

$i$  always refers to different species  
 $\alpha$  and  $\beta$  to resources

$$\frac{\partial R_\alpha}{\partial t} = \underbrace{K_\alpha}_{\text{external supply}} - \underbrace{r_\alpha}_{\text{dilution rates}} R_\alpha - \sum_i \underbrace{\frac{N_i c_{i\alpha} R_\alpha}{\text{total consumption of } \alpha \text{ by } i}}_{\text{consumption rate of } \alpha \text{ by } i, \text{ per unit concentrations of both}} + \sum_{i, \beta} \frac{w_\beta}{w_\alpha} \underbrace{D_{i\alpha\beta}}_{\text{determines the byproducts created when } i \text{ consumes } \beta} \underbrace{l_{i\beta}}_{\text{total leakage of } \beta \text{ when consumed by } i} \underbrace{N_i c_{i\beta} R_\beta}_{\text{total leakage of } \beta \text{ when consumed by } i} + \underbrace{D_{R_\alpha}}_{\text{diffusion constants}} \nabla^2 R_\alpha$$

$$\frac{\partial N_i}{\partial t} = \underbrace{g_i}_{\text{unit conversion factors}} N_i \left( \sum_\alpha (1 - \underbrace{l_{i\alpha}}_{\text{leakage/byproducts}}) \underbrace{w_\alpha}_{\text{unit conversion factors}} \underbrace{c_{i\alpha} R_\alpha}_{\text{cell upkeep energy consumption}} - \underbrace{m_i}_{\text{diffusion constants}} \right) + \underbrace{D_{N_i}}_{\text{diffusion constants}} \nabla^2 N_i$$

## Energetic model

- $N_i$  and  $R_\alpha$  are treated as forms of energy
- $g_i$  and  $w_\alpha$  facilitate conversion to biomass
- $K_\alpha$  are the energy sources and  $r_\alpha$  and  $m_i$  are the energy sinks
- everything else is an energy conversion

## Consumption with leakage and byproducts

