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SCHREYER HONORS COLLEGE

YOUR DEPARTMENT NAME

DYNAMICS OF FISH

YOUR NAME  
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A thesis  
submitted in partial fulfillment  
of the requirements  
for a baccalaureate degree  
in Engineering Science  
with honors in Engineering Science

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Date of Signature

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# Abstract

Some nonsense goes here.

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- $\alpha$  The first greek letter, p. 12
- $\alpha$  The first greek letter, p. 12
- $\alpha$  The first greek letter, but we should really add some more text, though we need it to go on two lines, p. 9
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# Acknowledgments

# Chapter 1 |

## A Few L<sup>A</sup>T<sub>E</sub>X Tips

### 1.1 This is a Section

After discussing the overall structure, let's throw in a table.

#### 1.1.1 A Table

Units are essential to any quantifiable measure. Newton's second law in scalar form,  $F = ma$ , provides for the formulation of a consistent and unambiguous system of units. We will use both U.S. Customary units and SI units (International System<sup>1</sup>) as shown in Table 1.1. This is a url <https://psu.instructure.com/courses/1929919>.

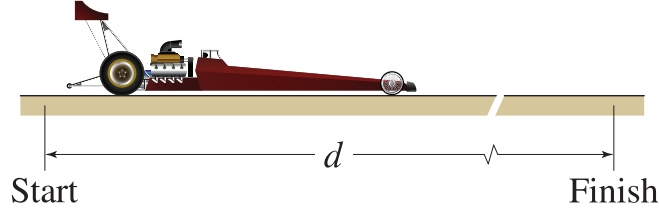
**Table 1.1.** U.S. Customary and SI unit systems.

Base dimension	System of units	
	<i>U.S. Customary</i>	<i>SI</i>
force	pound (lb)	newton <sup>a</sup> (N) $\equiv$ kg-m/s <sup>2</sup>
mass	slug <sup>a</sup> $\equiv$ lb-s <sup>2</sup> /ft	kilogram (kg)
length	foot (ft)	meter (m)
time	second (s)	second (s)

<sup>a</sup>derived unit

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<sup>1</sup>SI has been adopted as the abbreviation for the French *Le Système International d'Unités*.



**Figure 1.1.** This is a figure with a caption in a floating **figure** environment.

### 1.1.2 This is a Subsection

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. –That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, –That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. [1–15]

### 1.1.3 Some Equations

The FBD of the box as it tips over the corner is shown at the bottom right, where we have chosen to use polar coordinates for writing the Newton-Euler equations. The Newton-Euler equations corresponding to this FBD are

$$\sum F_r: \quad N - mg \cos \theta = ma_{Gr}, \quad (1.1)$$

$$\sum F_\theta: \quad mg \sin \theta - F = ma_{G\theta}, \quad (1.2)$$

$$\sum M_G: \quad F \frac{h}{2} = I_G \alpha, \quad (1.3)$$

where  $I_G = \frac{1}{12}m(h^2 + h^2) = \frac{1}{6}mh^2$ . Now writing the acceleration of  $G$  in polar coordinates, we have

$$\vec{a}_G = (\ddot{r} - r\dot{\theta}^2) \hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{u}_\theta = -\frac{h}{2}\dot{\theta}^2 \hat{u}_r + \frac{h}{2}\ddot{\theta} \hat{u}_\theta,$$

where we have used the fact that the distance from  $A$  to  $G$  is  $r = h/2$  and the fact that  $r$  is a constant. Substituting the acceleration components into Eqs. (1.1) and (1.2), we obtain

$$N - mg \cos \theta = -m \frac{h}{2} \dot{\theta}^2, \quad (1.4)$$

$$mg \sin \theta - F = m \frac{h}{2} \ddot{\theta}. \quad (1.5)$$

Solving Eq. (1.5) for  $F$  and substituting it into Eq. (1.3) and then solving for  $\ddot{\theta}$ , we obtain

$$\frac{h}{2} \left( mg \sin \theta - m \frac{h}{2} \ddot{\theta} \right) = \frac{1}{6} m h^2 \ddot{\theta} \Rightarrow \ddot{\theta} = \frac{6g}{5h} \sin \theta. \quad (1.6)$$

This expression can be substituted into Eq. (1.5) to obtain  $F$  as a function of  $\theta$  as

$$F = mg \sin \theta - m \frac{h}{2} \left( \frac{6g}{5h} \sin \theta \right) = \frac{2}{5} mg \sin \theta. \quad (1.7)$$

Equation (1.4) tells us that to find  $N$  as a function of  $\theta$ , we need to find  $\dot{\theta}$  as a function of  $\theta$ . This can be done either by applying the work-energy principle or by integrating the expression for  $\ddot{\theta}$  in Eq. (1.6)—we will do the latter.

Invoking the chain rule in Eq. (1.6) and integrating, we obtain

$$\begin{aligned} \ddot{\theta} = \dot{\theta} \frac{d\dot{\theta}}{d\theta} = \frac{6g}{5h} \sin \theta &\Rightarrow \int_0^{\dot{\theta}} \dot{\theta} d\dot{\theta} = \frac{6g}{5h} \int_0^{\theta} \sin \theta d\theta \\ \Rightarrow \frac{\dot{\theta}^2}{2} = -\frac{6g}{5h} \cos \theta \Big|_0^{\theta} = \frac{6g}{5h} (1 - \cos \theta) &\Rightarrow \dot{\theta}^2 = \frac{12g}{5h} (1 - \cos \theta). \end{aligned} \quad (1.8)$$

Substituting the expression for  $\dot{\theta}^2$  in Eq. (1.8) into Eq. (1.4) and then solving for  $N$ , we obtain

$$N = mg \cos \theta - m \frac{h}{2} \left[ \frac{12g}{5h} (1 - \cos \theta) \right] = \frac{1}{5} mg (11 \cos \theta - 6). \quad (1.9)$$

Now, the box will start to slip on the corner when  $F = \mu_s N$ . Using the expressions for  $F$  and  $N$  as functions of  $\theta$  in Eqs. (1.7) and (1.9) in this force law, we obtain

$$\begin{aligned} \frac{2}{5} mg \sin \theta = \frac{\mu_s}{5} mg (11 \cos \theta - 6) &\Rightarrow 2 \sin \theta = \mu_s (11 \cos \theta - 6) \\ \Rightarrow \frac{2}{\mu_s} \sin \theta = 11 \cos \theta - 6 &\Rightarrow \frac{2}{\mu_s} \sin \theta = 11 \cos \theta - 6 \end{aligned}$$

$$\Rightarrow \frac{2}{11} \left( \frac{1}{\mu_s} \sin \theta + 3 \right) = \cos \theta. \quad (1.10)$$

To solve the last of Eq. (1.10), we let  $\cos \theta = \sqrt{1 - \sin^2 \theta}$ , square both sides, and then rearrange to obtain a quadratic equation in  $\sin \theta$  as follows

$$\begin{aligned} \frac{4}{121} \left( \frac{1}{\mu_s} \sin \theta + 3 \right)^2 &= 1 - \sin^2 \theta \quad \Rightarrow \quad \frac{4}{121} \left( \frac{1}{\mu_s^2} \sin^2 \theta + \frac{6}{\mu_s} \sin \theta + 9 \right) = 1 - \sin^2 \theta \\ &\Rightarrow \left( \frac{4}{121\mu_s^2} + 1 \right) \sin^2 \theta + \frac{24}{121\mu_s} \sin \theta - \left( \frac{36}{121} - 1 \right) = 0 \\ &\Rightarrow (4 + 121\mu_s^2) \sin^2 \theta + 24\mu_s \sin \theta - 85\mu_s^2 = 0. \quad (1.11) \end{aligned}$$

Applying the quadratic formula, we obtain

$$\begin{aligned} \sin \theta &= \frac{-24\mu_s \pm \sqrt{(24\mu_s)^2 - 4(4 + 121\mu_s^2)(-85\mu_s^2)}}{2(4 + 121\mu_s^2)} \\ &= \frac{-24\mu_s \pm \sqrt{484\mu_s^2(4 + 85\mu_s^2)}}{2(4 + 121\mu_s^2)} \\ &= \frac{-24\mu_s \pm 22\mu_s \sqrt{4 + 85\mu_s^2}}{2(4 + 121\mu_s^2)} \\ &= \frac{\mu_s(-12 \pm 11\sqrt{4 + 85\mu_s^2})}{4 + 121\mu_s^2}. \end{aligned}$$

Therefore, the angle at which the box slips is

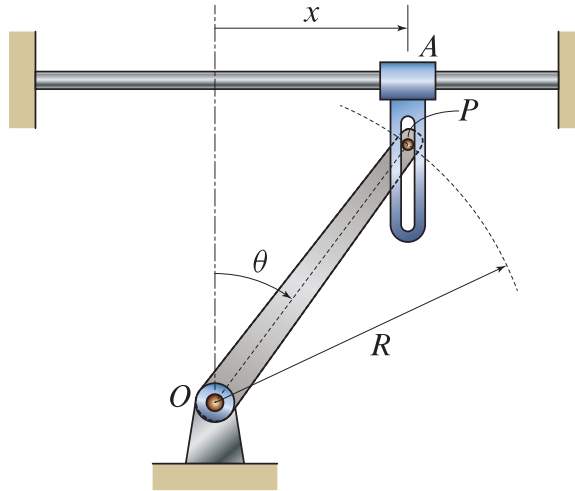
$$\theta = \sin^{-1} \left[ \frac{\mu_s(-12 \pm 11\sqrt{4 + 85\mu_s^2})}{4 + 121\mu_s^2} \right].$$

For example, for  $\mu_s = 1/3$ , the two angles are  $\theta = 32.78^\circ$  for the plus root and  $\theta = -90^\circ$  for the minus root. Obviously, the physically meaningful result is the angle corresponding to the plus root.

### 1.1.3.1 This is a Subsubsection

We hold these truths subsubsection 1.1.3.1 to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. That to secure these rights,





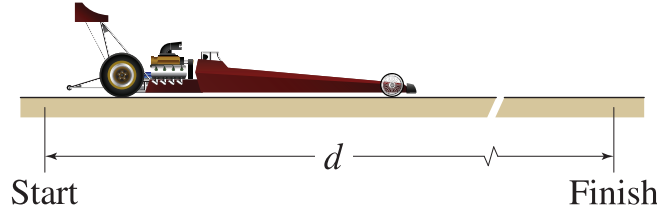
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## 1.2 More Declaration

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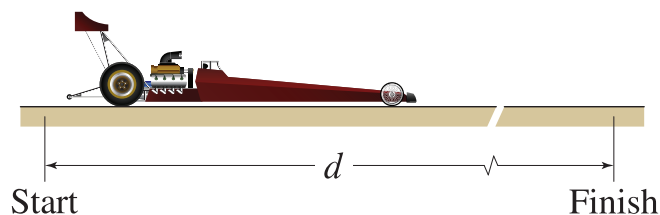
and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably



**Figure 1.3.** We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness.

the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their future security. –Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.

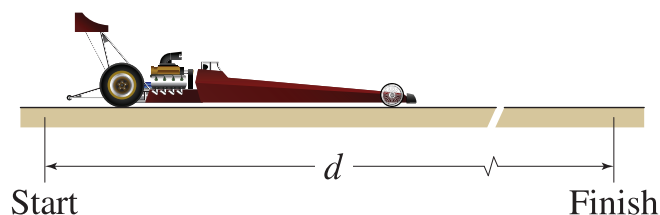
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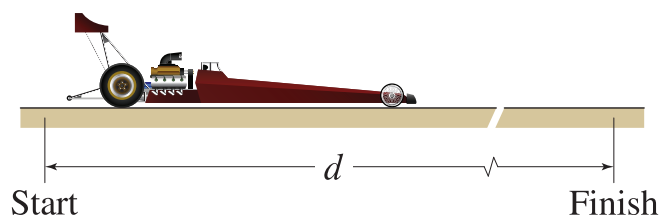


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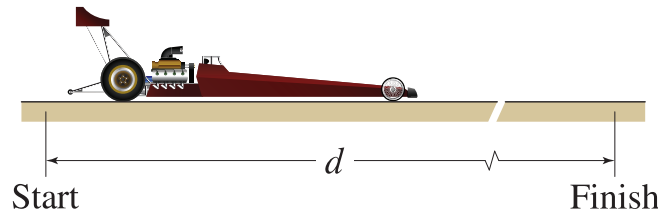


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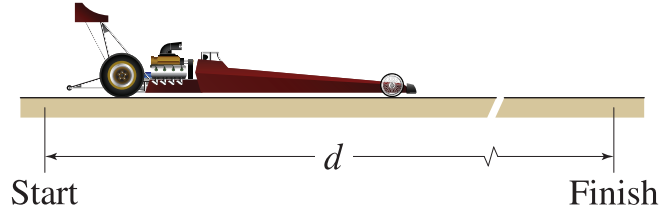
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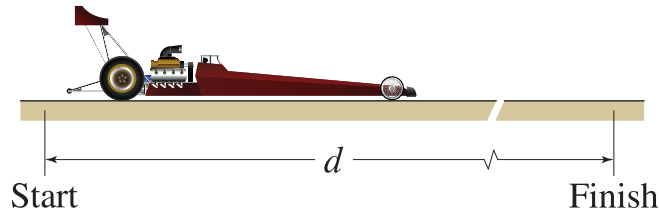
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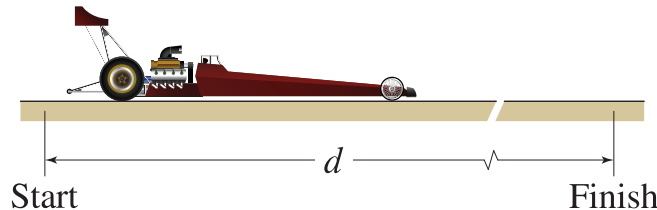


**Figure 1.9.** We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness.

the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their

future security. –Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably

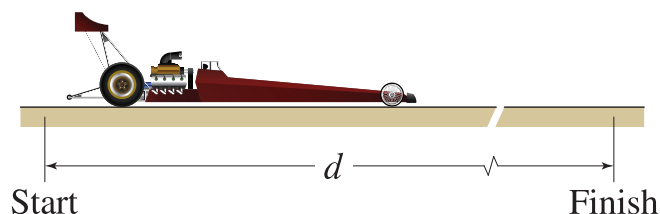


**Figure 1.10.** We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness.

the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their future security. –Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. That to secure these rights, Governments

are instituted among Men, deriving their just powers from the consent of the governed, That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably



**Figure 1.11.** We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness.

the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their future security. –Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.



# Appendix A |

## Title of the First Appendix

### A.1 Introduction

When in the Course of human events, it becomes necessary for one people to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them, a decent respect to the opinions of mankind requires that they should declare the causes which impel them to the separation.

### A.2 More Declaration

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. –That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, –That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness.

#### A.2.1 Some Subsection Title Here

Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and

usurpations, pursuing invariably the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their future security. –Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.

# Appendix B |

## Title of the Second Appendix

### B.1 Introduction

When in the Course of human events, it becomes necessary for one people to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them, a decent respect to the opinions of mankind requires that they should declare the causes which impel them to the separation.

### B.2 More Declaration

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. —That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, —That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their

future security. –Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.

# Appendix C |

## Title of the Third Appendix

### C.1 Introduction

When in the Course of human events, it becomes necessary for one people to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them, a decent respect to the opinions of mankind requires that they should declare the causes which impel them to the separation.

### C.2 More Declaration

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. —That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, —That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their

future security. –Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.

# Appendix D |

## Title of the Fourth Appendix

### D.1 Introduction

When in the Course of human events, it becomes necessary for one people to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them, a decent respect to the opinions of mankind requires that they should declare the causes which impel them to the separation.

### D.2 More Declaration

We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness. —That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, —That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness. Prudence, indeed, will dictate that Governments long established should not be changed for light and transient causes; and accordingly all experience hath shewn, that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed. But when a long train of abuses and usurpations, pursuing invariably the same Object evinces a design to reduce them under absolute Despotism, it is their right, it is their duty, to throw off such Government, and to provide new Guards for their

future security. –Such has been the patient sufferance of these Colonies; and such is now the necessity which constrains them to alter their former Systems of Government. The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world.



# Appendix E |

## Title of the Fifth Appendix

### E.1 Introduction

When in the Course of human events, it becomes necessary for one people to dissolve the political bands which have connected them with another, and to assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them, a decent respect to the opinions of mankind requires that they should declare the causes which impel them to the separation.

Some text.

```

      program chaos
c When a LS Fortran program has been compiled and linked into Mac
c application, all information written to the screen WRITE(6,...) or
c WRITE(*,...) appears in a standard Mac window, complete with basic
c menus.
      external fex, jac
      double precision atol, rtol, rwork, t, tout, h
      double precision tttotal, dtout
      dimension h(3), atol(3), rwork(70), iwork(23)
      character*8 tstart, tend
      neq = 3

      call time(tstart)
      write(6,*) "begin□integration□at□", tstart
      write(6,*)

c — Read in the total initial angular momentum. The total angular
c momentum H is always unity due to normalization.
      open(unit = 2, file = 'chaos.data', status = 'unknown')
      read(2,*) h(1), h(2), h(3)

c — The integration begins at t = 0 and the values are printed at
c every tout. tout is incremented below. tttotal is the length
c of the entire integration. The number of recorded values of
c the integration is given by npoints.
      t = 0.0d0
      tout = 0.0d0
      write(6,*) 'Duration□of□integration□interval□i.e.,□tfinal?'
      read(6,*) tttotal
      write(6,*)
      write(6,*) 'Number□of□points□for□trajectory□plot?'
      read(6,*) npoints
      write(6,*)
      dtout = tttotal/dfloat(npoints)
      tout = tout + dtout

c — Tolerance parameters used by lsoda.
      itol = 2
      rtol = 1.0d-9
      atol(1) = 1.0d-9
      atol(2) = 1.0d-9
```

```

    atol(3) = 1.0d-9

c ——— Other parameters used by lsoda. See below.
    itask = 1
    istate = 1
    iopt = 1
    lrw = 70
    liw = 23
    jt = 1

    do 11 kount = 5,10
        rwork(kount) = 0.0d0
        iwork(kount) = 0
11    continue
    iwork(6) = 100000

        open(unit = 3, file = 'traj.dat', disp = 'keep',
&          status = 'unknown')

c ——— The actual integration begins here. Loop on the value of iout.
    do 40 iout = 1, npoints

        call lsoda(fex,neq,h,t,tout,itol,rtol,atol,itask,istate,
&                iopt,rwork,lrw,iwork,liw,jdum,jt)

c ——— Write the output to the file traj.dat.
        write(3,20) t, h(1), h(2), h(3)
20    format(f9.1, 3e15.6)

        if (mod(tout,5000.0d0) .eq. 0.0d0) then
            write(6,*) tout
        end if

c ——— Check to see that things are going OK.
        if (istate .lt. 0) go to 80

c ——— Set the time at which the integration is next recorded and
c          continue the do-loop.
40    tout = tout + dtout

        write(6,*) 'number_of_steps_taken:', iwork(11)
        write(6,*) 'number_of_f_evaluations:', iwork(12)

```

```

write(6,*) 'number_of_Jacobian_evaluations:', iwork(13)
write(6,*) 'method_order_last_used:', iwork(14)
write(6,*) 'method_last_used(2==stiff):', iwork(19)
write(6,*) 'value_of_t_at_last_method_switch:', rwork(15)
write(6,*)

      call time(tend)
      write(6,*) "end_integration_at", tend
stop

c — If there is an error, given by istate < 0, write the following.
80 write(6,90) istate
90 format(///22h error halt.. istate =,i3)

stop
end

```

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**Vita**

**Your Name**

The details of my childhood are inconsequential.