CSC 585
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1 Assignment 9

1.1 Learning Objectives

1.2 Technical Task

In the Zip file, within the "Data" directory are the result of applying active contours to:

Easy Object / Easy Background
blueTop_result_0000.jpg
blueTop_result_0093.jpg
blueTop_result_0286.jpg

Irregular Object / Textured Background
glove_result_0000.jpg
glove_result_0062.jpg
glove_result_0189.jpg

1.3 Lecture Preparation

1. Rewrite by factoring out 1/z scaling factor from 3-d vector

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \frac{1}{z} \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \\ \frac{1}{z} \end{bmatrix} = \frac{1}{z} \begin{bmatrix} \frac{f \cdot x}{z} \\ \frac{f \cdot y}{z} \\ 1 \end{bmatrix}$$

yields a 2-d vector in normal form

2. Verify skew-symmetric matrix reprensentations equals cross products $u \times v$ and $-v \times u$

$$\begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -u_3 v_2 + u_2 v_3 \\ u_3 v_1 - u_1 v_3 \\ -u_2 v_1 + u_1 v_2 \end{bmatrix} = \begin{bmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{bmatrix}$$

$= u \times v$ which is the computational definition of a cross-product

2 cont.

$$\begin{bmatrix} 0 & v_3 & -v_2 \\ -v_3 & 0 & v_1 \\ v_2 & -v_1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} v_3 u_2 - v_2 u_3 \\ -v_3 u_1 + v_1 u_3 \\ v_2 u_1 - v_1 u_2 \end{bmatrix} = \begin{bmatrix} u_2 v_3 - u_3 v_2 \\ u_3 v_1 - u_1 v_3 \\ u_1 v_2 - u_2 v_1 \end{bmatrix}$$

$$= u \times v$$

3. Slope $= \frac{(n-t)}{(m-s)}$ from def $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

$$\overset{u}{\begin{bmatrix} m \\ n \\ 1 \end{bmatrix}} \times \overset{v}{\begin{bmatrix} s \\ t \\ 1 \end{bmatrix}} = \begin{bmatrix} n \cdot 1 - 1 \cdot t \\ 1 \cdot s - m \cdot 1 \\ m \cdot t - n \cdot s \end{bmatrix} = \begin{bmatrix} n - t \\ s - m \\ mt - ns \end{bmatrix}$$

$$(n-t)x + (s-m)y + (mt-ns) = 0$$

$$(s-m)y = -(n-t)x - (mt-ns)$$

$$y = \frac{-(n-t)x}{(s-m)} - \frac{(mt-ns)}{(s-m)}$$

$$y = \frac{(n-t)}{(m-s)}x + \frac{(mt-ns)}{(m-s)}$$

Slope is correct from above. Now, confirm y-intercept

$$y = \frac{(n-t)}{(m-s)}x + b \quad \text{or} \quad b = y - \frac{(n-t)}{(m-s)}x$$

Substituting first point (m, n)

$$b = n - \frac{(n-t)}{(m-s)}m$$

$$= \frac{(m-s)n - (n-t)m}{(m-s)}$$

$$= \frac{\cancel{mn} - sn - \cancel{nm} + tm}{(m-s)}$$

$$= \frac{(mt-ns)}{(m-s)}$$ Confirmed that cross product of two vectors in homogeneous coordinates gives a line.

4. Let f be the focal length of cameras C_1, C_2 and C_3. Let S be the scene depth which is the perpendicular distance from the dotted line running through point P and the X-axis

(a) From the pinhole model, the coordinates of the image plane pixel are

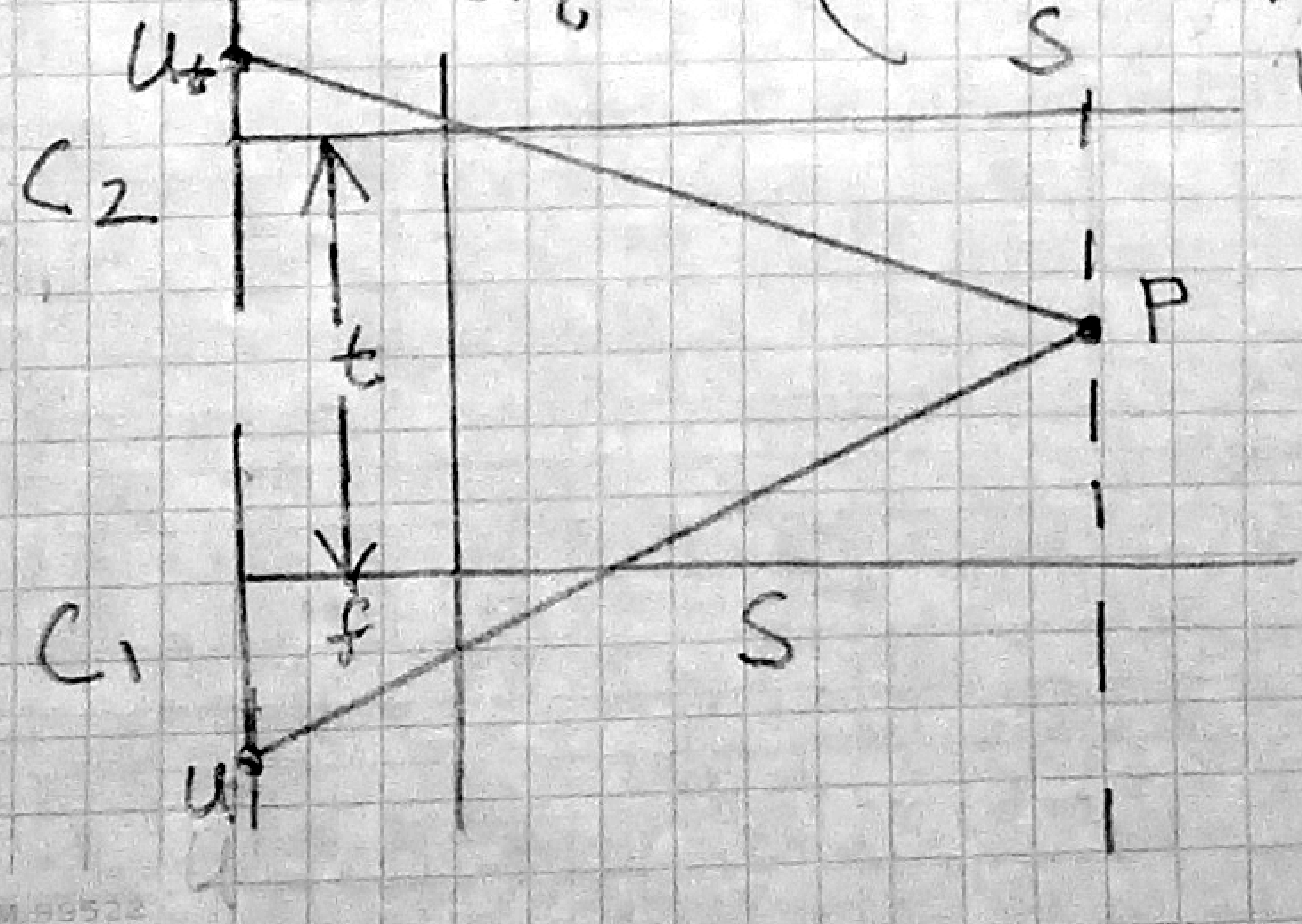
$$u = \Big(\underbrace{\frac{-f \cdot x}{S}}_{u_x}, \underbrace{\frac{-f \cdot y}{S}}_{u_y}, \underbrace{\frac{-f \cdot z}{S}}_{u_z}\Big)$$

The negative sign indicate the displacements x, y, z for the image and world points are opposite wrt to the coordinate axis. The rays cross in the pin hole

(b) The camera is only translated along the x-axis so y and z will not change. Also the image pixel moves in the <u>same</u> direction as the camer translation so

(c)

$$u_t = \Big(\frac{f \cdot t}{S} + \underbrace{u_x, u_y, u_z}_{\text{defined above}}\Big)$$



4 (d) the image point moves on the image plane in the opposite direction of the rotation of the camera C_3

(e) I would choose $R(-45°)$ because of the reason I just mentioned