 0$ , use that as a test and if it is failed go back and fix the extra - sign. To get a threshold consider

$$\underline{\underline{\Pi}}' \cdot \underline{\underline{\Pi}}' - \underline{\underline{\Pi}} = -\frac{1}{3}\underline{N}\underline{N} + \frac{4}{9}\delta + \underline{N}\underline{N} - \frac{2}{3}\delta = \frac{2}{3}\underline{N}\underline{N} - \frac{2}{9}\delta = \frac{1}{3}\frac{\underline{\underline{E}}}{\psi_1} \quad (7)$$

I haven't done the math but am quite confident that it is guaranteed that at least one of the components will be large, given the other should be precisely 0 we can just test for a relatively small threshold of say 0.1 or even less.