Forward Kinematics of the Stewart Platform

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Input

```
% Lengths of the legs (Input) in mm
leg_lengths =
[180.724259518072,180.546310004650,181.514478467853,180.852854760103,181.04887825540
3,180.468455208287];
```

Data

```

R1=130/2; % radius of the Fixed Platform in mm
R2=125/2; % radius of the Moving Platform in mmm
GAMMA=20; % half angle between to adjacent legs in Degree

% Define the fixed and moving platform geometry

        % % Theoretical Fixed platform leg points
        % B1=[R1*cosd(60-GAMMA) R1*sind(60-GAMMA) 0]';
        % B2=[R1*cosd(60+GAMMA) R1*sind(60+GAMMA) 0]';
        % B3=[-R1*cosd(GAMMA) R1*sind(GAMMA) 0]';
        % B4=[-R1*cosd(GAMMA) -R1*sind(GAMMA) 0]';
        % B5=[R1*cosd(60+GAMMA) -R1*sind(60+GAMMA) 0]';
        % B6=[R1*cosd(60-GAMMA) -R1*sind(60-GAMMA) 0]';

        % B = [B1 B2 B3 B4 B5 B6];

% Actual Fixed platform legs points
B1=[49.834,41.765,-10.52+11]';
B2=[11.327,63.988,-10.504+11]';
B3=[-61.058, 22.228,-10.654+11]';
B4=[-61.06,-22.233,-10.803+11]';
B5=[11.3,-64.009,-10.898+11]';
B6=[49.825,-41.784,-10.779+11]';

B = [B1 B2 B3 B4 B5 B6];

        % % Theoretical Moving platform legs points
```

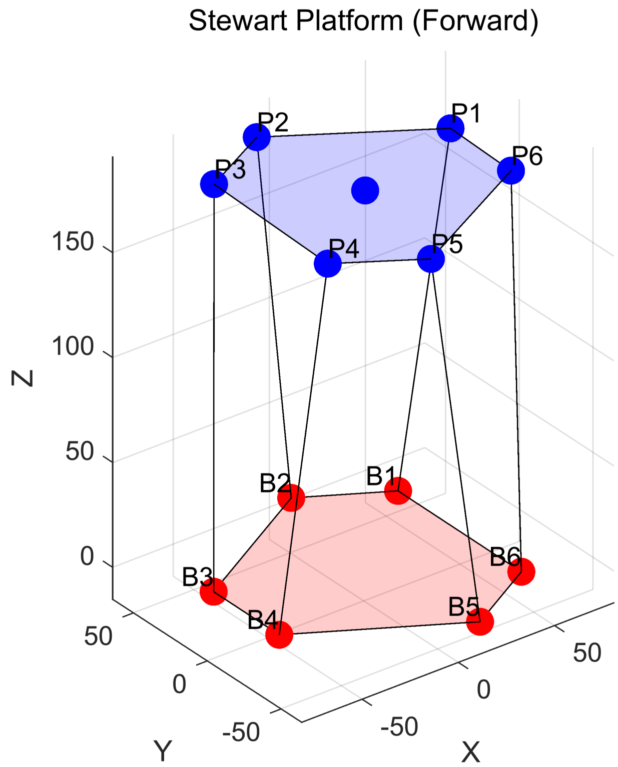
```
        % P1=[R2*cosd(GAMMA) R2*sind(GAMMA) 0]';
        % P2=[-R2*sind(30-GAMMA) R2*cosd(30-GAMMA) 0]';
        % P3=[-R2*sind(30+GAMMA) R2*cosd(30+GAMMA) 0]';
        % P4=[-R2*sind(30+GAMMA) -R2*cosd(30+GAMMA) 0]';
        % P5=[-R2*sind(30-GAMMA) -R2*cosd(30-GAMMA) 0]';
        % P6=[R2*cosd(GAMMA) -R2*sind(GAMMA) 0]';

        % P = [P1 P2 P3 P4 P5 P6];

% Actual Moving platform legs points
P1=[58.708,21.383,-11.171+11]';
P2=[-10.875,61.556,-10.94+11]';
P3=[-47.912,40.187,-10.888+11]';
P4=[-47.907,-40.162,-11.017+11]';
P5=[-10.873,-61.536,-11.135+11]';
P6=[58.704,-21.364,-11.243+11]';

P = [P1 P2 P3 P4 P5 P6];
```

Output

```
x = stewart_forward_kinematics(B,P,leg_lengths);
```

```
                                              Norm of      First-order   Trust-region
 Iteration  Func-count     ||f(x)||^2           step       optimality       radius
     0          7            6.43212                              23.7            1
     1         14           0.745336               1              2.65            1
     2         21        1.82527e-05        0.514502             0.243          2.5
     3         28        1.94636e-15      0.00163367           4.49e-06         2.5
     4         35        3.23117e-27     2.03222e-08           2.37e-12         2.5

Equation solved.

fsolve completed because the vector of function values is near zero
as measured by the value of the function tolerance, and
the problem appears regular as measured by the gradient.

<stopping criteria details>
Position of the platform (mm):
    x: 1.039763
    y: 0.421489
    z: 179.841882
orientation of the platform (degree):
    roll: 0.079101
    pitch: 0.119000
    yaw: -1.782172
```

```
% Plot the Stewart platform in 3D
[R, P_global, top_centre] = plot_stewart_platform(x, B, P);
```



Functions

Function of Non-linear Minimization solver

```
function [x] = stewart_forward_kinematics(B,P,leg_lengths)

    % Initial guess for platform position and orientation (x, y, z, roll, pitch,
yaw)
    x0 = [0, 0, mean(leg_lengths), 0, 0, 0];

    % Solve for the position and orientation

    % using fsolve (non linear solver)
    options = optimoptions('fsolve', 'Display', 'iter','FunctionTolerance',10^-9);
    % options = optimoptions('fsolve', 'Display', 'none','FunctionTolerance',10^-9);
    % options = optimoptions('fsolve','Display', 'none');

    %[x, fval, exitflag] = fsolve(@(x) kinematic_equations(x, B, P, leg_lengths),
x0, options);
    x = fsolve(@(x) kinematic_equations(x, B, P, leg_lengths), x0, options);

    % Print the result on display
    fprintf('Position of the platform (mm):\n');
    fprintf('\tx: %.6f\n', x(1));
    fprintf('\ty: %.6f\n', x(2));
    fprintf('\tz: %.6f\n', x(3));
```

```
    fprintf('orientation of the platform (degree):\n');
    fprintf('\troll: %.6f\n', rad2deg(x(4)));
    fprintf('\tpitch: %.6f\n', rad2deg(x(5)));
    fprintf('\tyaw: %.6f\n', rad2deg(x(6)));

end
```

Function to Calculate Position and orientation

```
function F = kinematic_equations(x, B, P, leg_lengths)
    % Extract position and orientation from x
    px = x(1);
    py = x(2);
    pz = x(3);
    roll = x(4);
    pitch = x(5);
    yaw = x(6);

    % Rotation matrix from roll, pitch, yaw
    % R = eul2rotm([yaw, pitch, roll]); % Traditional method
    R = Eul2rotm([roll, pitch, yaw]);


    % Compute the equations for each leg
    F = zeros(6, 1);
    for i = 1:6
        % Platform attachment point in global frame
        p_global = [px; py; pz] + R * P(:, i);

        % Compute the distance between base and platform attachment points
        d = norm(p_global - B(:, i));

        % error (computed leg length - actual leg_length)
        F(i) = d - leg_lengths(i);
    end

end
```

Function to create Rotational Matrix from Euler angles

```
% Traditional
    % function R = eul2rotm(eul)
    %     % Convert Euler angles to rotation matrix
    %     % eul = [yaw, pitch, roll]
    %     yaw = eul(1);
    %     pitch = eul(2);
    %     roll = eul(3);
```

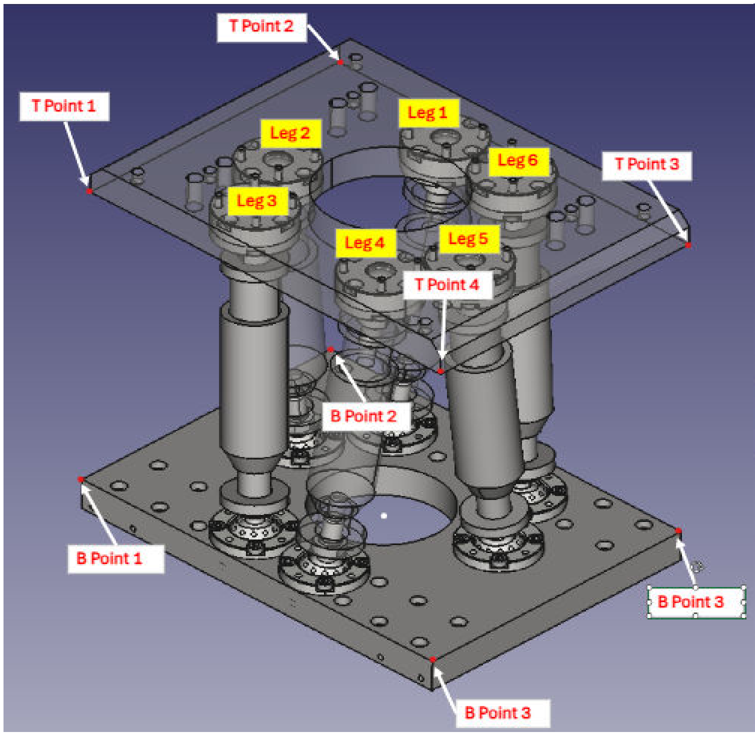
```
    %
    %     % Rotation matrix around z-axis (yaw)
    %     Rz = [cos(yaw), -sin(yaw), 0;
    %           sin(yaw),  cos(yaw), 0;
    %           0,         0,        1];
    %
    %     % Rotation matrix around y-axis (pitch)
    %     Ry = [cos(pitch), 0, sin(pitch);
    %           0,            1, 0;
    %           -sin(pitch), 0, cos(pitch)];
    %
    %     % Rotation matrix around x-axis (roll)
    %     Rx = [1, 0,        0;
    %           0, cos(roll), -sin(roll);
    %           0, sin(roll),  cos(roll)];
    %
    %     % Combined rotation matrix
    %     R = Rz * Ry * Rx;
    % end

function R = Eul2rotm(eul)
    % Euler angles are in the order of [roll, pitch, yaw]
    R = eul2rotm([eul(1), eul(2), eul(3)],"XYZ");
end
```

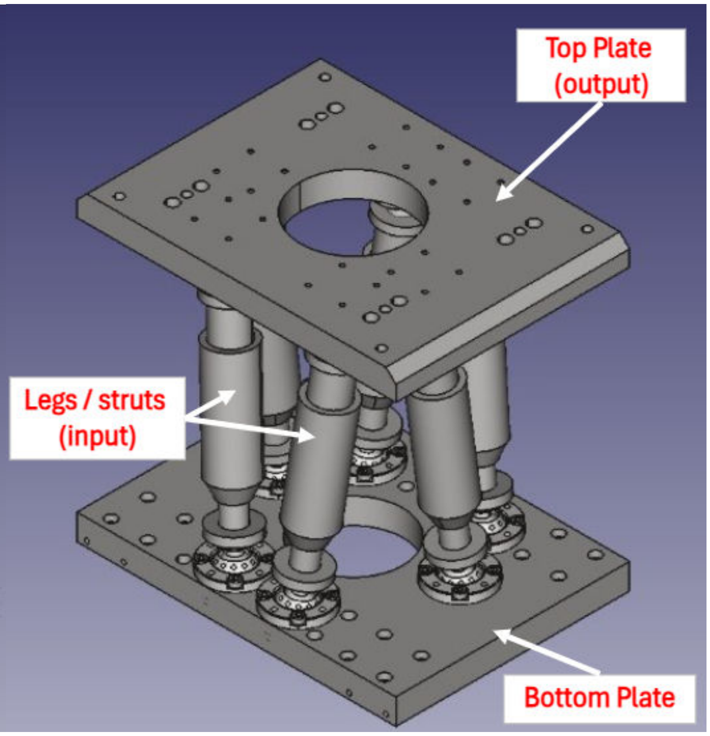
Function to Plot

```
function [R, P_global, top_centre] = plot_stewart_platform(x, B, P)
    % Extract position and orientation
    px = x(1);
    py = x(2);
    pz = x(3);
    roll = x(4);
    pitch = x(5);
    yaw = x(6);

    % Rotation matrix from roll, pitch, yaw
    R = Eul2rotm([roll, pitch,yaw]);

    % Compute the platform attachment points in the global frame
    P_global = [px; py; pz] + R * P;

    % Plot the base attachment points
    figure;
    hold on;
    plot3(B(1,:), B(2,:), B(3,:), 'ro', 'MarkerSize', 10, 'MarkerFaceColor', 'r');
    hold on
    fill3(B(1,:), B(2,:), B(3,:),'r', 'FaceAlpha', 0.2);
```

```
    text(B(1,:), B(2,:), B(3,:), {'B1', 'B2', 'B3', 'B4', 'B5',
'B6'},'VerticalAlignment', 'bottom', 'HorizontalAlignment', 'right');

    % Plot the platform attachment points
    plot3(P_global(1,:), P_global(2,:), P_global(3,:), 'bo', 'MarkerSize', 10,
'MarkerFaceColor', 'b');
    hold on
    fill3(P_global(1,:), P_global(2,:), P_global(3,:),'b','FaceAlpha', 0.2);
    text(P_global(1,:), P_global(2,:), P_global(3,:), {'P1', 'P2', 'P3', 'P4',
'P5', 'P6'},'VerticalAlignment', 'bottom', 'HorizontalAlignment', 'left');

    % Plot the legs
    for i = 1:6
        plot3([B(1,i), P_global(1,i)], [B(2,i), P_global(2,i)], [B(3,i),
P_global(3,i)], 'k-');
    end

    % Plot Centre
    top_centre = mean(P_global,2);
    plot3(top_centre(1),top_centre(2),top_centre(3),'bo', 'MarkerSize', 10,
'MarkerFaceColor', 'b');

    % Plot settings
    xlabel('X');
    ylabel('Y');
    zlabel('Z');
    title('Stewart Platform (Forward)');
    grid on;
    axis equal;
    view(3);
    hold off;

end
```

Reference Diagram


T Point 2
T Point 1
Leg 1
Leg 2
Leg 6
T Point 3
Leg 3
Leg 4
Leg 5
T Point 4
B Point 2
B Point 1
B Point 3
B Point 3


Top Plate
(output)
Legs / struts
(input)
Bottom Plate