Appendix: Mathematical description of the CovidSIM model (version 1.1)

Model dynamics

Number of susceptible individuals $$\frac{dS}{dt} = -\lambda(t)S$$

Number of individuals in the latent period $$\frac{dE_1}{dt} = \lambda(t)S - \varepsilon E_1$$

$$\frac{dE_k}{dt} = \varepsilon E_{k-1} - \varepsilon E_k \qquad (1 < k \leq n_E)$$

Number of individuals in the prodromal period $$\frac{dP_1}{dt} = \varepsilon E_{n_E} - \varphi P_1$$

$$\frac{dP_k}{dt} = \varphi P_{k-1} - \varphi P_k \qquad (1 < k \leq n_P)$$

Number of individuals in the early infectious period $$\frac{dI_1}{dt} = \varphi P_{n_P} - \gamma I_1$$

$$\frac{dI_k}{dt} = \gamma I_{k-1} - \gamma I_k \qquad (1 < k \leq n_I)$$

Number of individuals in the late infectious period $$\frac{dL_1}{dt} = \gamma I_{n_I} - \delta L_1$$

$$\frac{dL_k}{dt} = \delta L_{k-1} - \delta L_k \qquad (1 < k \leq n_L)$$

Number of recovered individuals $$\frac{dR}{dt} = \delta(1 - p_{Sick}\, p_{Death})L_{n_L}$$

Number of dead individuals $$\frac{dD}{dt} = \delta p_{Sick}\, p_{Death} L_{n_L}$$

Derived variables

Total number in latent period	$E_{Sum}(t) = \sum_{k=1}^{n_E} E_k(t)$
Total number in prodromal period	$P_{Sum}(t) = \sum_{k=1}^{n_P} P_k(t)$
Total number in early infectious period	$I_{Sum}(t) = \sum_{k=1}^{n_I} I_k(t)$
Total number in late infectious period	$L_{Sum}(t) = \sum_{k=1}^{n_L} L_k(t)$
Number of symptomatic cases	$f_{Sick}\left(I_{Sum}(t) + L_{Sum}(t)\right)$
Number of asymptomatic cases	$\left(1 - f_{Sick}\right)\left(I_{Sum}(t) + L_{Sum}(t)\right)$
Number of hospitalized cases	$f_{Sick}\, f_{Hosp} \int_0^{D_{Hosp}} \varphi P_{n_P}(t-\tau)\, d\tau$
Number of cases in ICU	$f_{Sick}\, f_{Hosp}\, f_{ICU} \int_0^{D_{ICU}} \varphi P_{n_P}(t-\tau)\, d\tau$

Intervention effects

Number of cases in the isolation unit

$$I_{Iso} = \begin{cases} f_{Sick} I_{Sum} & \text{if} \quad t_{Iso_1} \le t \le t_{Iso_2} \quad \text{and} \quad f_{Sick}\left(I_{Sum} + L_{Sum}\right) \le Q_{\max} \\ \dfrac{I_{Sum}}{I_{Sum} + L_{Sum}} Q_{\max} & \text{if} \quad t_{Iso_1} \le t \le t_{Iso_2} \quad \text{and} \quad f_{Sick}\left(I_{Sum} + L_{Sum}\right) > Q_{\max} \\ 0 & \text{if} \quad t < t_{Iso_1} \quad \text{or} \quad t > t_{Iso_2} \end{cases}$$

$$L_{Iso} = \begin{cases} f_{Sick} L_{Sum} & \text{if} \quad t_{Iso_1} \le t \le t_{Iso_2} \quad \text{and} \quad f_{Sick}\left(I_{Sum} + L_{Sum}\right) \le Q_{\max} \\ \dfrac{L_{Sum}}{I_{Sum} + L_{Sum}} Q_{\max} & \text{if} \quad t_{Iso_1} \le t \le t_{Iso_2} \quad \text{and} \quad f_{Sick}\left(I_{Sum} + L_{Sum}\right) > Q_{\max} \\ 0 & \text{if} \quad t < t_{Iso_1} \quad \text{or} \quad t > t_{Iso_2} \end{cases}$$

Number of cases in home isolation

$$I_{Home} = \begin{cases} I_{Sum} - \dfrac{I_{Sum}}{I_{Sum} + L_{Sum}} Q_{\max} & \text{if} \quad t_{Iso_1} \le t \le t \quad \text{and} \quad f_{Sick}\left(I_{Sum} + L_{Sum}\right) > Q_{\max} \\ 0 & \text{otherwise} \end{cases}$$

$$L_{Home} = \begin{cases} L_{Sum} - \dfrac{L_{Sum}}{I_{Sum} + L_{Sum}} Q_{\max} & \text{if} \quad t_{Iso_1} \le t \le t_{Iso_2} \quad \text{and} \quad f_{Sick}\left(I_{Sum} + L_{Sum}\right) > Q_{\max} \\ 0 & \text{otherwise} \end{cases}$$

General reduction of contacts

$$p_{Gen}(t) = \begin{cases} p_{Dist} & \text{if} \quad t_{Dist_1} \le t \le t_{Dist_2} \\ 0 & \text{otherwise} \end{cases}$$

Contact reduction triggered by number of cases

$$p_{Trig_{Sick}}(t) = \begin{cases} p_{Sick} & \text{if} \quad \left(I_{Sum}(t) + L_{Sum}(t)\right) \ge T_{Sick} N \\ 0 & \text{otherwise} \end{cases}$$

Contact reduction triggered by hospitalized

$$p_{Trig_{Hosp}}(t) = \begin{cases} p_{Hosp} & \text{if} \quad \left(I_{Sum}(t) + L_{Sum}(t)\right) f_{Sick} f_{Hosp} \ge T_{Hosp} N \\ 0 & \text{otherwise} \end{cases}$$

Contact reduction triggered by ICU patients

$$p_{Trig_{ICU}}(t) = \begin{cases} p_{ICU} & \text{if} \quad \left(I_{Sum}(t) + L_{Sum}(t)\right) f_{Sick} f_{Hosp} f_{ICU} \ge T_{ICU} N \\ 0 & \text{otherwise} \end{cases}$$

Contact rate and force of infection

Contact rate at time t

$$\beta(t)=\frac{R_0}{c_P D_P + D_I + c_L D_L}\cdot\left(1+a\cdot\cos\left(2\pi\cdot\frac{t-t_{R0_{\max}}}{365}\right)\right)\cdot\left(1-\max\left(p_{Gen}(t), p_{Trig_{Sick}}(t), p_{Trig_{Hosp}}(t), p_{Trig_{ICU}}(t)\right)\right)$$

Force of infection

$$\lambda(t)=\frac{\lambda_{Ext}+\beta(t)\left(c_P P_{Sum}(t)+\left(I_{Sum}(t)-I_{Iso}(t)-p_{Home}I_{Home}(t)\right)+c_L\left(L_{Sum}(t)-L_{Iso}(t)-p_{Home}L_{Home}(t)\right)\right)}{N}$$

Initial values

Number of susceptible individuals $S(0)=N-1$

Number of individuals in the latent period $E_1(0)=L_{init}$, $E_k=0$ for $1<k\le n_E$

Number of individuals in the prodromal period $P_k(0)=0$ for $1\le k\le n_P$

Number of individuals in the early infectious period $I_k(0)=0$ for $1\le k\le n_I$

Number of individuals in the late infectious period $L_k(0)=0$ for $1\le k\le n_L$

Number of recovered $R(0)=0$

Number of dead individuals $D(0)=0$

Parameters

N Population size

L_{Init} Number of initial infections

$Q_{\max}$ Maximum capacity of the isolation units

t_{Iso_1} Day when the case isolation measures start in the population

t_{Iso_2} Day when the case isolation measures end in the population

p_{Home} Prevented fraction of contacts of cases who are isolated at home

p_{Dist} Prevented fraction of contacts because of general social distancing measures

$t_{Dist\ 1}$ Day when social distancing measures start

$t_{Dist\ 2}$ Day when social distancing measures end

λ Total force of infection

λ_{Ext} Force of infection which originates from outside of the population (e.g. via travellers)

R_0 All-year average of the basic reproduction number

β Seasonally varying effective contact rate which includes the effect of general contact reduction measures

a Amplitude of the seasonal fluctuation of the basic reproduction number

$t_{R0_{\max}}$ Day when the seasonal fluctuation of the basic reproduction number reaches its maximum

D_E Average duration of the latent period

n_E Number of stages for the latent period

ε Stage transition rate for the latent period $(\varepsilon = n_E / D_E)$

D_P Average duration of the prodromal period

n_P Number of stages for the prodromal period

φ Stage transition rate for the prodromal period $(\varphi = n_P / D_P)$

c_P Contagiousness in the prodromal period (relative to the contagiousness in the early infectious period)

D_I Average duration of the early infectious period

n_I Number of stages for the early infectious period

γ Stage transition rate for the early infectious period $(\gamma = n_I / D_I)$

D_L Average duration of the late infectious period

n_L Number of stages for the late infectious period

δ Stage transition rate for the late infectious period $(\delta = n_L / D_L)$

c_L Contagiousness in the late infectious period (relative to the contagiousness in the early infectious period)

f_{Sick} Fraction of individuals in the (early and late) “infectious period” who have symptoms (i.e. who are sick)

$f_{Test_{Sick}}$ Fraction of sick cases who are tested for SARS-CoV-2 while the circulating infection is unknown

T_{Sick} Threshold fraction of sick cases which triggers a specific general contact reduction

p_{Sick} Prevented fraction of contacts while the number of sick cases exceeds the threshold T_{Sick}

$f_{Consult}$ Fraction of sick cases who seek medical help

f_{Hosp} Fraction of sick cases who are hospitalized

D_{Hosp} Average duration of hospitalization

$f_{Test_{Sick}}$ Fraction of hospitalized cases who are tested for SARS-CoV-2 while the circulating infection is unknown

T_{Hosp} Threshold fraction of hospitalized cases which triggers a specific general contact reduction

p_{Hosp} Prevented fraction of contacts while the number of hospitalized cases exceeds the threshold T_{Hosp}

f_{ICU} Fraction of hospitalized cases who need intensive care

D_{ICU} Average duration of stay at the ICU

T_{ICU} Threshold fraction of cases in ICU which triggers a specific general contact reduction

p_{ICU} Prevented fraction of contacts while the number of cases in ICU exceeds the threshold T_{ICU}

f_{Dead} Fraction of sick cases who die from the disease

$f_{Test_{Dead}}$ Fraction of dead cases who are tested for SARS-CoV-2 while the circulating infection is unknown

Incidences (number of new events in a given time interval)

Infections in time interval $[t_1, t_2]$ $$\int_{t_1}^{t_2} \lambda(t) S(t)\, dt$$

Sick cases in time interval $[t_1, t_2]$ $$f_{Sick} \int_{t_1}^{t_2} \varphi\, P_{n_P}(t)\, dt$$

Consultations in time interval $[t_1, t_2]$ $$f_{Sick}\, f_{Consult} \int_{t_1}^{t_2} \varphi\, P_{n_P}(t)\, dt$$

Hospitalizations in time interval $[t_1, t_2]$ $$f_{Sick}\, f_{Hosp} \int_{t_1}^{t_2} \varphi\, P_{n_P}(t)\, dt$$

ICU admissions in time interval $[t_1, t_2]$ $$f_{Sick}\, f_{Hosp}\, f_{ICU} \int_{t_1}^{t_2} \varphi\, P_{n_P}(t)\, dt$$

Detection probability

SARS-CoV-2 infections which are brought into the country may not be detected and may spread without being noticed because the symptoms of COVID-19 may easily be confused with other influenza-like illnesses (ILI). Few practitioners may decide to order a SARS-CoV-2 test for what they regard a normal ILI patient while no community-transmitted cases in the population have been reported. If we assume that fractions of ILI patients who (a) seek medical help or who (b) are hospitalized or who (c) die from the disease are tested for SARS-Cov-2, then the probability that *not one single test* has been performed on a COVID-19 patient by time *t* despite the ongoing transmission in the population is given by:

Probability not to detect any case $$\left(1 - f_{Test_{Sick}}\right)^{f_{Sick} \int_0^t \varphi P_{nP}(\tau) d\tau} \left(1 - f_{Test_{Hosp}}\right)^{f_{Sick} f_{Hosp} \int_0^t \varphi P_{nP}(\tau) d\tau} \left(1 - f_{Test_{Death}}\right)^{f_{Sick} f_{Death} \int_0^t \delta L_{n_L}(\tau) d\tau}$$

Probability detect at least one case $$1 - \left(1 - f_{Test_{Sick}}\right)^{f_{Sick} \int_0^t \varphi P_{nP}(\tau) d\tau} \left(1 - f_{Test_{Hosp}}\right)^{f_{Sick} f_{Hosp} \int_0^t \varphi P_{nP}(\tau) d\tau} \left(1 - f_{Test_{Death}}\right)^{f_{Sick} f_{Death} \int_0^t \delta L_{n_L}(\tau) d\tau}$$