1 Assignment 6

1.1 Learning Objectives

1.2 Written Questions

1. Computing a transformation matrix
   a. Find corresponding Keypoints between an object and an image using an appropriate algorithm like SIFT.
   b. Eliminate outliers among the point using an algorithm like RANSAC. At least 3 good point are required but over determination is better
   c. Solve this linear system

$$\underbrace{\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{a} = \underbrace{\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}}_{x'}$$

   where X is the linear transformation matrix, a is an original Keypoint, and x' is the mapped Keypoint
   d. In actuality, a numerical method would be used to find this:

$$\min(Xa - x')$$

2. Warping four corners
   Original four corners
   $(0,0)$, $(w-1, 0)$, $(w-1, h-1)$, $(0, h-1)$
   Warped four corners
   $(0,0)$, $T\begin{bmatrix} w-1 \\ 0 \\ 1 \end{bmatrix}$, $T\begin{bmatrix} w-1 \\ h-1 \\ 1 \end{bmatrix}$, $T\begin{bmatrix} 0 \\ h-1 \\ 1 \end{bmatrix}$, $T\begin{bmatrix} 0 \\ h-1 \\ 1 \end{bmatrix}$

1.3 Technical Task
See Zip file and contents of the "Data" folder

1.4 Lecture Preparation

a. SSD original $= \sum_{i=1}^{|W|} (x_i - y_i)^2$

Assuming uniform illumination $I_{new} - I_{orig} = \Delta i$

$$SSD_{new} = \sum_{i=1}^{|W|} \left[x_i - (y_i + \Delta i)\right]^2$$
$$= \sum x_i^2 - 2x_i(y_i + \Delta i) + y_i^2 + 2\Delta i + \Delta i^2$$
$$= \sum x_i^2 - 2x_iy_i + y^2 - 2x_i\Delta i + 2\Delta i + \Delta i^2$$
$$= SSD_{orig} + \underbrace{\sum\left[2\Delta i(1 - x_i) + \Delta i^2\right]}_{\text{Induced error}}$$

b. Show normalized Correlation Coefficient is invarient to changes in brihtness and contrast

$$\overline{ax_i + b} = \frac{1}{|W|}\sum ax_i + b = b + a\frac{\sum x_i}{|W|}$$
$$= b + a\overline{x}$$

$$\sigma^2_{ax_i+b} = \frac{1}{|W|}\sum\left[ax_i + b - (b + a\overline{x})\right]^2$$
$$= \frac{1}{|W|}\sum\left[a(x_i - \overline{x})\right]^2$$
$$= \frac{1}{|W|}\cdot a^2\sum(x_i - \overline{x})^2$$
$$= a^2 \cdot \sigma^2_{x_i} \text{ or } \sigma_{ax_i+b} = a \cdot \sigma_{x_i}$$

$$nCC_{ax_i+b} = \frac{1}{|W|} \sum \frac{[ax_i+b-(b-a\bar{x})][ay_i+b-(b-a\bar{y})]}{a^2\sigma_{x_i} \cdot a^2\sigma_{y_i}}$$

$$= \frac{1}{a^4} \cdot \frac{1}{|W|} \sum \frac{a^2(x_i-\bar{x})(y_i-\bar{y})}{\sigma_x \sigma_y}$$

$$= \frac{1}{|W|} \sum \frac{(x_i-\bar{x})(y_i-\bar{y})}{\sigma_x \sigma_y}$$

$\therefore nCC_{ax_i+b} = nCC_{x_i}$ and the normalized Correlation Coefficient is invarient to change in brightness and contrast

c. The goal of Newton-Raphson method is to find roots for a function - values in the domain at which the function equals zero.

Finding the line tangent to $f(x)$ at x_0

$$y = mx + b$$
$$m = f'(x_0)$$
$$y = f(x_0)$$
$$f(x_0) = f'(x_0)x_0 + b$$
$$b = f(x) \cdot f'(x_0)$$

$\therefore y = f'(x_0)x + f(x_0) - f'(x_0)x_0$ is the equation for the tangent line

d. Brush up on Eigen values