This notebook goes through all 88 nonequivalent rules grouped by their "equilibrium parents" and summarizes what their attracting components (ACs) look like, including images

Note that looking only at the acs we naturally miss a lot of detail, including that not all states within an ac will be equally likely to be occupied in the steady state etc.

Equilibrium rules

204

108 (201)

156 (198)

105

150

60 (102, 153, 195)

54 (147)

57 (99)

51

Skipping rule 204 as its boring

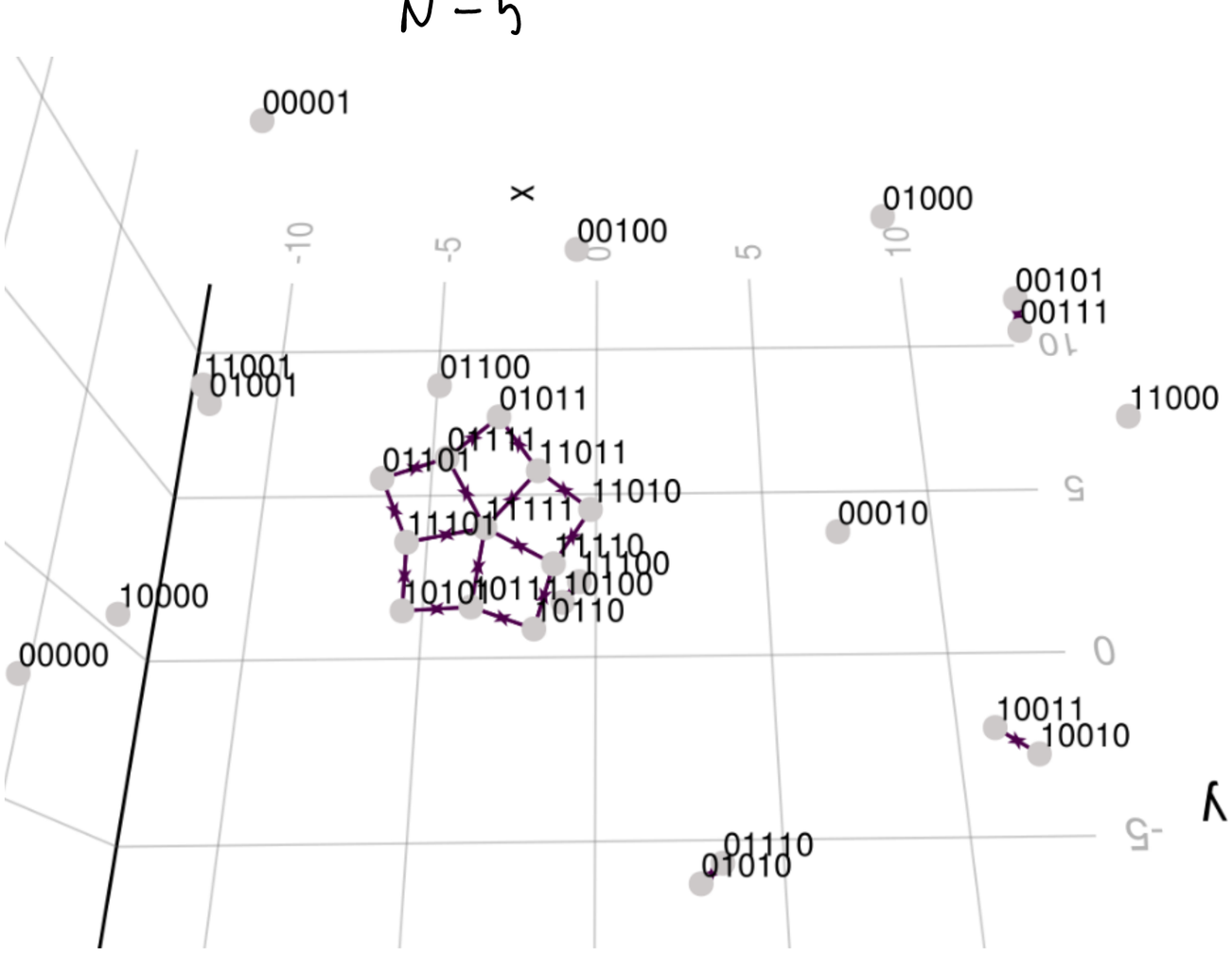
Eq rule 108

0 0 0
0 1 0 1

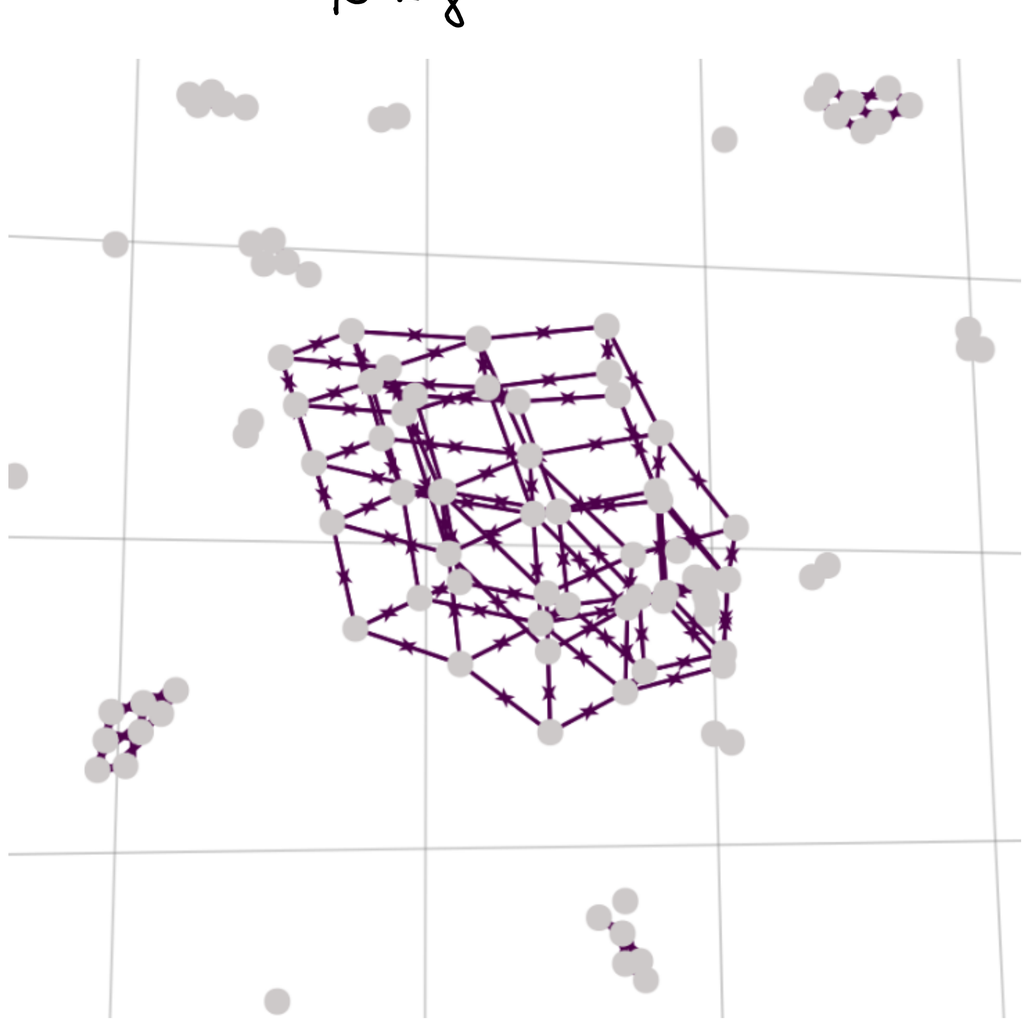
Creates complex structures around states with domains of 1s, in particular the all 1s state

otherwise all isolated states

$N = 5$



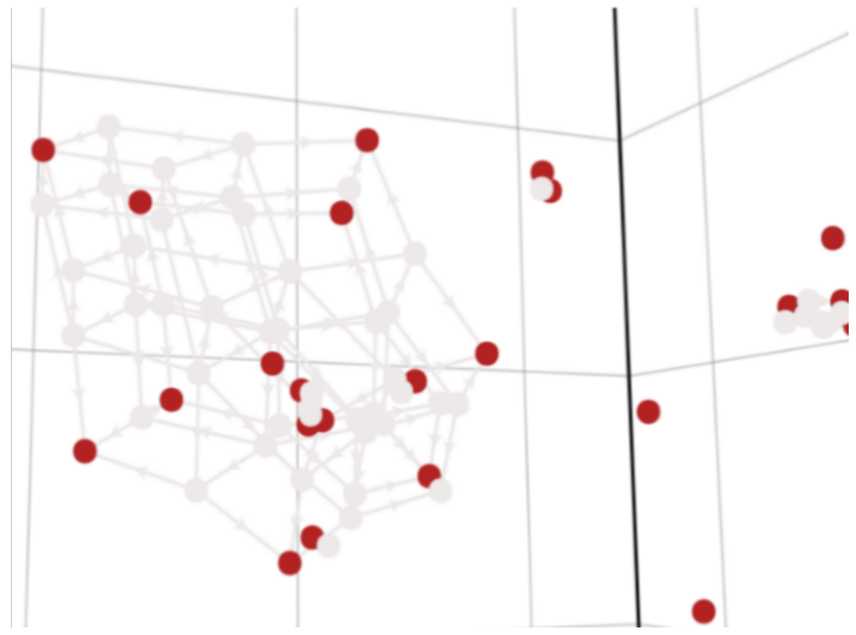
$N = 8$



Non eq subrules

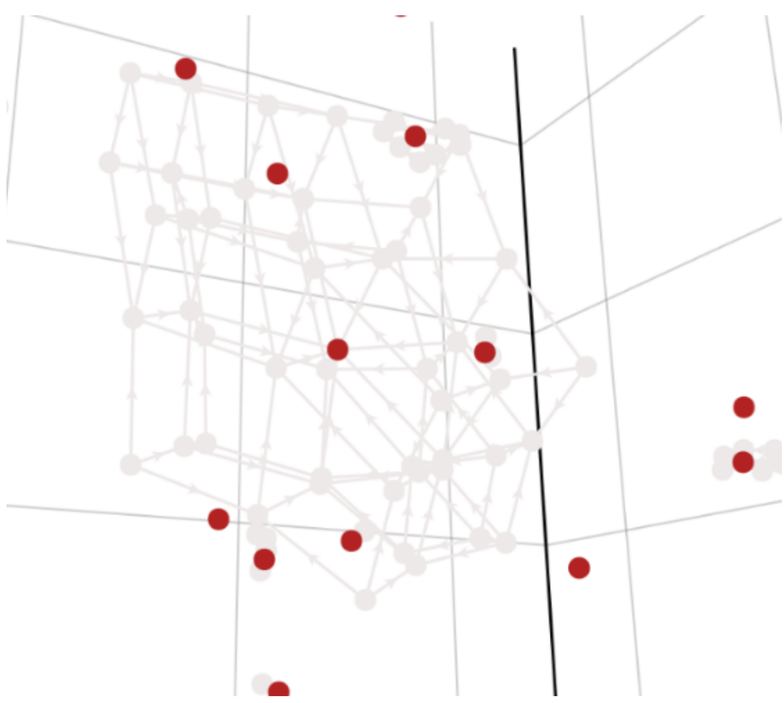
76 - domains of 1s can be irreversibly broken, only single acs, there are multiple acs around the edges of the structures

0 0 0 0
0 0 0 1



236 - domains can be completed but not broken - every structure has 1 single ac

0 0 0 0
0 1 0 0



Eg rule 156

00 00
10 10

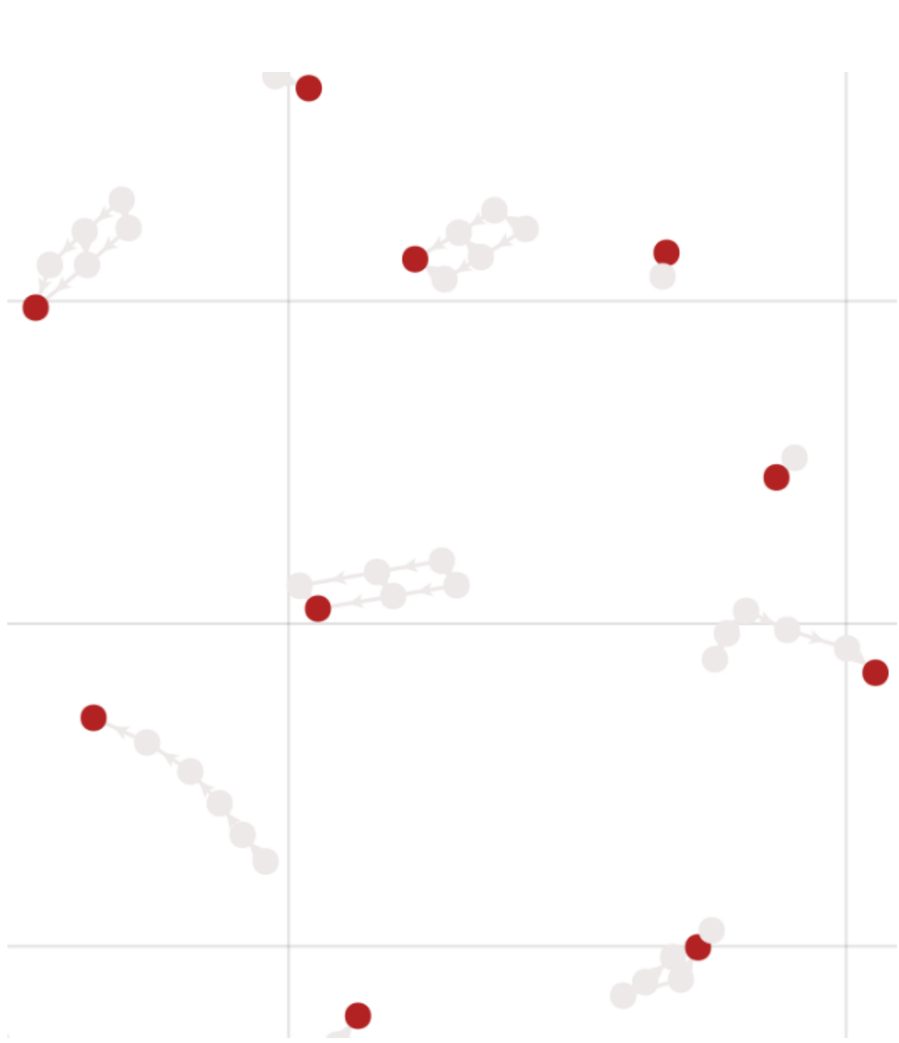
Boundaries can grow/shrink on one side but not propagate!

we see many single state lines of states and at higher N
also their "cartesian products"

N=7

Only 1 subrule

140 - Biased towards one side, naturally
00 01
00 10 results in 1 single ac at the end
or side of each group



Eg rule 105

10 10
01 01

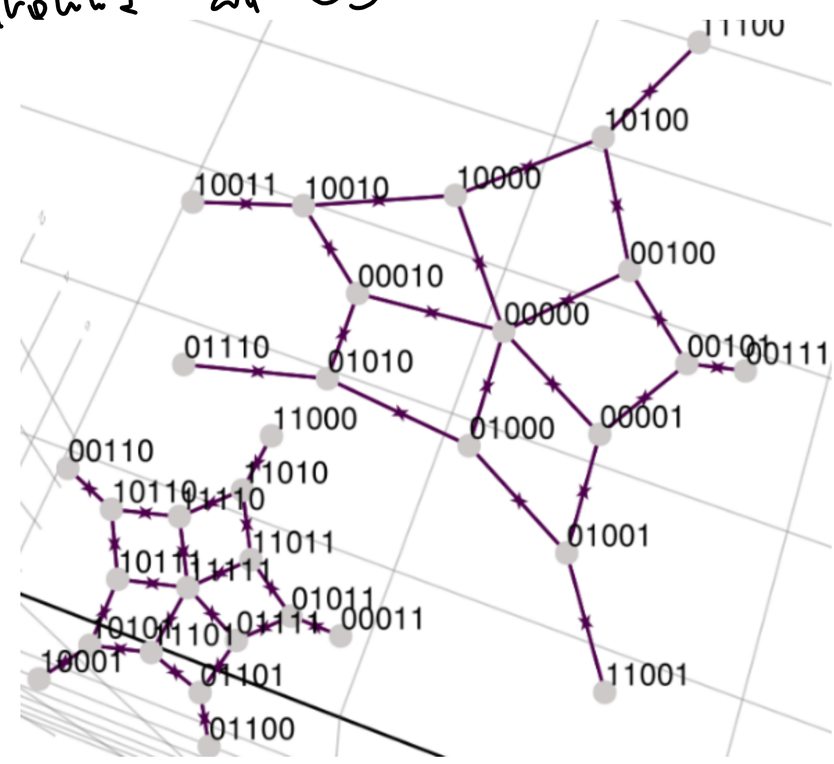
All domains can be seeded and completed and while they cannot move directly, we get a 2 step move by

... 00011 ... → ..01011... → ..01111..

Hence we get much more connections

- ~~011 N~~ we get 2 components one around all 0s and one around all 1s

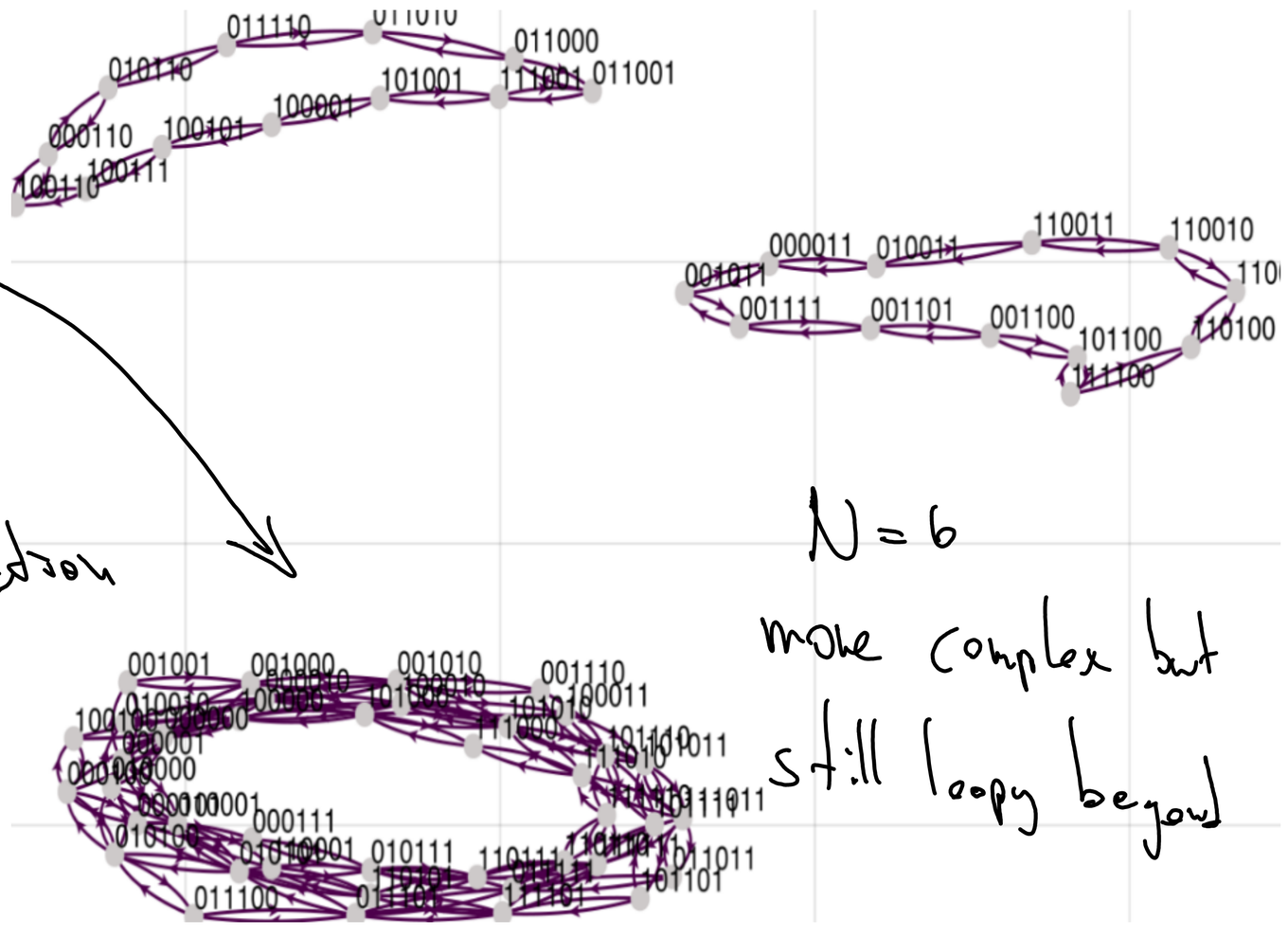
(they are vaguely plane-like)



- ~~Even N~~ We get loops, all states are in loops besides $\overline{0011}$ which only happen in N divisible by 4

There is always one loop containing both all 0s and all 1s

The rest seem to mostly rely on the 2 step propagation mechanism described above



N = 6

More complex but still loopy beyond

2 emergent subrules

77 ⟷ 232 (T)

10 00
0 0 0 1

0 0 1 0
0 1 0 0

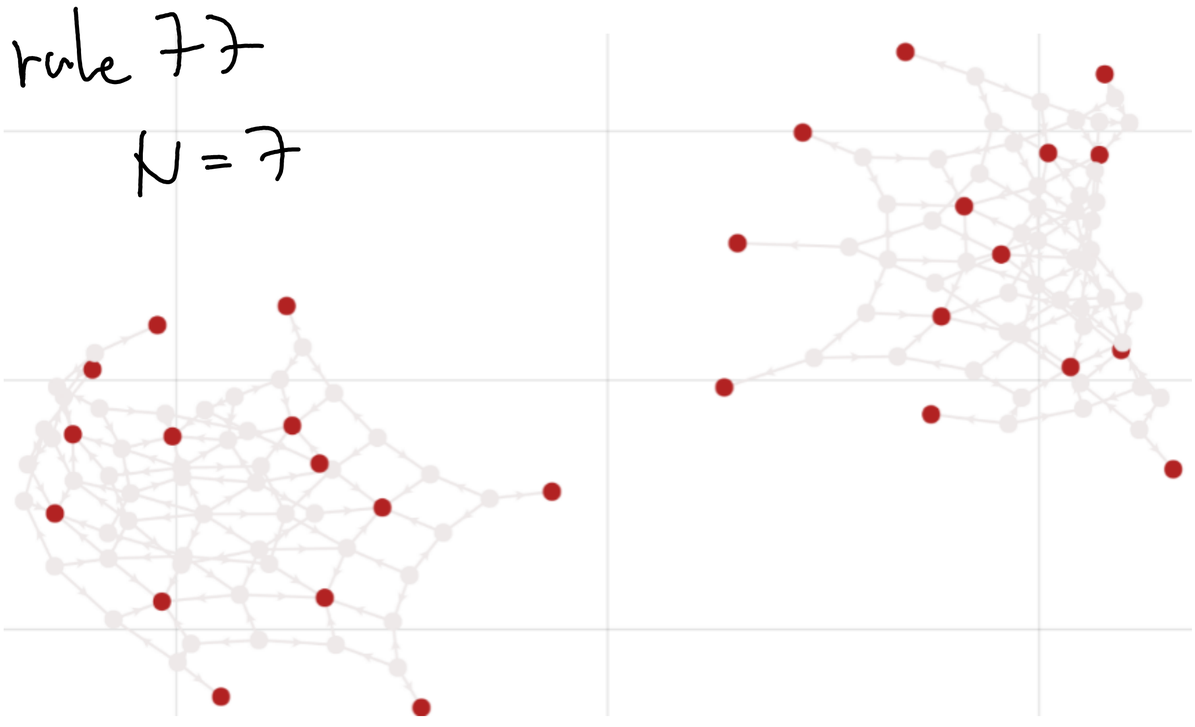
72 — fully biased

0 0 1 0
0 0 0 1

each have a bunch of single acs

rule 77

N = 7



3 emergent subrules

73 ⟷ 104 (XOR)

1 0 1 0
0 0 0 1

0 0 1 0
0 1 0 1

each break the propagating mechanism

104 — cannot seed 1s and hence gets stuck at either all 0s or packets of --0110--

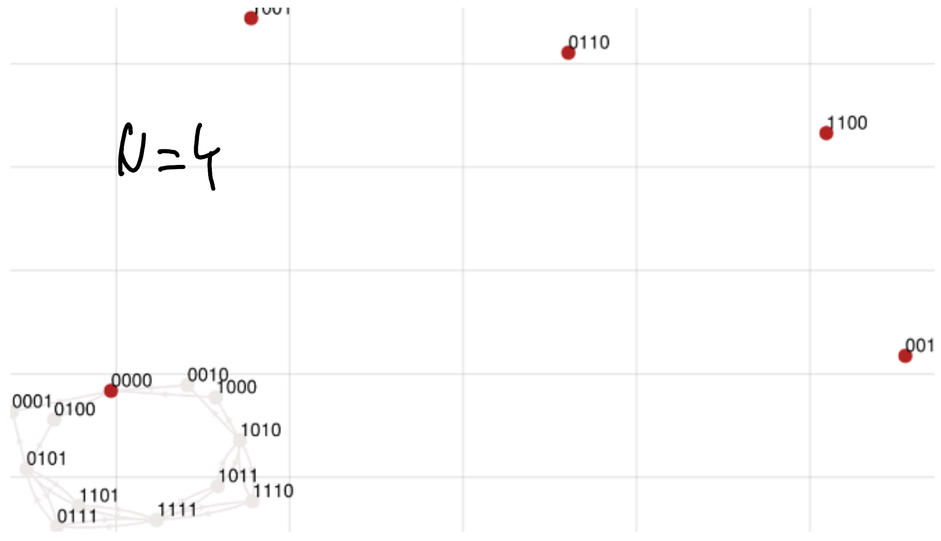
0 0 1 0
0 1 0 1

73 — cannot complete 1s but has non-single diffusive acs

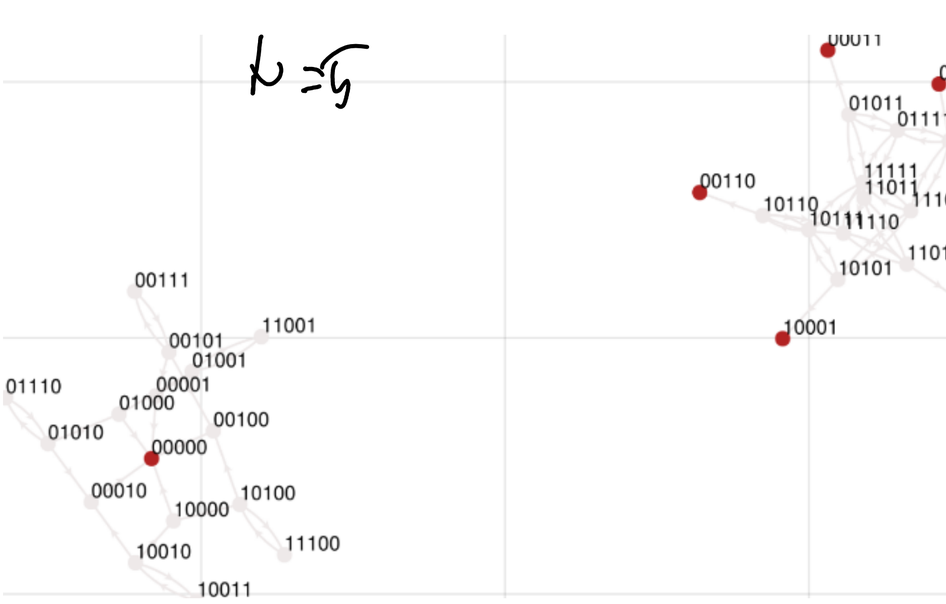
1 0 1 0
0 0 0 1

*

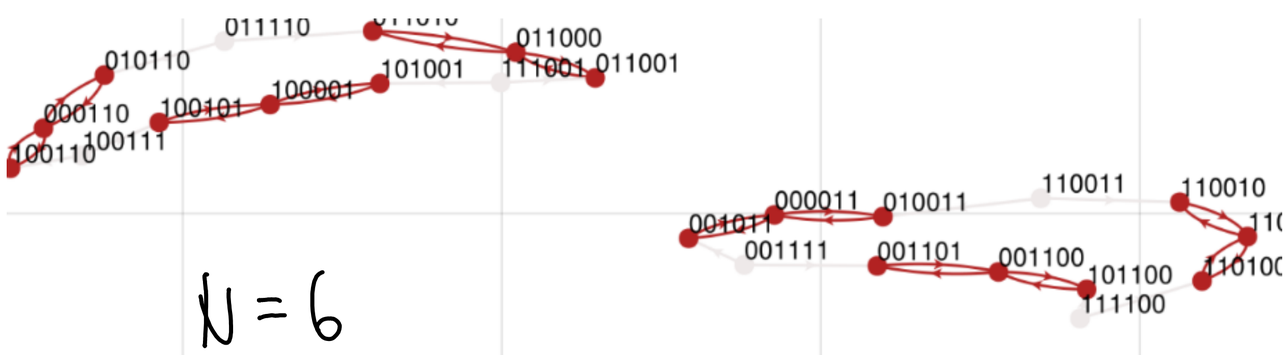
N = 4



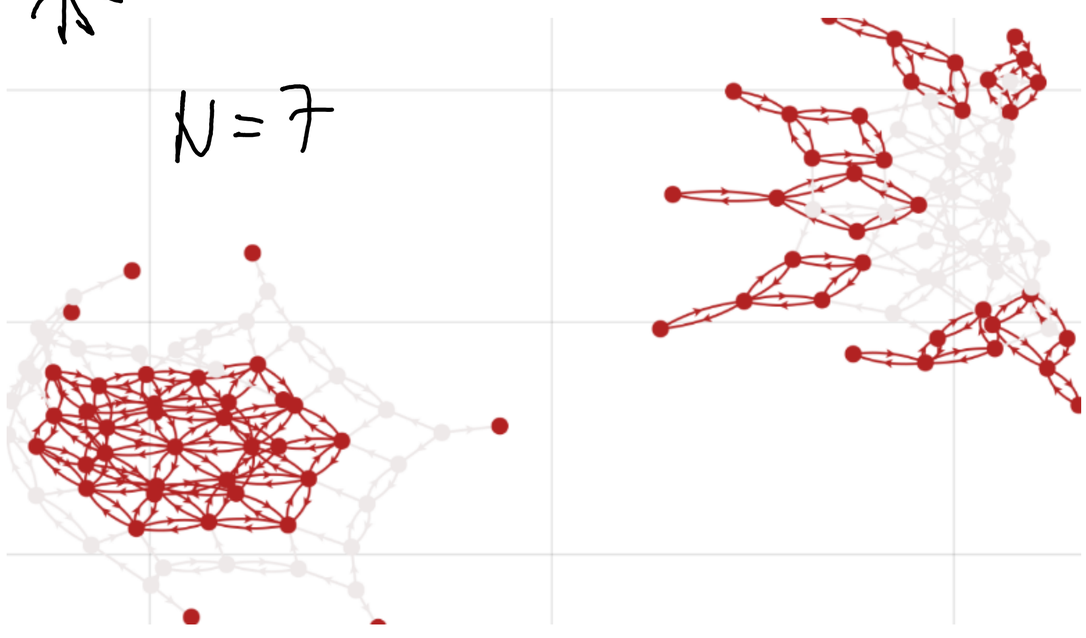
N = 5



N = 6



N = 7



Eq rule 150

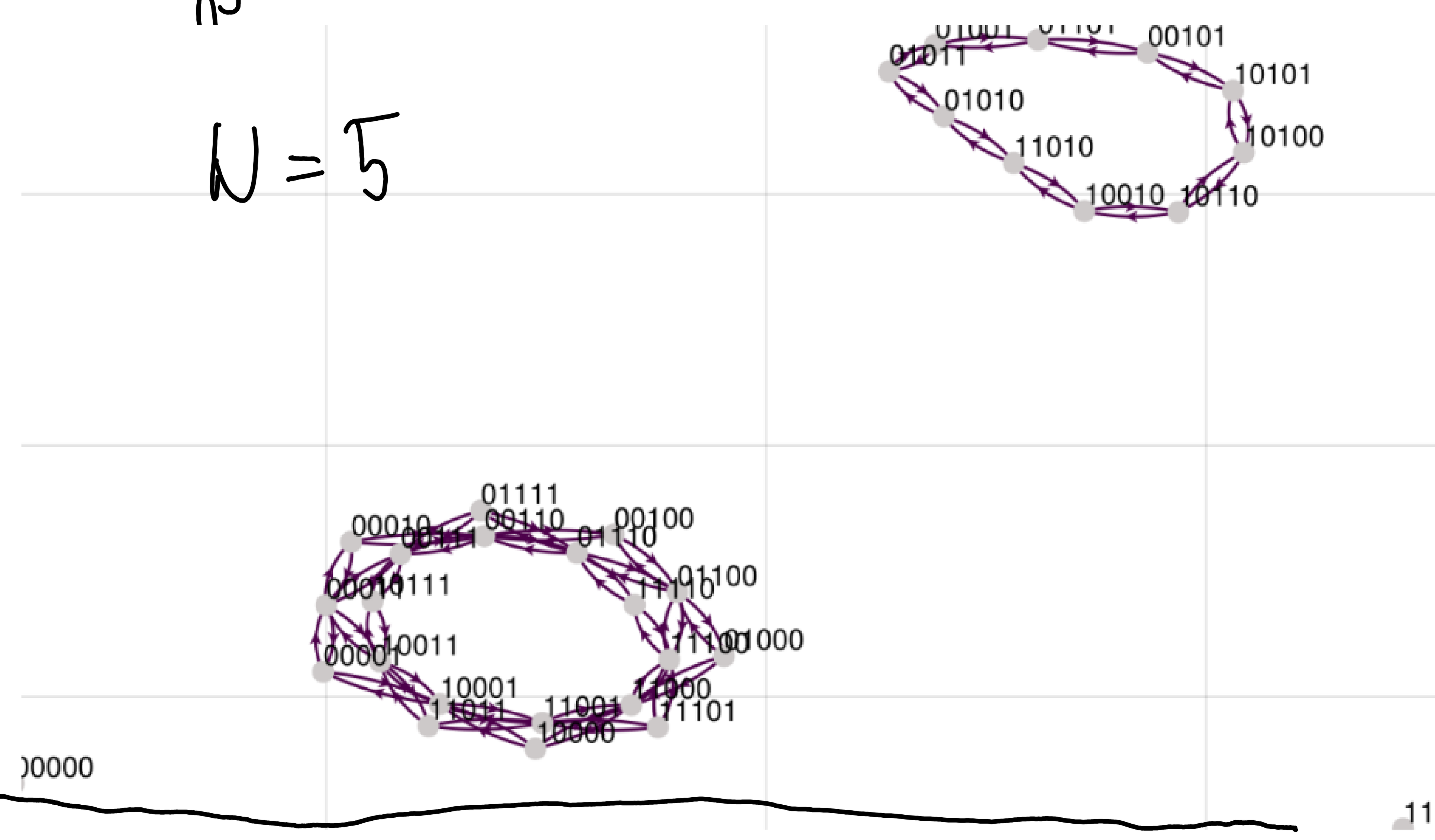
01 01
10 10

all boundaries can move both ways but
no seeding/completion

"canonical" loops where a wave moves around ! states $\overline{01}$
gets isolated all 0s and all 1s and also alternating if they exist

most loops branch but if N is odd there's always
1 single state loop

N = 5



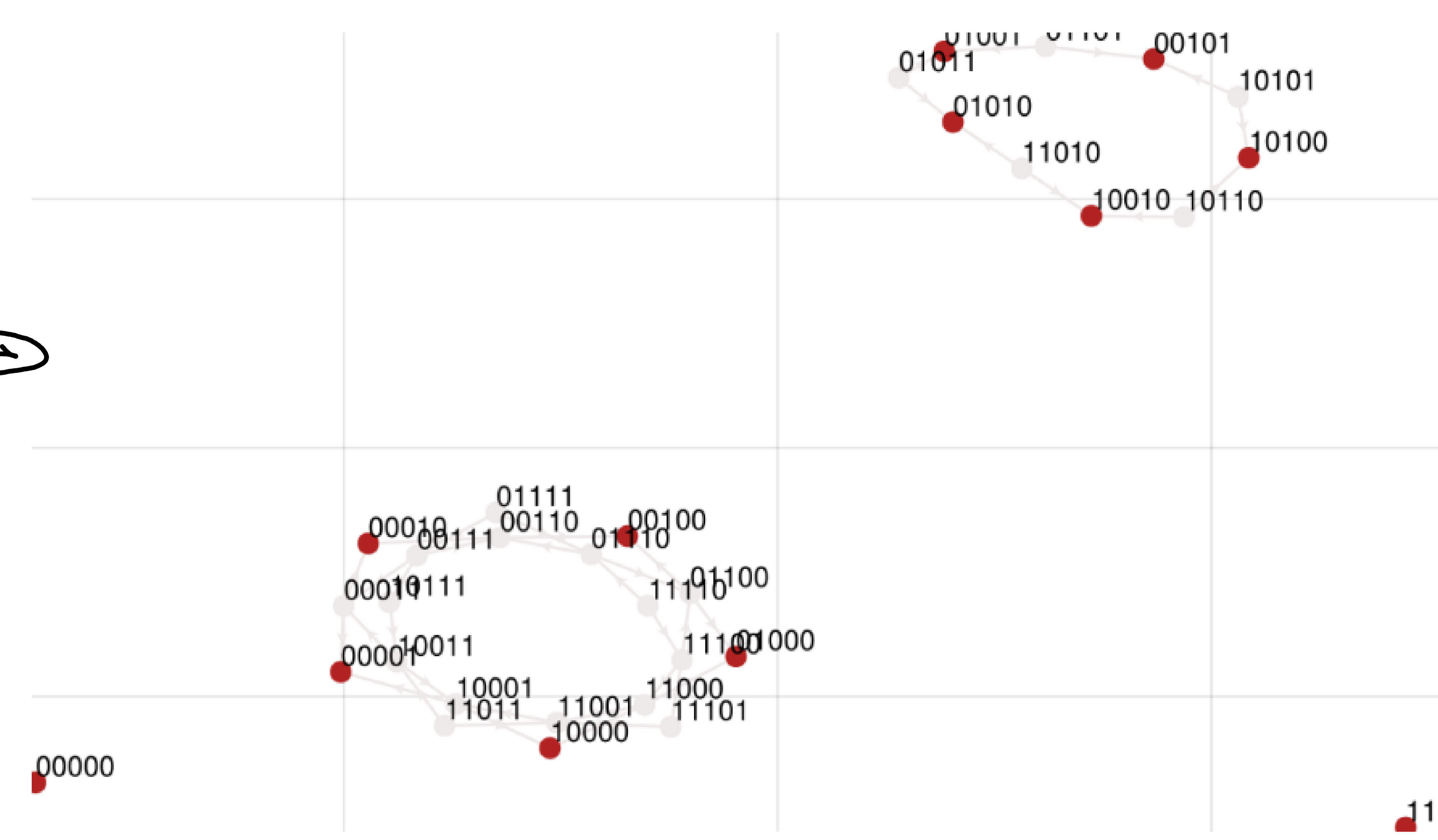
2enzyme subrules

142 – directed moving wave ! all arrows directed

0100
0010

137 – breaks apart the arcs into single states ! →

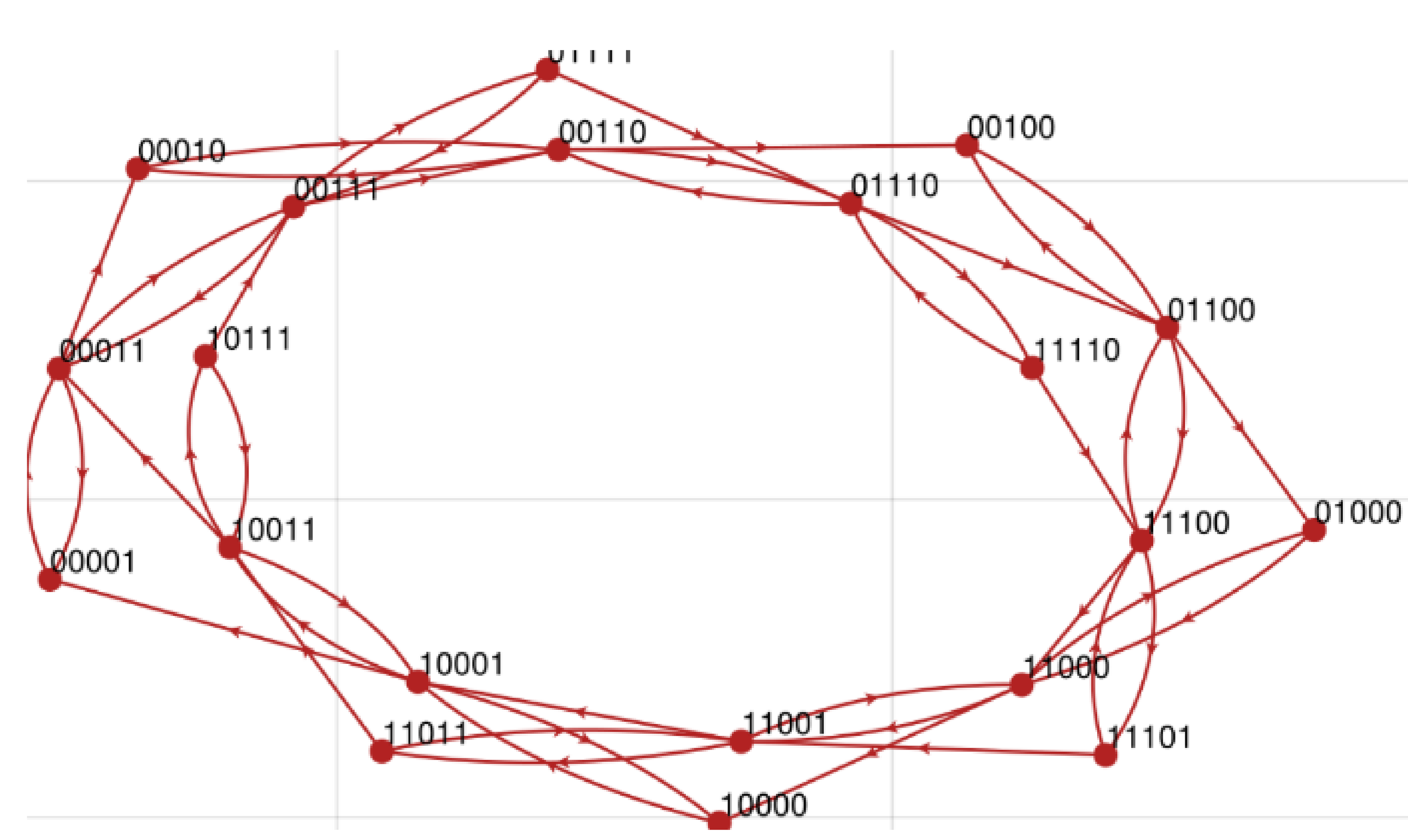
0001
0010



3enzyme subrules

134 – partially directed loops, every other arrow is directed, works even for branched loops →

0101
0010

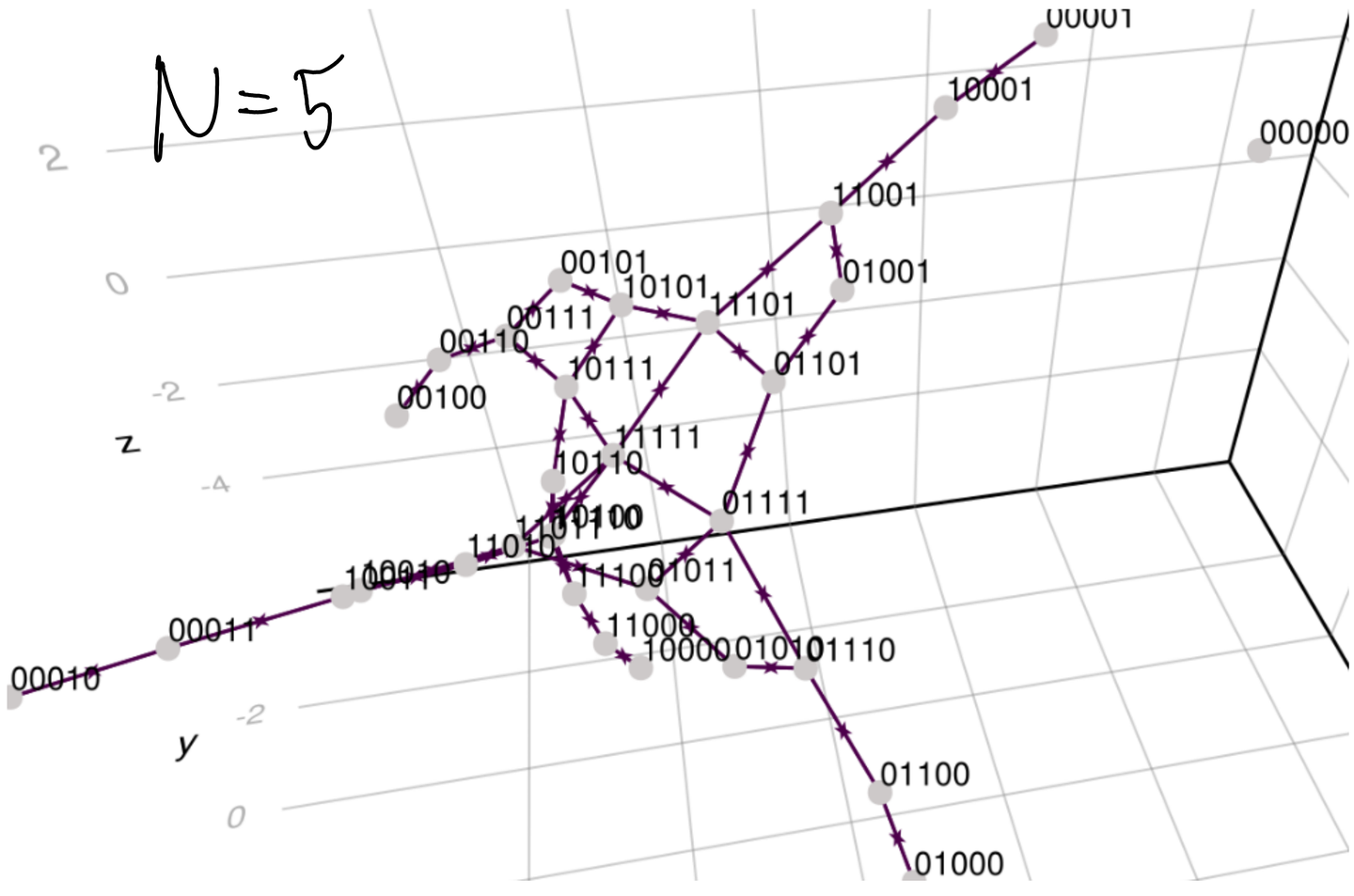


Eg rule 60

00 00
11 11

Forms "nets" expanding from all 1s (all 0s is isolated)

the net having distinct "tips" each ending in a state with one isolated 1



Legume subrules

12 ⟷ 252

12	T	252
00 00		00 00
00 11		11 00

strongly biased, all single acs

252 only acs are all 1s and all 0s

12 acs are interspersed and include network tips

172 ⟷ 92

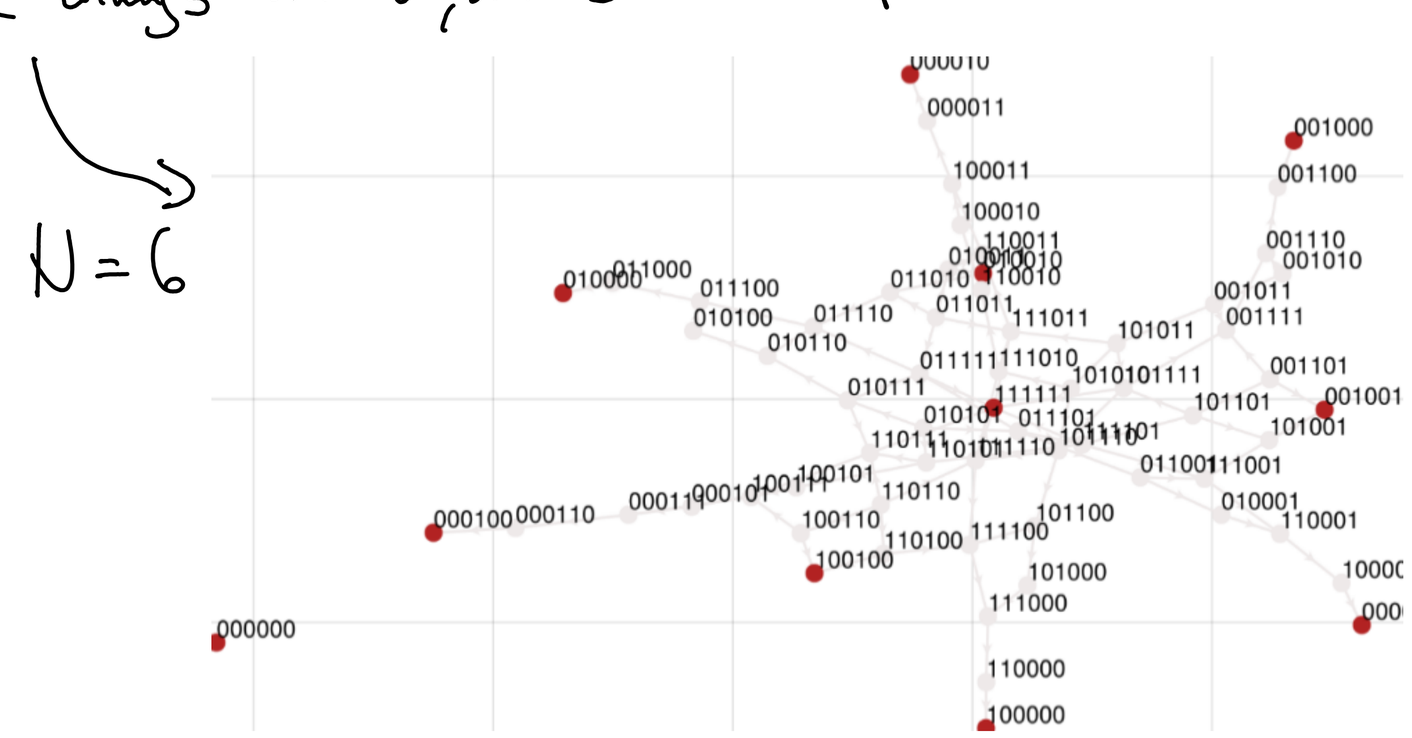
172	T	92
00 00		00 00
01 10		10 01

also only single acs

92 interspersed close to middle

172 always all 0s, all 1s and tips

N = 6



Benzene substrates

188 – all 1s and all 0s only

0000
1110

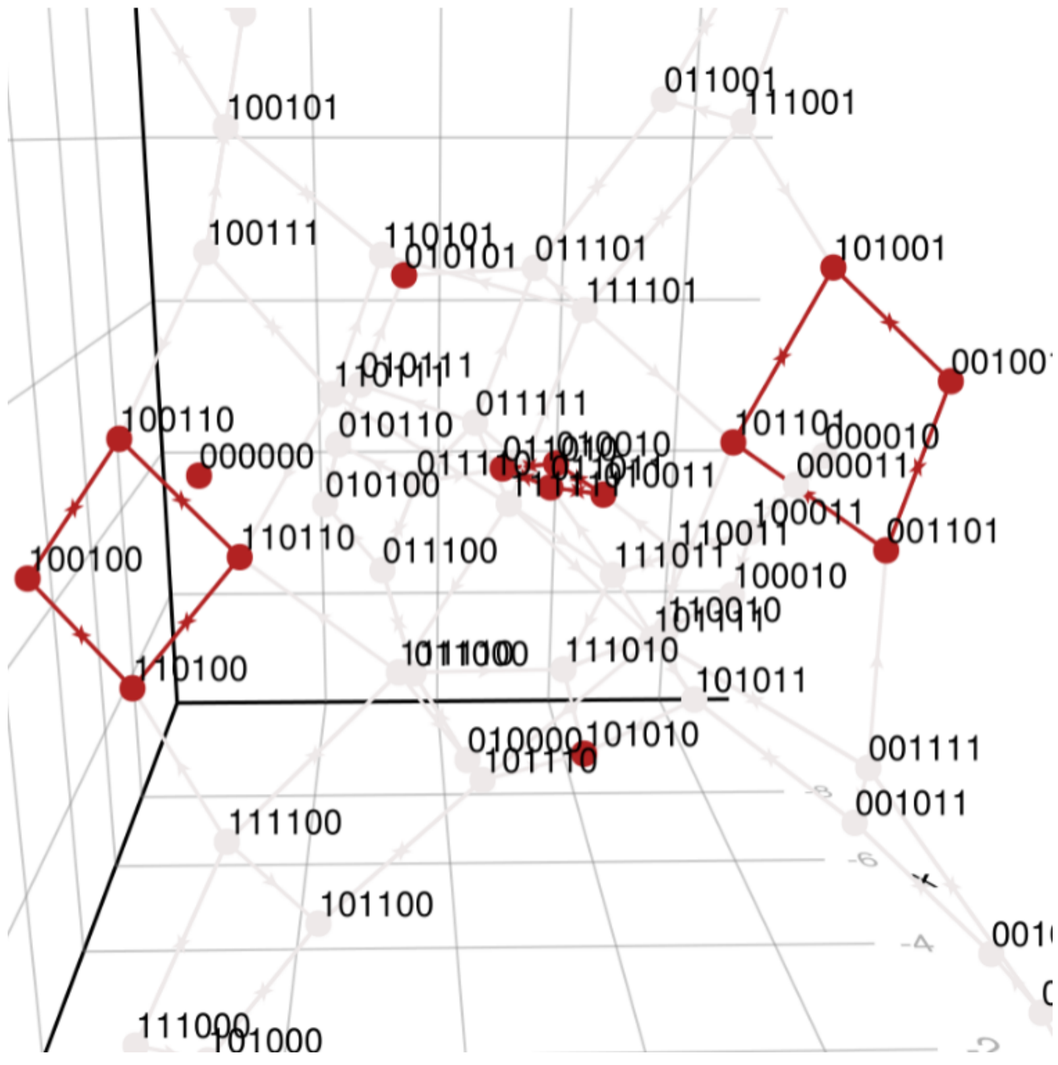
two acs

↓T

78 – has many small diffusive

0000
1011

non-single acs due to a 1100 boundary shifting back and forth



44 – bunch of single acs including the het tips

0000
0111

↕

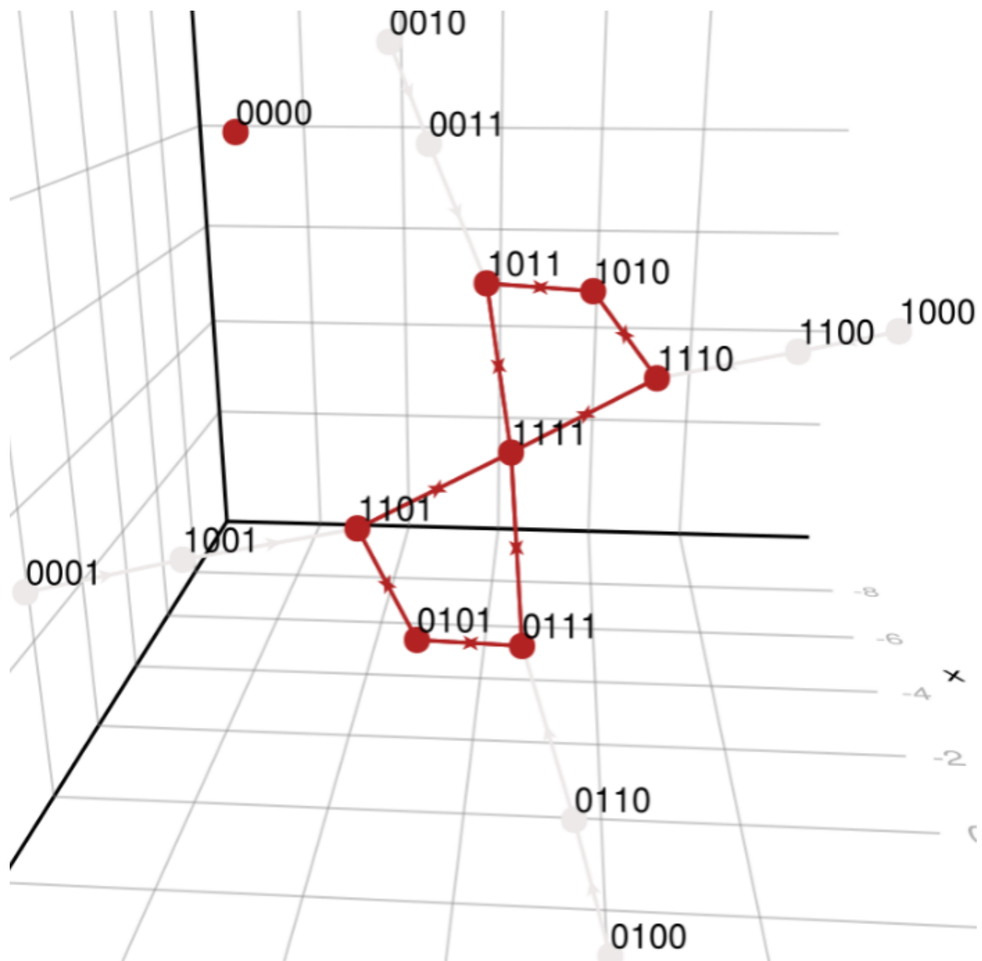
124 – has a diffusive housing

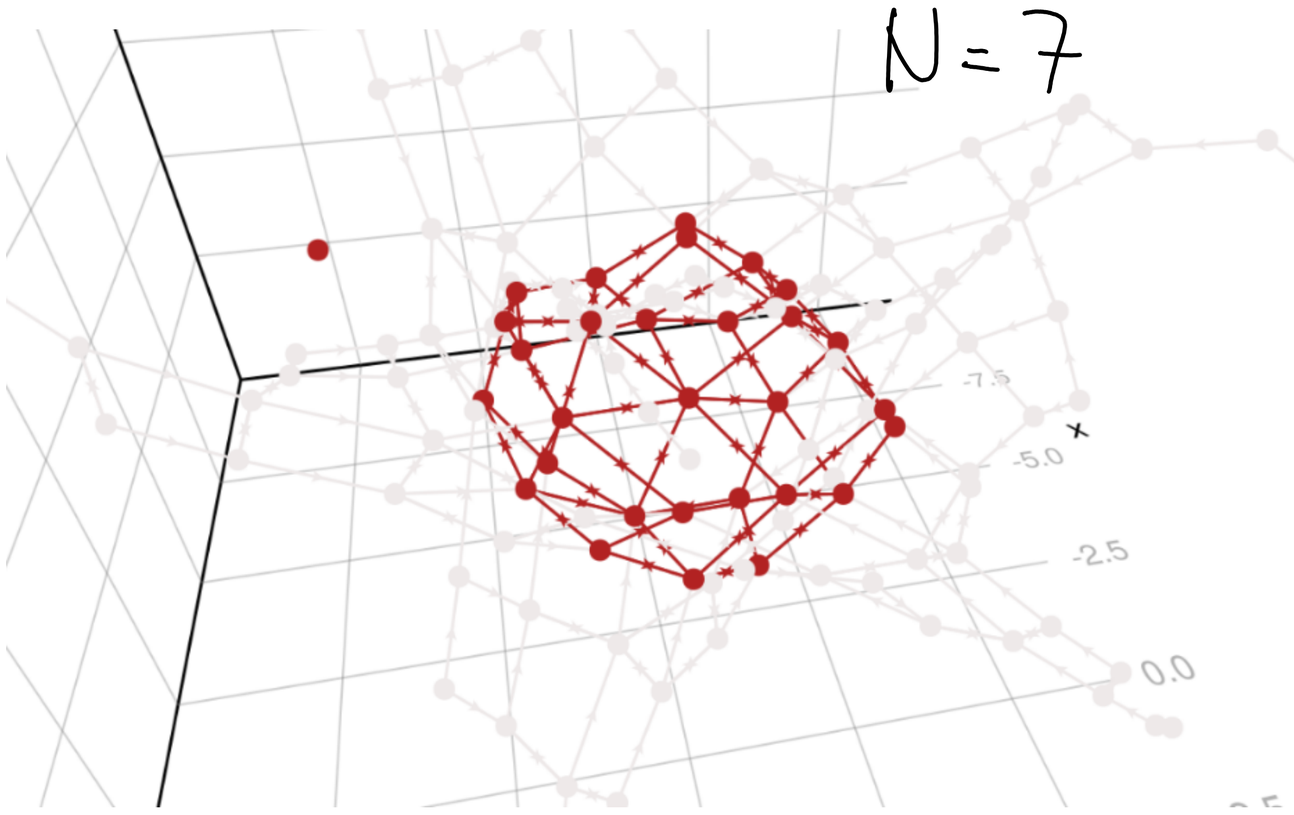
0000
1101

ac around all 1s →

↓

N = 7



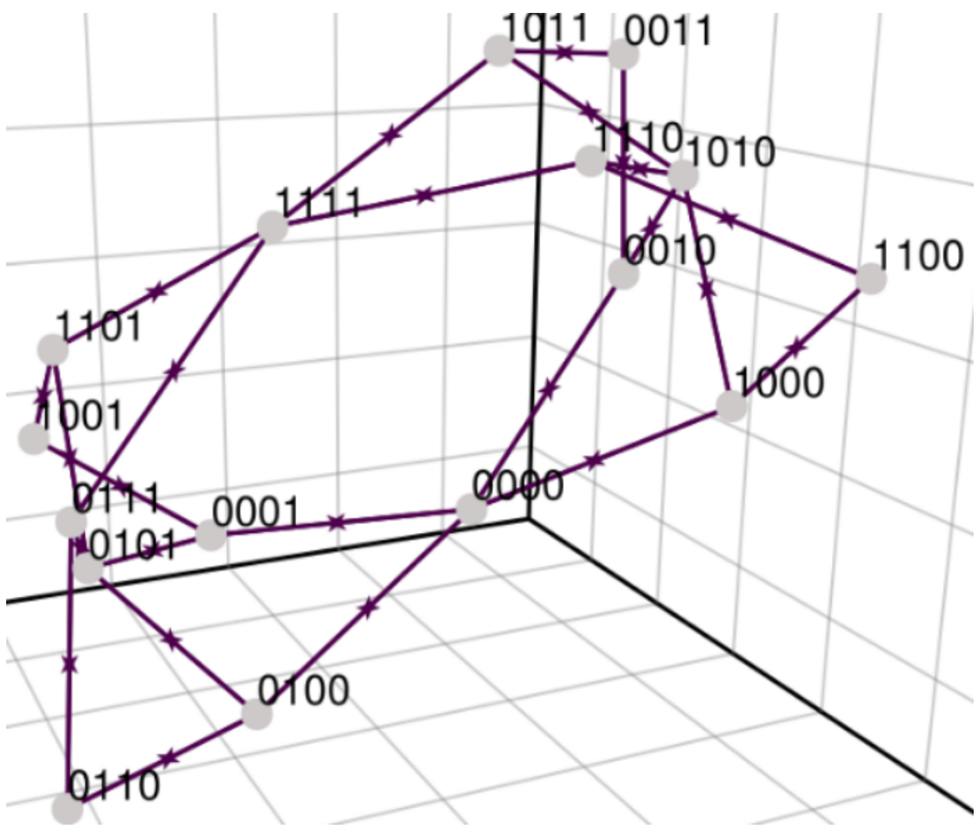


Eq rule 57

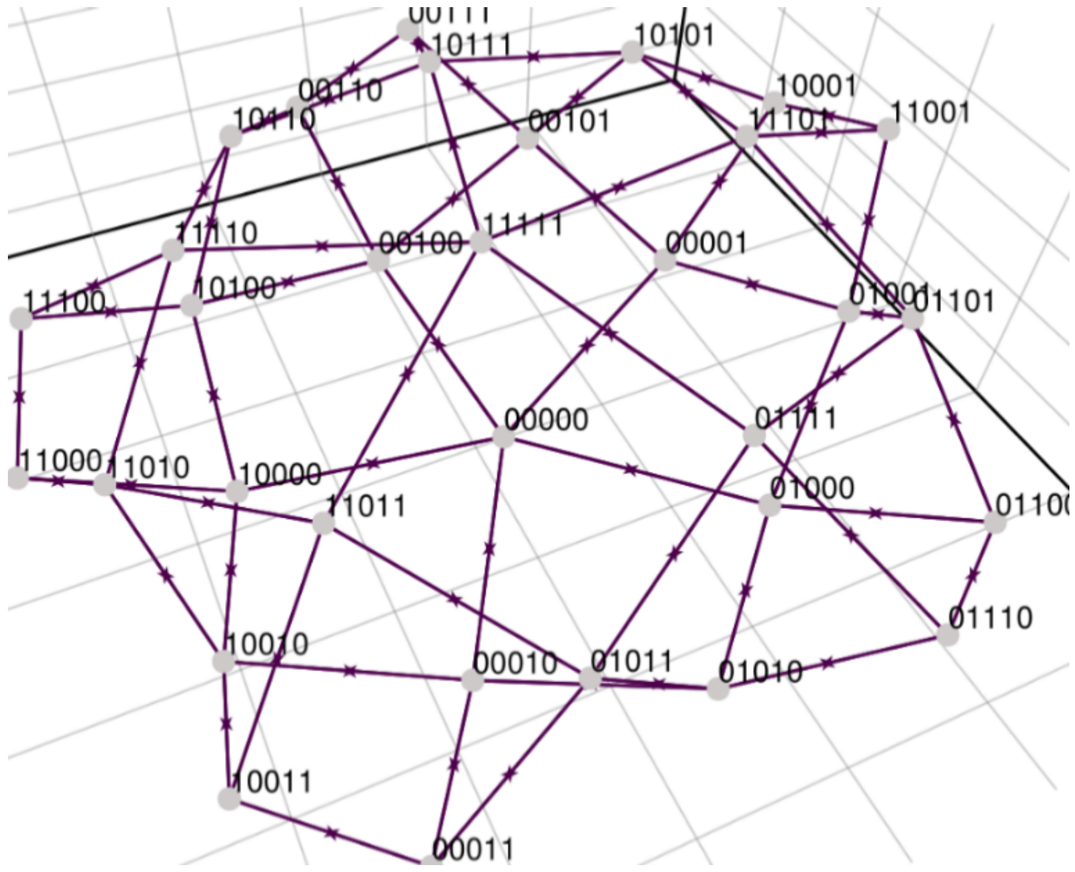
10 10
11 11 - everything but moving a ..0011.. boundary

Forms very pretty symmetrical structures with all 0s and all 1s "opposite"

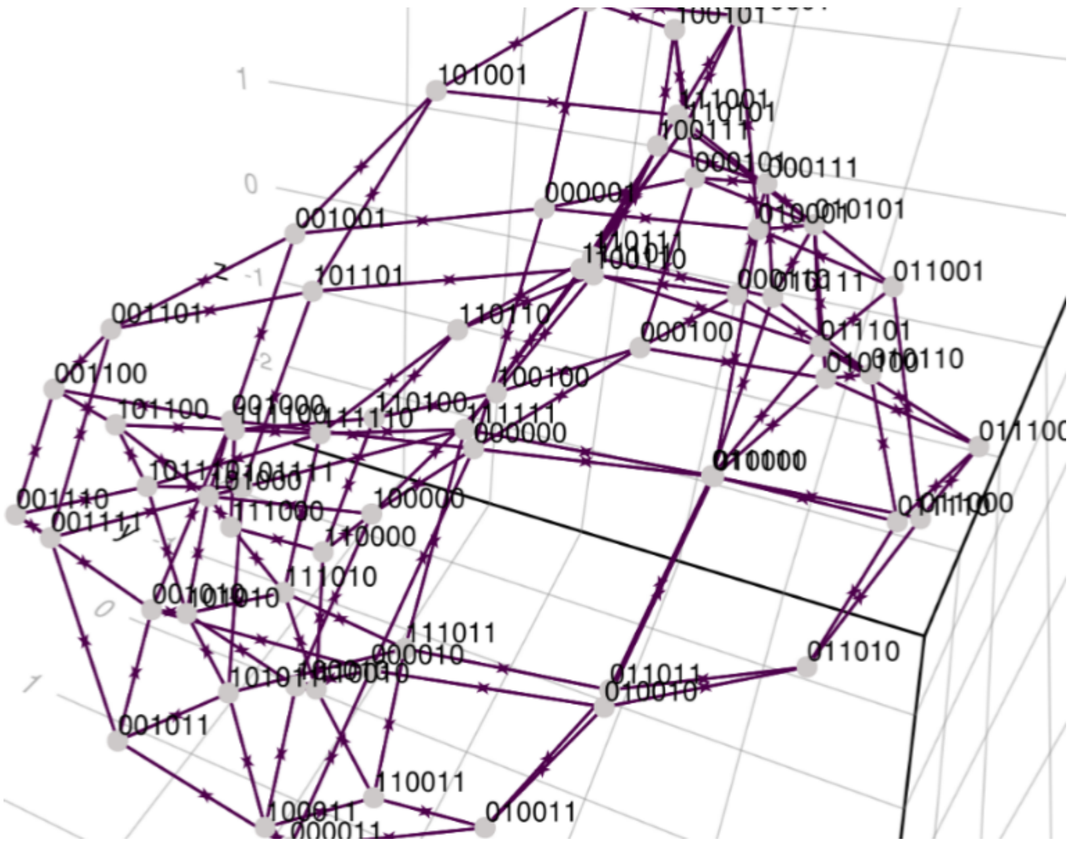
N = 4



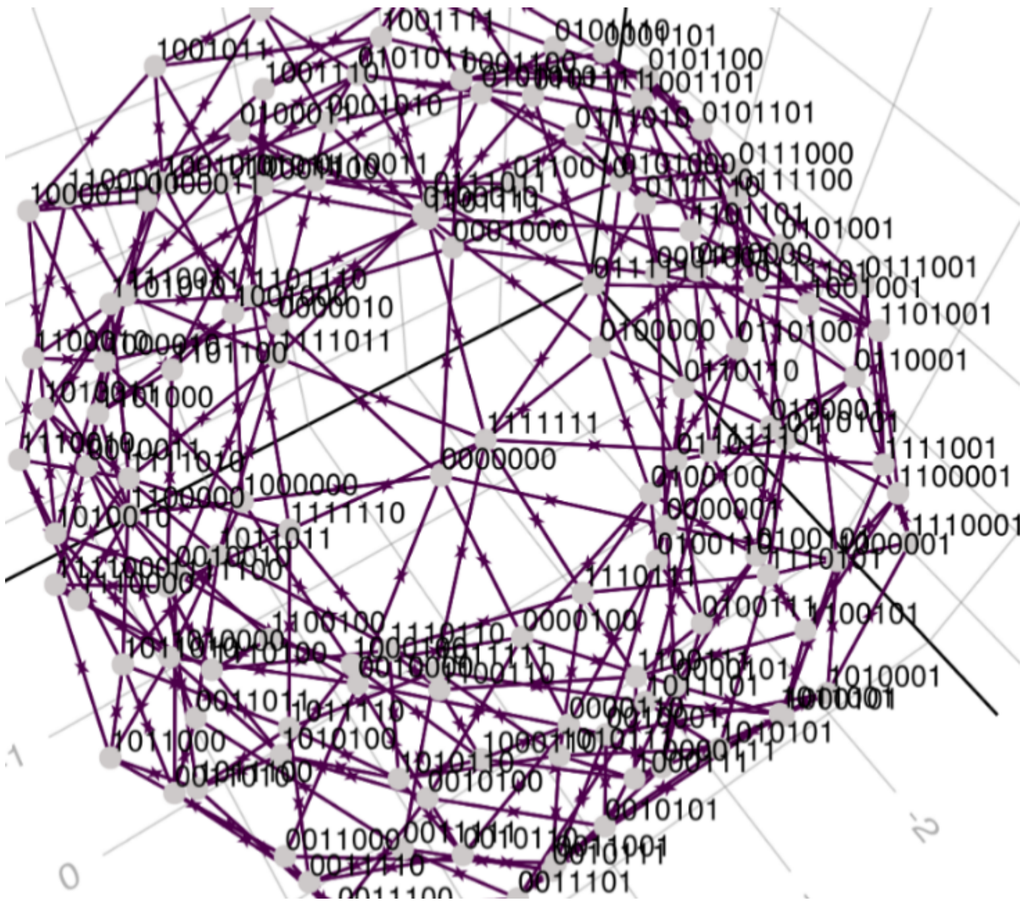
N = 5



N = 6

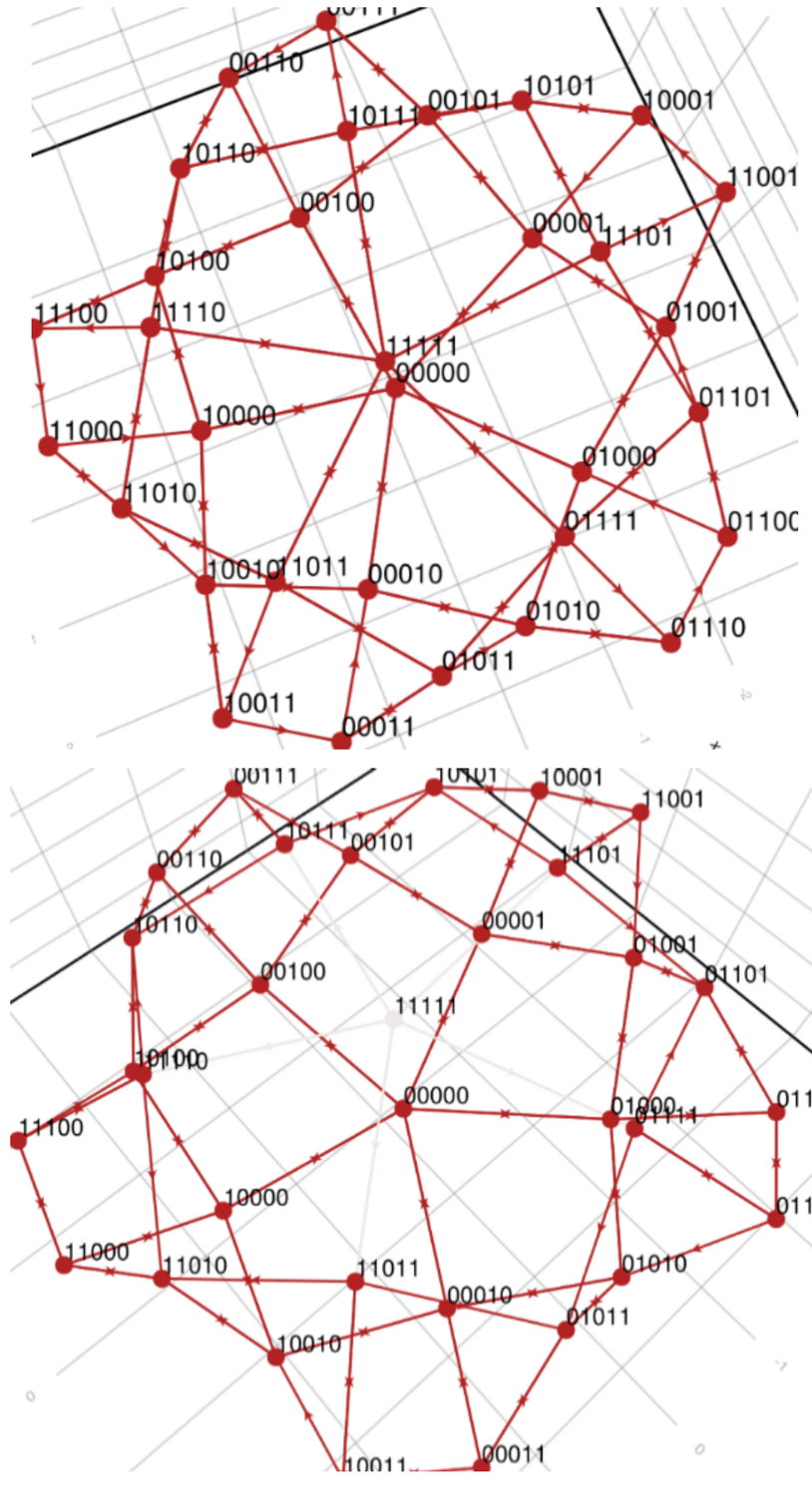


N = 8



5 enzyme subrules

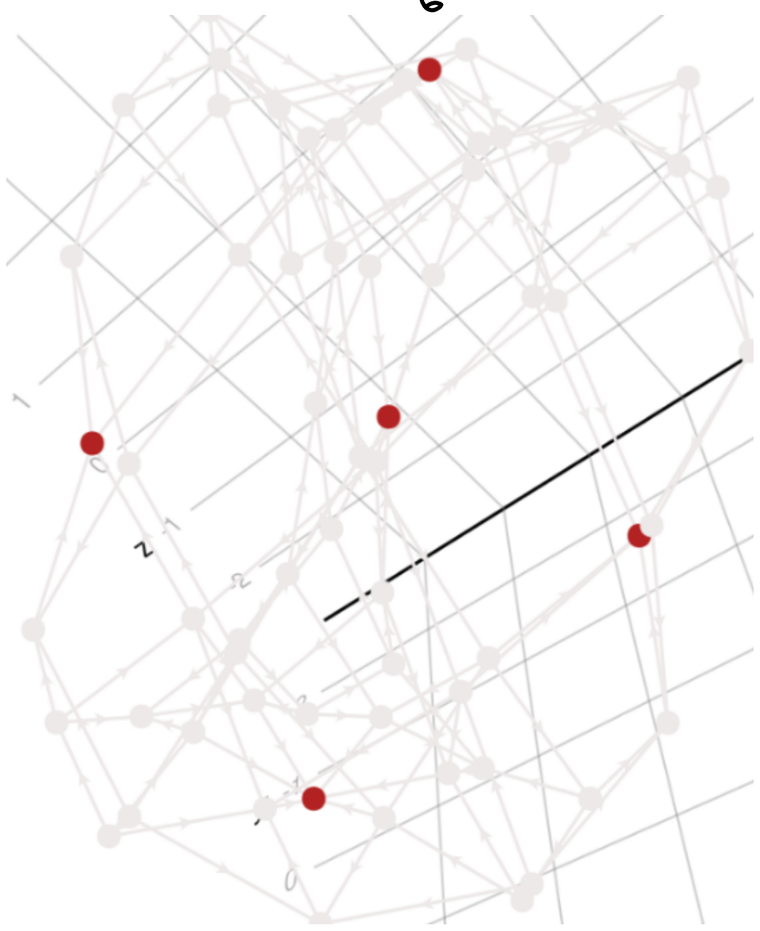
41 – biased towards all 0s
10 10 (as 0 domains cannot shrink)
01 11 but remains a single large ac



36 – makes all 0s inescapable
00 10 all 0s is only ac
11 11
↕ T+01+LR

25 – somewhat similar to 7, has 1
10 10 ac but flow are some directed
10 11 arrows, this time near all 1s
which cannot be reached

3 enzyme subrules



D – all 0s only ac
00 10
00 11

88 – all 0s + if N div by 3
00 10 also repeats of $\overline{110}$
10 01

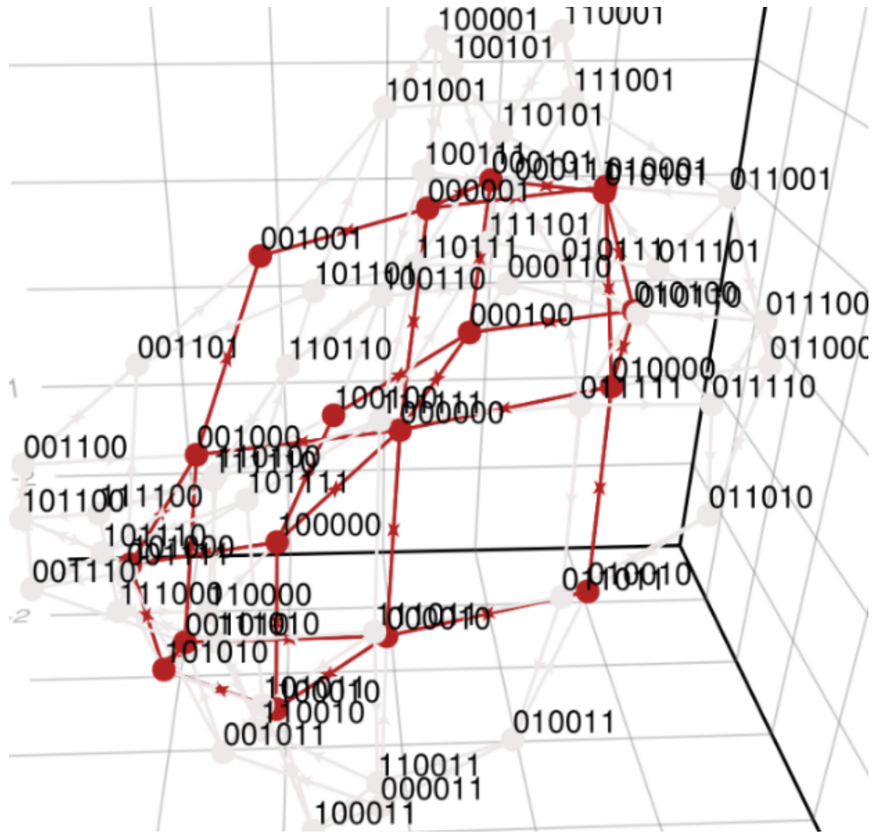
13 – always a couple of elects
10 00 dispersed acs
00 11
↕ T+01+LR

168 – all 0s and all 1s
00 10 are always the only
01 10 2 acs

Enzyme subrules

40
00 10
01 11
} T++

- all 0s only ac

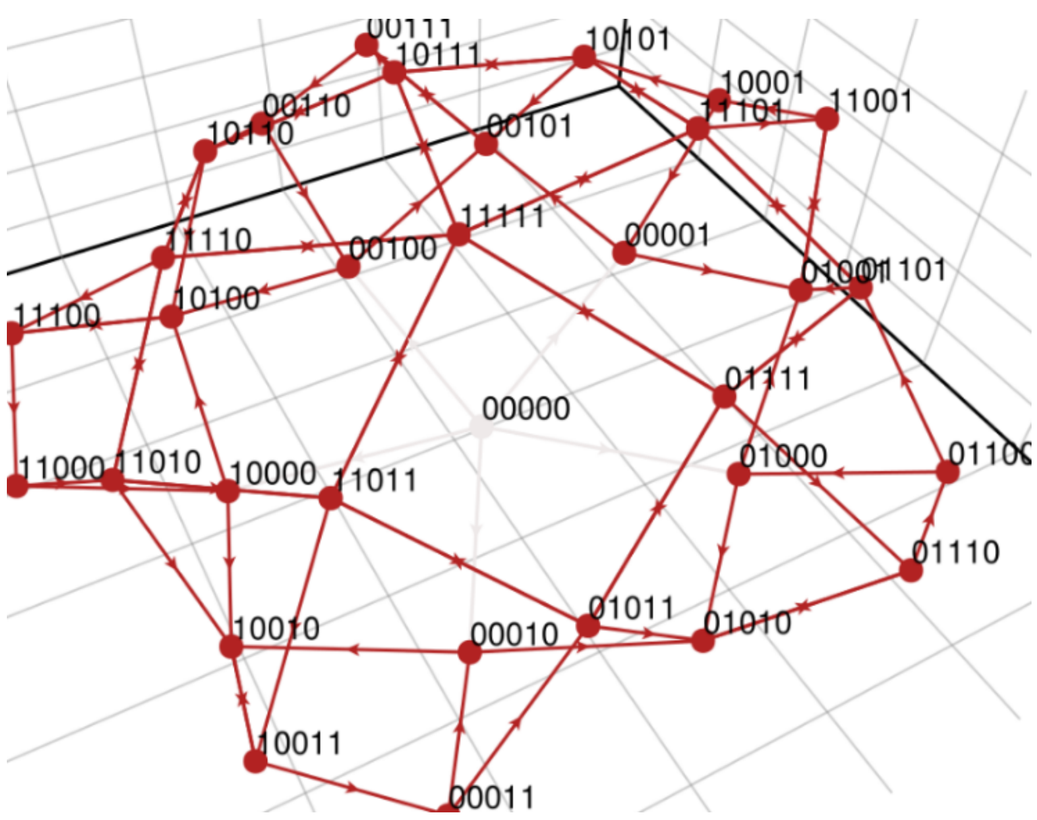


9
10 10
00 11

- 1 complex diffusive ac around all 0s

24
00 10
10 11

- all 0s only ac

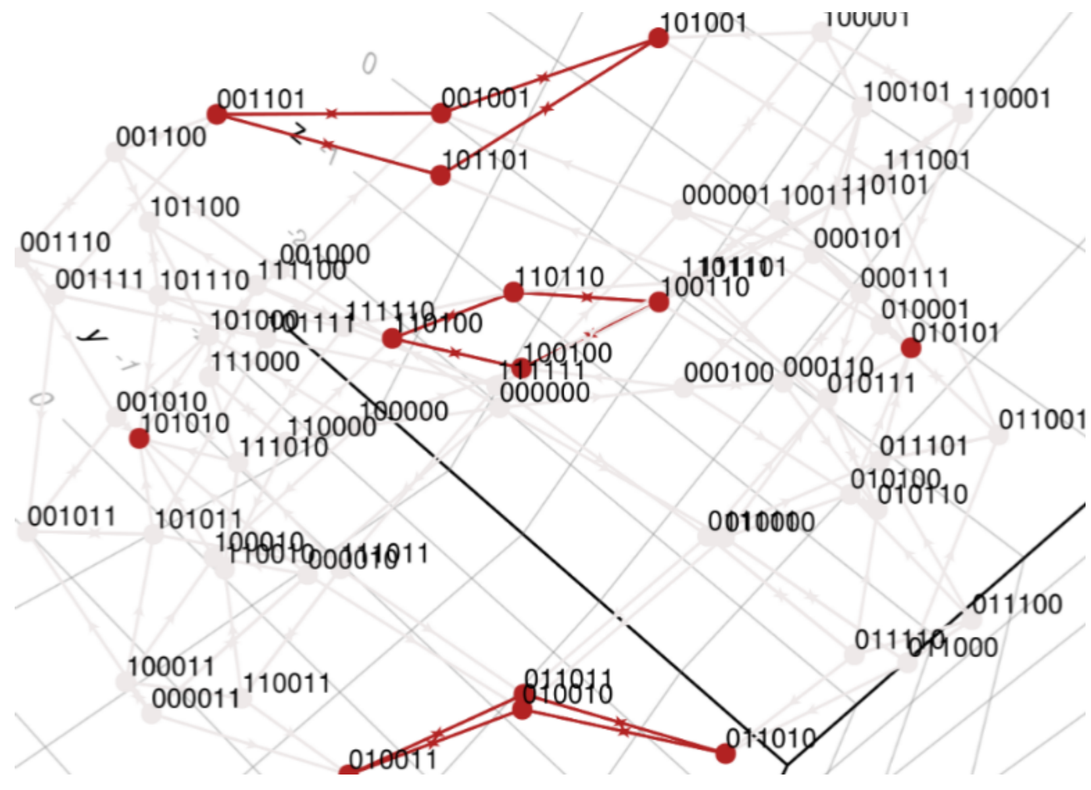


120
00 10
11 01
} T

- all 0s only ac

45
10 00
01 11

- only ac is everything except all 0s unless N div by 3 → flow gets stuck at 001

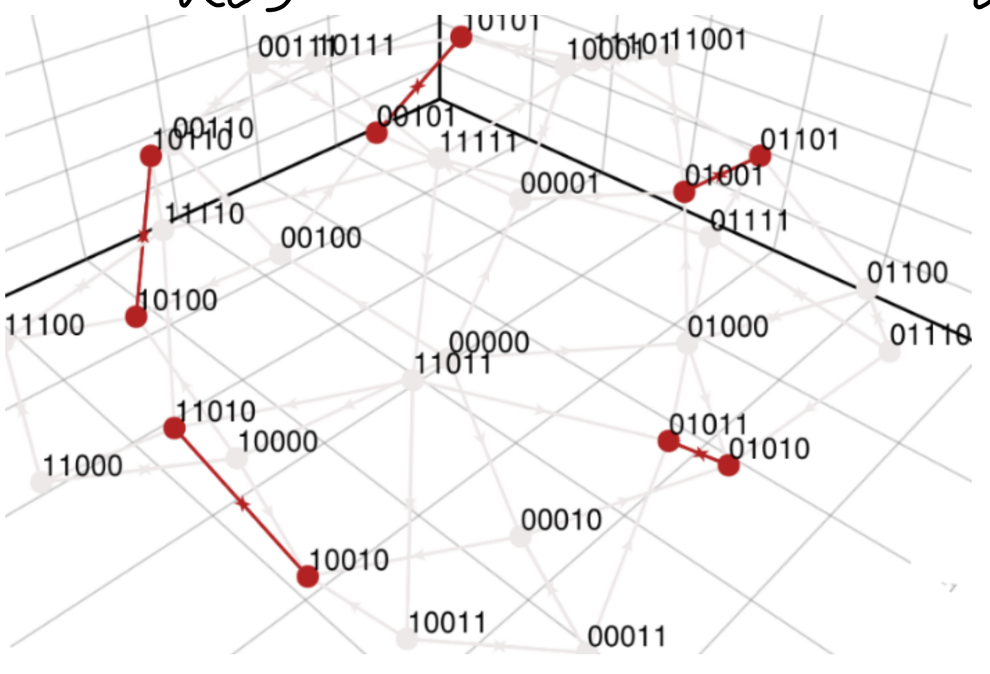


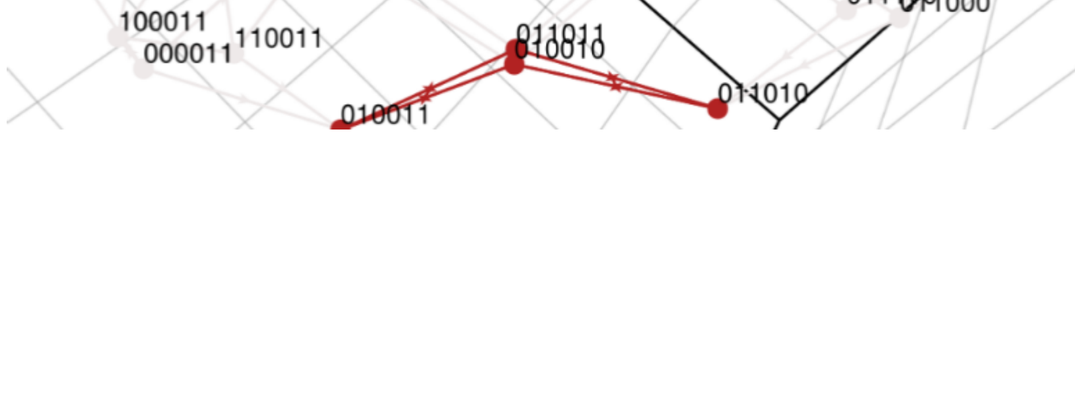
184
00 10
11 10
} T

- all 0s and all 1s only acs

29
10 00
10 11

- multiple small diffusive acs



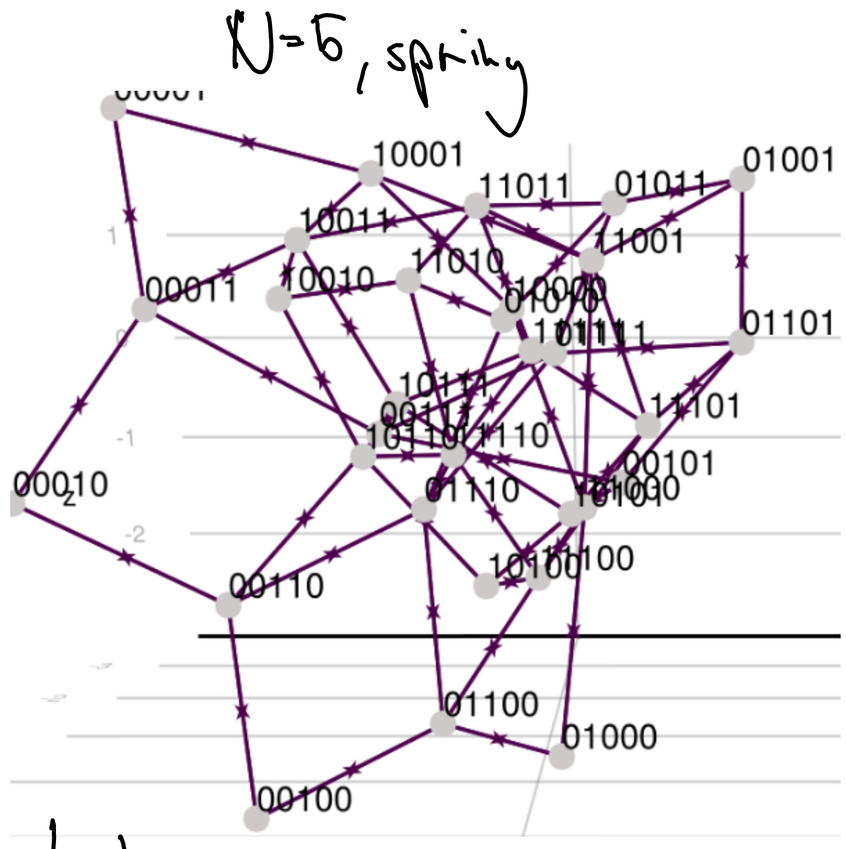


Eg rule 54

0101
1111 — cannot seed or complete a 0 domain

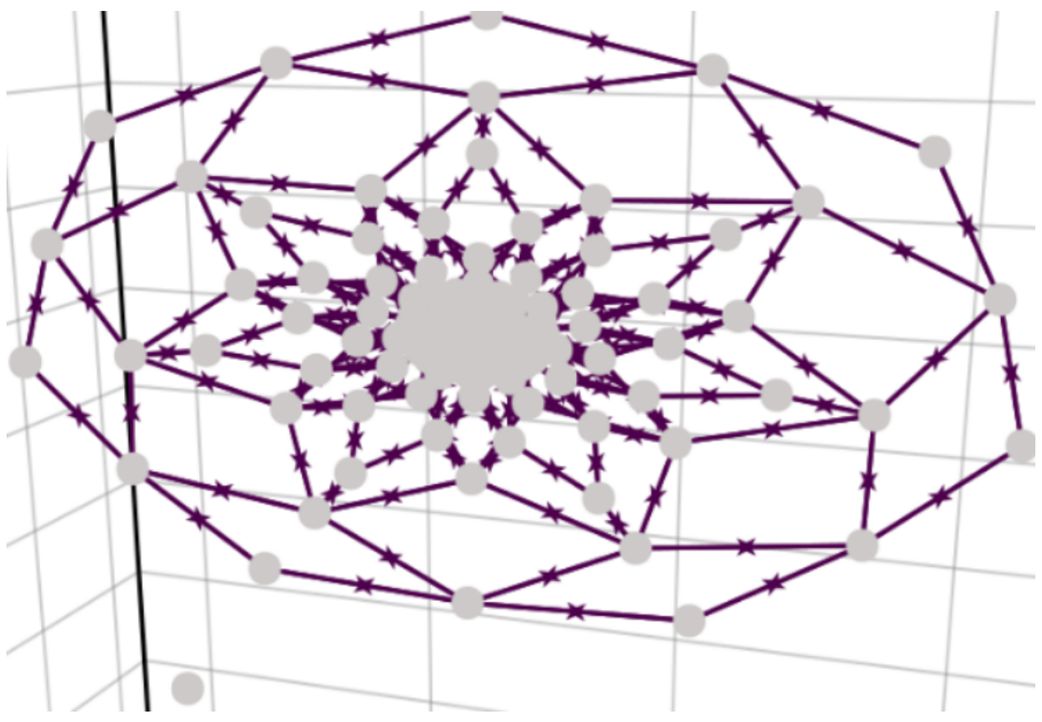
all 0s is always isolated

in Spring it looks like a structured blob →

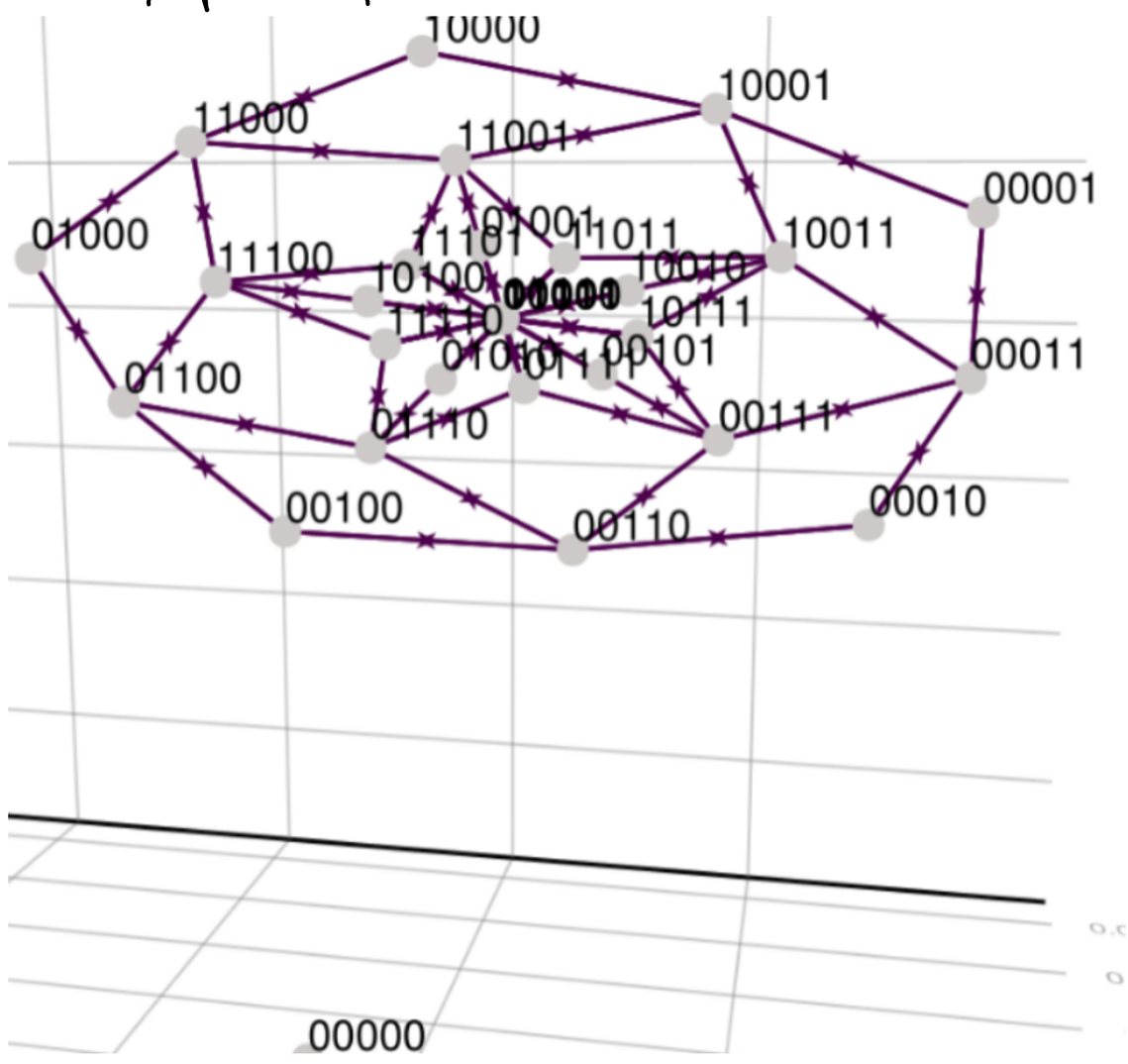


Spectral always gives a star/disk shape

N = 7



N = 5, spectral



5 enzyme subrules

182 - all 0s and all 1s only acs

01 01
1 1 10

↕ T

22 - loops. At odd N has one non-single ac which is a single state loop

01 01
10 11

At even N gets stuck in $\overline{01}$ states

38 - both loop the entire structure as one complex ac but looking at ss makes it clear there are biases

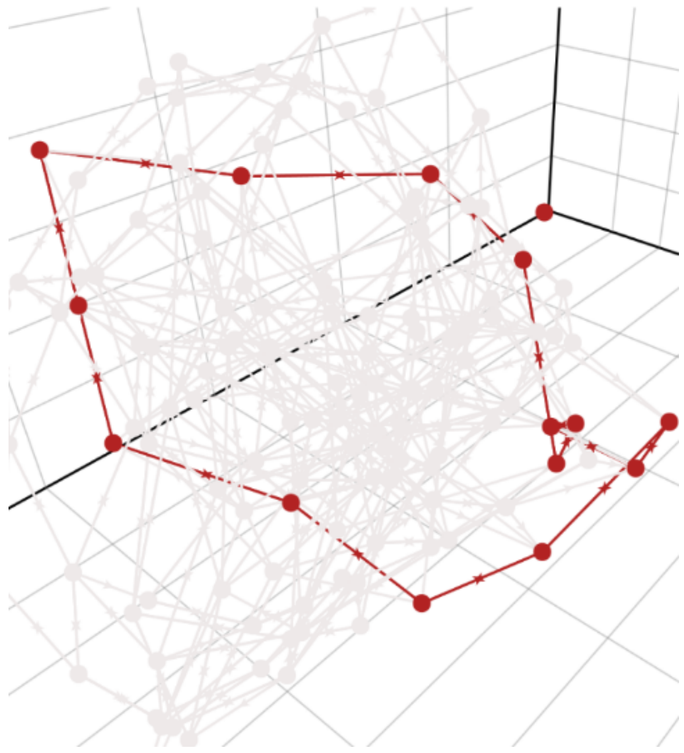
01 01
01 1 1

↕ T+LR

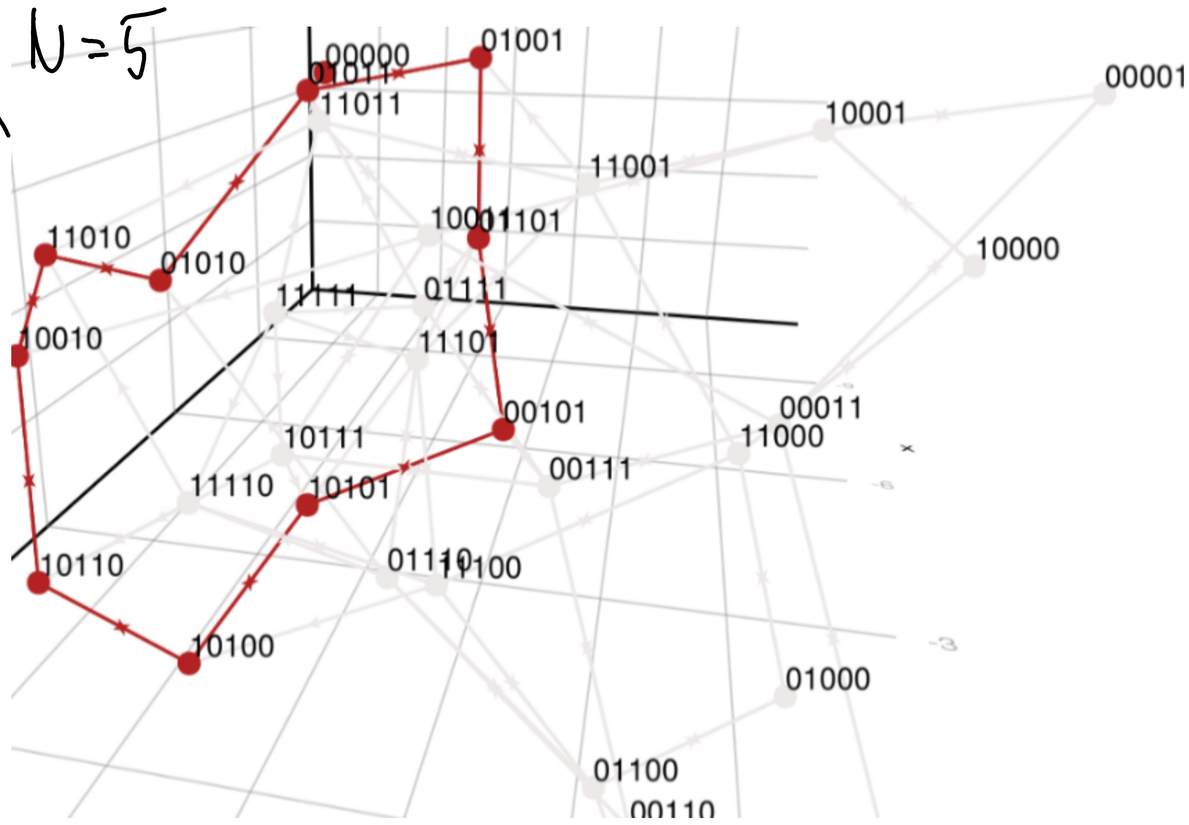
62

01 00
1 1 1 1

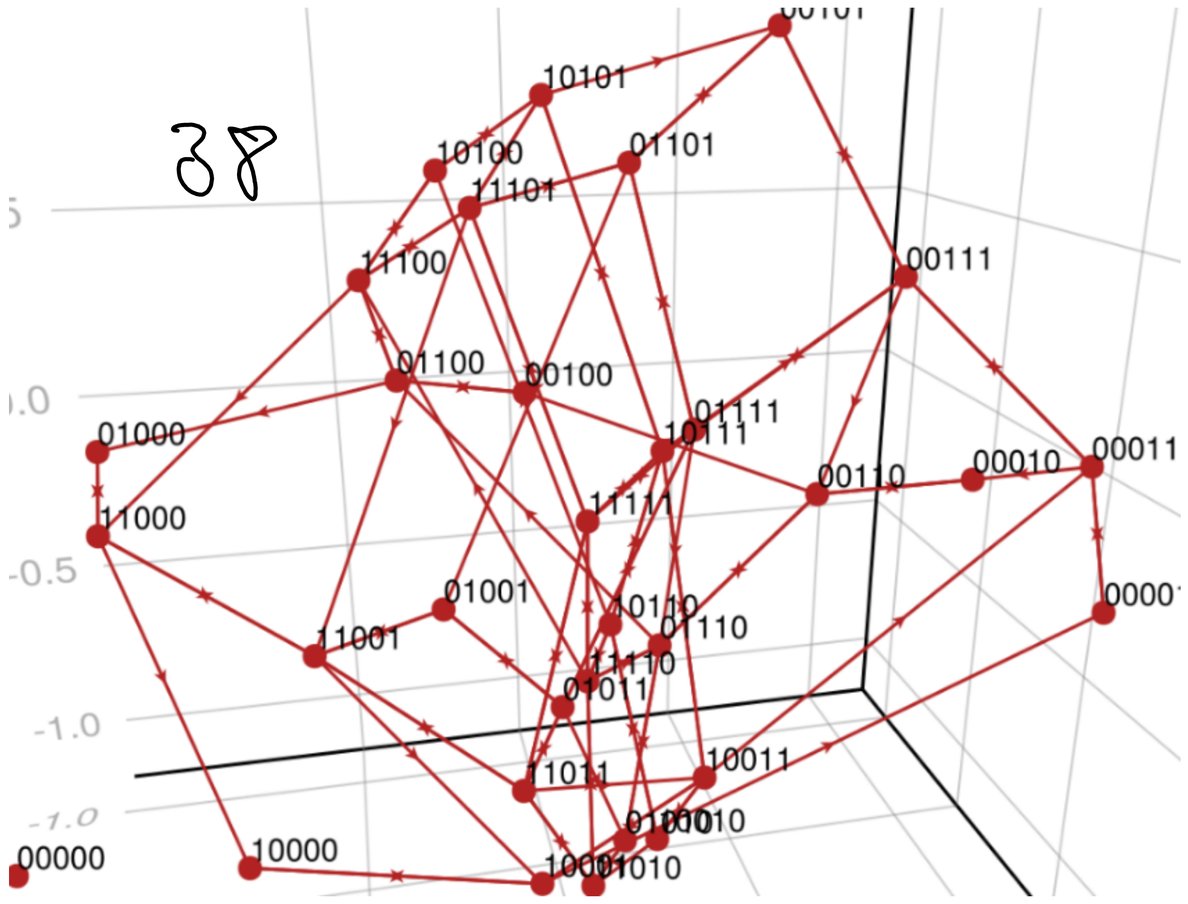
N=7



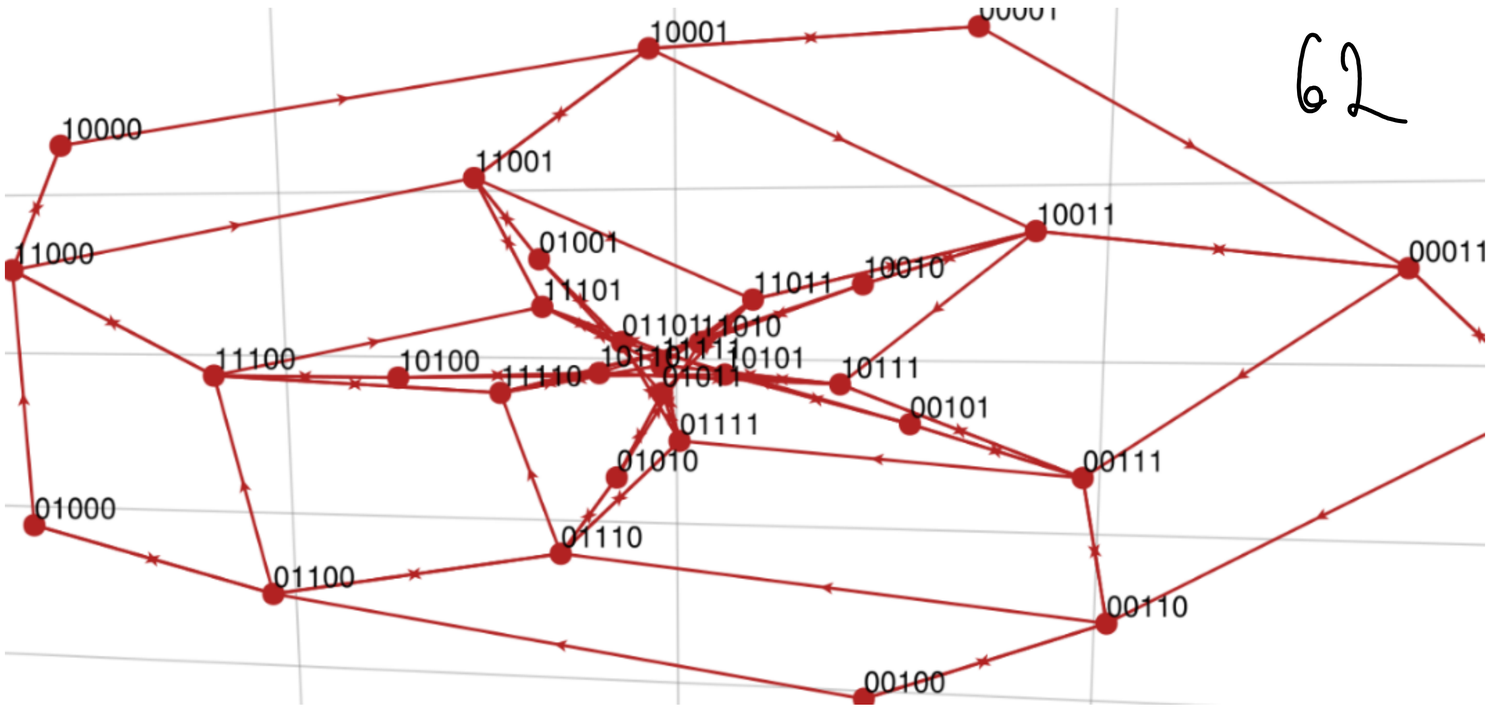
N=5



38



62

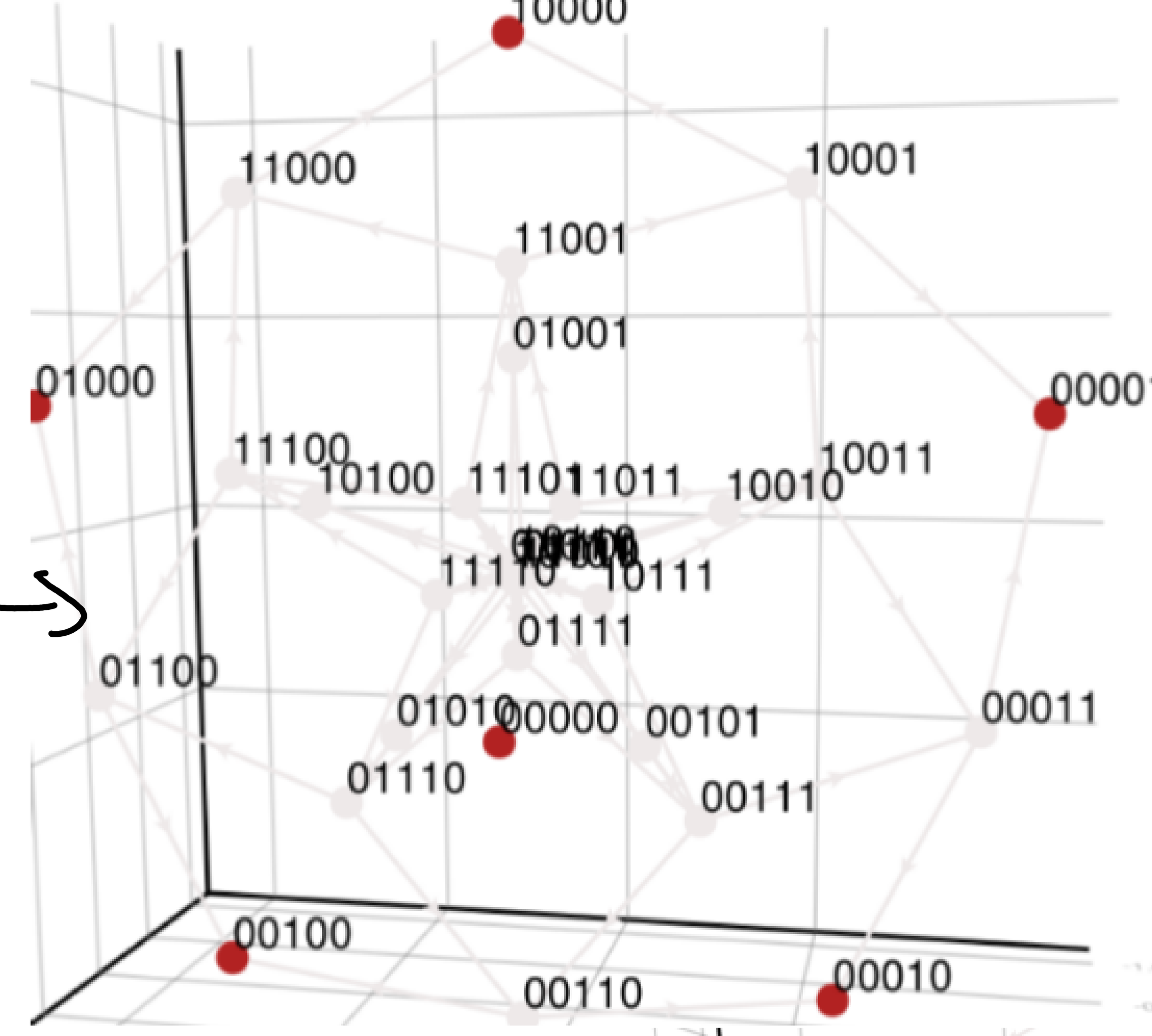


both in Spectral

4 enzyme subrules

36 — multiple, symmetric single acs
all with isolated 1s eg ...00100...

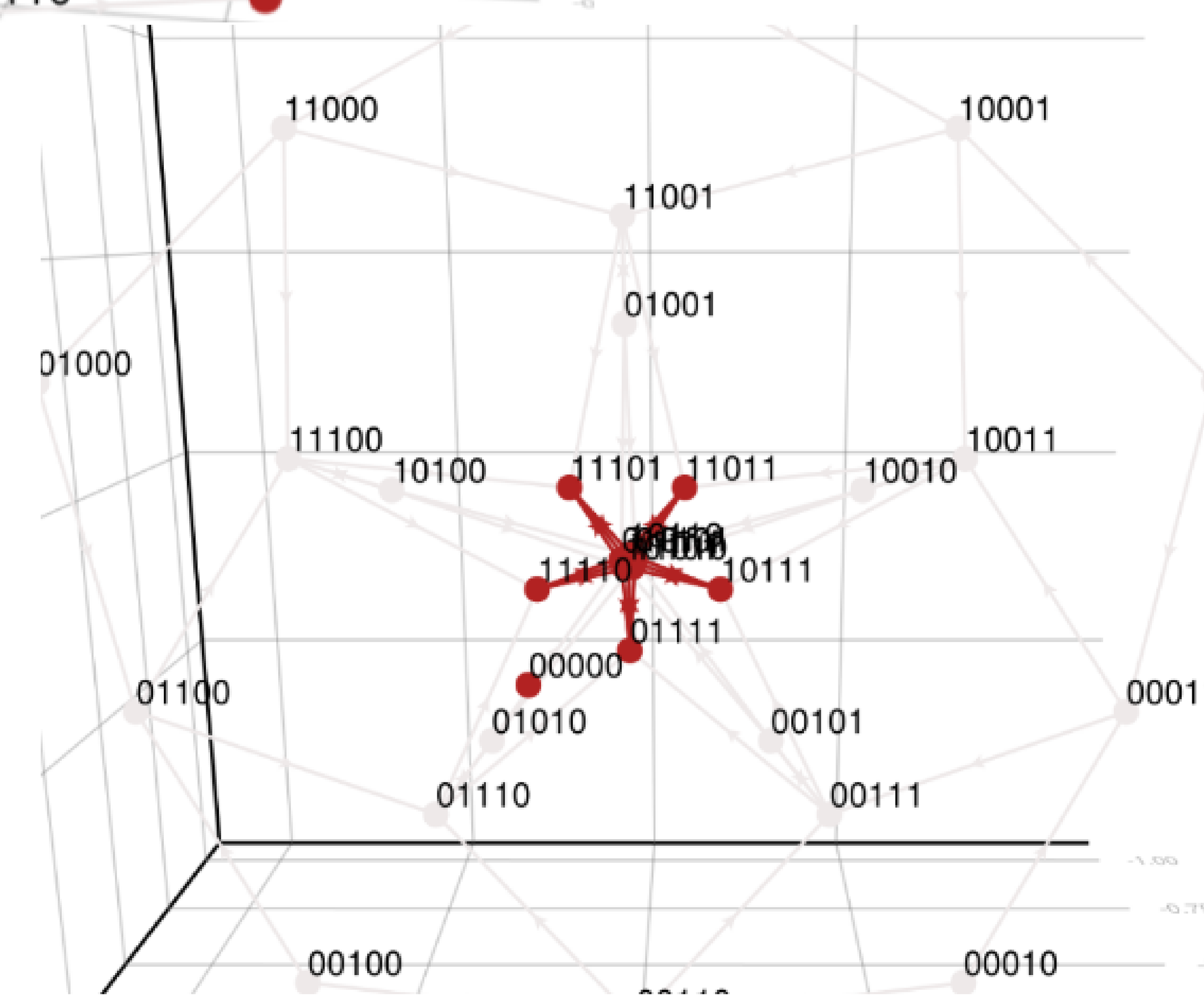
00 01
01 11



↕ T

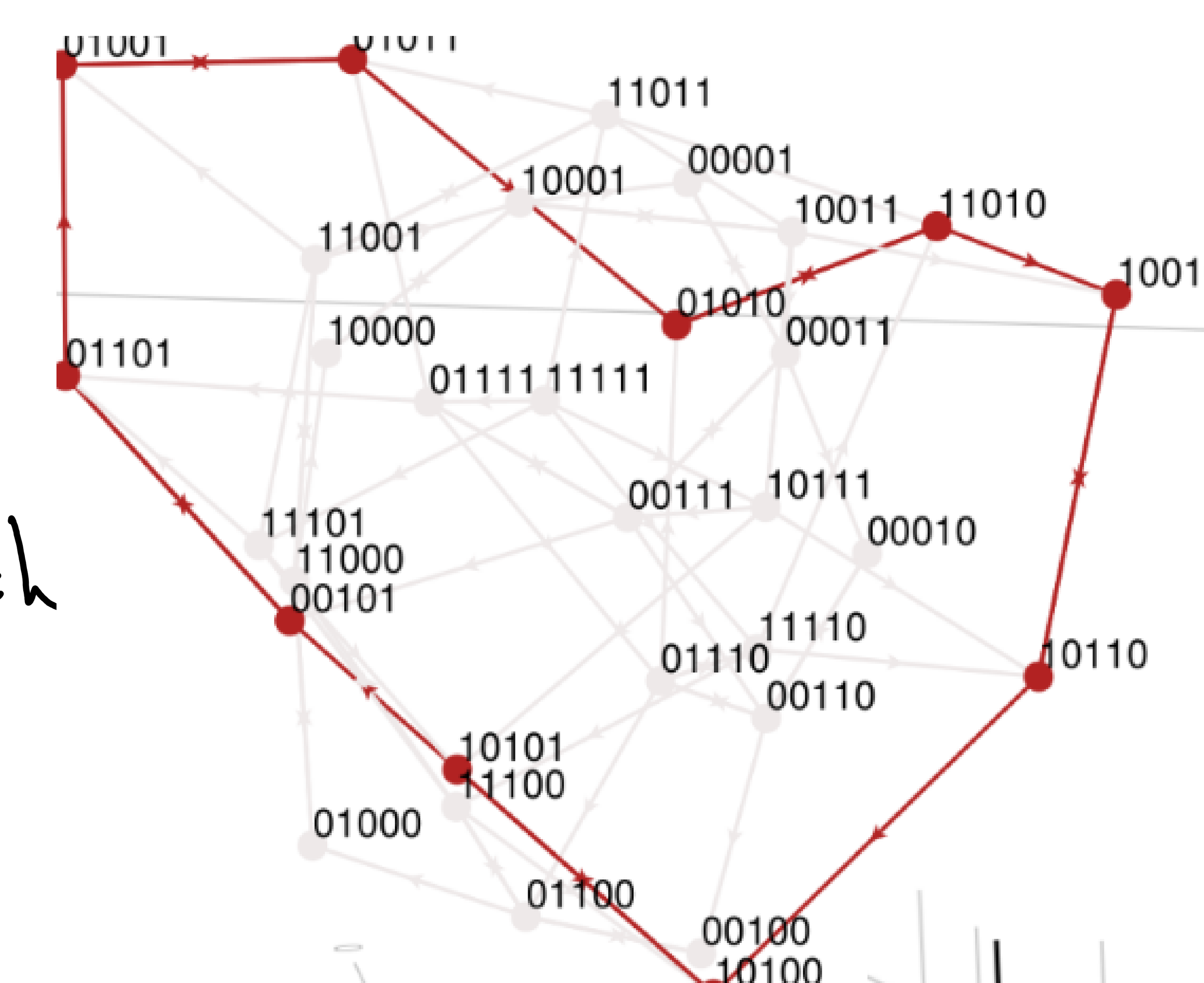
126 — similar, 1 diff. ac all one step away from all 1s, so isolated 0s

01 00
11 01



6 * — loops at odd N
at even N gets stuck at $\overline{01}$ states

01 01
00 11



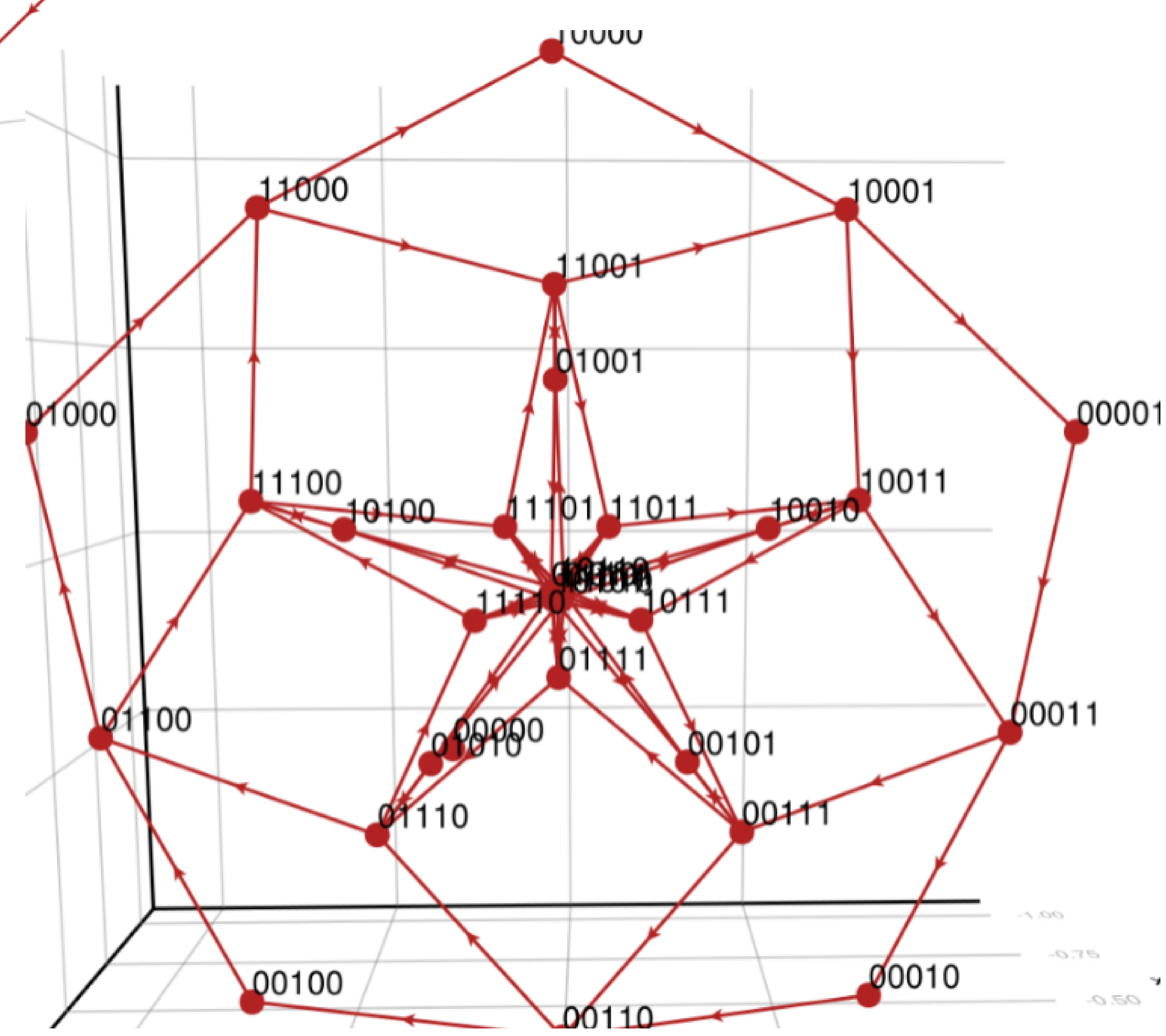
↕ T++

190 — all 1s and all 0s only acs

01 00
11 10

46 * — all states remain in one complex ac which is however clearly directed

01 00
01 11



166 — all 1s and all 0s only acs

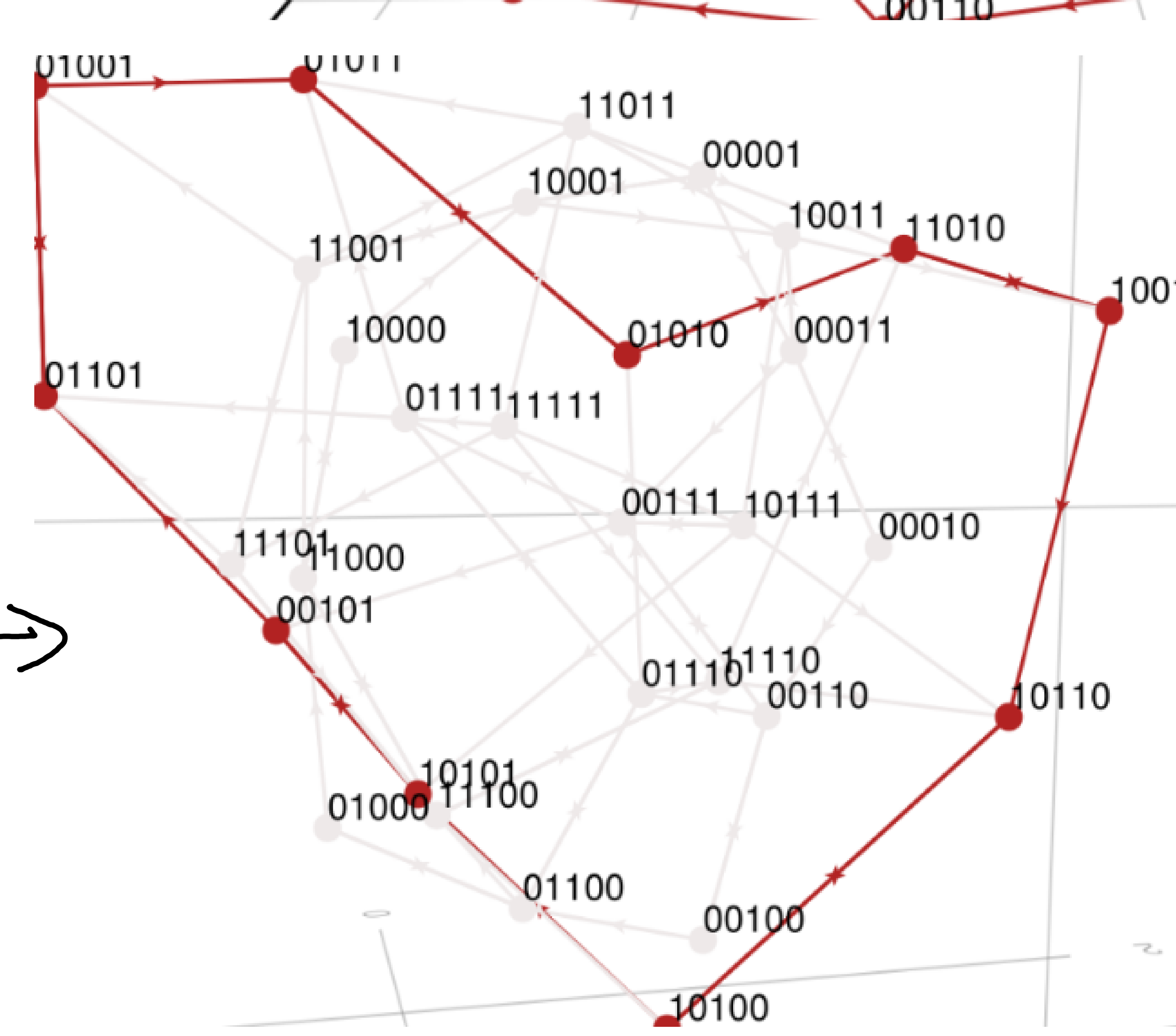
01 01
01 10

↕ T++

* same loops as 46 but
30 — the other arrows are directed

01 00
10 11

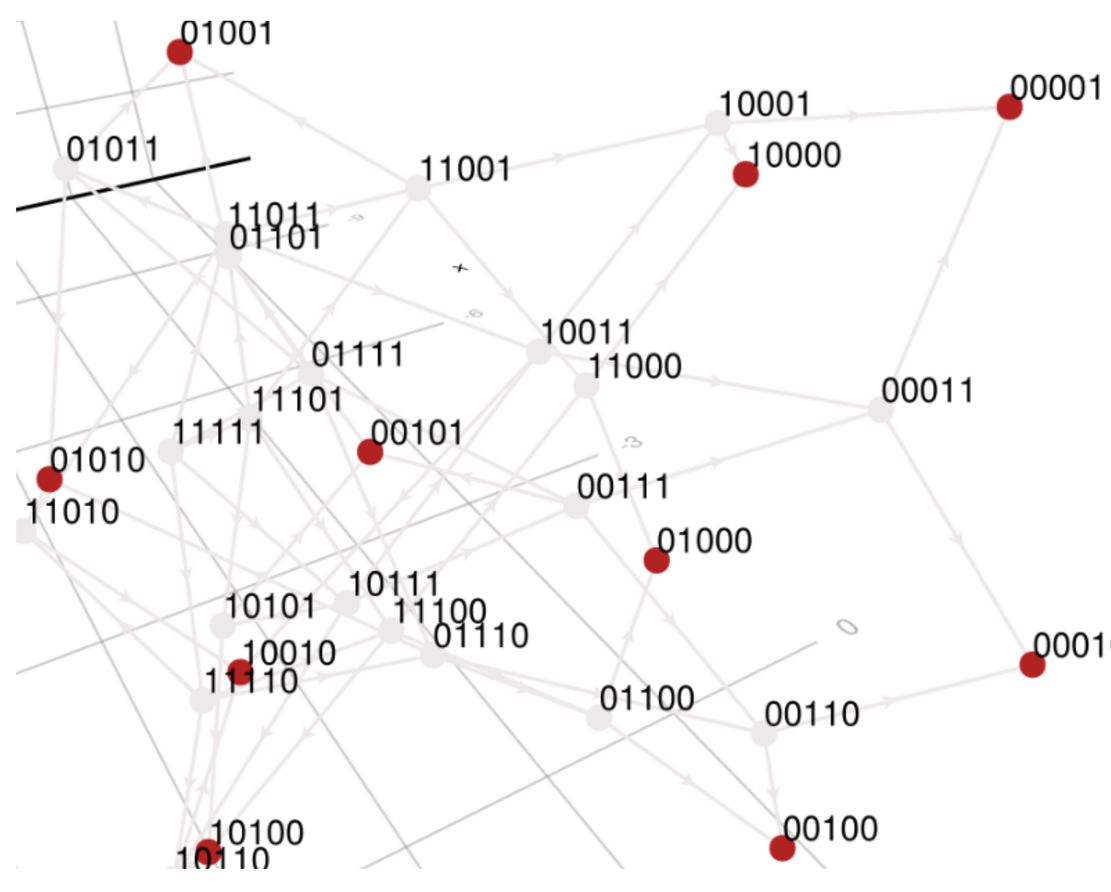
!



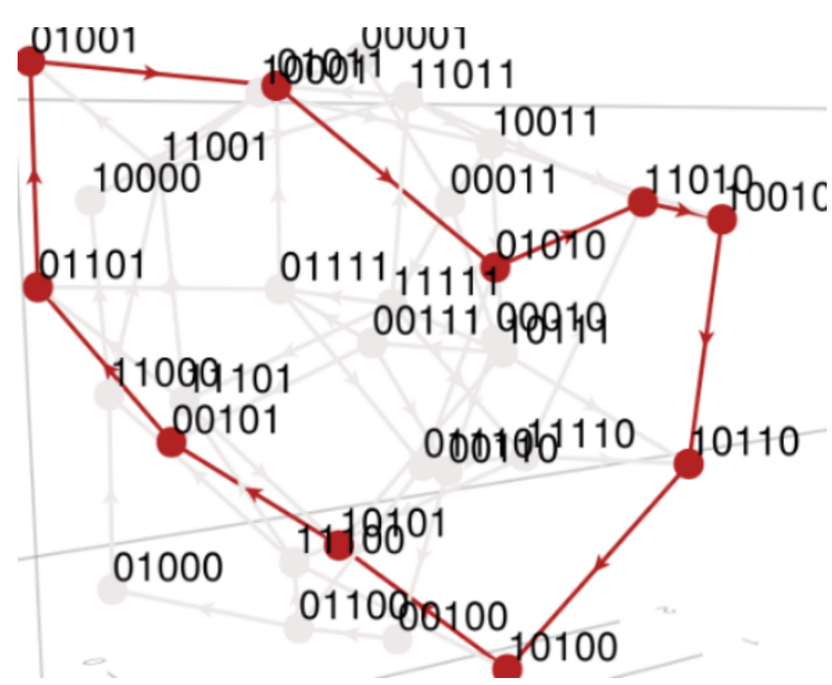
Benzene subrules

4 — many single state acs,
00 01 can only go towards 0s
00 11 but gets stuck at isolated 1s
↓T

254 — all 0s and all 1s
01 00 only acs
11 00

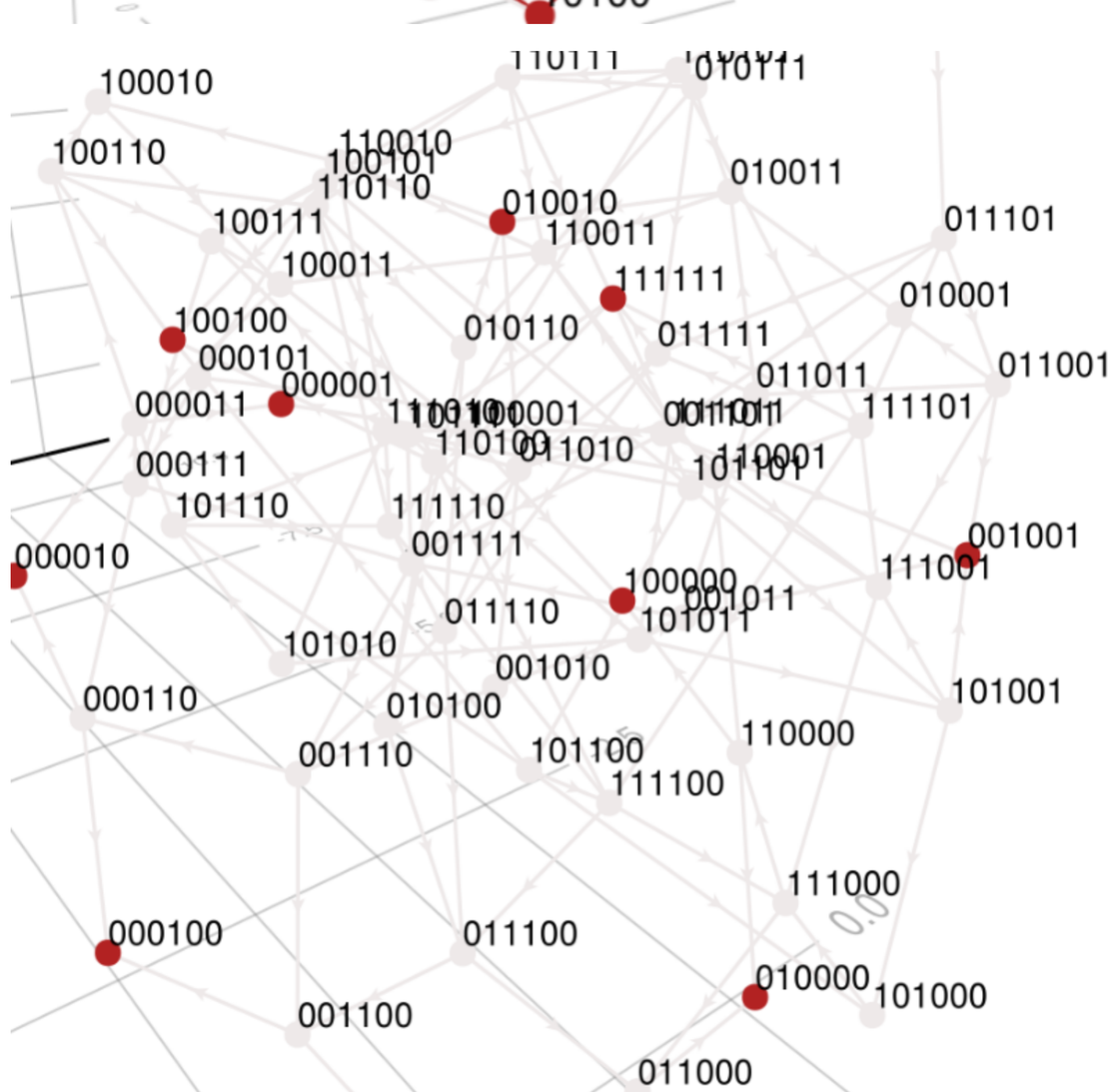


14 — at odd N has th
01 00 save loops as 6, 30 but
00 11 all arrows are directed
↑
↕Tth at even N gets stuck at 01

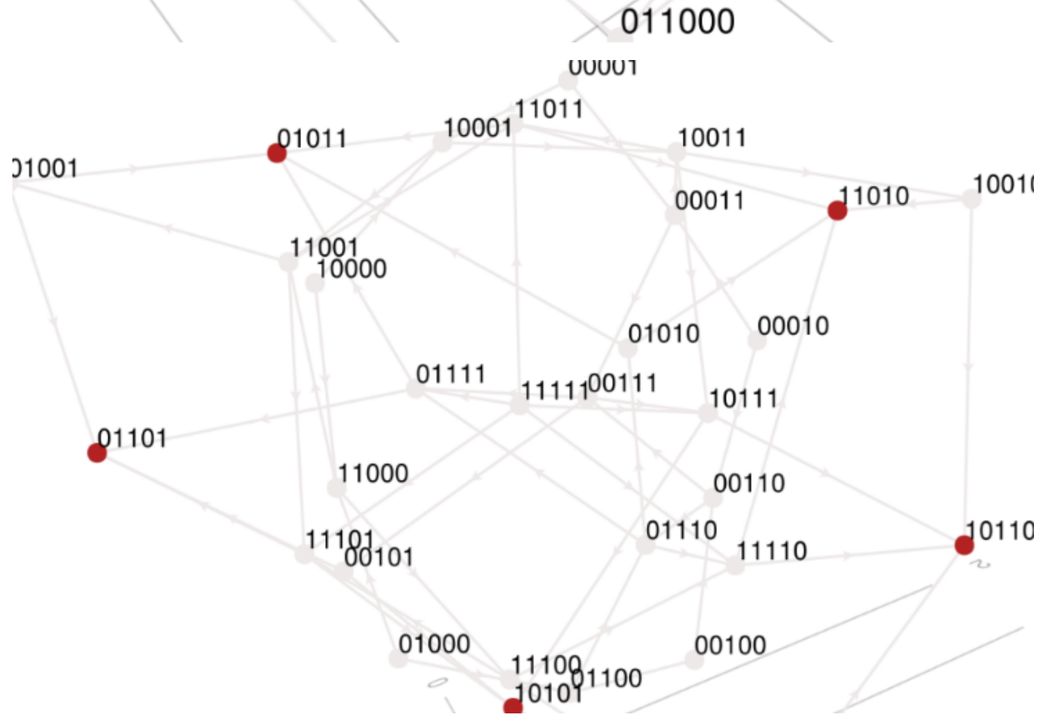


174 — all 0s and all 1s
01 00 only acs
01 10

only 0 domains can spread
but cannot be completed



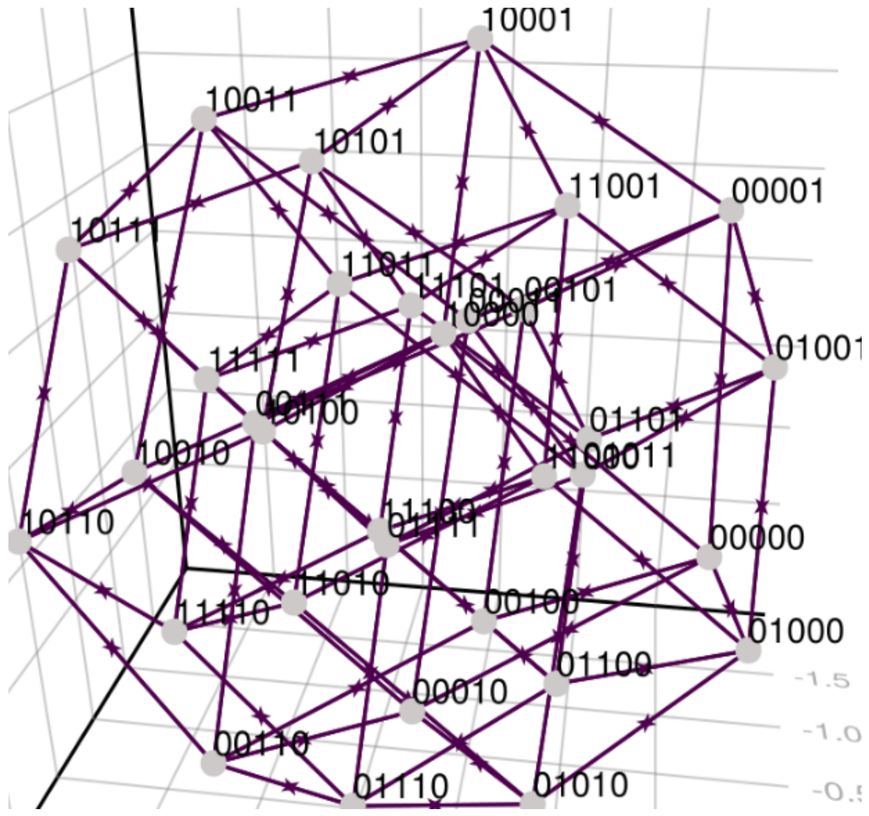
164 — but 1 domains cannot be
00 01 broken either — many
01 10 small acs include all 0s,
↕T all 1s and isolated 1s
94



01 00
10 01 1s can be broken but
cannot ever shrink bunch
of single acs each packet
of isolated 0s

Eg rule 51

Spring layout



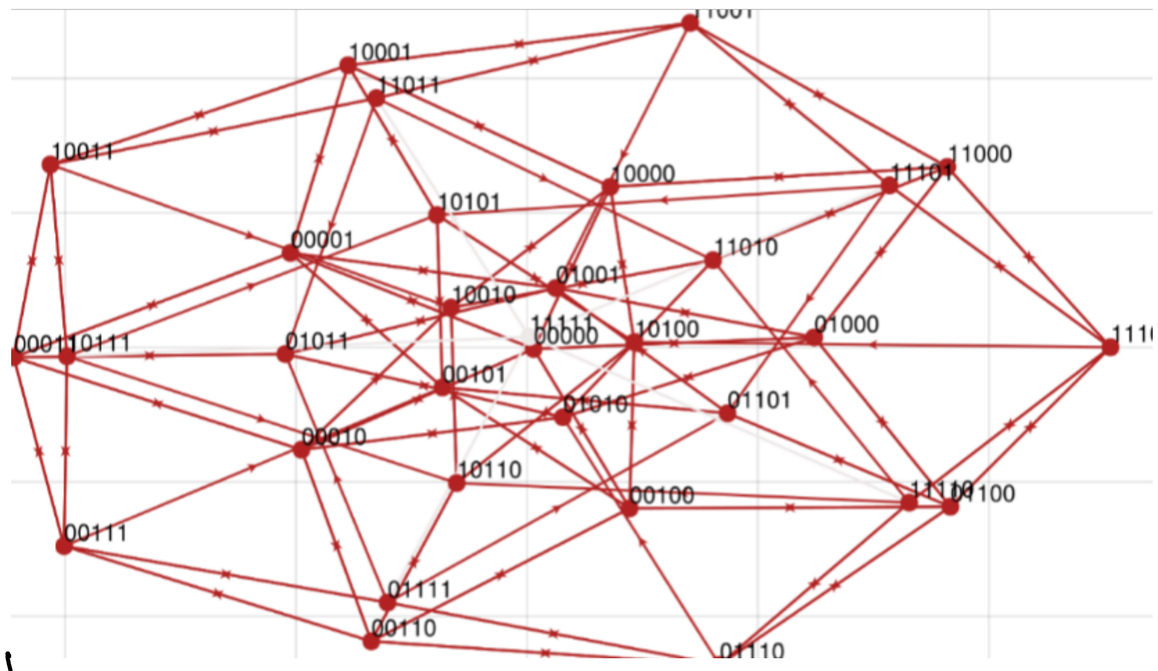
1111
1111 - not much to say, everything is allowed

Spectral looks similar

7 enzyme subrules

50 - all 0s only cc,

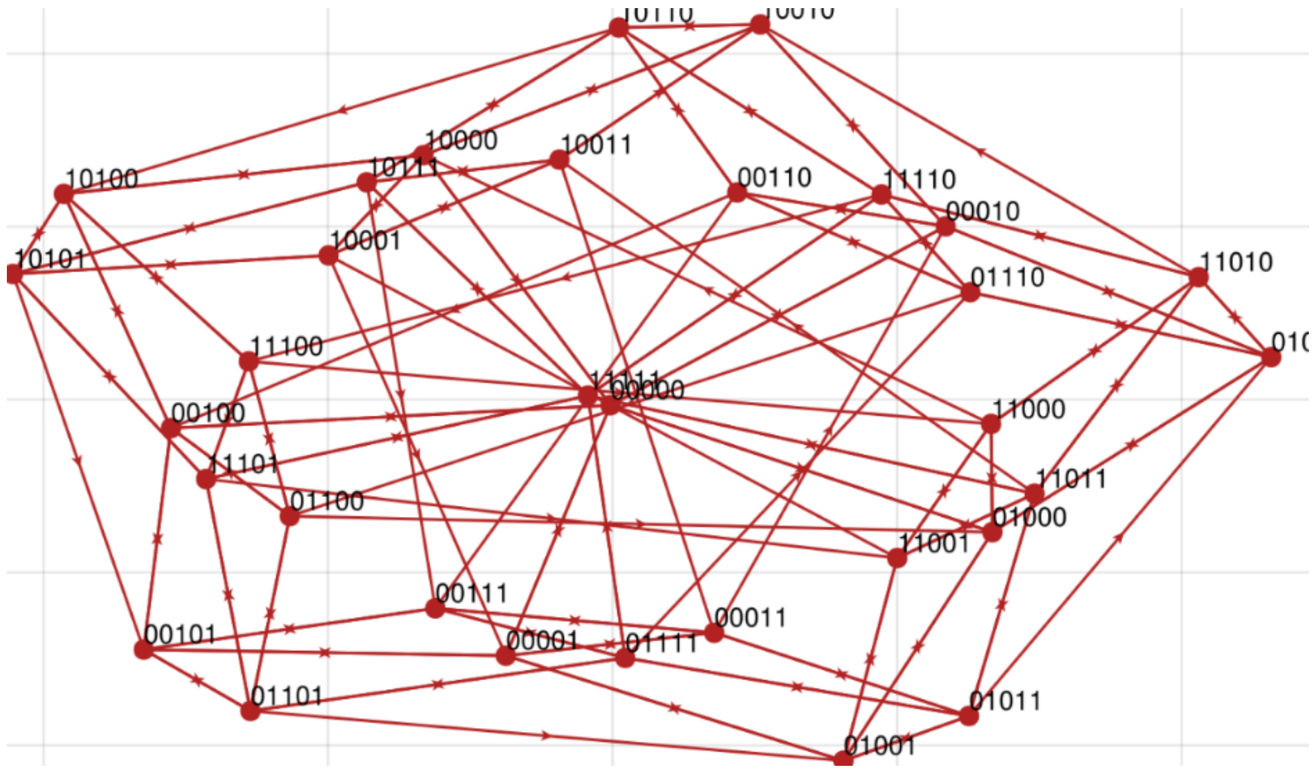
0111
1111 makes sense as cannot
↕ T+t be escaped
19



1111
1011 - all except all 1s is in one big cc, note that cc is not fully diff. as it is biased towards 0s

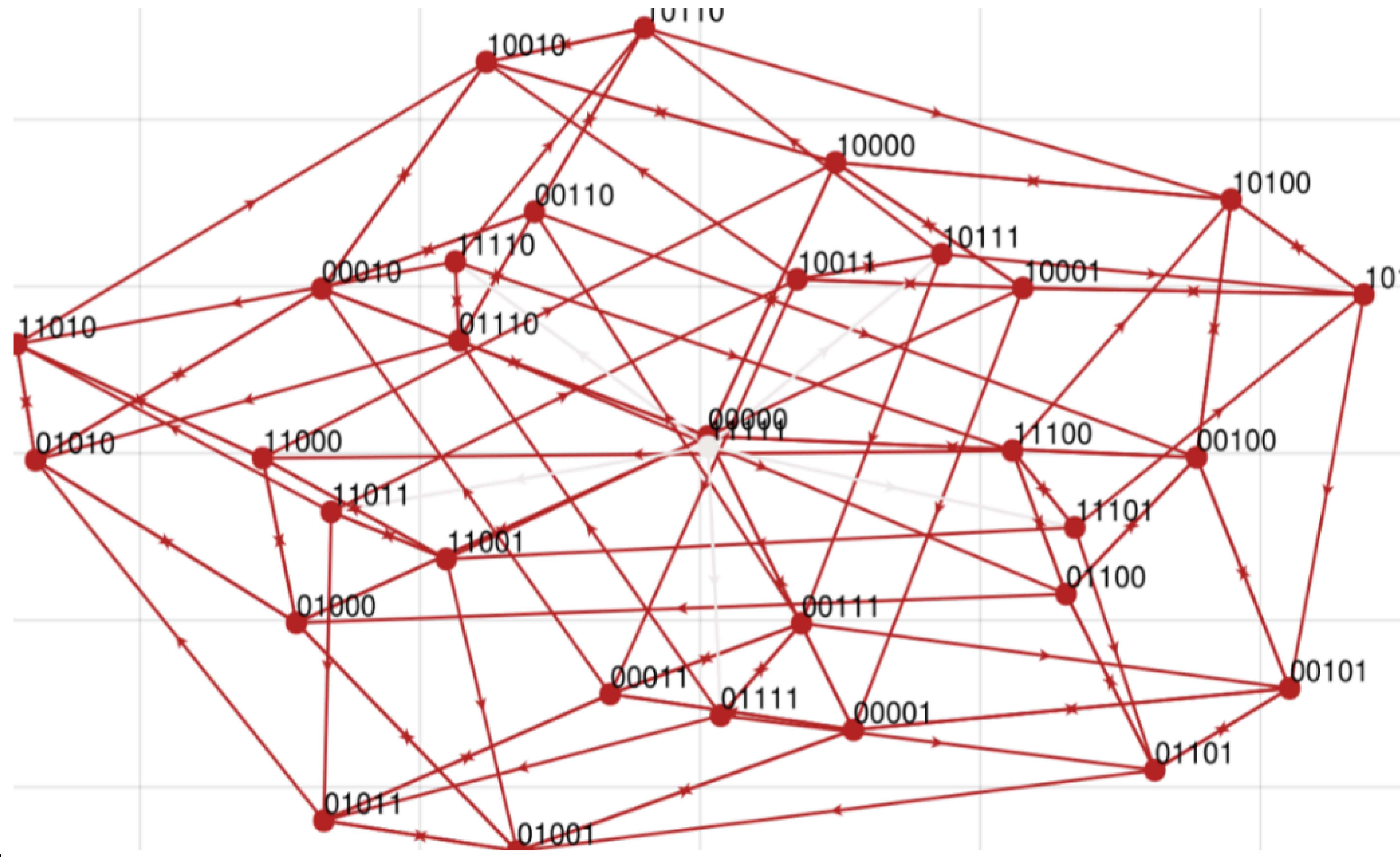
35 - all one big cc biased (but only partly) towards 0s and seemingly somewhat chaotic

0111
0111



Genzyme subrules

starting with the more biased ones



34 – all 0s only ac

01 11
01 11

↕ T##

3 – logically, mix of 19 and 35, has one complex non-diff. ac which includes everything except all 1s

11 11
00 11

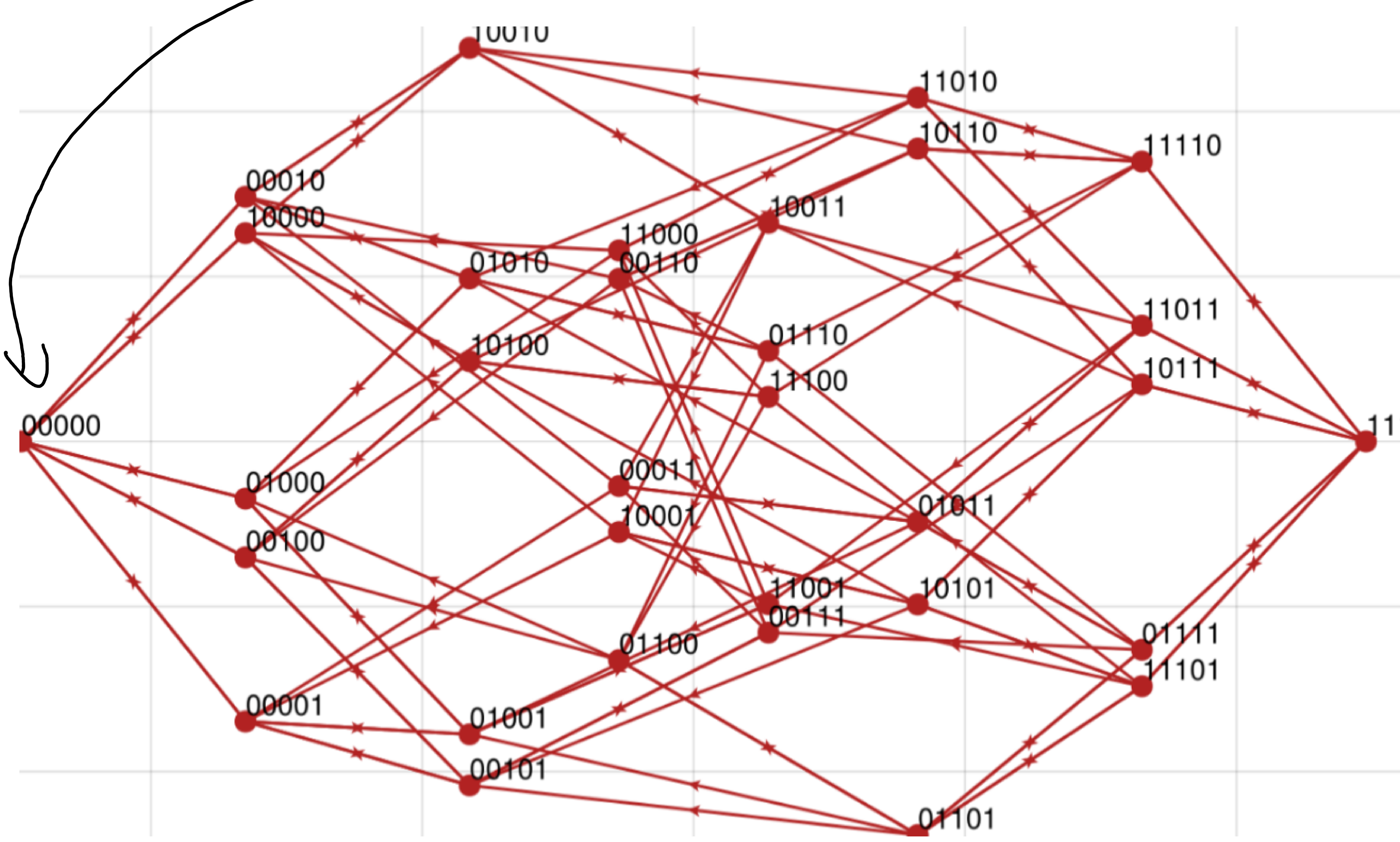
18 – all 0s only ac

01 11
10 11

one complex ac includes all states

33 – Spectral layout changed significantly though how all 1s and all 0s are opposing each other!

10 11
01 11

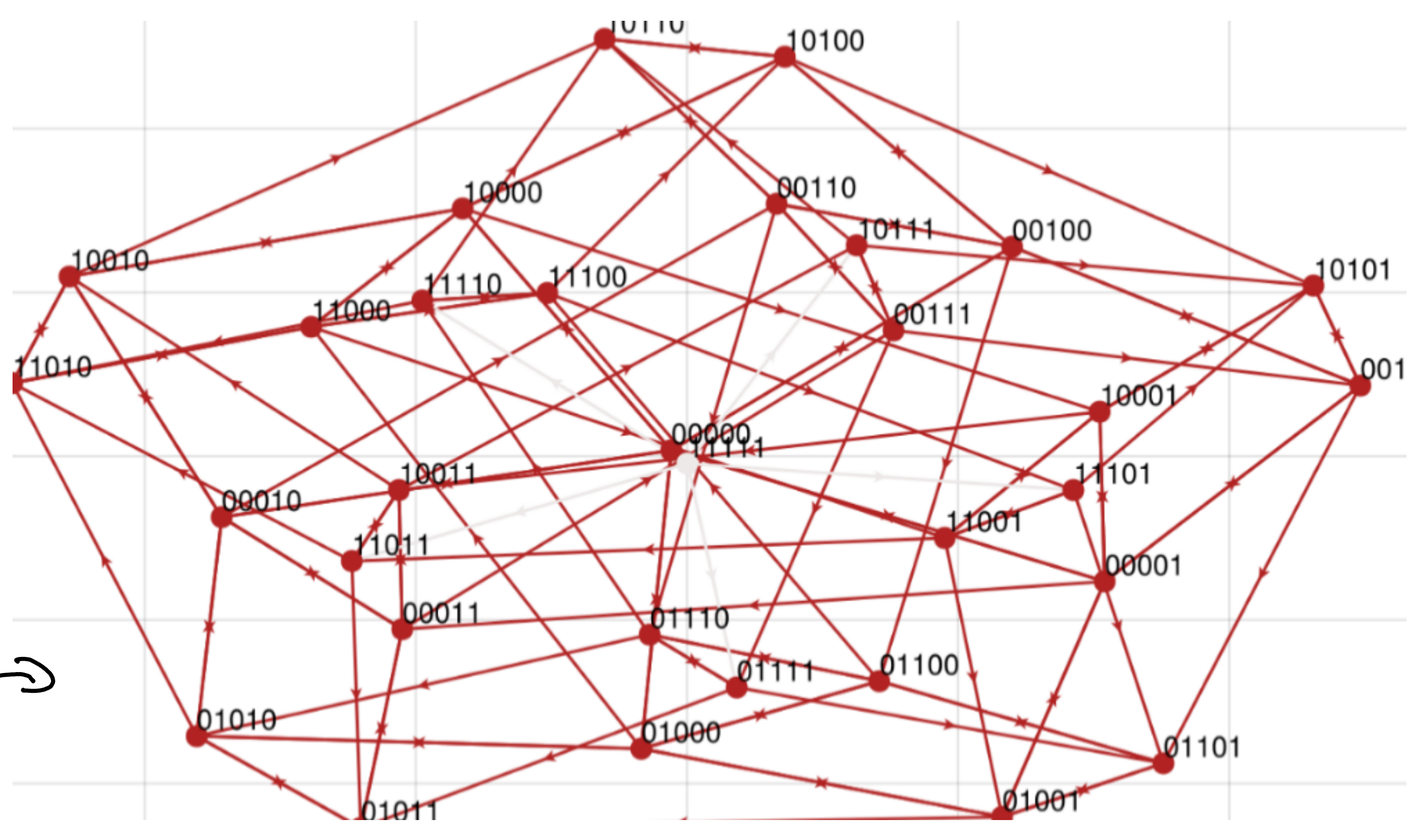


58 – all 0s only cc

01 10
11 11
⇕T+L

27 – one complex cc including everything but all 1s, also looks chirally →

11 10
10 11



178 – all 0s and all 1s only ccs

01 11
11 10
⇕T+T

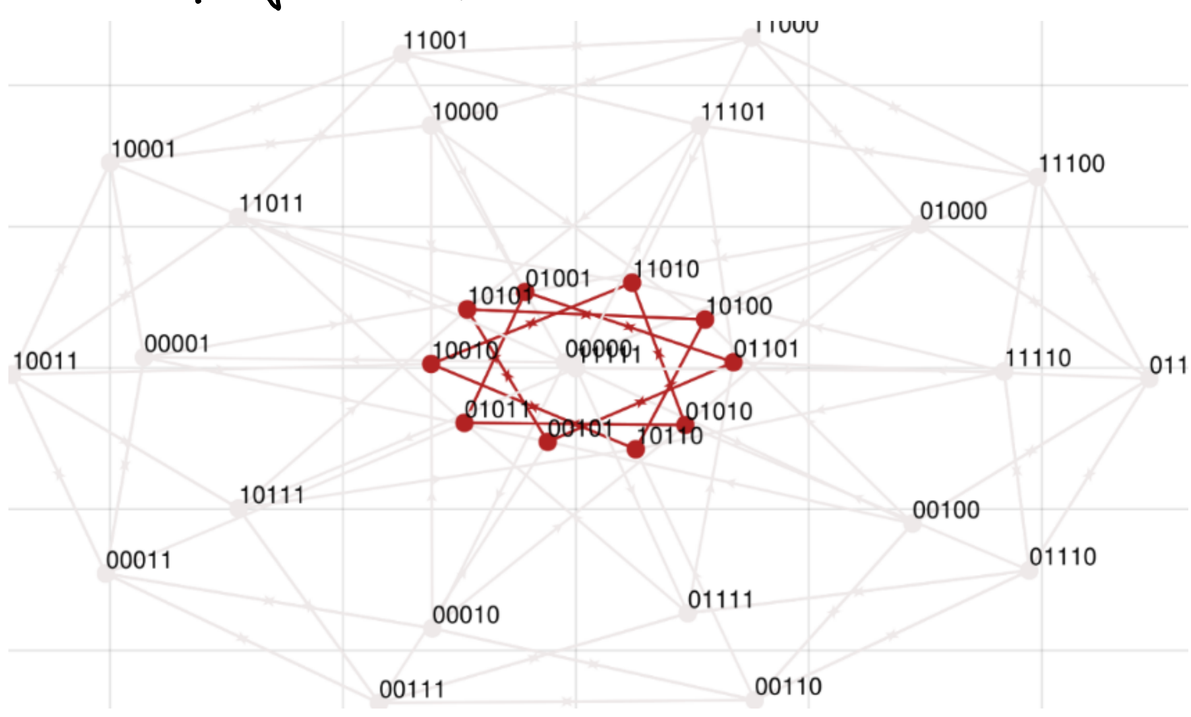
at odd N makes a single 2 3 – single-state loop cc and a very interesting one ∀

11 01
10 11

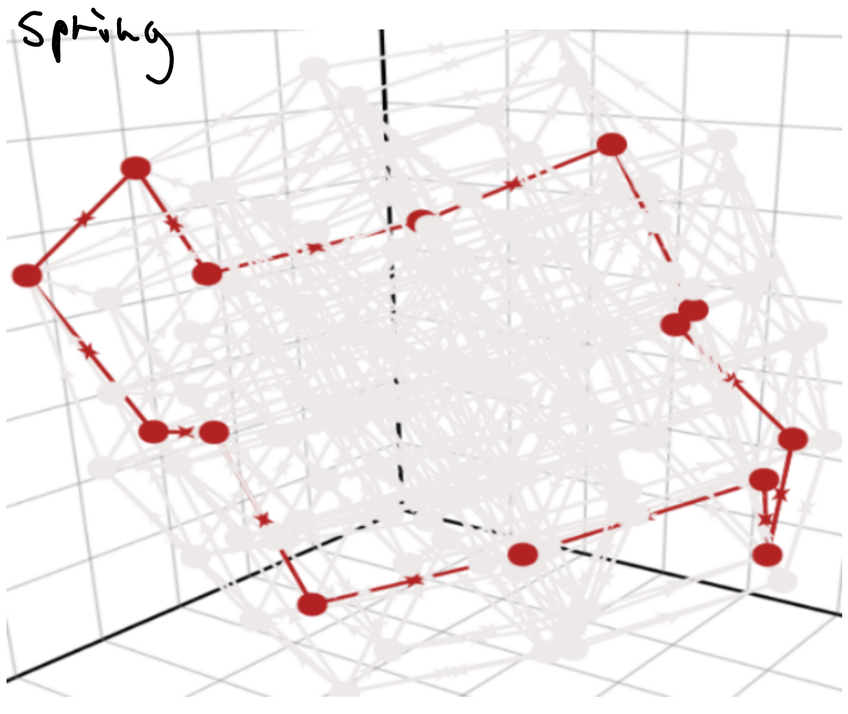
the system gets almost stable full of many domains and they shift one after another (diffusive though)

at even N gets stuck in $\overline{01}$

N=5, spectral →



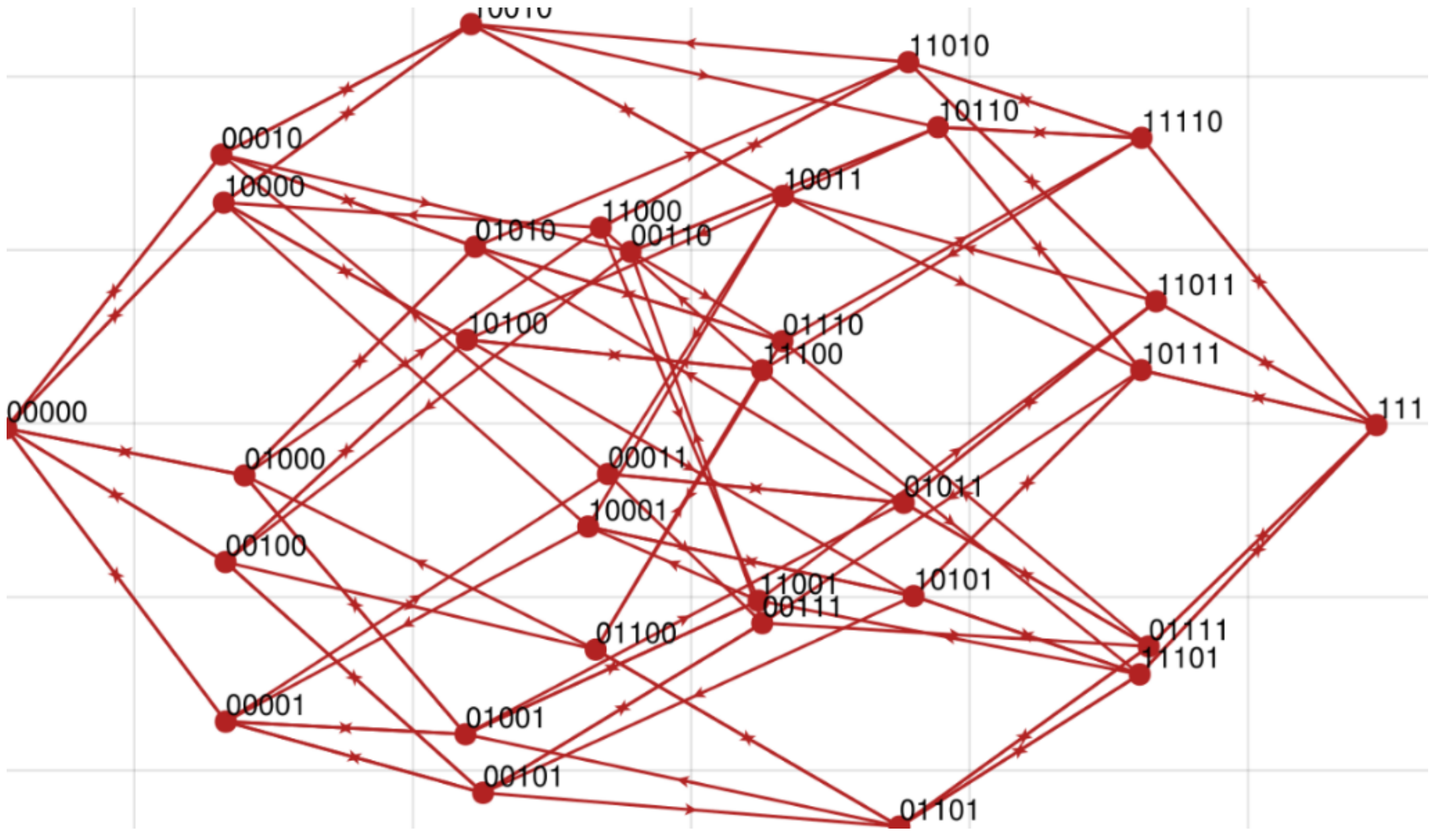
N=7, spring →



43 – very similar to 33

11 10
01 11

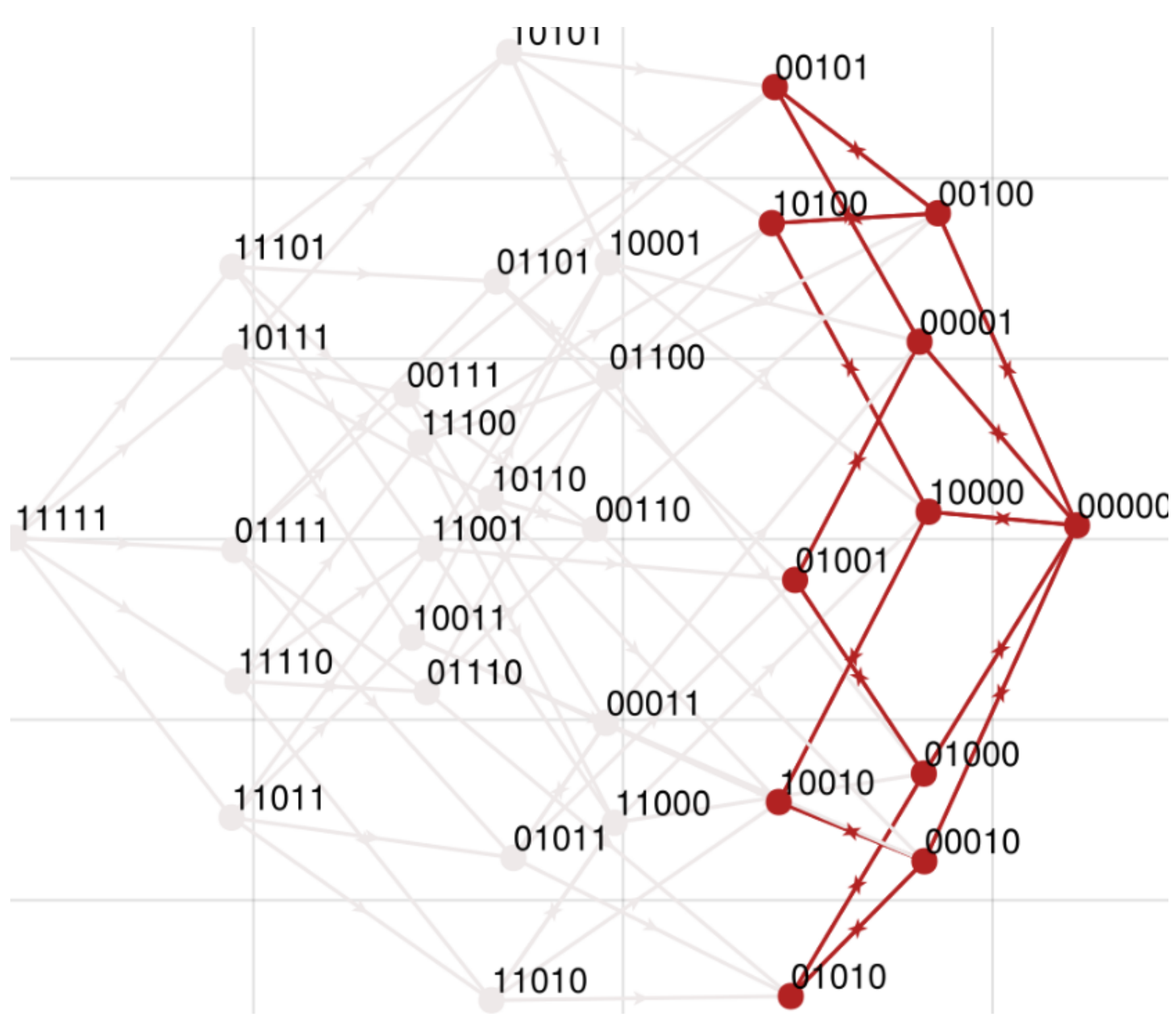
but enriched →



5 elzgme subrules

1 — 1 diff ac connecting
10 11
00 11
↕ Tff
to all 0s

32 — all 0s only ac
00 11
01 11



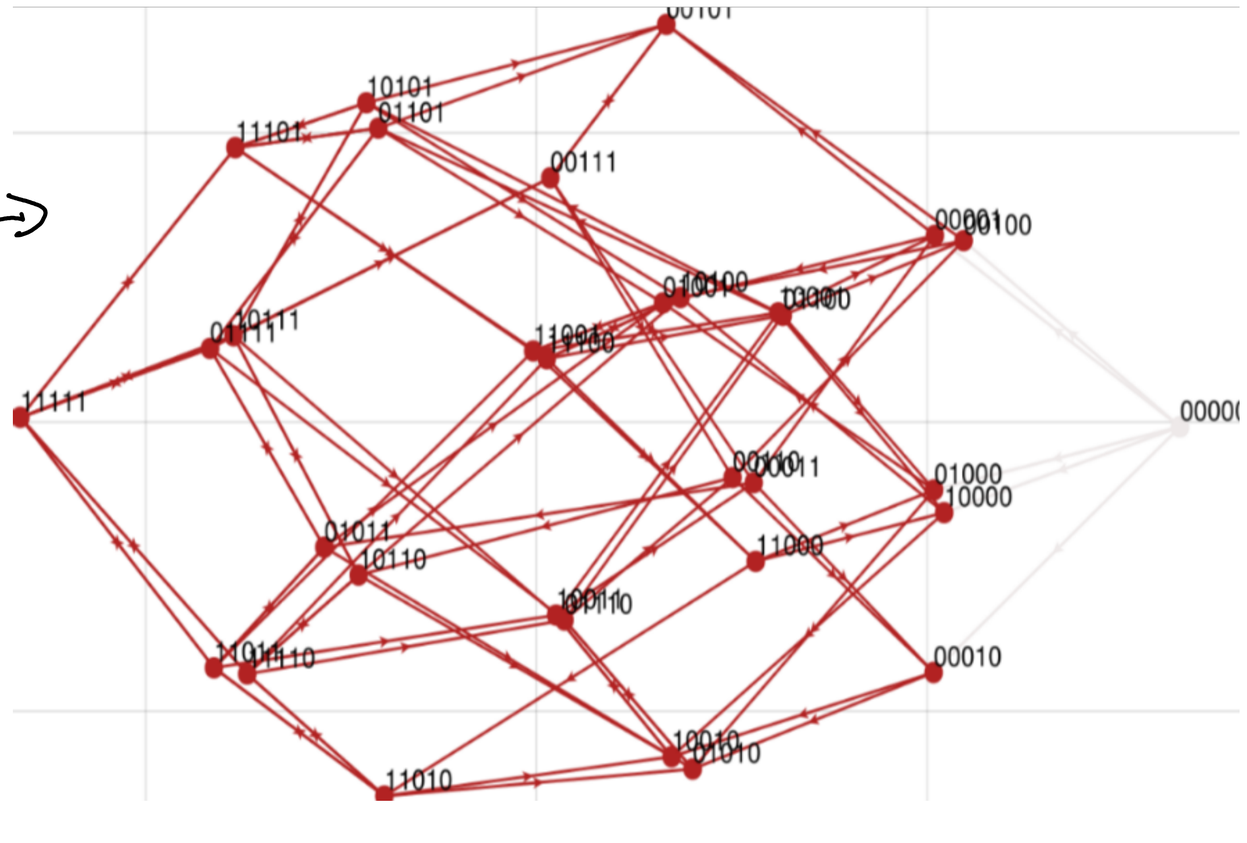
2 — all 0s only ac
01 11
00 11

at .IIN makes one big →

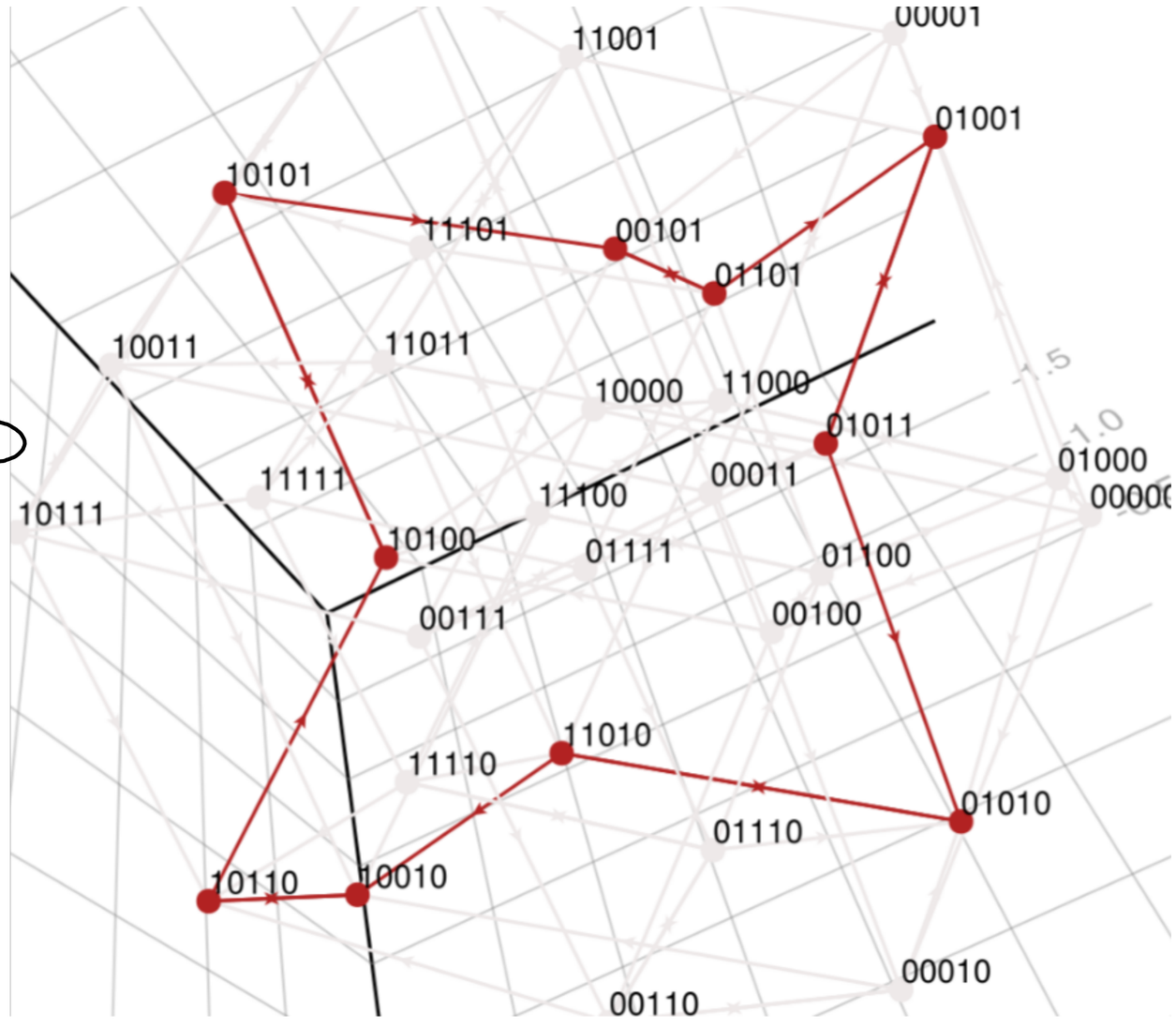
37 — ac including all but all 0s

10 01
01 11
↕ T
at elemN gets stuck in $\overline{001}$

12 — all 0s only ac
01 10
11 01



7 — interesting loops at odd N
gets stuck in $\overline{01}$ at elemN
very similar to 23 but
every other arrow in loop →
is directed ✳
11 01
00 11
↑ Tff



162 — all 0s and all 1s
only acs
01 11
01 10

42 – all 0s only ac

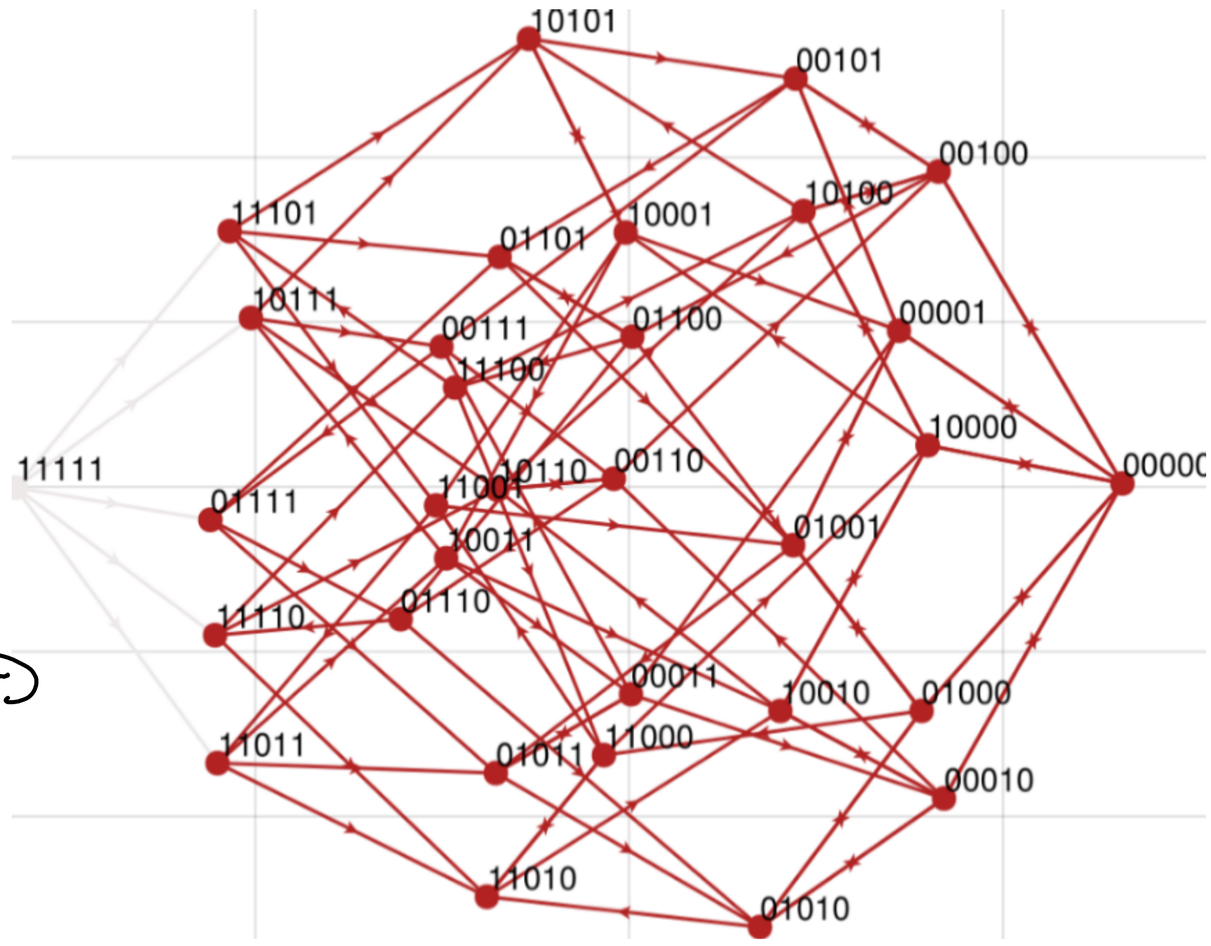
01 10
01 11

↓ TH

11 – long similar to 37, has one

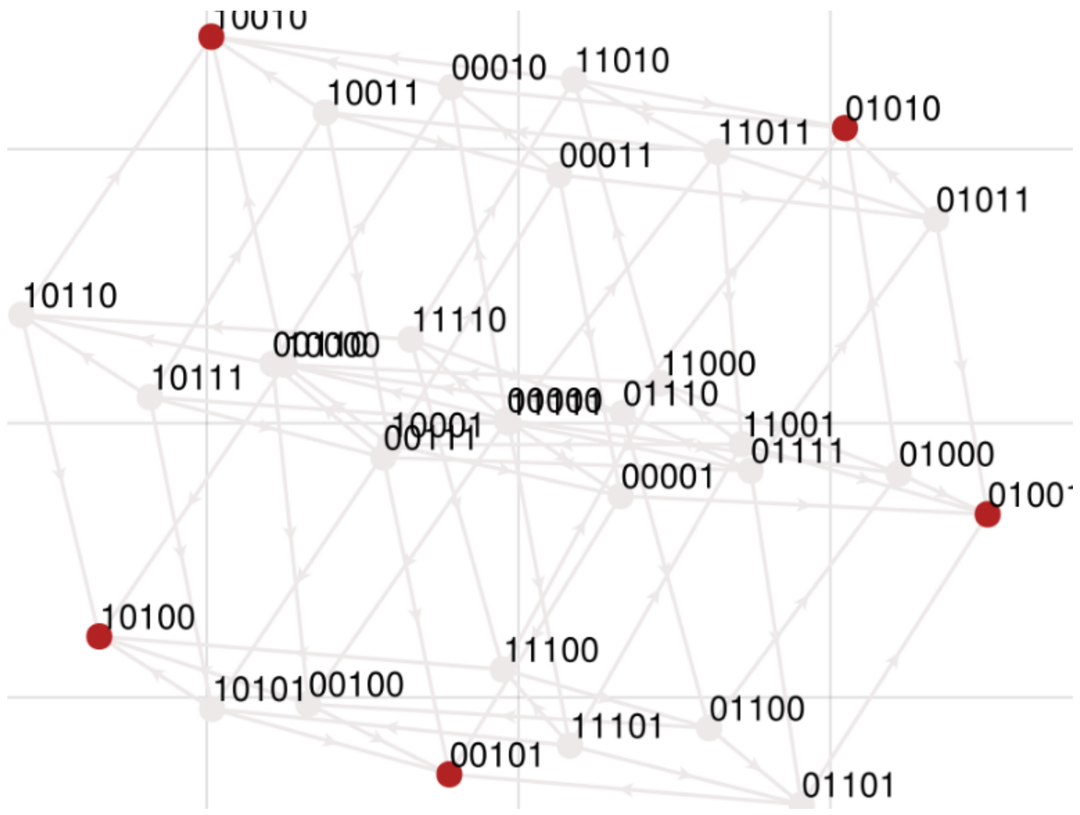
11 10
00 11

complex ac connecting all but →
all 1s



26 – all 0s only ac

01 10
10 11



4 engine subrules

0 – all 0s only ac

00 11
0 0 1 1

10 – all 0s only ac

01 10
0 0 1 1

5 – multiple single-state
acs – all full of isolated
1s

10 01
0 0 1 1

↓ T++

160 – all 0s and all 1s only acs

00 11
01 10

120 - all 0s and all 1s only ccs

0 1 10
0 1 10
↓ T++

15 - final continuation of

11 00
00 11

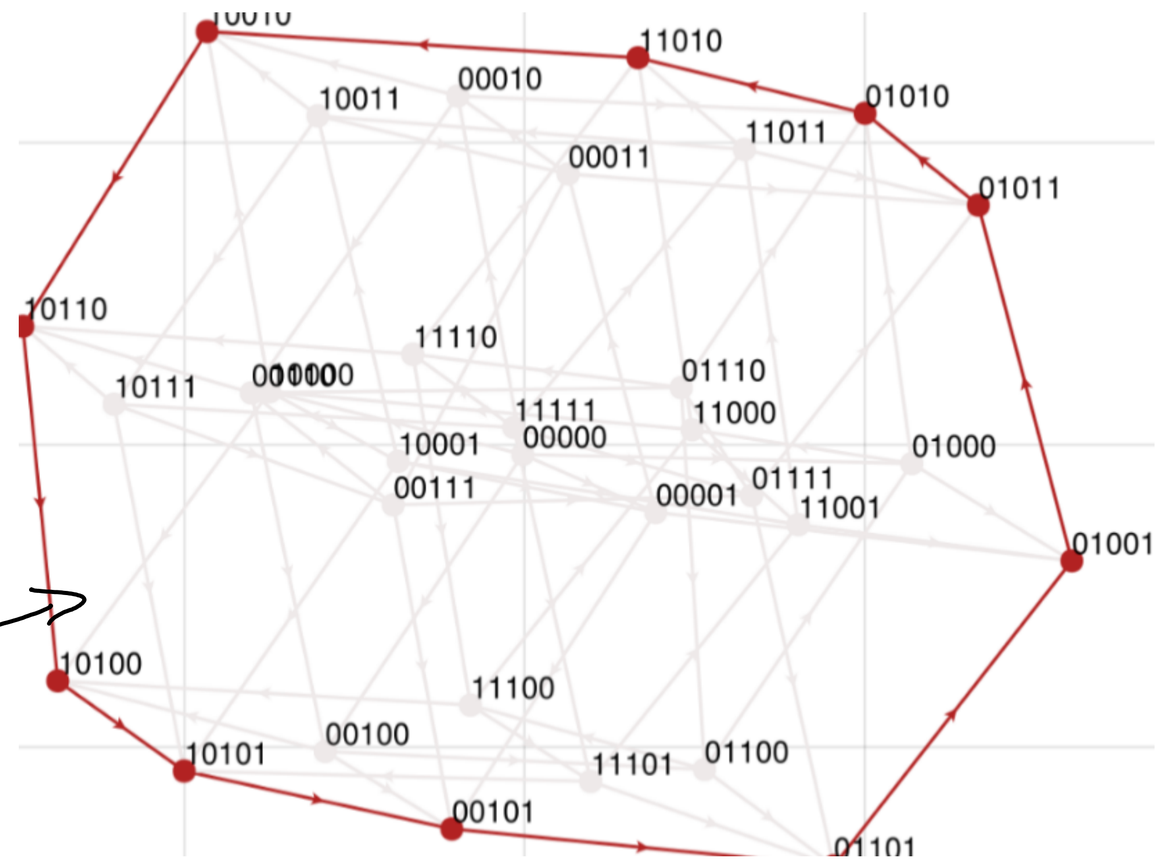
23 and 7,

has now fully directed

loops at odd N

and gets stuck at $\overline{01}$

in even N



90 - all 0s only cc wires

0 1 10
10 0 1

N is divisible by

3 in which case

it gets stuck at $\overline{110}$

which are 3 additional

ccs