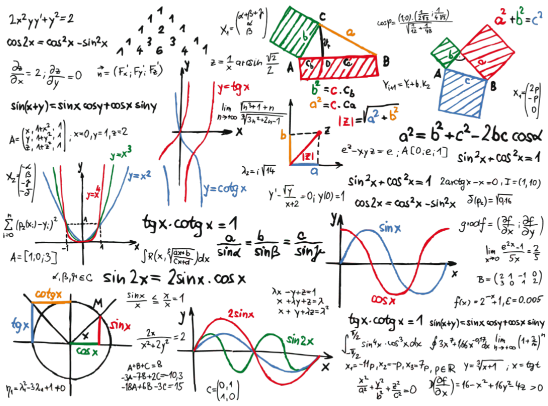
B5 - Mathematics

B-MAT-500

306radiator

Plumbing and the Gauss Curve



2.1

306radiator

binary name:	306radiator
repository name:	306radiator_*$ACADEMIC_YEAR*
repository rights:	ramassage-tek
language:	everything working on "the dump"
compilation:	when necessary, via Makefile, including re, clean and fclean rules

- Your repository must contain the totality of your source files, but no useless files (binary, temp files, obj files,...).
- All the bonus files (including a potential specific Makefile) should be in a directory named *bonus*.
- Error messages have to be written on the error output, and the program should then exit with the 84 error code (0 if there is no error).

A radiator is placed in a square room. We want to know how the temperature is distributed in the room based on the radiator's location. The temperature in the room verifies the very famous heat equation:

$$\frac{\partial^2 T}{\partial x^2}(x,y) + \frac{\partial^2 T}{\partial y^2}(x,y) = f(x,y)$$

where:

- T is the room temperature,
- $\partial^2 T/\partial u^2$ is the second partial derivative of T relative to u,
- f is the heating power emitted by the radiator.

To simplify things, we assume that the temperature along the room's walls is non-existent (these are the boundary conditions), and that the radiator is regular; the f function is therefore null everywhere except in this point.

It is impossible to analytically solve this equation. Therefore, you must look for an approximation. In order to do this, we map the room on a grid of N^2 points, by only considering N values of x (named $x_0 = 0, x_1, \ldots, x_{N-1}$) and N values of y (named $y_0 = 0, y_1, \ldots, y_{N-1}$) and defining h as the distance between two points.

It is therefore possible to approximate the second derivative in x and y by using the following formulas, with $1 \leq i,j \leq N-2$:

$$\frac{\partial^2 T}{\partial x^2}(x_i, y_j) = \frac{T(x_{i-1}, y_j) - 2T(x_i, y_j) + T(x_{i+1}, y_j)}{h^2}$$

$$\frac{\partial^2 T}{\partial y^2}(x_i, y_j) = \frac{T(x_i, y_{j-1}) - 2T(x_i, y_j) + T(x_i, y_{j+1})}{h^2}$$

The boundary conditions are written, with $0 \leq i, j \leq N-1$:

$$T(x_0, y_j) = T(x_{N-1}, y_j) = 0$$
$$T(x_i, y_0) = T(x_i, y_{N-1}) = 0$$

Therefore, we obtain the following system of N^2 equations:

$$\begin{cases}
T(x_0, y_0) & = 0 \\
\dots & \\
T(x_{N-1}, y_0) & = 0 \\
& \\
T(x_0, y_1) & = 0 \\
(T(x_1, y_0) + T(x_0, y_1) - 4T(x_1, y_1) + T(x_2, y_1) + T(x_1, y_2))/h^2 & = f(x_1, y_1) \\
(T(x_2, y_0) + T(x_1, y_1) - 4T(x_2, y_1) + T(x_3, y_1) + T(x_2, y_2))/h^2 & = f(x_2, y_1) \\
\dots & \\
T(x_{N-1}, y_1) & = 0 \\
& \\
T(x_0, y_2) & = 0 \\
(T(x_1, y_1) + T(x_0, y_2) - 4T(x_1, y_2) + T(x_2, y_2) + T(x_1, y_3))/h^2 & = f(x_1, y_2) \\
(T(x_2, y_1) + T(x_1, y_2) - 4T(x_2, y_2) + T(x_3, y_2) + T(x_2, y_3))/h^2 & = f(x_2, y_2) \\
\dots & \\
T(x_{N-1}, y_2) & = 0 \\
& \\
\dots & \\
& \\
T(x_0, y_{N-2}) & = 0 \\
(T(x_1, y_{N-3}) + T(x_0, y_{N-2}) - 4T(x_1, y_{N-2}) + T(x_2, y_{N-2}) + T(x_1, y_{N-1}))/h^2 & = f(x_1, y_{N-2}) \\
(T(x_2, y_{N-3}) + T(x_1, y_{N-2}) - 4T(x_2, y_{N-2}) + T(x_3, y_{N-2}) + T(x_2, y_{N-1}))/h^2 & = f(x_2, y_{N-2}) \\
\dots & \\
T(x_{N-1}, y_{N-2}) & = 0 \\
& \\
T(x_0, y_{N-1}) & = 0 \\
\dots & \\
T(x_{N-1}, y_{N-1}) & = 0
\end{cases}$$

Now we can define the two following vectors:

$$X = (T(x_0, y_0), \dots, T(x_{N-1}, y_0), T(x_0, y_1), \dots, T(x_{N-1}, y_{N-1}))$$
$$Y = (f(x_0, y_0), \dots, f(x_{N-1}, y_0), f(x_0, y_1), \dots, f(x_{N-1}, y_{N-1}))$$

If X_k and Y_k are the k^{th} coordinates of X and Y, the system becomes:

$$
\begin{cases}
X_0 & = Y_0 \\
\dots & \\
X_{N-1} & = Y_{N-1} \\
& \\
X_N & = Y_N \\
(X_1 + X_N - 4X_{N+1} + X_{N+2} + X_{2N+1})/h^2 & = Y_{N+1} \\
(X_2 + X_{N+1} - 4X_{N+2} + X_{N+3} + X_{2N+2})/h^2 & = Y_{N+2} \\
\dots & \\
X_{2N-1} & = Y_{2N-1} \\
& \\
X_{2N} & = Y_{2N} \\
(X_{N+1} + X_{2N} - 4X_{2N+1} + X_{2N+2} + X_{3N+1})/h^2 & = Y_{2N+1} \\
(X_{N+2} + X_{2N+1} - 4X_{2N+2} + X_{2N+3} + X_{3N+2})/h^2 & = Y_{2N+2} \\
\dots & \\
X_{3N-1} & = Y_{3N-1} \\
& \\
X_{N^2-2N} & = Y_{N^2-2N} \\
(X_{N^2-3N+1} + X_{N^2-2N} - 4X_{N^2-2N+1} + X_{N^2-2N+2} + X_{N^2-N+1})/h^2 & = Y_{N^2-2N+1} \\
(X_{N^2-3N+2} + X_{N^2-2N+1} - 4X_{N^2-2N+2} + X_{N^2-2N+3} + X_{N^2-N+2})/h^2 & = Y_{N^2-2N+2} \\
\dots & \\
X_{N^2-N-1} & = Y_{N^2-N-1} \\
& \\
X_{N^2-N} & = Y_{N^2-N} \\
\dots & \\
X_{N^2-1} & = Y_{N^2-1}
\end{cases}
$$

The heat equation can now be written as a $AX = Y$ matrix, a form on which can be applied the Gaussian elimination.

> It's up to you to determine the A matrix...

We write i_r and j_r the radiator's coordinates, with $1 \leq i_r, j_r \leq N - 2$.

> We fix the following values: $h = 0.5$ and $f(x_{i_r}, y_{i_r}) = -300$

Your program must either display the A matrix and the X vector on the standard output, or the temperature at a given point in the room.

Usage

```
Terminal
~/B-MAT-500> ./306radiator -h
USAGE
    ./306radiator n ir jr [i j]

DESCRIPTION
    n            size of the room
    (ir, jr)     coordinates of the radiator
    (i, j)       coordinates of a point in the room
```

We will limit ourselves to a precision at 10^{-1}.

Suggested bonuses

- A graphical display representing the temperature distribution in the room,
- Different boundary conditions,
- A second radiator.

Examples

Terminal

```
~/B-MAT-500> ./306radiator 4 1 1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	4	0	0	4	-16	4	0	0	4	0	0	0	0	0	0
0	0	4	0	0	4	-16	4	0	0	4	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	4	0	0	4	-16	4	0	0	4	0	0
0	0	0	0	0	0	4	0	0	4	-16	4	0	0	4	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

0.0
0.0
0.0
0.0
0.0
21.9
6.3
0.0
0.0
6.3
3.1
0.0
0.0
0.0
0.0
0.0
```

The matrix's coefficients are separated by tabs.
A blank line separates the matrix from the vector result.

```
Terminal
~/B-MAT-500> ./306radiator 4 1 1 2 2
3.1
```

```
Terminal
~/B-MAT-500> ./306radiator 5 1 2 3 2
3.3
```

```
Terminal
~/B-MAT-500> ./306radiator 8 4 6 3 6
9.4
```

```
Terminal
~/B-MAT-500> ./306radiator 12 3 9 1 6
2.5
```