Notes on Master Equation Methods

1 Transition and Other Matrices

Standard master equation has the form of

$$\partial_t p_i = \sum_j (R_{ij} p_j - R_{ji} p_i) \tag{1}$$

where R_{ij} corresponds to the $j \to i$ transition. For a steady state we must have $\partial_t p_i = 0$ and if each term in the sum of eq. (1) is independently 0 we have detailed balance. In general however to solve for 0 it is convenient to put the equation in a eigenvalue like form such as

$$\partial_t p_i = \sum_j W_{ij} p_j \tag{2}$$

for some matrix $\underline{\underline{W}}$. This matrix can be expressed in terms of $\underline{\underline{R}}$ as

$$W_{ij} = R_{ij} - \delta_{ij} \sum_k R_{ki} \tag{3}$$

as this leads to

$$\begin{aligned}
\partial_t p_i = \sum_j W_{ij} p_j = \sum_j \left(R_{ij} - \delta_{ij} \sum_k R_{ki} \right) p_j = & \qquad (4)\\
= \sum_j R_{ij} p_j - \sum_{j,k} \delta_{ij} R_{ki} p_j = & \qquad (5)\\
= \sum_j R_{ij} p_j - \sum_k R_{ki} p_i = & \qquad (6)\\
= \sum_j (R_{ij} p_j - R_{ji} p_i) \quad \text{as required} & \qquad (7)
\end{aligned}$$