1 Assignment 1 (100 points)

1.1 Learning Ob jectives

1.2 Technical Task (50 points)

In Lab 4, you were provided with code that takes two images (an exemplar and a testing image), finds SIFT keypoints, extracts SIFT descriptors, and then performs matching using OpenCV's brute force matcher.

The brute force matching algorithm takes every descriptor found in the exemplar image and finds its nearest match in the testing image, no matter how bad that match is. You can use the slider on the top of the window so that the program will only display matches where the difference between the descriptors is less than an adjustable threshold. Collect input files and produce output files showing the feature matching working as described below.

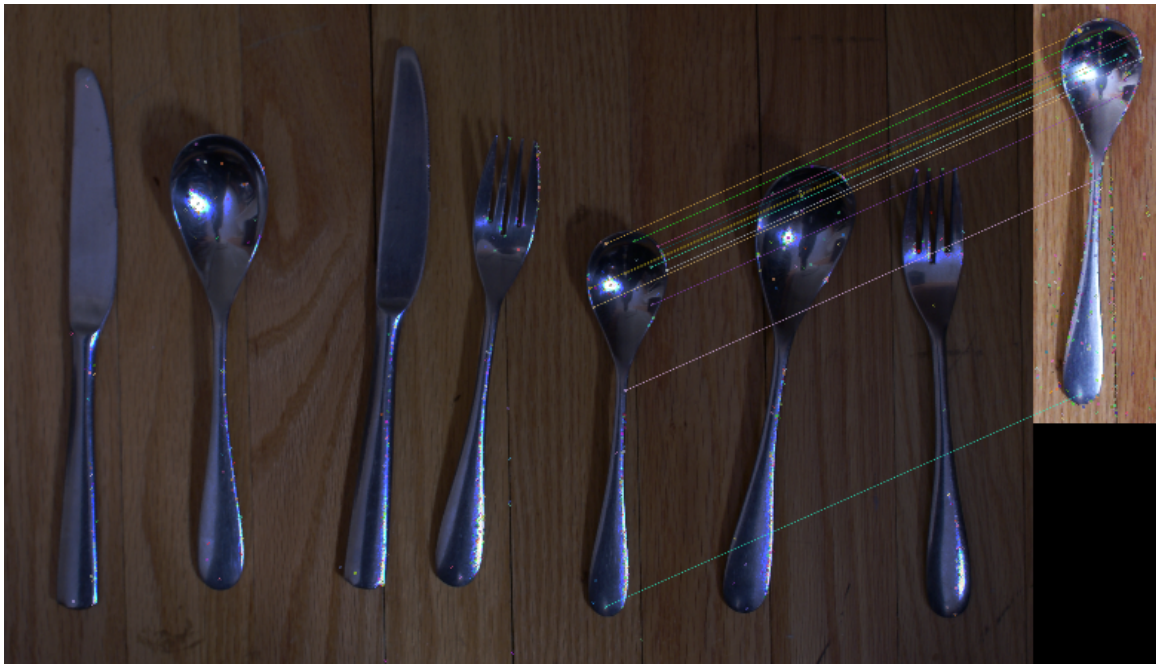
1. (3 pts) Take a clear picture of some object in your environment and crop out a thumbnail of the object of interest. Use the program from Lab to perform matching of features from the thumbnail to the larger image. Save your output as "matchingYourObject01.png".

matchingYourObject01.png

Many keypoints match – Successful.

2. (24 pts)

(a) Change (Darkened) the brightness of the image.



matchingYourObject02.png

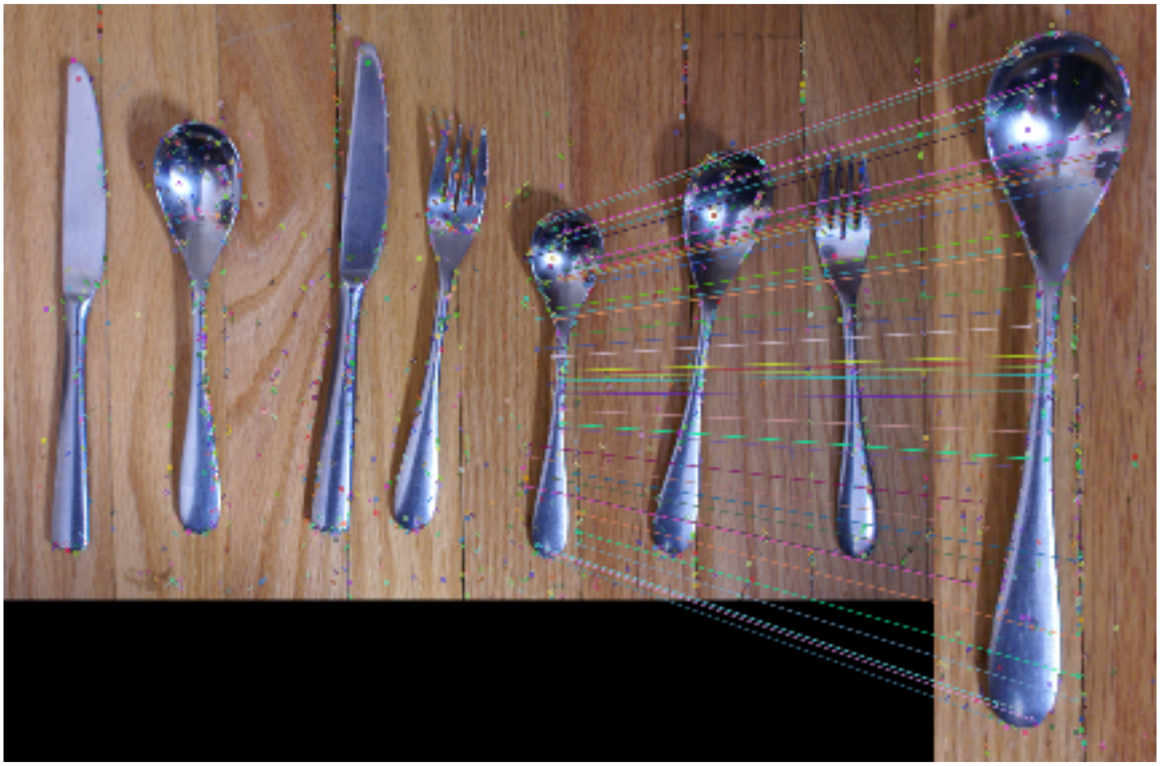
Nine keypoints match – Successful.

(b) Change (Reduced) the contrast of the image

matchingYourObject03.png

Five keypoints match – Successful.

(c) Change (Reduced by 2X) the size of the image (bigger or smaller)



matchingYourObject04.png

Five keypoints match – Successful.

(d) Blur the image

matchingYourObject05.png

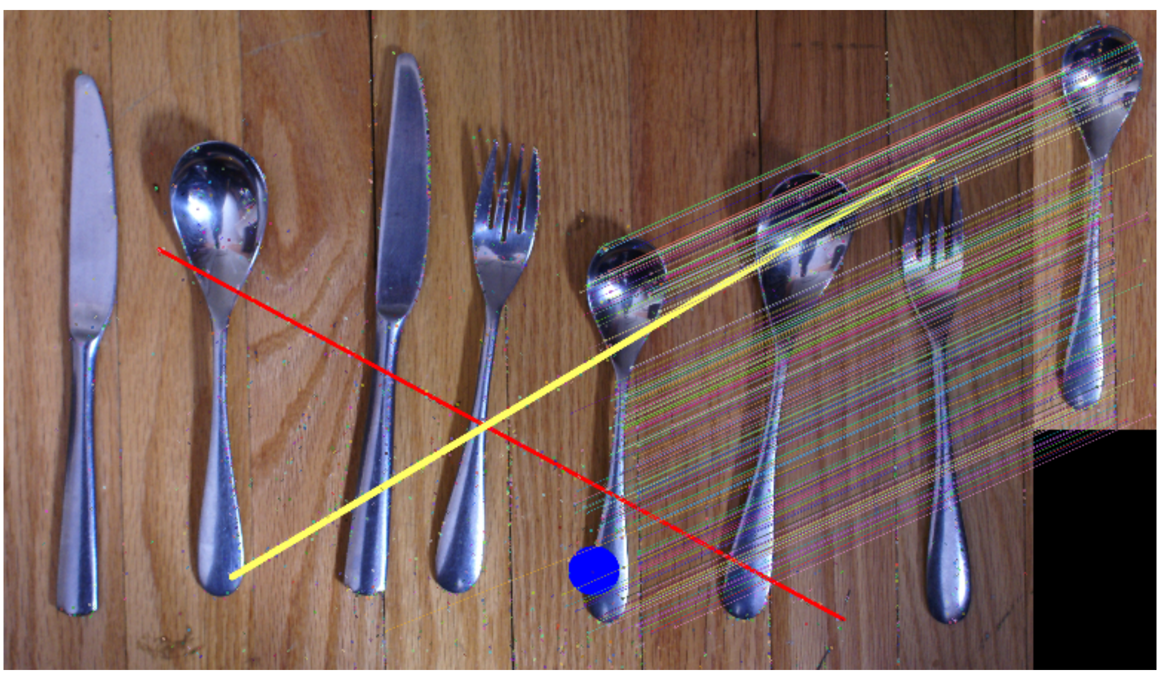
Two keypoints match – Unsuccessful.

(e) Add noise to the image

matchingYourObject06.png

Five keypoints match – Successful.

(f) Draw some marks across the region of interest in the image to simulate occlusion



matchingYourObject07.png

Many keypoints match – Successful.

Based on your outputs, write a paragraph summarizing your findings. You should write approximately one sentence per image manipulation. (6 pts)

Everything is a matter of degree. I could have occluded, blurred, etc. the images to an extent that even a human could not correlate the exemplar image and an object in it. However, the SIFT algorithm was able to find the small spoon under moderate amounts of manipulation:

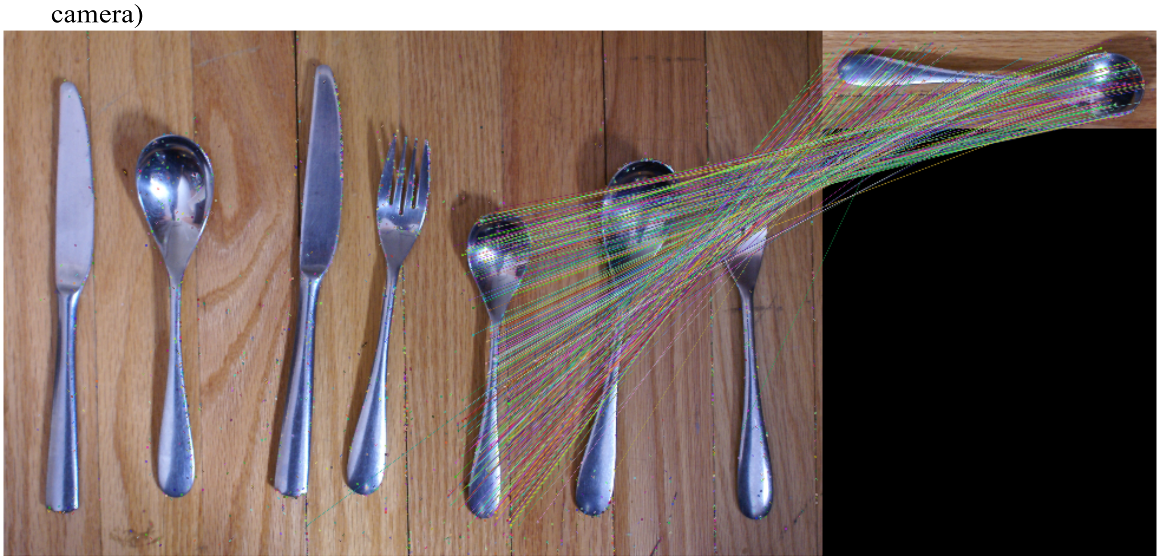
Very Well: Under scale change and occlusion. Obviously, any of the obstructed keypoints in the occluded image could not be correlated.

Moderately Well: Under reduced brightness, reduced contrast and noise.

Unsuccessful: Under a significant amount of blurring.

3. (23 pts)Take several pictures of the same object, crop a thumbnail from one of the images, and try to perform matching in one of the other images. You can use **Part** 3 from Lab 3 to capture a sequence of images from a webcam. Save your output as ”matchingYourObject08.png”,”matchingYourObject09.png”,etc.(3 pts each)

   (a) Use a planar (flat) object and rotate it in-plane (keeping the face of it towards the camera)



matchingYourObject08.png
90° Planar rotation of object – Successful.

(b) Use a planar object and rotate it out-of-plane (rotating the face of the object away from the camera)

matchingYourObject09.png

Spoon Out of Plane – Unsuccessful.

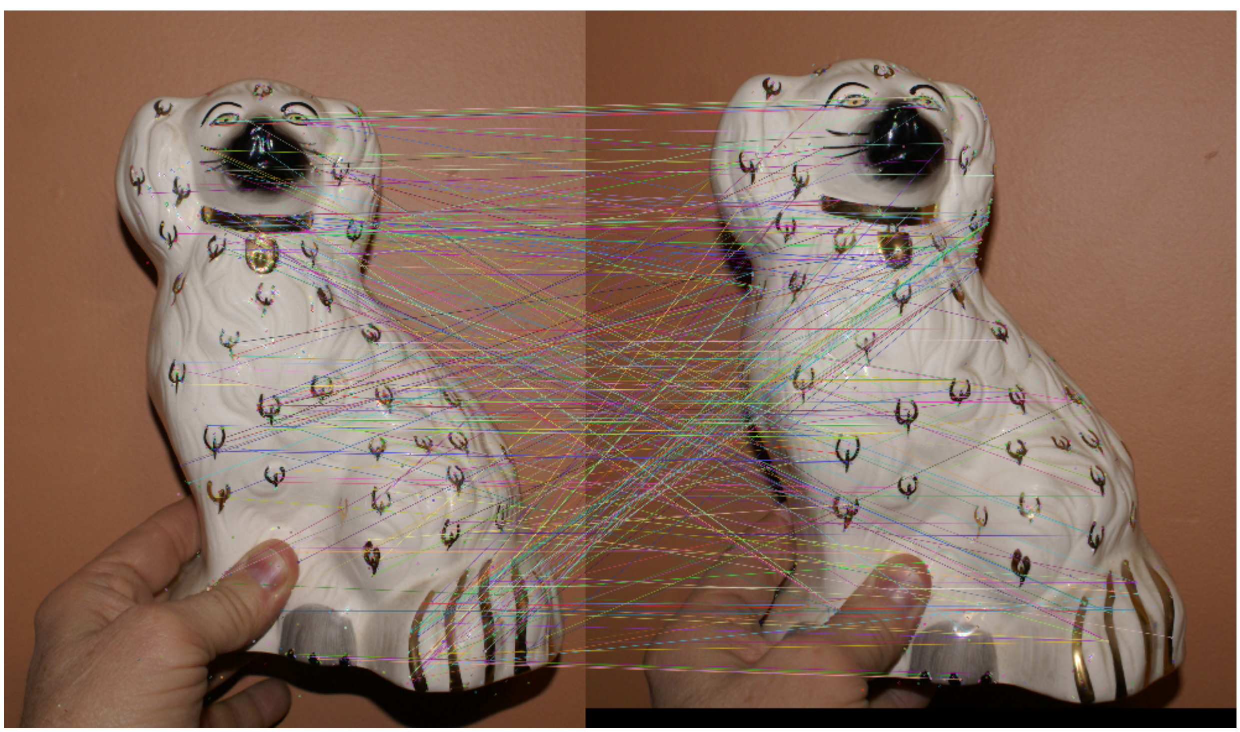
(c) Use a planar object and move it closer to and further away from the camera

matchingYourObject10.png

Far Away Spoon – Unsuccessful.

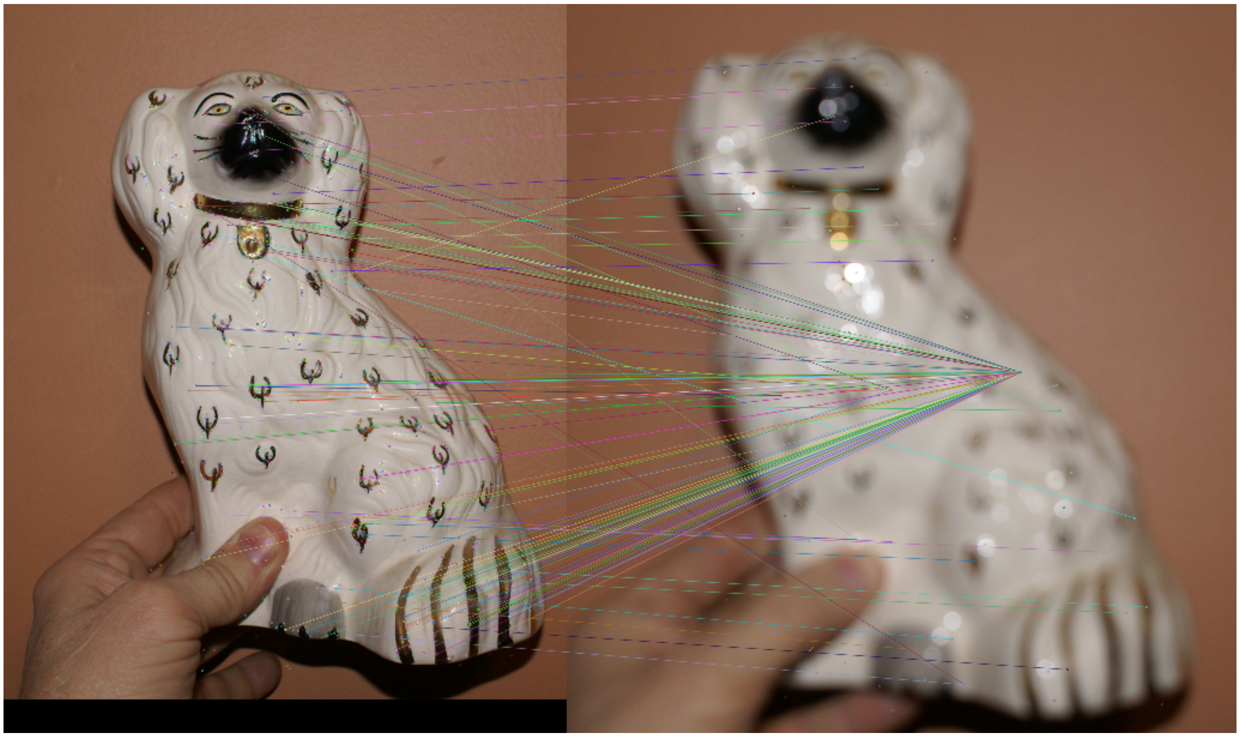
matchingYourObject11.png
Close Spoon – Unsuccessful.

(d) Use a 3D object and rotate it in front of the camera



matchingYourObject12.png
3D Object with 20 deg Rotation –Successful.

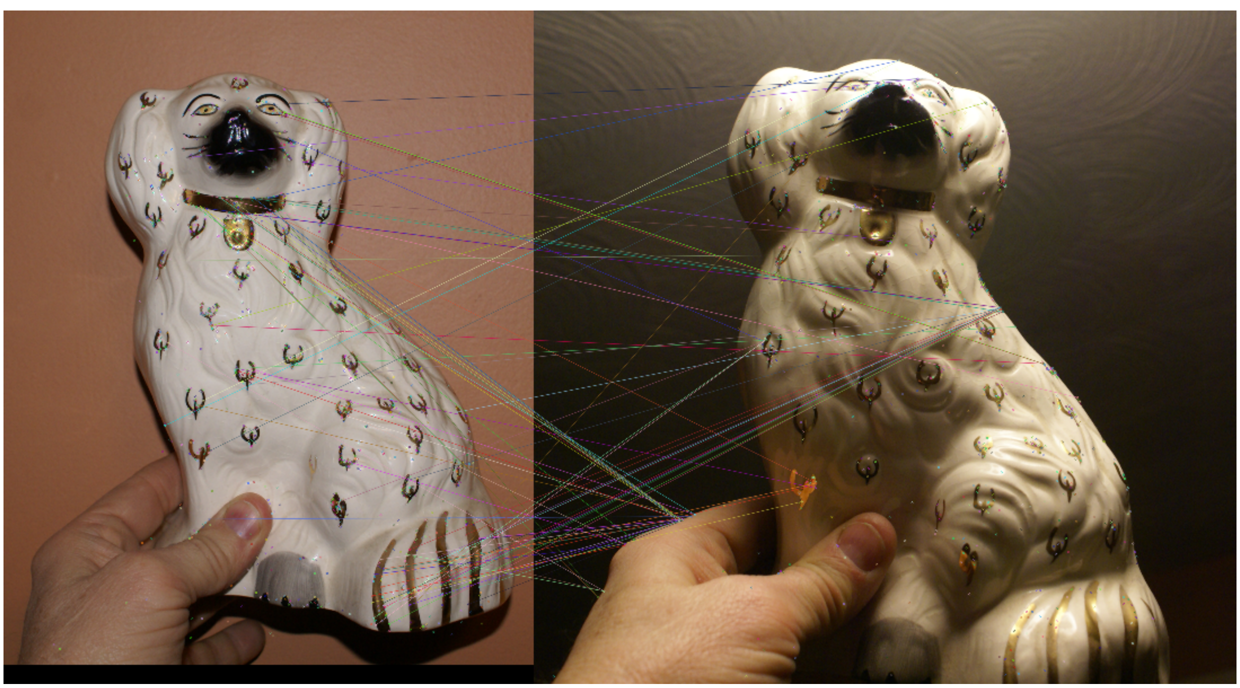
(e) Use a 3D object and move it closer to and further away from the camera. See if you can catch images before the camera's auto-focus catches up with you.



matchingYourObject13.png

3D Object with Heavy Blurring –Some Keypoint Matching

(f) Use a 3D object and have a friend turn the lights in the room on or off (the effect of this should be different than simply brightening a single image in photoshop, since the angle of the light will probably change.)



matchingYourObject14.png

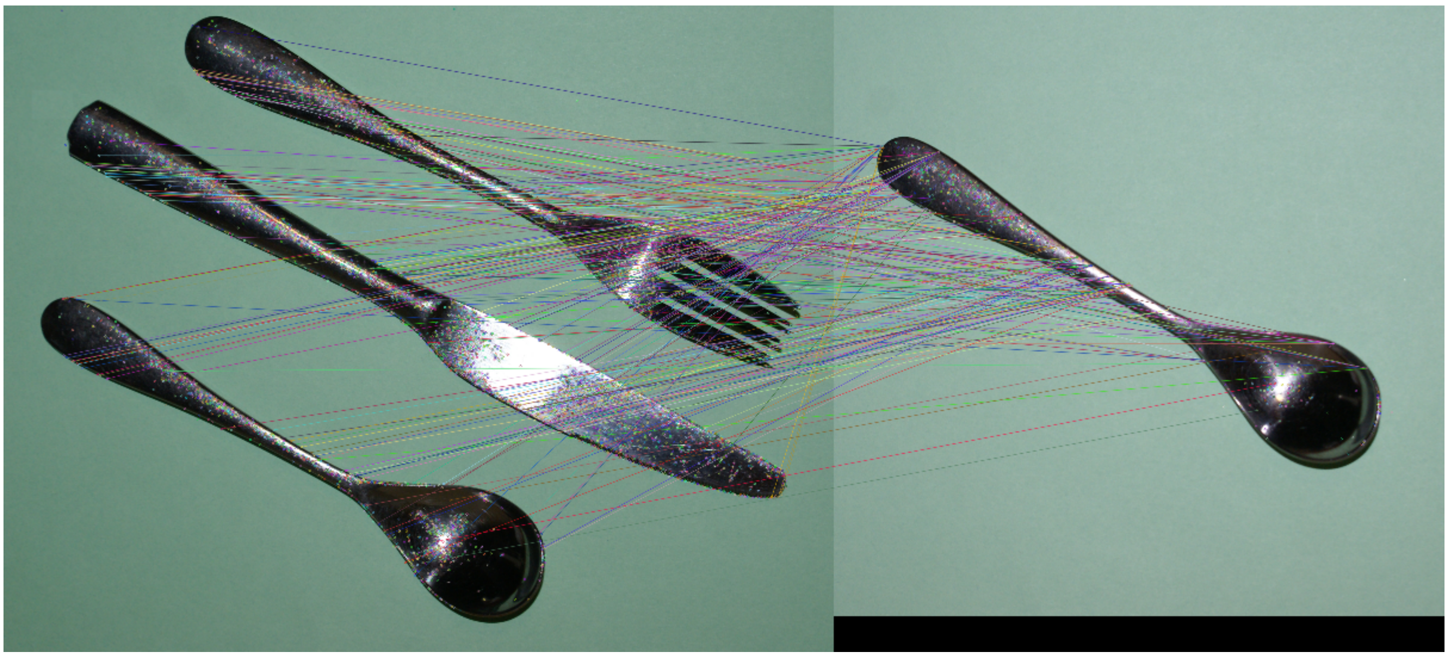
3D Object with Top Lighting –Unsuccessful

Based on your outputs, write a paragraph summarizing your findings. You should write approximately one sentence per object manipulation. (5 pts)

My spoon is a mostly planar item, but it has no texture or interesting features that can server as keypoints. The edge features suffer from the aperture problem. However, I think that the SIFT algorithm did so well in the previous section is because the algorithm was not correlating any part of the spoon, but instead was correlating reflections in the spoon and also the interesting patterns in the wood floor near the edge of the spoon. I think in diffused lighting on a plain color background, the SIFT algorithm would totally fail on my silverware.

When I tilted the spoon out of the plane to normal to the optical axis, the algorithm failed. Likewise under indirect lighting was it unsuccessful. I was surprised that under blurring, some of the keypoints matched, but I think it was for the same reasons that I gave in the earlier paragraph.

This image illustrates my point. There are as many keypoint matches between the fork and the spoon as there are spoon to spoon. Consequently, SIFT would not be a good algorithm for classifying (and counting) texture-less objects like silverware.



1.3 Written Questions (20 points)

1. (3 pts)List four different types of invariance that we would like to have when matching feature descriptors (i.e. we would like to be able to match features even when images are different in what ways.)

Scale, Lighting (brightness), Rotation and Translation. It would also be nice to have some degree of invariance when the object changes its angle in relation to the optical axis, but this is difficult.

2. (17 pts)Histograms are an extremely useful tool for summarizing information about a set of numbers. SIFT descriptors are just weighted histograms of the orientation of gradients in a window surrounding the keypoint. (Other uses are possible.)

   Below is a table of the orientations of some gradients in a 5×5 window. The magnitude of the gradient is also provided. Format:(orientation,magnitude)

$(5^\circ,1)$	$(-5^\circ,2)$	$(-80^\circ,1)$	$(-150^\circ,4)$	$(-160^\circ,5)$
$(-44^\circ,1)$	$(-10^\circ,2)$	$(-70^\circ,1)$	$(-150^\circ,3)$	$(-160^\circ,5)$
$(-88^\circ,2)$	$(-45^\circ,2)$	$(-75^\circ,1)$	$(-148^\circ,3)$	$(-160^\circ,4)$
$(-91^\circ,3)$	$(-75^\circ,1)$	$(-90^\circ,2)$	$(-145^\circ,2)$	$(-155^\circ,4)$
$(-92^\circ,3)$	$(-85^\circ,1)$	$(-93^\circ,2)$	$(-140^\circ,2)$	$(-150^\circ,3)$

   (a) (5 pts)Using histogram bins of B=45 degrees [-180 to 180) construct a histogram of the orientation of the gradients by quantizing the orientation into bins using the floor operation (so an orientation of 89 degrees will go into the bin from [45 – 90), and an orientation of 90 degrees will go into the bin from [90 – 135)).

Count	10	3.5	7	3.5	1	0	0	0	0
Math	1+1+ 1+1+ 1+1+ 1+1+ 1+1	1+0.5+ 1+1	1+ 1+ 1+0.5+1 1+0.5+ 1	1+ 1+1+ 0.5	1				
Degree Interval	-180° to -135°	-135° to -90°	-90° to -45°	-45° to -0°	0° to -45°	1° to 45°	45° to 90°	90° to 135°	135° to 180°

   (b) (6 pts)Using histogram bins of B=45 degrees, use linear interpolation to construct a histogram where the weight of the samples is divided according to the distance between the data point and the center of each bin. To do this, you should identify the two bin centers c_1 and c_2 on either side of the sample x, compute the distances between the sample point and the bin centers d_1 and d_2, and then divide the weight for the sample point using the ratio $(B - d_1)/B$ and $(B - d_2)/B$ (So, a sample whose value was 90 degrees would be divided equally between the bins for [45–90) and [90 –135), and a sample whose value was 22.5 degrees would be wholly assigned to the bin for [0 –45).)

Count	17.86	26.84	8.82	3.42	3.06	0	0	0
Bin No.	1	2	3	4	5	6	7	8
Bin Center	-157.5	-112.5	-67.5	-22.5	22.5	66.5	112.5	157.5
Degree Interval	-180° to -135°	-135° to -90°	-90° to -45°	-45° to -0°	0° to -45°	45° to 90°	90° to 135°	135° to 180°

(c) (6 pts)Using histogram bins of 45 degrees, create a weighted histogram where each sample is weighted by the corresponding magnitude of the gradient using linear interpolation.

Count	35	9	9	6	1	0	0	0	0
Math	4+5+3+5+3+4 +2+4+2+3	3+1+3+2	1+ 1+ 2+1+1 1+1+ 1	2+1+2+1	1				
Degree Interval	-180° to -135°	-135° to -90°	-90° to -45°	-45° to -0°	0° to -45°	1° to 45°	45° to 90°	90° to 135°	135° to 180°

1.4 Lecture Preparation

1. $$AX = Y$$
$$A^{-1}(AX) = A^{-1}Y$$
$$IX = A^{-1}Y$$
$$X = A^{-1}Y$$

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\det(A) = ad - bc$

Since $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$X = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= \frac{1}{\det(A)} \begin{bmatrix} d \cdot y_1 - b y_2 \\ -c \cdot y_1 + a y_2 \end{bmatrix}$$

$$X_1 = \frac{d y_1 - b y_2}{ad - bc} \qquad x_2 = \frac{-c y_1 + a y_2}{ad - bc}$$

2. To prove that matrix multiplation doesn't commute have to find one counter example

$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 1\cdot1 + 1\cdot0 & 1\cdot1 + 1\cdot1 \\ 1\cdot1 + 0\cdot0 & 1\cdot1 + 0\cdot1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 1\cdot1 + 1\cdot1 & 1\cdot1 + 1\cdot0 \\ 0\cdot1 + 1\cdot1 & 0\cdot1 + 1\cdot0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A \cdot B \neq B \cdot A$$

$\therefore$ Matrix multiplication doesn't commute

(It does if A, B are diagonalizable)

8. $x^T \cdot y = \underset{1 \times n}{\begin{bmatrix} x_1 & x_2 \end{bmatrix}} \cdot \underset{n \times 1}{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}} = x_1 y_1 + x_2 y_2$

(#3 is below)

#3 $\Leftrightarrow$ #8

$= y_1 \cdot x_1 + y_2 x_2$ by commutative of mult.

$= \begin{bmatrix} y_1 & y_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$= y^T \cdot x$

4. $A = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$ $Y = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ $\det(A) = 3 \cdot 5 - 1 \cdot 1$
$= 14$

$X = A^{-1} Y$

$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix}$ Prop of 2x2 Matrix for finding inverse

$X = \frac{1}{14} \begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$

$= \frac{1}{14} \begin{bmatrix} 24 \\ -2 \end{bmatrix}$

$= \begin{bmatrix} 12/7 \\ -1/7 \end{bmatrix}$

5. $6x_1 + 2x_2 = 10$

equals $2(3x_1 + x_2 = 5)$

so there are infinite solutions lying on the line $x_2 = -3x_1 + 5$

6. $(AB)^T = \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \right)^T$

$= \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}^T$ Def of matrix mult.

$= \begin{bmatrix} ae + bg & ce + dg \\ af + bh & cf + dh \end{bmatrix}$ Def of transpose

$= \begin{bmatrix} ea + gb & ec + gd \\ fa + hb & fc + hd \end{bmatrix}$ Commutative of mult.

$= \begin{bmatrix} e & g \\ f & h \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ Def of matrix mult.

$= B^T A^T$ Def of transpose

7. $(AX)^T = \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right)^T$ Def of matrix mult

$= \begin{pmatrix} ax_1 + bx_2 \\ cx_2 + dx_2 \end{pmatrix}^T$

$= \begin{pmatrix} x_1 a + x_2 b \\ x_2 c + x_2 d \end{pmatrix}^T$ Commutative of mult.

$= [x_1 \; x_2] \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Def of transpose

$= X^T A^T$

3. (#8 is above) $X \cdot Y = [x_1 y_1 + x_2 y_2]$ Def of dot product

#8 $\Leftrightarrow$ #3

$= [x_1 \; x_2] \cdot \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ Def of matrix mult.

$= X^T \cdot Y$

9. You can prove that

$x^T A^T A x$ is a scalar by just looking at the dimensionality

$$\underset{(1\times 2)}{X^T} \quad \underset{(2\times 2)}{A^T} \quad \underset{(2\times 2)}{A} \quad \underset{(2\times 1)}{X}$$

Matrix multiplication is associative so

$$\Big[\underset{(1\times 2)}{[x^T A^T]} \ \underset{(2\times 2)}{A}\Big] \ \underset{(2\times 1)}{X}$$

$$\underset{(1\times 2)}{[X^T A^T A]} \quad \underset{(2\times 1)}{X}$$

$$\underset{(1\times 1)}{[X^T A^T A X]}$$ which is scalar

If you actually multiply it out

$$A^T A = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Let
$e = a^2 + c^2$
$f = b^2 + d^2$
$g = ab + cd$

$$= \begin{bmatrix} a^2+c^2 & ab+cd \\ ab+cd & b^2+d^2 \end{bmatrix}$$

$$= \begin{bmatrix} e & g \\ g & f \end{bmatrix}$$

$$X^T A^T A = [x_1 \ x_2] \begin{bmatrix} e & g \\ g & f \end{bmatrix}$$

$$X^T A^T A X = [x_1 e + x_2 g \quad x_1 g + x_2 f] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$X^T A^T A X = [x_1(x_1 e + x_2 g) + x_2(x_1 g + x_2 f)]$$

$$= e x_1^2 + g x_1 x_2 + g x_1 x_2 + f x_2^2$$

$$= e x_1^2 + 2g x_1 x_2 + f x_2^2$$

10. $$\frac{\partial q}{\partial x_1} = \frac{\partial(ex_1^2 + 2gx_1x_2 + fx_2^2)}{\partial x_1}$$

$$= 2ex_1 + 2gx_2 + ?$$

$$= 2(a^2+c^2)x_1 + 2(ab+cd)x_2$$

$$\frac{\partial q}{\partial x_2} = \frac{\partial(ex_1^2 + 2gx_1x_2 + fx_2^2)}{\partial x_2}$$

$$= 2fx_2 + 2gx_1$$

$$= 2(b^2+d^2)x_2 + 2(ab+cd)x_1$$

$$\frac{\partial q}{\partial x_1 \partial x_2} = 2 \cdot \begin{bmatrix} (a^2+c^2)x_1 + (ab+cd)x_2 \\ (b^2+d^2)x_2 + (ab+cd)x_1 \end{bmatrix}$$

$$= 2 \begin{bmatrix} (a^2+c^2) + (ab+cd) & (b^2+d^2) + (ab+cd) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(dimensionality change)

$$= 2 \cdot \begin{bmatrix} a^2+c^2 & ab+cd \\ ab+cd & b^2+d^2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= 2 \cdot \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$ Using results from #9

$$= 2A^TAX$$

11. $s = x^T y = [x_1 \;\; x_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$= [x_1 y_1 + x_2 y_2]$

$\frac{\partial s}{\partial x_1} = \frac{\partial (x_1 y_1 + x_2 y_2)}{\partial x_1}$

$= y_1$

$\frac{\partial s}{\partial x_2} = \frac{\partial (x_1 y_1 + x_2 y_2)}{\partial x_2}$

$= y_2$

$\frac{\partial s}{\partial x_1 \partial_2} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = Y$

12. $\overset{2\times 4}{\begin{bmatrix} x_1 & x_2 & 0 & 0 \\ 0 & 0 & x_1 & x_2 \end{bmatrix}} \overset{4\times 1}{\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}}$

$= \overset{2\times 1}{\begin{bmatrix} a x_1 + b x_2 \\ c x_1 + d x_2 \end{bmatrix}}$

$= \overset{2\times 2}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \overset{2\times 1}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$

13. $y' = \underset{3\times3}{T}\,\underset{3\times1}{x'}$

$$= \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 + t_x \\ x_2 + t_y \\ 1 \end{bmatrix}$$

A translation $(x_1, x_2) \xrightarrow{(t_x, t_y)} (y_1, y_2)$

$$= \begin{matrix} x_1 + t_x = y_1 \\ x_2 + t_y = y_2 \end{matrix}$$

So $\begin{bmatrix} x_1 + t_x \\ x_2 + t_y \\ 1 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = y'$