Machine Learning for Computer Vision

Exercise 8

Kodai Matsuoke, Yuyan Li

June 27, 2017

1 Random Walker

1.1 1D Random walker

Suppose x_i is the probability that a random walk particle started from position i first reaches to Seed A. Obviously, for i $\in \{0, 1, 2, 3\}, x_i = 1.$ also for i $\in \{8, 9, 10, 11, 12\}, x_i = 0.$
So now, we want to calculate x_4, x_5, x_6, x_7. We formulate this problem as follows.

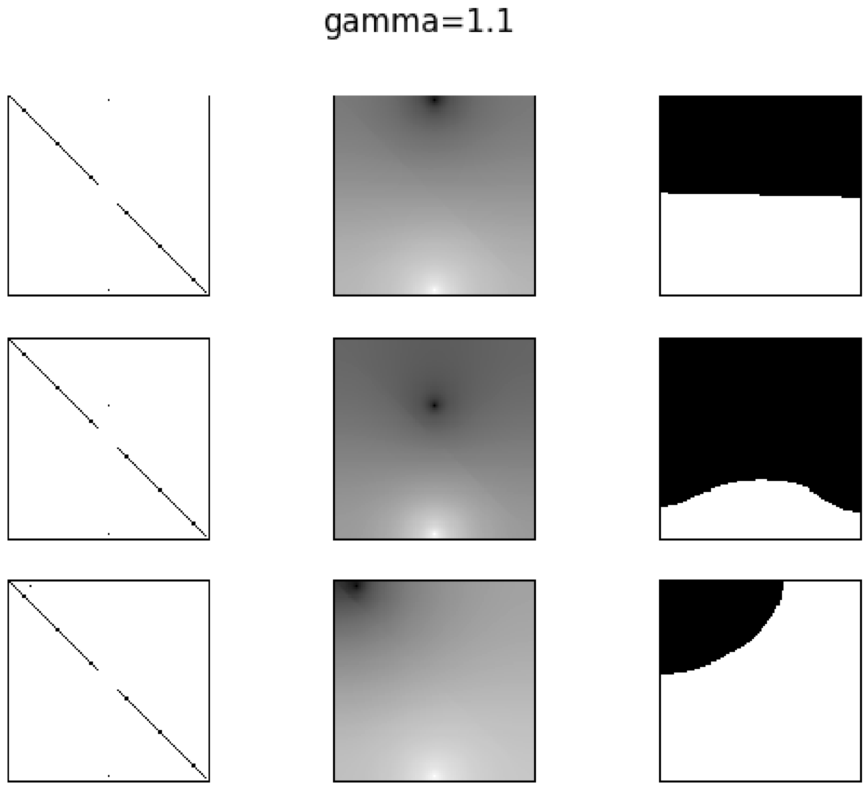
$$x = (x_4, x_5, x_6, x_7) \quad x_m = (x_3, x_8) \quad A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}$$

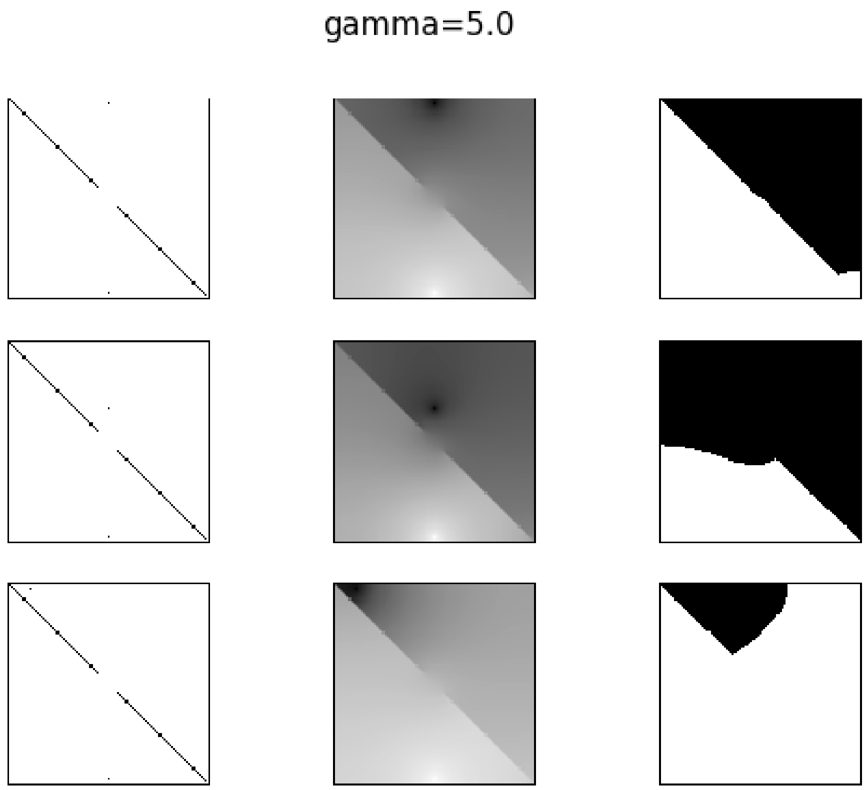
By solving $Ax = -Bx_m$, we obtain $x = (0.8, 0.6, 0.4, 0.2)$. As well we can calculate the probability for Seed B. The overall probability is as shown below.

1_1.png

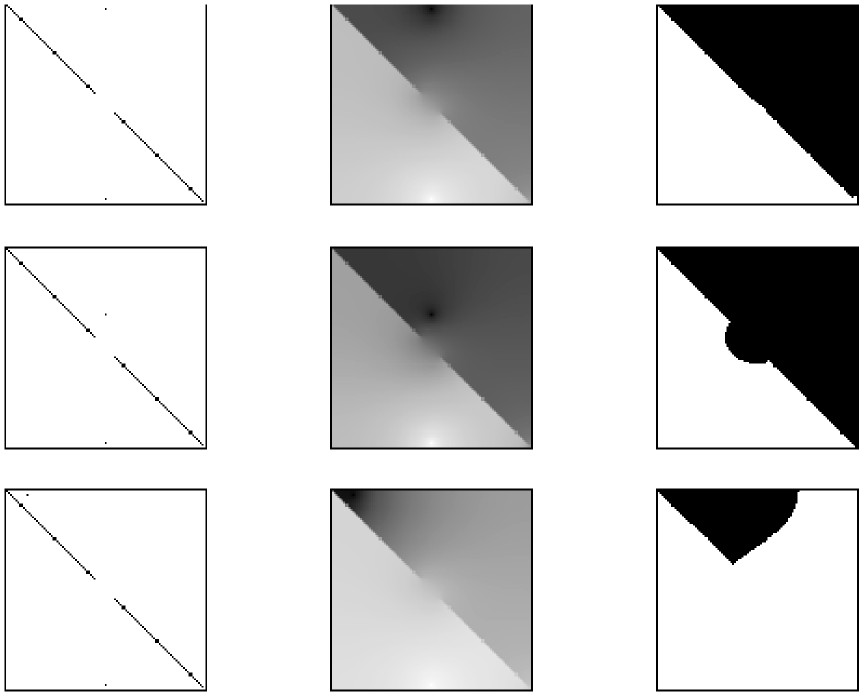
1.2 Implementation

Below is our simple implementation of the random walker. We experimented with different seed positions and different values of γ. One can see that for small γ the random walker doesn't properly respect the borders in the picture. For sufficiently large values of γ the algorithm gives a satisfying result.

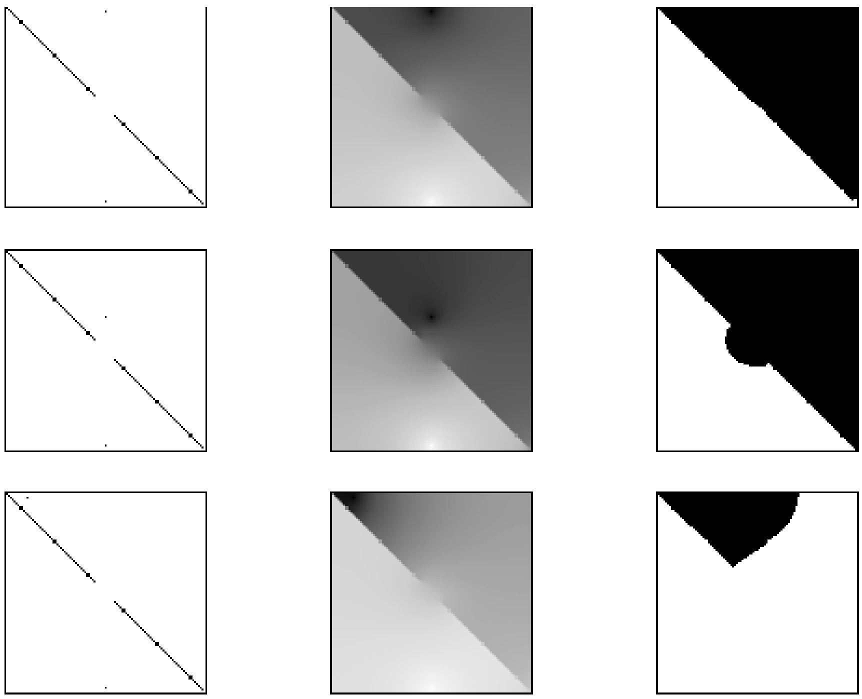

gamma=1.1


gamma=5.0

gamma=10.0



gamma=100.0



Listing 1: random$_{w}alker.py$

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.sparse
import scipy.sparse.linalg
import time


def buildFancyQ(im, gamma):
    start = time.time()

    nrows, ncols = shape = im.shape
    size = nrows * ncols

    Q = scipy.sparse.lil_matrix((size,size), dtype='float')

    # off diagonal elements
    def getBeta(c0, c1):
        return - np.exp(-gamma * np.linalg.norm(c0 - c1))

    # vectorized index
    def getInd(x, y):
        return y + x * ncols

    for x in range(nrows):
        for y in range(ncols):
            c0 = im[x,y]
            i = getInd(x,y)

            if x < ncols-1:
                j = getInd(x+1, y)

                beta = getBeta(c0, im[x+1, y])
                Q[i,j] = beta
                Q[j,i] = beta

                # add values to diagonal elements in same row
                Q[i,i] += abs(beta)
                Q[j,j] += abs(beta)

            if y < nrows-1:
                j = getInd(x, y+1)

                beta = getBeta(c0, im[x, y+1])
                Q[i,j] = beta
                Q[j,i] = beta

                Q[i,i] += abs(beta)
                Q[j,j] += abs(beta)


    print("Fancy Q built in", time.time()-start)
```

```
    return Q.tocsc()


def simple_random_walker(img, fg, bg, gamma=1.0):
    """
    A simple random walker that takes one seed for foreground and
        background each.
    img: image array
    fg: indices of foreground seed
    bg: indices of background seed
    gamma: weight parameter

    return: probability image, segmentation image

    Using buildFancyQ a laplacian qmat is made
    Bringing qmat into the proper form one can just solve the equation
    A x = B xm
    for x
    (in the code A=a, B=b)
    """
    nrows, ncols = shape = img.shape
    size = nrows * ncols


    def getInd(x, y):
        return y + x * ncols


    # Seed indices
    fg_ind = getInd(fg[0], fg[1])
    bg_ind = getInd(bg[0], bg[1])

    label = np.zeros(size)
    label[fg_ind] = 1
    label[bg_ind] = 2

    indizes = np.arange(size)
    unseeded_indizes = indizes[label == 0]
    seeded_indizes = indizes[label > 0]


    qmat = buildFancyQ(img, gamma)
    a = qmat[unseeded_indizes][:, unseeded_indizes]
    b = qmat[unseeded_indizes][:, seeded_indizes]


    xms = []
    for i in range(1,3):
        mask = (label[seeded_indizes] == i)
        xm = scipy.sparse.csr_matrix(mask)
        xm = xm.transpose()
```

```
        xms.append(xm)


    xs = []
    for xm in xms:
        x = scipy.sparse.linalg.spsolve(a, -b.dot(xm))
        xs.append(x)

    label[bg_ind] = 0

    probs = []
    for x in xs:
        prob = np.zeros(size)
        prob[seeded_indizes] = label[seeded_indizes]
        prob[unseeded_indizes] = x
        probs.append(prob)

    pred = np.zeros(size)
    pred[seeded_indizes] = label[seeded_indizes]
    fg_indizes = indizes[probs[0] > probs[1]]
    pred[fg_indizes] = 1


    return probs[0].reshape(img.shape), pred.reshape(img.shape)


def experiment(gamma):
    """
    Experiment with different foreground seed positions

    return: figure with images
    """
    n = 100
    n2 = int(n/2)
    image = np.zeros((n+1, n+1))

    # Add a diagonal line
    for i in range(n):
        image[i,i] = 1

    # Add an opening in the line
    o = int(n/20)
    for i in range(n2-o, n2+o):
        image[i,i] = 0

    # Foreground and background seed
    fgs = [(2, n2), (int(n/3), n2), (2,10)]
    bg = (n-2, n2)

    probs, preds = [], []
    images = []
    for fg in fgs:
```

```
        prob, pred = simple_random_walker(image, fg, bg, gamma)

        # Add a border
        prob = np.lib.pad(prob, ((1,1),(1,1)), 'constant', constant_values
            =(1))
        pred = np.lib.pad(pred, ((1,1),(1,1)), 'constant', constant_values
            =(1))
        probs.append(prob)
        preds.append(pred)

        # Draw seeds
        img = image.copy()
        img[fg] = 1
        img[bg] = 1
        img = np.lib.pad(img, ((1,1),(1,1)), 'constant', constant_values
            =(1))
        images.append(img)


    fig, ax = plt.subplots(3, 3)

    ax[0,0].imshow(images[0], cmap='Greys', interpolation='nearest')
    ax[0,0].axis('off')

    ax[0,1].imshow(probs[0], cmap='Greys')
    ax[0,1].axis('off')

    ax[0,2].imshow(preds[0], cmap='Greys')
    ax[0,2].axis('off')

    ax[1,0].imshow(images[1], cmap='Greys', interpolation='nearest')
    ax[1,0].axis('off')

    ax[1,1].imshow(probs[1], cmap='Greys')
    ax[1,1].axis('off')

    ax[1,2].imshow(preds[1], cmap='Greys')
    ax[1,2].axis('off')

    ax[2,0].imshow(images[2], cmap='Greys', interpolation='nearest')
    ax[2,0].axis('off')

    ax[2,1].imshow(probs[2], cmap='Greys')
    ax[2,1].axis('off')

    ax[2,2].imshow(preds[2], cmap='Greys')
    ax[2,2].axis('off')

    fig.suptitle(f"gamma={gamma}")

    return fig
```

```
if __name__ == "__main__":
    for gamma in [1.1, 5.0, 50.0, 100.0]:
        fig = experiment(gamma)
        g = int(gamma)
        fig.savefig(f"walker{g}")
```

2 Random walker as Anisotropic Diffusion

$$\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2}$$

When a large number of particles make randomwalk, diffusion equation describes the time developement of the particle density. I am going to show it.
Let's suppose that the particle at position x makes a random walk such that after time Δt, it moves to x+Δx with probability 1/2, x-Δx with probability 1/2.
Now, the following equation goes with regard to the particle density ϕ

$$\phi(x, t + \Delta t) = \frac{1}{2}\phi(x - \Delta x, t) + \frac{1}{2}\phi(x + \Delta x, t)$$

Using Taylor expansion, it can be transformed as follows.

$$\frac{\partial \phi}{\partial t} + O(\Delta t) = \frac{\Delta x^2}{\Delta t}\frac{\partial^2 \phi}{\partial x^2} + O(\frac{\Delta x^3}{\Delta t})$$

Take colloids in the water as a example of diffusion. They do brownian movement and their mean travel distance is proportional to the square root of time. For this reason, we can say that $\frac{\Delta x^2}{\Delta t}$ is constant if we take Δx and Δt in proper range. Call constant D, by bringing Δx and Δt close to 0, we obtain a diffusion equation.