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Lab 2 - Statistics

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22.2.2022

Classical definition

Classical definition

Probability of event A is the ratio of its occurrence to the number of experiments:

$$P(A) = \frac{N_A}{N}$$

Limit definition

Probability of event A is its relative occurrence in an infinite number of experiments:

$$P(A) = \lim_{n \to \infty} \frac{N_A}{N}$$

Axiomatic definition- *sigma*-algebra

σ-algebra

A field of events $\mathcal{A}$ is a σ-algebra over Ω e.g:

$$\Omega \in \mathcal{A} \tag{1}$$

$$A \in \mathcal{A} \implies A^c \in \mathcal{A} \tag{2}$$

$$(\forall n \in \mathbb{N})(A_n \in \mathcal{A}) \implies \bigcup_{n \in \mathbb{N}} A_n \in \mathcal{A} \tag{3}$$

Elementary event

Elementary event is an event $\in \Omega$ which cannot be subdivided into other events.

Axiomatická definícia - probability

Probability

Probability is a function $P : \mathcal{A} \to \langle 0, 1 \rangle$, such that:

$$P(\Omega) = 1 \tag{4}$$

$$P\left(\bigcup_{n\in\mathbb{N}} A_n\right) = \sum_{n\in\mathbb{N}} P(A_n), \tag{5}$$

if A_n is a sequence of mutually exclusive events.

Probability space

Probability space is a triplet $(\Omega, \mathcal{A}, P)$.

Operations

Union

The union $A \cup B$ occurs if at least one of them occurs

Prienik

The intersection $A \cap B$ occurs if both of them occur.

Complementary event

The complementary event to A is A^c.

Disjoint events

If $A \cap B = \emptyset$ then they are disjoint.

Example 1

What is the probability that when throwing a dice:

- We get a number X
  - positive elementary events
  - total elementary events
- We roll an even number
  - positive elementary events
  - total elementary events

Example 1

What is the probability that when throwing a dice:

- We get a number X
  - positive elementary events = 1
  - total elementary events = 6
- We roll an even number
  - positive elementary events
  - total elementary events

Example 1

What is the probability that when throwing a dice:

- We get a number $X = \frac{1}{6}$
  - positive elementary events = 1
  - total elementary events = 6
- We roll an even number
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  - total elementary events

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What is the probability that when throwing a dice:

- We get a number X $= \frac{1}{6}$
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- We roll an even number
  - positive elementary events = 3
  - total elementary events = 6

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What is the probability that when throwing a dice:

- We get a number $X = \frac{1}{6}$
  - positive elementary events = 1
  - total elementary events = 6
- We roll an even number $= \frac{1}{2}$
  - positive elementary events = 3
  - total elementary events = 6

Example 2

Our shipment contains 50 nuts and 150 screws. Half of the nuts and screws are corroded. If we randomly select a component what is the chance that it will be a nut or that it will be corroded.

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- A the item is corroded - 100.
- B the item is a nut - 50.

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- A the item is corroded - 100.
- B the item is a nut - 50.
- $A \cap B$ item is a corroded nut - 25.

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Our shipment contains 50 nuts and 150 screws. Half of the nuts and screws are corroded. If we randomly select a component what is the chance that it will be a nut or that it will be corroded.

The experiment has 200 outcomes

- A the item is corroded - 100.
- B the item is a nut - 50.
- $A \cap B$ item is a corroded nut - 25.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{100}{200} + \frac{50}{200} - \frac{25}{200} = \frac{5}{8}$

Conditional probability

Probability that A occurs under the condition that B occured:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Example 3

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- B at least one is male $\{(m,m),(m,f),(f,m)\}$

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- $\Omega = \{(m, m), (m, f), (f, m), (f, f)\}$
- A both are male $\{(m, m)\}$.
- B at least one is male $\{(m, m), (m, f), (f, m)\}$
- $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

Theorem of total probability

Partition of a sample space

$\{B_1, B_2, ...\}$ is a finite or countably infinite partition of a sample space Ω when they are all pairwise disjoint and $\cup_{i \in \hat{n}} B_i = \Omega$.

Law of total probability

Let $\{B_1, ..., B_n\}$ is a finite partition of a sample space Ω. Then for event $A \in \Omega$:

$$P(A) = \sum_{k \in \hat{n}} P(A|B_i)P(B_i)$$

Example 4

Electrical lightbulbs are produced in 3 factories. The first produces 25%, the second produces 40% and the third 35% of the shipment. Out of the total production of the first factory 88% of the first factory, 75% of the second, 85% of the third are standard. What is the probability that a randomly selected lightbulb is standard.

Example 4

Electrical lightbulbs are produced in 3 factories. The first produces 25%, the second produces 40% and the third 35% of the shipment. Out of the total production of the first factory 88% of the first factory, 75% of the second, 85% of the third are standard. What is the probability that a randomly selected lightbulb is standard.

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- B_i is the probability that the lightbulb was manufactured in the i-th factory
- $P(B_1) = 0.25, P(B_2) = 0.4, P(B_3) = 0.35$

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- $P(B_1) = 0.25, P(B_2) = 0.4, P(B_3) = 0.35$
- $P(A|B_1) = 0.88, P(A|B_2) = 0.75, P(A|B_3) = 0.85$

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- $P(A) = \sum_{k \in \hat{3}} P(A|B_i)P(B_i)$
- $P(A) = 0.88 \cdot 0.25 + 0.75 \cdot 0.4 + 0.85 \cdot 0.35 = 0.8175$

Example 5

A vacuum tube in a TV can come from 4 manufacturers with probabilities 0,2; 0,3; 0,35; 0,15. The probabilities that the tubes will last the proper amount of time before breaking are 0,45; 0,60; 0,75; 0,30 respectively. What is the probability that a randomly selected tube will last the proper amount of time?

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A vacuum tube in a TV can come from 4 manufacturers with probabilities 0,2; 0,3; 0,35; 0,15. The probabilities that the tubes will last the proper amount of time before breaking are 0,45; 0,60; 0,75; 0,30 respectively. What is the probability that a randomly selected tube will last the proper amount of time?

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- B_i is the probability that the vacuum tube was manufactured in the i-th factory
- $P(B_1) = 0.2, P(B_2) = 0.3, P(B_3) = 0.35, P(B_4) = 0.15$

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- B_i is the probability that the vacuum tube was manufactured in the i-th factory
- $P(B_1) = 0.2, P(B_2) = 0.3, P(B_3) = 0.35, P(B_4) = 0.15$
- $P(A|B_1) = 0.45, P(A|B_2) = 0.6, P(A|B_3) = 0.75, P(A|B_4) = 0.3$

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- B_i is the probability that the vacuum tube was manufactured in the i-th factory
- $P(B_1) = 0.2, P(B_2) = 0.3, P(B_3) = 0.35, P(B_4) = 0.15$
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- $P(A|B_1) = 0.45, P(A|B_2) = 0.6, P(A|B_3) = 0.75, P(A|B_4) = 0.3$
- $P(A) = \sum_{k \in \hat{4}} P(A|B_i)P(B_i)$
- $P(A) = 0.45 \cdot 0.2 + 0.6 \cdot 0.3 + 0.75 \cdot 0.35 + 0.3 \cdot 0.15 = 0.5775$

Example 6

In a field test of reliability 10 bikes of first series remained which showed reliability of 85%, 8 bikes of second series with reliability rate of 75%, 5 bikes of the third series with a reliability rate of 60%. What is the probability that a randomly selected bike will prove to not be reliable the next day?

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In a field test of reliability 10 bikes of first series remained which showed reliability of 85%, 8 bikes of second series with reliability rate of 75%, 5 bikes of the third series with a reliability rate of 60%. What is the probability that a randomly selected bike will prove to not be reliable the next day?

- Let A be the probability that the bike will not be reliable.

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- B_i is the reliability of a bike of the i-th series.

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- $P(A) = \sum_{k \in \hat{3}} P(A|B_i)P(B_i)$
- $P(A) = 0.85 \cdot \frac{8}{23} + 0.75 \cdot \frac{8}{23} + 0.6 \cdot \frac{5}{23} = 0.24$

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- B_i is the reliability of a bike of the i-th series.
- $P(A) = \sum_{k \in \hat{3}} P(A|B_i)P(B_i)$
- $P(A) = 0.85 \cdot \frac{8}{23} + 0.75 \cdot \frac{8}{23} + 0.6 \cdot \frac{5}{23} = 0.24$
- Výsledok = $P(A^c) = 1 - P(A) = 0.76$

Example 7

We have a dark cellar with Kompots in shelves. The first shelf contains 6 blueberry and 10 raspberry kompots. The second shelf contains 7 blueberry and 5 raspberry kompots. Marienka moved one kopmot from the first shelf onto the second one. Janko has then chosen a kompot a) from the first shelf, b) from the second shelf. What is the probability that he chose a raspberry kompot?
Riešenie a)

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- B_1 - 5 B a 10 R on the first shelf
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- $P(A|B_1) = \frac{10}{15}, P(A|B_2) = \frac{9}{15}$

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- B_1 - 5 B a 10 R on the first shelf
- B_2 - 6 B a 9 R on the first shelf
- $P(A|B_1) = \frac{10}{15}, P(A|B_2) = \frac{9}{15}$
- $P(A) = \frac{10}{15} \cdot \frac{6}{16} + \frac{9}{15} \cdot \frac{10}{16} = 0.625$

Example 8

Katarína A., Lenka B. a Monika C. study together for maturita. Out of 30 questions they only studied for 20 questions. What is the probability that a) Lenka b) Monika selects a question she knows if the go in alphabetical order?
Riešenie a)

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Riešenie a)

- B_1 - Katka chooses a question she knows
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- $P(A|B_1) = \frac{20}{30}, P = \frac{10}{30}$

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Riešenie a)

- B_1 - Katka chooses a question she knows
- B_2 - Katka chooses a question she doesn't know
- $P(A|B_1) = \frac{20}{30}, P = \frac{10}{30}$
- $P(A) = \frac{19}{29} \cdot \frac{20}{30} + \frac{20}{29} \cdot \frac{10}{30} = \frac{2}{3}$

Bayes law

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Example 9

Karol had a chest x-ray with a positive result for cancer. What is the probability of Karol having cancer if we know that the test is falsely positive in 5% of cases and falsely negative in 40% of cases. We also know that only one in 500 workers in Karols job have cancer.

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- $P(A_p) = 0.002$ - cancer, $P(A_n) = 0.998$ - healthy
- B - positive test

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- $P(A_p) = 0.002$ - cancer, $P(A_n) = 0.998$ - healthy
- B - positive test
- $P(B|A_p) = 0.6, P(B|A_n) = 0.05$

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- $P(A_p) = 0.002$ - cancer, $P(A_n) = 0.998$ - healthy
- B - positive test
- $P(B|A_p) = 0.6, P(B|A_n) = 0.05$
- $P(A_p|B) = \frac{0.6 \cdot 0.002}{0.6 \cdot 0.002 + 0.05 \cdot 0.998} = 0.024$

Example 10

Peter had the same test done and he also got a positive result. However, Peter worked in a mine for 20 years. Peter knows that 15% of his former colleagues have cancer. What is the probability that Peter has cancer.

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- $P(A_p) = 0.15$ - cancer, $P(A_n) = 0.85$ - healthy
- B - positive test
- $P(B|A_p) = 0.6, P(B|A_n) = 0.05$
- $P(A_p) = \frac{0.6 \cdot 0.15}{0.6 \cdot 0.15 + 0.05 \cdot 0.85} = 0.679$

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Let us assume that we have a school with 60% boys and 40% girls. All the boys wear trousers and so do half of the girls. What is the probability that a random student in trousers is a girl?

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- $P(A_c) = 0.6$ - boy, $P(A_d) = 0.4$ - girl
- B - trousers

Example 11

Let us assume that we have a school with 60% boys and 40% girls. All the boys wear trousers and so do half of the girls. What is the probability that a random student in trousers is a girl?

- $P(A_c) = 0.6$ - boy, $P(A_d) = 0.4$ - girl
- B - trousers
- $P(B|A_c) = 1, P(B|A_d) = 0.5$

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- $P(A_c) = 0.6$ - boy, $P(A_d) = 0.4$ - girl
- B - trousers
- $P(B|A_c) = 1, P(B|A_d) = 0.5$
- $P(A_p) = \frac{0.5 \cdot 0.4}{0.5 \cdot 0.4 + 0.6} = 0.25$

Random variable

Random variable

A function whose value is determined as an outcome of a random experiment.
Maps a numerical value to each event.

Distribution function

Describes the distribution of probability of a random variable defined on the probability space.

Example 12

We have 4 independent machines. The first breaks with probability of 10%, the second with 15%, the third 30% and the fourth with 50%. The random variable X determines the number for broken machines. Describe this random variable. E.g. calculate the probabilities for all of the possible values.

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- $P(X = 4) = 0.1 \cdot 0.15 \cdot 0.3 \cdot 0.5 = 0.00225$

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- $P(X = 4) = 0.1 \cdot 0.15 \cdot 0.3 \cdot 0.5 = 0.00225$
- $P(X = 0) = 0.9 \cdot 0.85 \cdot 0.7 \cdot 0.5 = 0.26775$

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E.g. calculate the probabilities for all of the possible values.

- $P(X = 4) = 0.1 \cdot 0.15 \cdot 0.3 \cdot 0.5 = 0.00225$
- $P(X = 0) = 0.9 \cdot 0.85 \cdot 0.7 \cdot 0.5 = 0.26775$
- $P(X = 1) = P(S_1 \cap S_2^c \cap S_3^c \cap S_4^c) + P(S_1^c \cap S_2 \cap S_3^c \cap S_4^c) + +P(S_1^c \cap S_2^c \cap S_3 \cap S_4^c) + P(S_1^c \cap S_2^c \cap S_3^c \cap S_4) = 0.4595$

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- $P(X = 4) = 0.1 \cdot 0.15 \cdot 0.3 \cdot 0.5 = 0.00225$
- $P(X = 0) = 0.9 \cdot 0.85 \cdot 0.7 \cdot 0.5 = 0.26775$
- $P(X = 1) = P(S_1 \cap S_2^c \cap S_3^c \cap S_4^c) + P(S_1^c \cap S_2 \cap S_3^c \cap S_4^c) + +P(S_1^c \cap S_2^c \cap S_3 \cap S_4^c) + P(S_1^c \cap S_2^c \cap S_3^c \cap S_4) = 0.4595$
- $P(X = 2) = 0.23, P(X = 3) = 0.0405$