Ones

1 The Ones Model

This is another graph model where our microstates consist of essentially binary strings of zeros and ones. We then add transitions to the according to set rules, the main type of rule we consider is adding transition cycles which affect a subset of the digits. For example we may have a 2 digit transition cycle going from $00 \to 10 \to 11 \to 01 \to 00$. We then add transitions to the graph by looking at all 2 digit susbequences of all the microstates and whenever they match the start of the cycle we add all the relevant transitions stemming from that cycle.

With this being the base of the method, additional complexities can be added but the simplest method is to consider a model with

List 1:

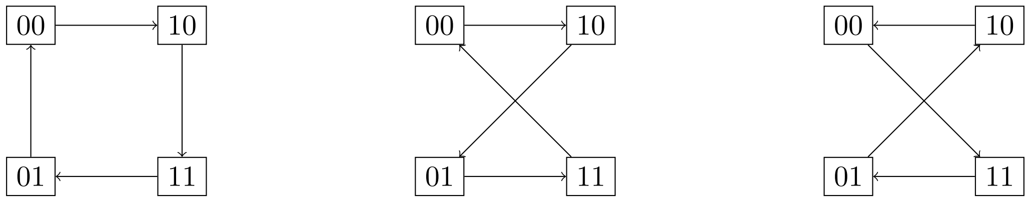
1. only one such futile cycle
2. all the transition rates around the futile cycle to be equal
3. the futile cycle can take place at any subsequence of the string (except potentially at boundaries).

Section 2 discusses all such simple models with futile cycles affecting 2 digits only. It is worth noting that item 2 from list 1 guarantees that all steady states will be (piecewise) uniform as it guarantees that each microstate has the equal "outgoing and incoming rates".

Another thing to consider with this method, how do we deal with the ends of the sequence. We consider two geometries here, that of a chain where the ends are actually ends with no neighbors beyond them. And a loop geometry corresponding essentially to periodic boundary conditions where the two ends are considered to be connected to each other.

2 Single Two Digit Cycles with Equal Weights

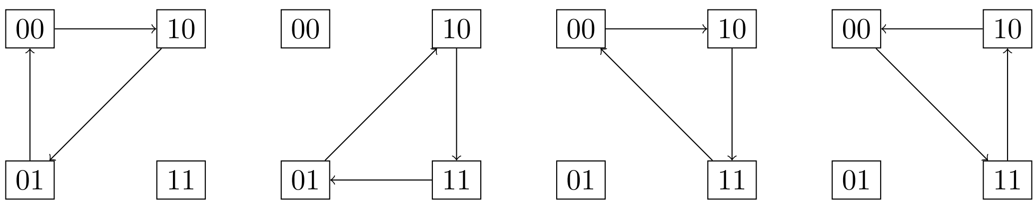
What are all the different (as in they lead to different behaviours) 2 digit cycles we can implement in the model? In the diagrams as shown in fig. 1 we first note that the diagrams lead to the same behaviour if they are the same under reflection along the top-left to bottom-right axis which corresponds to reflecting the sequence (physically the same system if working with a loop geometry). We first focus on such cycles that visit all possible 2 digit sequences. We either get diagrams with or without a crossing, and there is only one diagram without a crossing which is fig. 1a. For diagrams with a crossing that visit all four 2 digit sequences we always the motif as in figs. 1b and 1c which are the only such diagrams up to the mentioned reflection. Also note that the two cycles with crossings are reverses of each other!



(a) simple (b) crossing with $00 \to 10$ (c) crossing with $00 \to 11$

Figure 1: All the different 2 digit simple cycles that visit all four such sequences, these are unique up to right-left reflection.

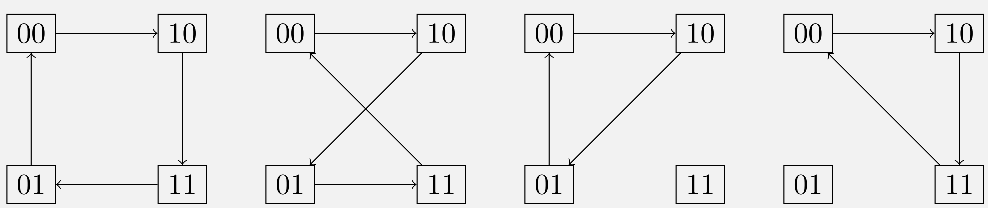
Following the same logic, we only get four cycles which visit 3 of the sequences, shown in fig. 2. With the last two also being reveres of each other. We don't examine cycles which visit only 2 of the sequences as it's difficult to imagine anything interesting taking place.



(a) (b) (c) (d)

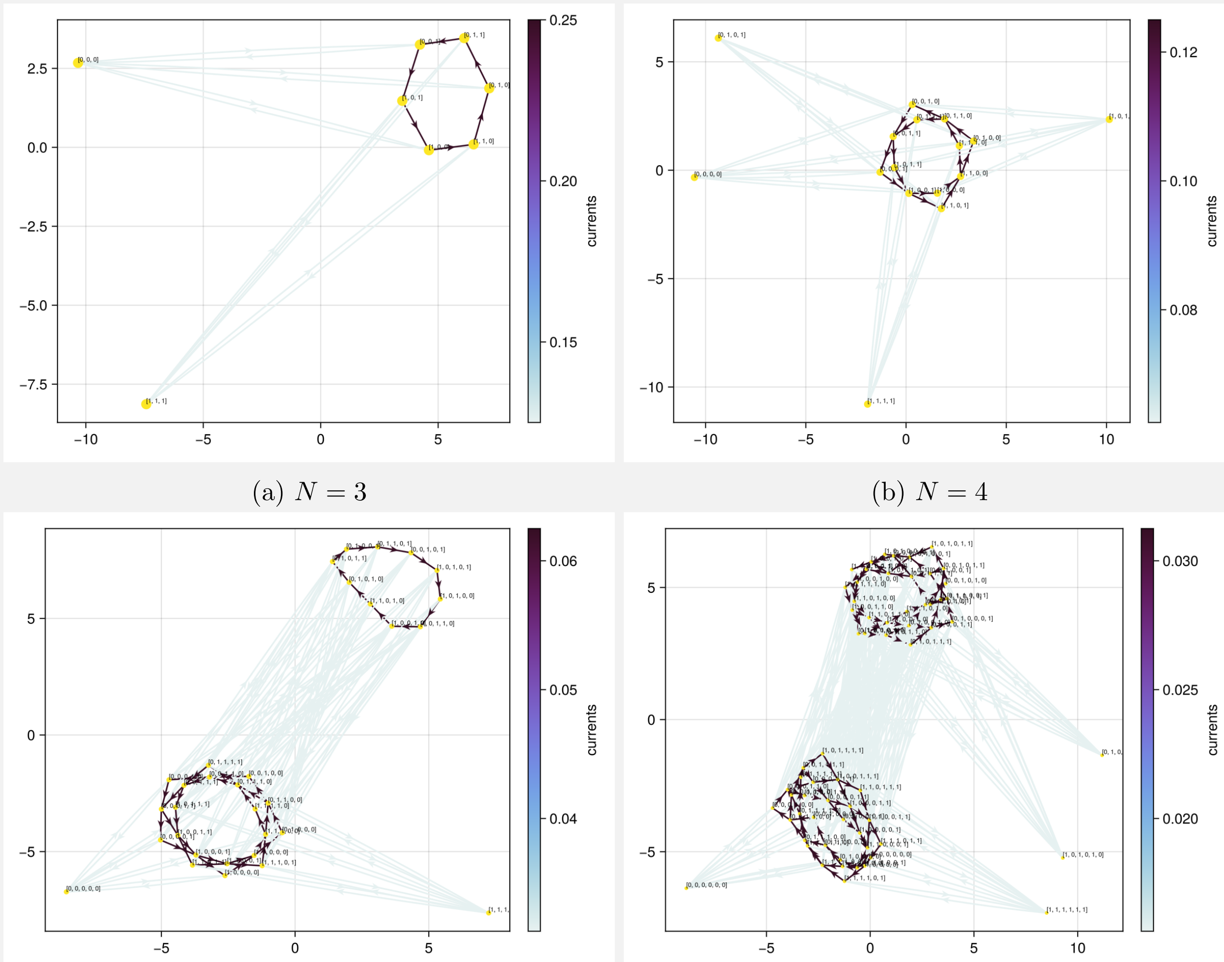
Figure 2: All the different 2 digit simple cycles that do not visit all such sequences, these are unique up to right-left reflection.

As reverses of cycles will give exactly the same behaviour just revered in time, we can focus on only one of each such cycles. In addition, it is reasonable to suspect that replacing all 0s by 1s and vice versa should also be some symmetry of the system and so we only look at one cycle for each of such classes. This gives only the cycles shown in fig. 3 to look at.



(a) (b) (c) (d)

Figure 3: The only 2 digit simple cycles that are worth investigating. Of which only the first only affects 1 digit at a time – thus this is the main object of interest.



(a) $N = 3$ (b) $N = 4$

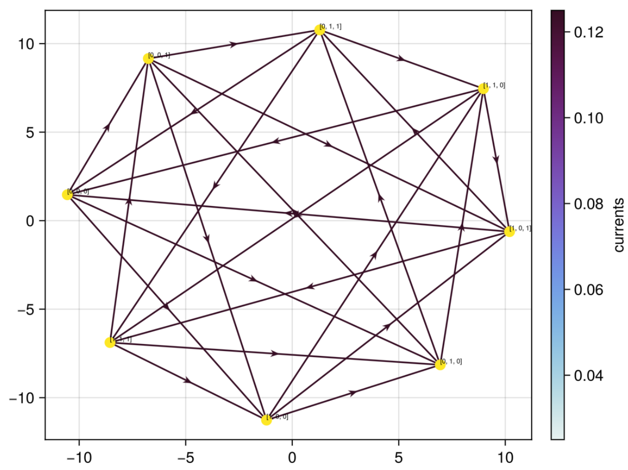
(c) $N = 5$ (d) $N = 6$

Figure 4: Resulting transition graphs when using only the cycle from fig. 3a for different N in the loop geometry. The node color shows the steady state probability of that node, which notably is always uniform! Transition colors designate the probability current.

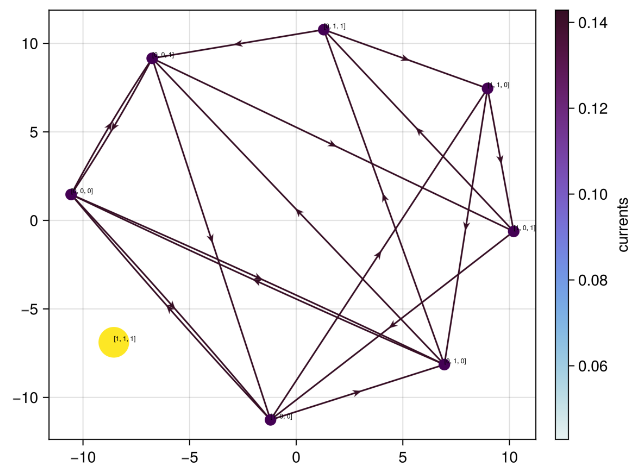
2.1 Loop geometry graphs

The most interesting of these is the first as that is the only one which does not change two digits at once, a perhaps reasonable physical constraint. Figure 4 shows the resulting graphs for a couple of different N, where we get very interesting loop patterns, though the steady states are still always uniform!

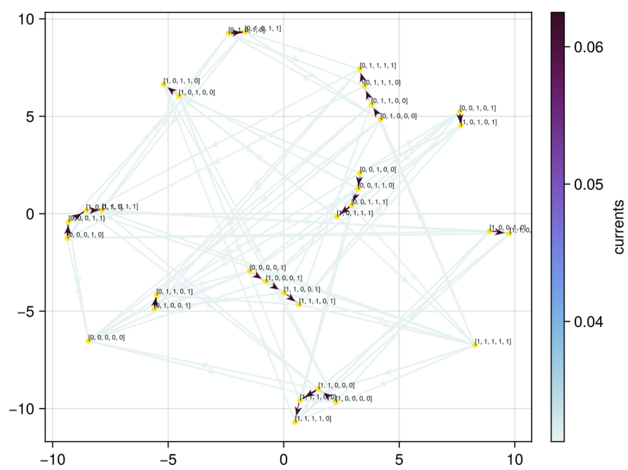
The other base cycles all give rise to not quite as interesting patterns, some of which are shown in . And it is also worth noting that all of these also gave rise only to uniform steady states.



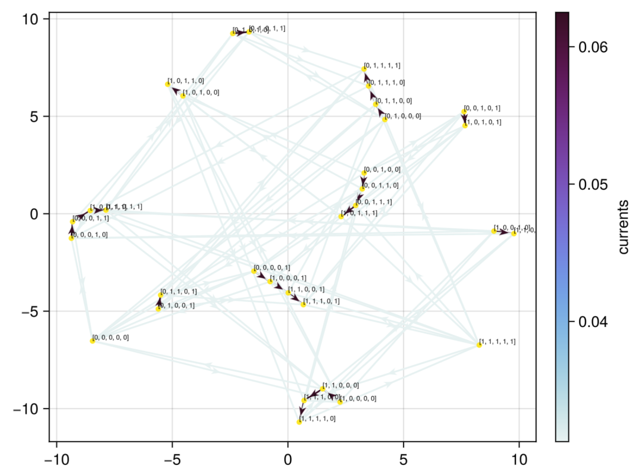
(a) Second cycle, $N = 3$



(b) Third cycle, $N = 3$



(c) Fourth cycle, $N = 3$



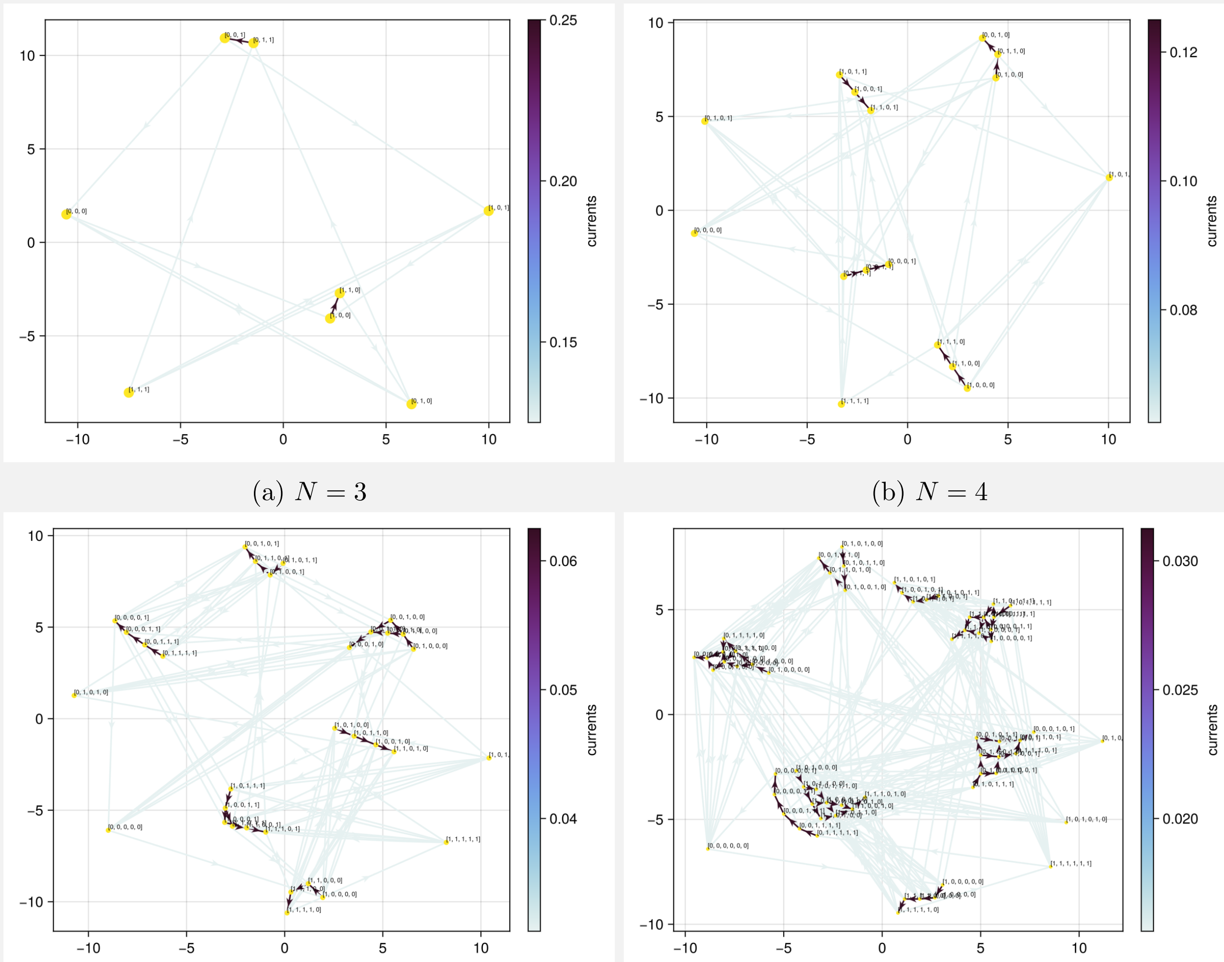
(d) Fourth cycle, $N = 5$

Figure 5: Some example transition graphs for the other 2 digit base cycles from fig. 3.

2.2 Chain geometry graphs

First a couple of thoughts, as we're thinking of 2 digit cycles, switching from a loop to a chain geometry will always break precisely those cycles taking place on the last and first digits wrapped around. There is one such cycle for any sequence of the intermediate digits so switching to a chain geometry will always break precisely 2^{N-2} cycles and thus remove $4 * 2^{N-2} = 2^N$ "edges" from the graph some of which may have contributed to the doubled up edges from figs. 4 and 5.

Some resulting graphs for the most interesting 2 digit cycle are shown in fig. 6. The loops from before entirely disappear and we only get lines which at higher N can have doubled up segments much like the cycles had before. For the resulting 2 digit cycles the graphs naturally changes, but not in any simple or interesting way, they got a bit less regular but the nature of each of the cycles stayed the same.



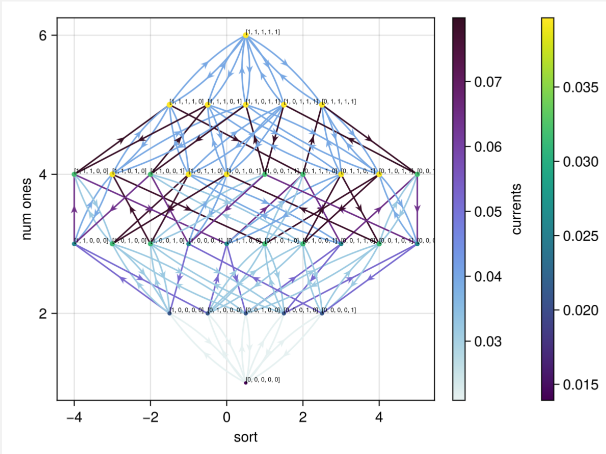
(a) $N = 3$ (b) $N = 4$

(c) $N = 5$ (d) $N = 6$

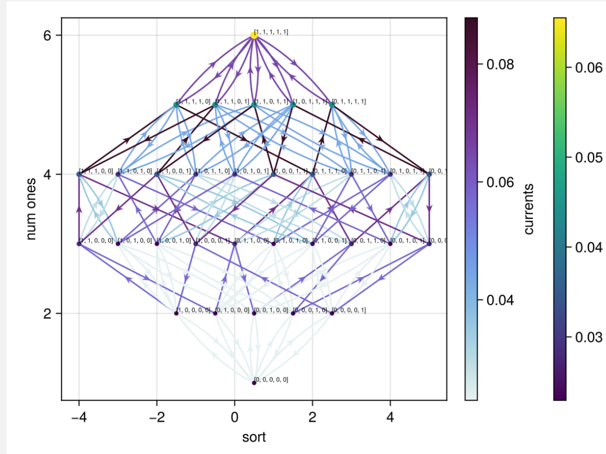
Figure 6: Resulting transition graphs when using only the cycle from fig. 3a for different N in the chain geometry. The node color shows the steady state probability of that node, which notably is always uniform! Transition colors designate the probability current.

2.3 Different transitions rates within the cycle

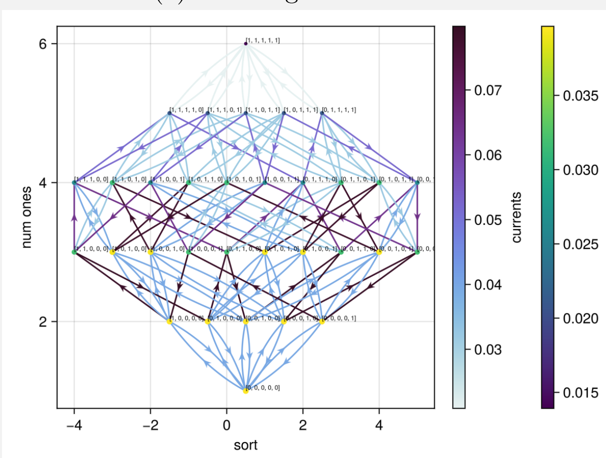
For a quick look at relaxing item 2 from list 1, we look at the base cycle fig. 3a again. We now instead of making all the rates in each futile cycle to be equal and 1.0, we boost one of them to be 1.5 instead. This gives an idea of what boosting any of the transitions would do to the graph, the results are shown in fig. 7 for $N = 5$ with the same patterns appearing in other cases as well.



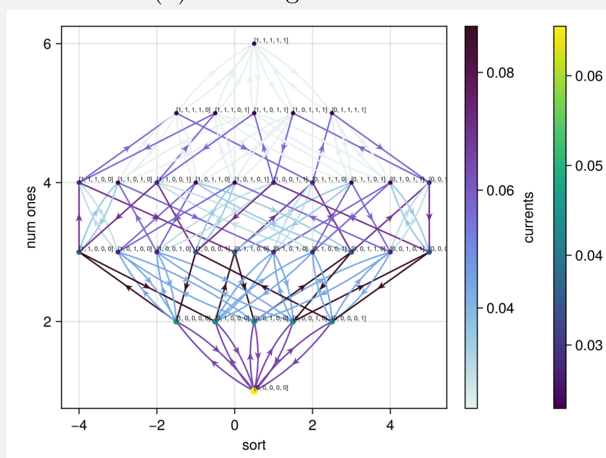
(a) Boosting 1st transition



(b) Boosting 2nd transition



(c) Boosting 3th transition



(d) Boosting 4th transition

Figure 7: Transition graphs using the cycle from fig. 3a with $N = 5$ and all the transition rates around the cycle being 1.0 except one which is boosted to be 1.5 instead.