Machine Learning for Computer Vision

Exercise

Kodai Matsuoke, Yuyan Li

date

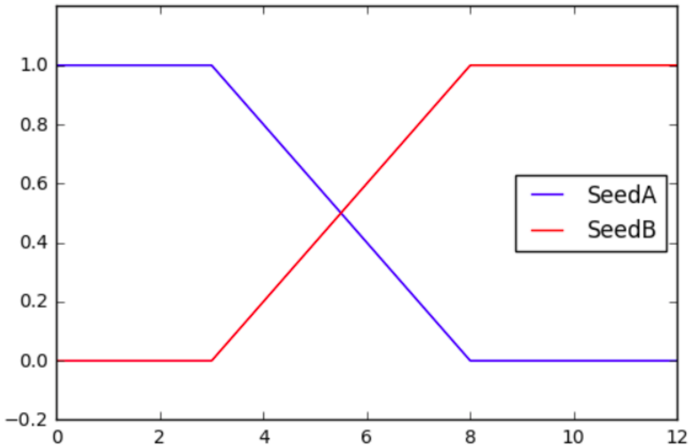
1 Random Walker

1.1 1D Random walker

Suppose x_i is the probability that a random walk particle started from position i first reaches to Seed A. Obviously, for i $\in \{0, 1, 2, 3\}, x_i = 1$. also for i $\in \{8, 9, 10, 11, 12\}, x_i = 0$.
So now, we want to calculate x_4, x_5, x_6, x_7. We formulate this problem as follows.

$$x = (x_4, x_5, x_6, x_7) \quad x_m = (x_3, x_8) \quad A = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}$$

By solving $Ax = -Bx_m$, we obtain $x = (0.8, 0.6, 0.4, 0.2)$. As well we can calculate the probability for Seed B. The overall probability is as shown below.



2 Random walker as Anisotropic Diffusion

$$\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2}$$

When a large number of particles make randomwalk, diffusion equation describes the time developement of the particle density. I am going to show it.
Let's suppose that the particle at position x makes a random walk such that after time Δt, it moves to x+Δx with probability 1/2, x-Δx with probability 1/2.
Now, the following equation goes with regard to the particle density ϕ

$$\phi(x, t+\Delta t) = \frac{1}{2}\phi(x-\Delta x, t) + \frac{1}{2}\phi(x+\Delta x, t)$$

Using Taylor expansion, it can be transformed as follows.

$$\frac{\partial \phi}{\partial t} + O(\Delta t) = \frac{\Delta x^2}{\Delta t}\frac{\partial^2 \phi}{\partial x^2} + O(\frac{\Delta x^3}{\Delta t})$$

Take colloids in the water as a example of diffusion. They do brownian movement and their mean travel distance is proportional to the square root of time. For this reason, we can say that $\frac{\Delta x^2}{\Delta t}$ is constant if we take Δx and Δt in proper range. Call constant D, by bringing Δx and Δt close to 0, we obtain a diffusion equation.