

(A) Imagine a furniture company that makes tables and chairs. A table requires 40 board feet of wood and a chair requires 30 board feet of wood. Wood costs \$1 per board foot and 40,000 board feet of wood are available. It takes 2 hours of skilled labor to make an unfinished table or an unfinished chair. Three more hours of labor will turn an unfinished table into a finished table; two more hours of skilled labor will turn an unfinished chair into a finished chair. There are 6000 hours of skilled labor available. (Assume that you do not need to pay for this labor.) The prices of output are given in the table below:

Product	Price
unfinished table	\$70
finished table	\$140
Unfinished chair	\$60
finished chair	\$110

(1) Formulate an LP that describes the production plans that the firm can use to maximize its profits.

Solution :

$x_1$  = number of unfurnished tables

$x_2$  = number of furnished tables

$x_3$  = number of unfurnished chairs

$x_4$  = number of furnished chairs

Maximize  $z = 70x_1 + 140x_2 + 60x_3 + 110x_4$

subject to

$$40x_1 + 40x_2 + 30x_3 + 30x_4 \leq 40000.$$

$$2x_1 + 5x_2 + 2x_3 + 4x_4 \leq 6000.$$

(2) What would happen if the price of unfinished chairs went up?

Solution :

We can see from the reports that the cost of unfinished chairs would not effect the profit because, the values of  $x_3$  values i.e unfinished chairs contributing to the profit is 0(zero).

And from reports we could see that the price could be still increased by \$50, but still it doesn't effect the profit.

(3) What would happen if the price of unfinished tables went up?

Solution :

We can see from reports that though the cost of unfinished tables could be increased by 76.67(approx..) there won't be any change in the profit for the company.

(4) What if the price of finished chairs fell to \$100?

Solution :

If the price of finished chairs fell to \$100, the profit decreases from 146666.667 to 140000.

As we can see the allowable decrease of the cost for the finished chairs only \$5, as we reduced \$10, the optimal solution changed and there would no value for the finished chairs variable anymore, and the value is being added to finished table variable.

(5) How would profit change if lumber supplies changed?

Solution :

With the current supplier, the cost of each wood block is 3.666667(approx..),

The supplier of wood could be increased till 45000 blocks, the cost per each wood block gets decreased to \$3.5, and with the current constraints we will have total profit raised to \$165000.

(6) How much would you be willing to pay an additional carpenter?

Solution :

(7) Suppose that industrial regulations complicate the finishing process, so that it takes one extra hour per chair or table to turn an unfinished product into a finished one. How would this change your plans?

Solution :

Objective	x1	x2	x3	x4			
	70	140	60	110			
z	3.13E-13	1000	0	0	140000		
c1	40	40	30	30	40000	<=	40000
c2	2	6	2	5	6000	<=	6000

We could see that no. of unfinished table going into negative values, and the profit has fallen from 146666.6667 to 140000. We already saw that spending time on x1, x2, x3 have no impact on the profit, because we are not manufacturing them , so though we spend extra time on them, it would increase increase our profit rather it would result in loss.

(8) The owner of the firm comes up with a design for a beautiful hand-crafted cabinet. Each cabinet requires 250 hours of labor (this is 6 weeks of full time work) and uses 50 board feet of lumber. Suppose that the company can sell a cabinet for \$200, would it be worthwhile?

Solution :

Objective	x1	x2	x3	x4			
	70	140	60	110	200		
z	0	637.5	0	485	0	142600	
c1	40	40	30	30	50	40050	<= 40050
c2	2	6	2	5	250	6250	<= 6250

Previously we used to get profit of 146666.6667 approx, but after adding the extra work of making cabinet, we found that the profit is reduced to 142600, but we can see that no. of number of furnished tables changed from 0 to 637.5 , which shows are positive effect that, if there is change in resources or any other features there are chances that the company may end up in profits.