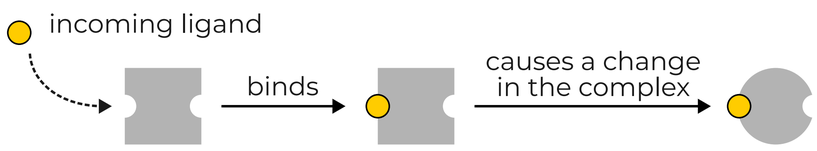
# Functional dynamics in out-of-equilibrium allosteric assemblies

Jan Kocka, Jaime Agudo-Canalejo, Kabir Husain
*Department of Physics and Astronomy, UCL*

**Allostery** is the communication between distant sites of a macromolecule, such as binding sites on a protein



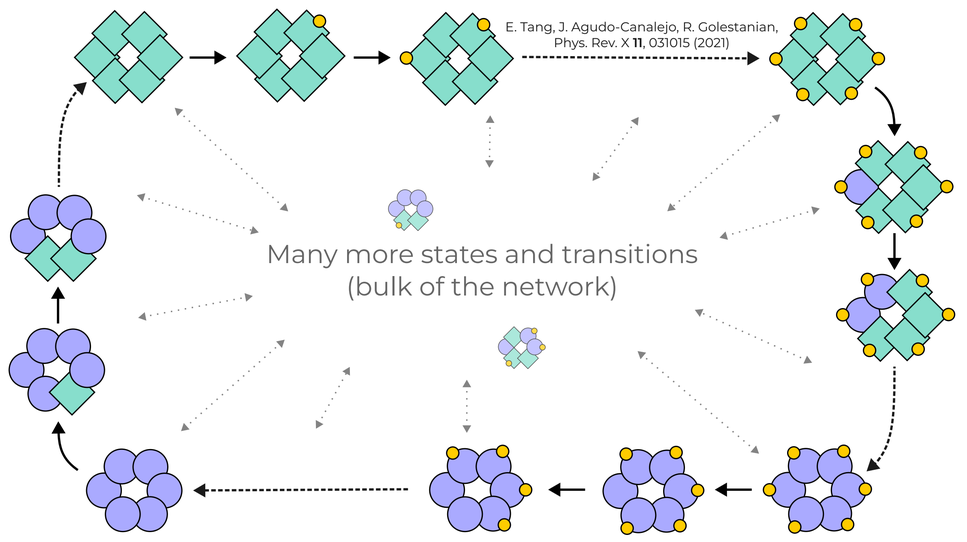

Equilibrium allostery

- MWC, KNF and the comprehensive Ensemble Allostery Model
- Cooperative binding, allosteric regulation/signalling

But there are **out-of-equilibrium** allosteric complexes such as AAA ATPases (*e.g.* cyanobacterial circadian clock KaiC or the DNA clamp loader)

## What new behaviour is possible out of equilibrium?

- Dynamic steady states
- Oscillations (such as in KaiC or other circadian clocks)
- Sensitivity to initial conditions (memory/spontaneous symmetry breaking)
- Dimensionality reduction (constraining the dynamics to part of the state space)
- Topologically protected states
- Modified cooperative binding




## We build models to identify classes of behaviour in biology and guide synthetic designs

- Models with **identical subunits** (polymer like)
- Statistical physics and graph theory methods
- **Local (nearest-neighbour) interactions**
- **Thermodynamically consistent** transitions between system states (satisfying local detailed balance)

$$\text{system} + \mathrm{R}_1 + \cdots \underset{r_\mathrm{b}}{\overset{r_\mathrm{f}}{\rightleftharpoons}} \text{changed system} + \mathrm{P}_1 + \cdots$$

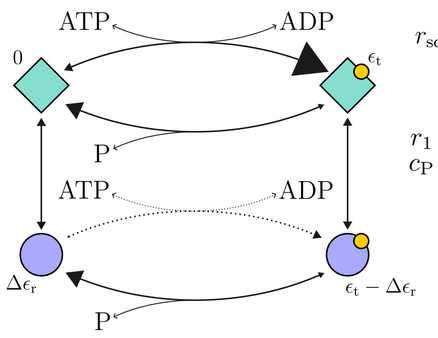
$$\frac{r_\mathrm{f}}{r_\mathrm{b}} = \exp((\Delta\epsilon + \mu_{\mathrm{R}_1} + \cdots - \mu_{\mathrm{P}_1} - \cdots)/\mathrm{kT})$$

change in system energy — chemical potentials of all reactants and products

# Towards topological edge currents in non-equilibrium assemblies

**2 conformation states**: phosphorylate, dephosphorylate

Including two different reactions brings the system out of equilibrium and allows individual subunits to perform **futile cycles**



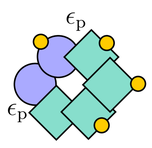

$$r_{\text{square phosphorylation}} = r_1 \frac{c_\mathrm{P} e^{\frac{\Delta\epsilon}{\mathrm{kT}}}}{c_\mathrm{P} e^{\frac{\Delta\epsilon}{\mathrm{kT}}} + 1} + r_2 \frac{c_\mathrm{R} e^{\frac{\Delta\epsilon}{\mathrm{kT}}}}{c_\mathrm{R} e^{\frac{\Delta\epsilon}{\mathrm{kT}}} + 1}$$

with $c_\mathrm{R} = \frac{c_\mathrm{ATP}}{c_\mathrm{ADP}}$

$r_1$ and $r_2$ are reaction speeds
$c_\mathrm{P}$ and $c_\mathrm{R}$ are effective concentrations

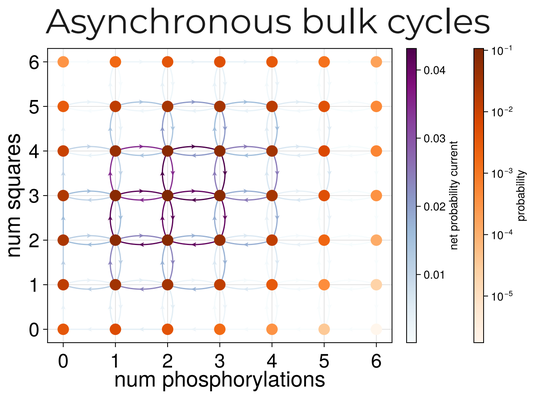
equilibrium iff $c_\mathrm{P} = c_\mathrm{R}$

Subunits then interact in an equilibrium manner through an energy penalty of $\epsilon_\mathrm{p}$ for each square-circle boundary

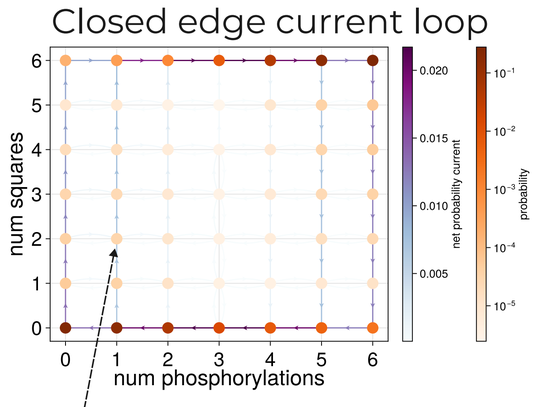



## What types of dynamical steady states can we get?

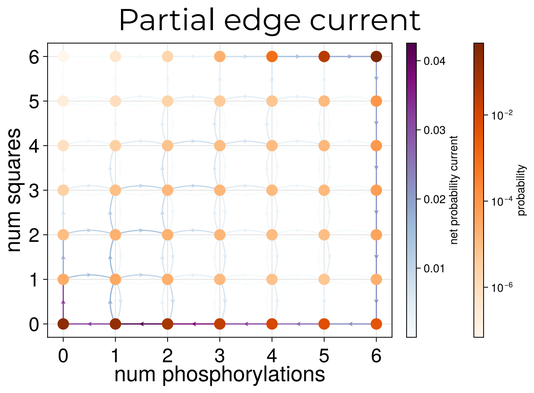
Asynchronous bulk cycles




Closed edge current loop



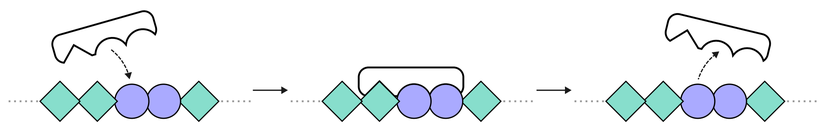

Partial edge current




each node is a group of system states with the same number of bound ligands and the same number of squares/circles

# Realizing molecular automata with site-specific enzymes

- **Bring out-of-equilibrium drive directly into the nearest-neighbour interactions**
- Equivalent to tochastic dynamics on **binary strings**
- Potential implementation: context dependent enzymes



$$\text{enzyme } E + \overline{\cdots \mathrm{i0j} \cdots} \underset{r_{E,1\to0,ij}}{\overset{r_{E,0\to1,ij}}{\rightleftharpoons}} \text{changed } E + \overline{\cdots \mathrm{i1j} \cdots}$$

$$r_{E,0\to1,ij} = K_{E,ij} \frac{\exp(\beta\mu_E)}{\exp(\beta\mu_E) + 1}$$

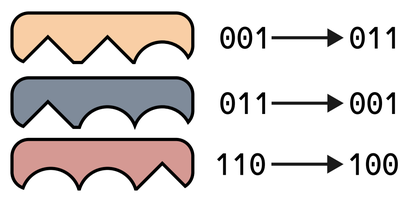
$$r_{E,1\to0,ij} = K_{E,ij} \frac{1}{\exp(\beta\mu_E) + 1}$$

- For each enzyme $\mu_E$ determines its bias toward 0s or 1s and $K_E$ which combination of neighbours it may act on

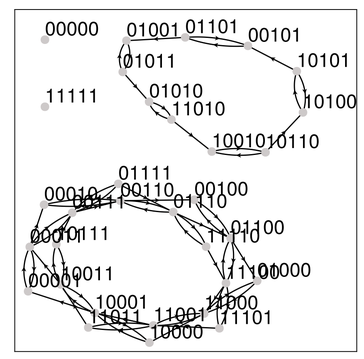
**Maximally driven case:** $\mu_\mathrm{E} \to \pm\infty$

- Fully specified by the presence of **up to 8 different enzymes**
- Can be **mapped to a cellular automata rule**

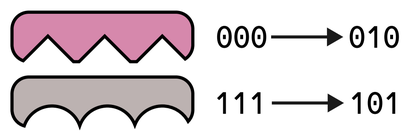
**Rule 134**




1 dimensional, directed loops
at 5 subunits




**Rule 77**




disjoint planar structures
at 5 subunits