

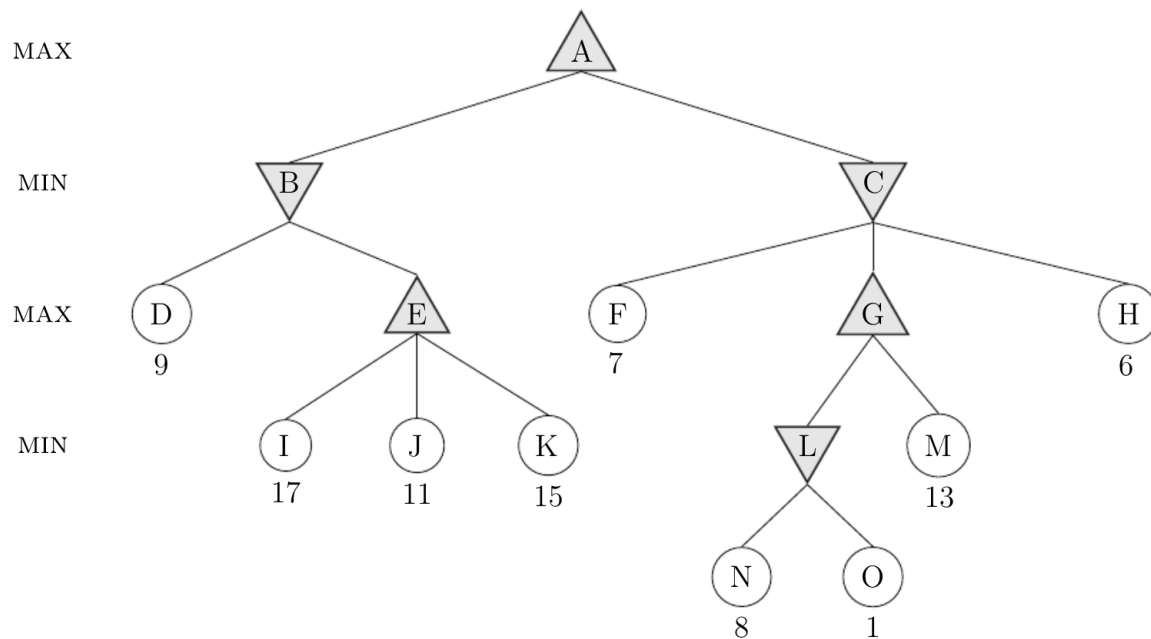
Foundations of Artificial Intelligence: Homework 2

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Problem 1

(10 points)

Consider the MAX-MIN game tree shown below where the numbers underneath the leaves of the tree are utility values from the first player's point of view (**MAX**).



a) Draw a copy of the tree on paper and perform the **minimax** algorithm on it by hand. Write the resulting minimax values next to every node

b) Do the same, but with **left-to-right alpha-beta** pruning. Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning.

c) Do the same, but with **right-to-left alpha-beta** pruning. Write the final values for α and β next to every node, and indicate which nodes are not examined due to pruning.

MAX

MIN

MAX

MIN

9

17

11

15

7

8

1

13

6

9

The diagram illustrates a minimax search tree with the following structure and values:

- Root Node (MAX):** A (Value: 9)
- Level 1 (MIN):**
 - B (Value: 9)
 - C (Value: 7)
- Level 2 (MAX):**
 - D (Value: 9)
 - E (Value: 17)
 - F (Value: 7)
 - G (Value: 13)
 - H (Value: 6)
- Level 3 (MIN):**
 - I (Value: 17)
 - J (Value: 11)
 - K (Value: 15)
 - L (Value: 8)
 - M (Value: 13)
- Level 4 (MAX):**
 - N (Value: 8)
 - O (Value: 1)

Handwritten annotations indicate the minimax process:

- Alpha and Beta values:**
 - At A: $\alpha = -\infty$, $\beta = \infty$
 - At B: $\alpha = -\infty$, $\beta = 9$
 - At C: $\alpha = 9$, $\beta = \infty$
 - At E: $\alpha = -\infty$, $\beta = 9$
 - At F: $\alpha = 9$, $\beta = \infty$
 - At G: $\alpha = -\infty$, $\beta = \infty$
 - At I: $\alpha = -\infty$, $\beta = \infty$
 - At L: $\alpha = 9$, $\beta = \infty$
- Optimal Path:** A-B-E-I-K (indicated by a thick line and arrows).

$V_F = 7 < \alpha = 9$
 G, H, L, M, N, O not examined

MAX

MIN

MAX

MIN

$\alpha = 6$
 $\beta = 9$

$\alpha = 9$
 $\beta = \infty$

$\alpha = 6$
 $\beta = 17$

$\alpha = 17$
 $\beta = \infty$

$\alpha = -\infty$
 $\beta = 6$

$\alpha = -\infty$
 $\beta = \infty$

$\alpha = 15$
 $\beta = \infty$

$\alpha = 15$
 $\beta = \infty$

$\alpha = 6$
 $\beta = \infty$

$\alpha = -\infty$
 $\beta = 6$

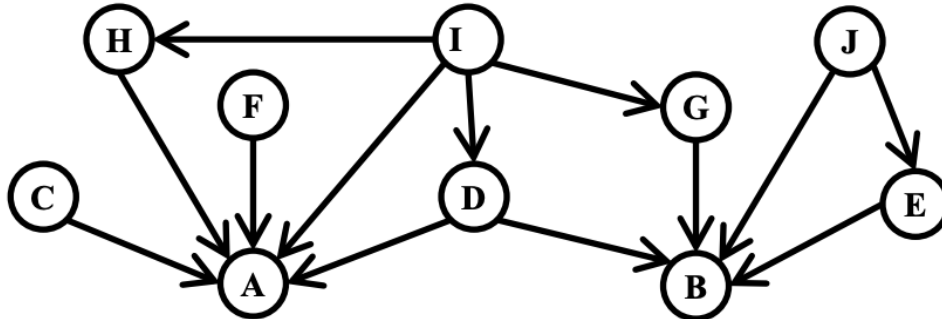
$\alpha = -\infty$
 $\beta = \infty$

$\Rightarrow L, N, O$ not examined

Problem 2

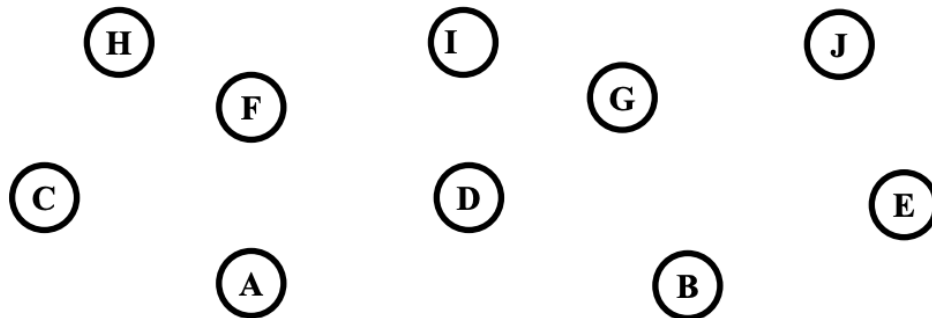
(10 points)

- (a) Write down the factored joint probability distribution according to the following Bayesian Network.

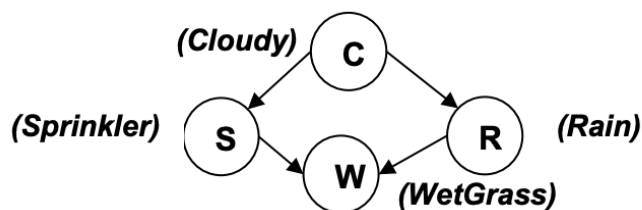


- (b) Draw the Bayesian Network that corresponds to this conditional probability:

$$P(A|C, D, F, H)P(B|D, E, J)P(C|H)P(D|G, J)P(E)P(F|G, I)P(G|I, J)P(H)P(I)P(J)$$



- (c) Below is the Bayesian network for the WetGrass problem.



P(C)
.5

C	P(S)
t	.1
f	.5

C	P(R)
t	.8
f	.2

S	R	P(W)
t	t	.99
t	f	.90
f	t	.90
f	f	.00

Write down an expression that will evaluate to

$$P(C = f \wedge R = f \wedge S = t \wedge W = t).$$

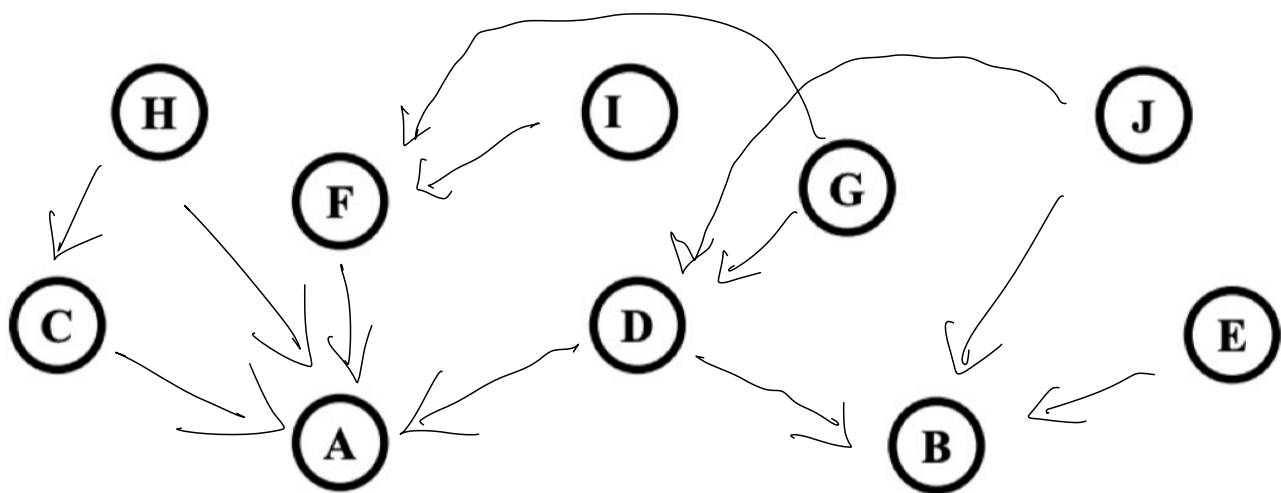
You do not need to carry out the multiplication to produce a single number (probability).

Q2

(a) $P(A|C, H, F, I, D) P(B|D, G, J, E) P(C) P(D|I) P(E|J) P(F) P(G|I) P(H|I) P(Z|J) P(Y)$

(b)

$$P(A|C, D, F, H) P(B|D, E, J) P(C|H) P(D|G, J) P(E) P(F|G, I) P(G|I, J) P(H) P(I) P(J)$$



(c) $P(W=t | S=t \wedge R=f) P(S=t | C=f) P(R=f | C=f) P(C=f)$

Q3

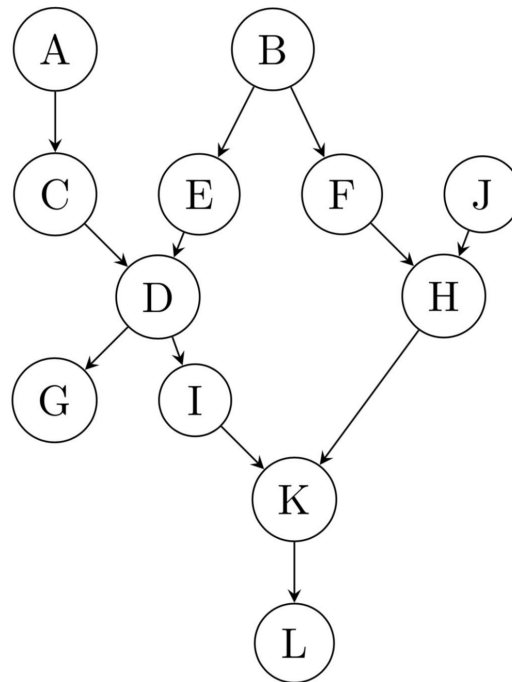
(a) A, C, G, D, I

(b) L

Problem 3

(10 points)

According to the following Bayesian Network,



- (a) List all the variables that are d-separated from F given E.
- (b) List all the variables that are d-separated from F given E and K.

Problem 4

(10 points)

Draw a Bayes net with four states $\{A, B, C, D\}$, that follows all of the independence constraints below.

- (a) $A \perp\!\!\!\perp B$
- (b) $A \not\perp\!\!\!\perp D \mid B$
- (c) $A \perp\!\!\!\perp D \mid C$
- (d) $A \not\perp\!\!\!\perp C$
- (e) $B \not\perp\!\!\!\perp C$
- (f) $A \not\perp\!\!\!\perp B \mid D$
- (g) $B \perp\!\!\!\perp D \mid A, C$

