

$$3.1 \quad (a) \quad x_1(t) = y(t), \quad x_2(t) = \frac{dy(t)}{dt} \quad F(t) = u(t)$$

$$(b) \quad M \frac{d^2 y(t)}{dt^2} + b \frac{dy(t)}{dt} + k y(t) = F(t)$$

$$M \dot{x}_2(t) + b x_2(t) + k x_1(t) = F(t)$$

$$M \dot{x}_2(t) = -b x_2(t) - k x_1(t) + F(t)$$

$$(c) \quad \dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F(t)$$

$$Y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} X(t)$$

3.3

$$x_1(t) = i_L(t) \quad x_2(t) = V_C(t) \quad V = V_2 - V_C \quad u(t) = \begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix}$$

$$Y(t) = V_R(t)$$

$$KVL: -V_1(t) + L \frac{di_L(t)}{dt} - V_C(t) + V_2(t) = 0$$

$$-V_1(t) + L \dot{i}_L(t) - x_2(t) + V_2(t) = 0$$

$$\dot{x}_1(t) = \frac{1}{L} x_2(t) + \frac{1}{L} V_1(t) - \frac{1}{L} V_2(t)$$

$$KCL: C \frac{dV_C(t)}{dt} = i_R - i_L \rightarrow C \dot{x}_2(t) = \frac{V}{R} - x_1(t) = x_1(t) - \frac{x_2}{R} + \frac{V_2}{R}$$

$$\dot{x}_2(t) = -\frac{1}{C} x_1(t) - \frac{1}{RC} x_2(t) + \frac{1}{RC} V_2(t)$$

$$\dot{x}_1(t) = \frac{1}{L} x_2(t) + \frac{1}{L} V_1(t) - \frac{1}{L} V_2(t)$$

$$\dot{x}_2(t) = -\frac{1}{C} x_1(t) - \frac{1}{RC} x_2(t) + \frac{1}{RC} V_2(t) \quad Y(t) = V_R(t) = V = V_2 - V_C$$

$$\dot{X}(t) = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} X(t) + \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} \\ 0 & \frac{1}{RC} \end{bmatrix} V(t)$$

$$Y(t) = \begin{bmatrix} 0 & -1 \end{bmatrix} X(t) + \begin{bmatrix} 1 & 0 \end{bmatrix} V(t)$$

3.5

$$V_1(s) = \frac{s+2}{s+3} (R(s) - Y(s)) \quad V(s) = \frac{1}{s-3} V_1(s)$$

$$Y(s) = \frac{1}{3} V(s) \quad T(s) = \frac{Y(s)}{R(s)} = \frac{s+2}{s^3 + 5s^2 - 23s + 2}$$

$$Y(s) = s Z(s) + 2 Z(s) \rightarrow Y(t) = \dot{z}(t) + 2 z(t)$$

$$R(s) = s^3 Z(s) + 5s^2 Z(s) - 23s Z(s) + 2 Z(s) \rightarrow r(t) = \ddot{z}(t) + 5 \dot{z}(t) - 23 z(t) + 2 z(t)$$

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} z \\ \dot{z} \\ \ddot{z} \end{bmatrix}$$

$$R(t) = u(t)$$

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 23 & -5 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$Y(t) = \begin{bmatrix} 2 & 1 & 0 \end{bmatrix} X(t)$$

3.12

$$Y(s) = 8Z(s) + 40Z(s) \quad B(s) = s^3 Z(s) + 12s^2 Z(s) + 44sZ(s) + 48Z(s)$$

$$Z = y(t) = 8\dot{z}(s) + 40z(s) \quad z = r(t) = \ddot{z}(t) + 12\dot{z}(t) + 44z(t) + 48z(t)$$

$$X(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} z \\ \dot{z} \\ \ddot{z} \end{bmatrix} \quad r(t) = u(t)$$

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -48 & -44 & -12 \end{bmatrix} X(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$Y(t) = [40 \quad 8 \quad 0] X(t)$$

$$\Phi(s) = (sI - A)^{-1} =$$

$$\left(\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -48 & -44 & -12 \end{bmatrix} \right)^{-1} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 48 & 44 & s+12 \end{bmatrix}^{-1}$$

$$M_{11} = s^2 + 12s + 44 \quad M_{12} = -48 \quad M_{13} = -48s$$

$$M_{21} = s + 12 \quad M_{22} = s^2 + 12s \quad M_{23} = 44s - 48 = \frac{1}{(s+6)(s+4)(s+2)}$$

$$M_{31} = 1 \quad M_{32} = s \quad M_{33} = s^2$$

$$\Phi(t) = \begin{pmatrix} e^{-6t} - 3e^{-4t} + 3e^{-2t} & \frac{3}{4}e^{-6t} - 2e^{-4t} + \frac{5}{4}e^{-2t} & \frac{1}{8}e^{-6t} - \frac{1}{4}e^{-4t} + \frac{1}{2}e^{-2t} \\ -6e^{-6t} + 12e^{-4t} - 6e^{-2t} & -\frac{9}{2}e^{-6t} + 8e^{-4t} - \frac{5}{2}e^{-2t} & -\frac{3}{4}e^{-6t} + e^{-4t} - \frac{1}{4}e^{-2t} \\ 36e^{-6t} - 48e^{-4t} + 12e^{-2t} & 27e^{-6t} - 32e^{-4t} + 5e^{-2t} & \frac{9}{2}e^{-6t} - 4e^{-4t} + \frac{1}{2}e^{-2t} \end{pmatrix}$$

3.17

$$sX(s) = AX(s) + BU(s) \Rightarrow X(s) = (sI - A)^{-1}BU(s)$$

$$Y(s) = CX(s) \Rightarrow Y(s) = C\Phi(s)BU(s)$$

$$\Rightarrow G(s) = \frac{Y(s)}{U(s)} = C\Phi(s)B$$

$$\Phi(s) = \begin{pmatrix} s-1 & -1 & 1 \\ -4 & s-3 & 0 \\ 2 & -1 & s+1 \end{pmatrix}^{-1}$$

$$M_{11} = (s-3)(s+1) \quad M_{12} = 4s-4 \quad M_{13} = -2s+1$$

$$M_{21} = s-1 \quad M_{22} = s^2-11s+8 \quad M_{23} = s-3 = \frac{1}{s^3-14s^2+37s+20}$$

$$M_{31} = -s+3 \quad M_{32} = -4 \quad M_{33} = s^2-4s-1$$

$$\begin{bmatrix} s^2-13s+9 & s-11 & -s+3 \\ 4s-4 & s^2-11s+8 & -4 \\ -2s+10 & s-3 & s^2-4s-1 \end{bmatrix}$$

$$G(s) = C\Phi(s)B = \frac{1}{s^3-14s^2+37s+20} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$

$$= \frac{-4s+12}{s^3-14s^2+37s+20}$$