Latent Variable Multivariate Mixed-type Response Regression

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Installation

The package can be installed from GitHub, using devtools.

```
  # Currently private repository
  # devtools::install_github("koekvall/lvmmrPQL")
```

Notation

The matrix of responses, Y, has n rows and r columns. The matrix of predictors, X, has nr rows and p columns; the first r rows of X are the design matrix for the r responses in the first row of Y, the next r rows of X are the design matrix for the second row of Y, and so on. Thus, \texttt{matrix(X %*% Beta, nrow = n, ncol = r, byrow = TRUE)} gives an $n \times r$ matrix whose ith row is the mean of the ith latent vector.

Example with normal responses

```
set.seed(4)
n <- 500
type <- rep(1, 5) # Only normal responses
r <- length(type)

# Each observation has its own intercept
X <- Matrix::kronecker(rep(1, n), diag(r))
Beta_true <- (1:r) / r

# Variance parameters, psi treated as known
Sigma_true <- 0.5^abs(outer(1:r, 1:r, FUN = "-"))
psi_true <- rep(0.5, r)

Y <- lvmmrPQL::generate_lvmmr(X = X, Beta = Beta_true, R = chol(Sigma_true),
                    type = type, psi = psi_true)
# No restrictions with normal responses
M <- matrix(NA, r, r)

# Compute MLEs
fit_MLE <- lm(Y ~ 1)
Beta_MLE <- c(coef(fit_MLE))
Sigma_MLE <- crossprod(residuals(fit_MLE)) / n - diag(psi_true)

# Does MLE exist? That is, is maximizer PD?
min(eigen(Sigma_MLE)$values)
```

```
## [1] 0.249554
```

```
# Skip W update; obj. fun, does not depend on W with mult. norm. resp.
# MLE of Beta does not depend on Sigma, so expect correct MLE for Beta
# regardless of whether algorithm finds MLE of Sigma.
fit <- lvmmrPQL::lvmmr_PQL(Y = Y, X = X, type = type, M = M,
                           relative = T,
                           quiet = c(F, T, T, T),
                           maxit = c(100, 100, 500, 0),
                           tol = c(1e-12, 1e-8, 1e-12, 1e-8),
                           psi = psi_true)
```

```
## Change in parameters:  1153.165
## Change in parameters:  5.128422e-08
## Change in parameters:  4.94989e-08
## Change in parameters:  9.554761e-09
## Change in parameters:  4.744042e-08
## Change in parameters:  2.081875e-14
```

```
# Difference to MLEs
fit$Beta - Beta_MLE
```

```
##
##  1.110223e-16  2.942091e-15 -4.218847e-15  1.554312e-15 -1.110223e-14
```

```
fit$Sigma - Sigma_MLE
```

```
##               [,1]          [,2]          [,3]          [,4]          [,5]
## [1,] -1.220655e-06  1.433181e-06  1.792936e-06 -5.529616e-08 -9.912511e-07
## [2,]  1.433181e-06  7.382859e-08  5.867502e-07  1.154297e-06 -3.624168e-07
## [3,]  1.792936e-06  5.867502e-07 -3.937836e-07  2.548942e-07  1.845730e-06
## [4,] -5.529616e-08  1.154297e-06  2.548942e-07  2.304977e-07  1.848131e-06
## [5,] -9.912511e-07 -3.624168e-07  1.845730e-06  1.848131e-06 -9.591571e-07
```

```
# With MLE as starting value
fit <- lvmmrPQL::lvmmr_PQL(Y = Y, X = X, type = type, M = M,
                           relative = T,
                           quiet = c(F, T, T, T),
                           maxit = c(100, 100, 500, 0),
                           tol = c(1e-12, 1e-8, 1e-12, 1e-8),
                           Beta = Beta_MLE,
                           Sigma = Sigma_MLE,
                           psi = psi_true)
```

```
## Change in parameters:  2.304933e-14
```

```
fit$iter
```

```
## [1] 1
```

```
# Difference to MLEs
fit$Beta - Beta_MLE
```

```
##
## -2.164935e-15 -1.004752e-14 -1.632028e-14 -2.409184e-14 -2.020606e-14
```

```
fit$Sigma - Sigma_MLE
```

```
##               [,1]          [,2]          [,3]          [,4]          [,5]
## [1,]  1.554312e-15  1.110223e-16 -1.665335e-15  2.220446e-16 -1.387779e-16
## [2,]  1.110223e-16 -4.551914e-15  4.218847e-15 -3.275158e-15  1.110223e-16
## [3,] -1.665335e-15  4.218847e-15 -9.547918e-15  6.328271e-15 -1.942890e-15
## [4,]  1.665335e-16 -3.219647e-15  6.328271e-15 -9.769963e-15  1.221245e-15
## [5,] -2.220446e-16  1.110223e-16 -2.053913e-15  1.221245e-15 -2.220446e-15
```

```
# See that objective is correct
D1 <- t(lvmmrPQL:::get_cumulant_diffs(t(fit$W), type, 1))
D2 <- t(lvmmrPQL:::get_cumulant_diffs(t(fit$W), type, 2))

lvmmrPQL:::working_ll(Y = Y, X = X, Beta = fit$Beta, Sigma = fit$Sigma,
                      W = fit$W, psi = psi_true, D1 = D1, D2 = D2)
```

```
## [1] -3921.977
```

```
lvmmrPQL:::working_ll(Y = Y, X = X, Beta = Beta_MLE, Sigma = Sigma_MLE,
                      W = fit$W, psi = psi_true, D1 = D1, D2 = D2)
```

```
## [1] -3921.977
```

```
# Double check w. multivariate normal likelihood
Xb <- matrix(X %*% fit$Beta, nrow = n, ncol = r, byrow = T)
sum(mvtnorm::dmvnorm(x = Y - Xb, sigma = fit$Sigma + diag(psi_true), log = TRUE))
```

```
## [1] -3921.977
```

```
sum(mvtnorm::dmvnorm(x = Y - predict(fit_MLE), sigma = Sigma_MLE + diag(psi_true), log = TRUE))
```

```
## [1] -3921.977
```

Example with mixed-type responses

```
set.seed(4)
n <- 500
type <- 2:3
r <- length(type)

# Each observation has one uniform predictor (SUR)
X <- as.matrix(Matrix::KhatriRao(matrix(runif(n * r, -1, 1), n, r),
                                 diag(1, r)))
Beta_true <- (r:1) / r

# Variance parameters, psi treated as known
Sigma_true <- 0.9^abs(outer(1:r, 1:r, FUN = "-"))
psi_true <- rep(1, r) # psi \neq 1 currently not supported Ber and Poi

Y <- lvmmrPQL::generate_lvmmr(X = X, Beta = Beta_true, R = chol(Sigma_true),
                      type = type, psi = psi_true)
# No restrictions with normal responses
M <- matrix(NA, r, r)
diag(M)[type == 2] <- 1 # For identifiability (try)

fit <- lvmmrPQL::lvmmr_PQL(Y = Y, X = X, type = type, M = M,
                           relative = T,
                           quiet = c(F, F, F, T),
```

```
                            maxit = c(1000, 1000, 5000, 1000),
                            tol = c(1e-5, 1e-7, 1e-12, 1e-10),
                            psi = psi_true,
                            )
```

```
## Change from Beta update:  -3.25601e-06
## 1620.792
## 1488.031
## 1382.466
## 1279.104
## 1193.596
## 1128.094
## 1079.712
## 1037.994
## 1007.587
## 987.6615
## 975.8121
## 968.7973
## 966.1053
## 966.2984
## 968.0302
## 970.3189
## 972.5366
## 974.3306
## 975.549
## 976.1847
## 976.3347
## 976.1658
## 975.8786
## 975.6601
## 975.6216
## 975.7313
## 972.5805
## 969.356
## 966.2473
## 963.5029
## 961.3921
## 960.1476
## 959.9043
## 960.6506
## 962.2071
## 964.2437
## 966.3409
## 968.0868
## 969.1785
## 969.4861
## 969.0546
## 968.0505
## 966.6884
## 965.1713
## 963.6614
## 962.2763
## 961.1013
## 960.203
```

```
## 959.6375
## 959.4481
## 959.6536
## 960.2279
## 961.0822
## 962.0598
## 962.9579
## 963.5766
## 963.7821
## 963.55
## 962.9665
## 962.1864
## 961.3743
## 960.6581
## 960.1092
## 959.7462
## 959.5511
## 959.4877
## 959.5168
## 959.6048
## 959.7271
## 959.8678
## 960.0162
## 960.1632
## 960.2974
## 960.4033
## 960.4624
## 960.4566
## 960.3736
## 960.2123
## 959.9859
## 959.7208
## 959.452
## 959.2156
## 959.0413
## 958.947
## 958.9364
## 958.9999
## 959.118
## 959.2656
## 959.4165
## 959.5473
## 959.6399
## 959.6837
## 959.6754
## 959.6189
## 959.5241
## 959.4049
## 959.2771
## 959.1559
## 959.0544
## 958.9813
## 958.9405
## 958.9309
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## 958.9471
## 958.9814
## 959.0246
## 959.0687
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## 959.1358
## 959.1528
## 959.1578
## 959.1518
## 959.1362
## 959.1127
## 959.083
## 959.0493
## 959.0137
## 958.979
## 958.9483
## 958.9243
## 958.9095
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## Change from Sigma update:  -0.0002949913
## Change from Beta update:  -2.478837e-07
```

```
## 957.7838
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## Change from Sigma update:  -1.344688e-08
## Change in parameters:  3140.237
## Change from Beta update:  -0.09921083
## 635.8881
## 566.3889
## 542.3598
## 567.3326
## 706.8236
## 1004.723
## 1078.59
## 884.5951
## 755.9636
## 691.3003
## 669.7117
## 667.3266
## 670.1334
## 677.1185
## 685.996
## 695.0928
## 703.4084
## 660.3496
## 574.1872
## 666.1035
## 604.2446
## 566.4474
## 548.3296
## 542.2014
## 542.8349
## 547.4117
## 553.642
## 560.1556
## 561.8704
## 558.4316
## 554.5208
## 550.4914
## 546.7692
## 543.8345
## 542.1705
## 542.0712
## 543.2397
## 544.6286
## 544.9972
## 543.9737
## 542.3524
## 541.2257
## 541.1468
## 542.0367
## 543.4883
```

```
## 545.0453
## 546.3453
## 547.1641
## 547.4143
## 547.1282
## 546.435
## 545.5332
## 544.6508
## 543.9871
## 543.6398
## 543.544
## 543.478
## 543.1771
## 542.5108
## 541.5791
## 540.6404
## 539.9443
## 539.6116
## 539.62
## 539.8572
## 539.8998
## 539.96
## 540.058
## 540.2136
## 540.4312
## 540.6903
## 540.9472
## 541.1488
## 541.2544
## 541.2519
## 541.2335
## 541.2031
## 541.1643
## 541.0683
## 540.9401
## 540.7994
## 540.6559
## 540.3649
## 540.0017
## 539.6944
## 539.547
## 539.5644
## 539.6893
## 539.923
## 540.2349
## 540.5921
## 540.9616
## 541.3123
## 541.6177
## 541.8566
## 541.7025
## 541.2242
## 540.5904
## 540.0019
```

```
## 539.6212
## 539.526
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## Change from Sigma update:  -6.147047e-08
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## Change from Beta update:  -4.774847e-12
## Change from Sigma update:  -2.046363e-10
## Change in parameters:  0.7225206
## Change from Beta update:  -0.006371078
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## Change from Sigma update:  -0.0520231
## Change from Beta update:  -9.59429e-06
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## Change from Sigma update:  -3.192781e-09
## Change in parameters:  0.03062994
## Change from Beta update:  -4.12384e-06
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## 536.1181
## Change from Sigma update:  -0.0005253711
## Change from Beta update:  -7.921471e-08
## Change from Sigma update:  -4.774847e-12
## Warning in update_W(Y = Y, X = X, W = W, Beta = new_Beta, Sigma = new_Sigma, : w_8 update did not co
## Warning in update_W(Y = Y, X = X, W = W, Beta = new_Beta, Sigma = new_Sigma, : w_141 update did not
## Warning in update_W(Y = Y, X = X, W = W, Beta = new_Beta, Sigma = new_Sigma, : w_267 update did not
## Warning in update_W(Y = Y, X = X, W = W, Beta = new_Beta, Sigma = new_Sigma, : w_270 update did not
## Warning in update_W(Y = Y, X = X, W = W, Beta = new_Beta, Sigma = new_Sigma, : w_300 update did not
## Warning in update_W(Y = Y, X = X, W = W, Beta = new_Beta, Sigma = new_Sigma, : w_373 update did not
## Warning in update_W(Y = Y, X = X, W = W, Beta = new_Beta, Sigma = new_Sigma, : w_438 update did not
## Change in parameters:  0.003140683
## Change from Beta update:  -2.908314e-08
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## Change from Sigma update:  -3.215345e-06
## Change in parameters:  0.0002102233
## Change from Beta update:  -1.057515e-09
## Change from Sigma update:  -2.078195e-10
## Change in parameters:  8.768309e-06
```

```r
# Difference to truth
fit$Beta - Beta_true
```

```
##
## -0.12567621 -0.01970939
```

```r
fit$Sigma - Sigma_true
```

```
##            [,1]       [,2]
## [1,]  0.0000000 -0.1610301
## [2,] -0.1610301 -0.1006721
```

```r
# Predict
n_pred <- 1e4
X_new <- as.matrix(Matrix::KhatriRao(matrix(runif(n_pred * r, -1, 1), n_pred, r),
                                     diag(1, r)))
Y_new <- lvmmrPQL::generate_lvmmr(X = X_new, Beta = Beta_true, R = chol(Sigma_true),
                    type = type, psi = psi_true)
Beta_GLM <- c(
 coef(glm(Y[, 1] ~ 0 + X[seq(1, 2 * n,  by = 2), 1], family = "binomial")),
  coef(glm(Y[, 2] ~ 0 + X[seq(2, 2 * n,  by = 2), 2], family = "poisson")))
Xb_GLM <- matrix(X_new %*% Beta_GLM, nrow = n_pred, ncol = r, byrow = T)
pred_GLM <- t(lvmmrPQL:::get_cumulant_diffs(t(Xb_GLM), type, 1))


# We win (sometimes and often small).
colMeans((Y_new - lvmmrPQL::predict_lvmmr(X = X_new,
                                          Beta = fit$Beta,
                                          sigma = sqrt(diag(fit$Sigma)),
                                          type = type,
                                          num_nodes = 15))^2)
```

```
## [1] 0.2364115 6.7945671
```

```r
colMeans((Y_new - pred_GLM)^2)
```

```
## [1] 0.2361706 7.1866357
```