Lecture 1: Preliminaries

Games, Competition and Markets 2023/24

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Introduction

Introduction

- Welcome!
- This is a new course.
- To graduate, you *either* need this course *or* the course *Introduction to Mathematical Economics*.

- We study topics related to markets and competition on markets
- using game theory.
- Also known as Industrial Organization.
- Topics are mainly related to new technologies and online markets.

Topics

1. Preliminaries

Introductory lecture. Review of game-theoretic concepts. Some basic models of competition.

2. Consumer Search

What if consumers have to engage in costly search to find out about products and/or prices?

3. Advertising

What if producers have to inform consumers about their products and/or prices?

4. Menu Pricing

What if firms design different products and different prices for different consumers?

5. Durable Goods

What if a monopolist sells a durable good and cannot commit to future quantities?

6. Switching Costs

What if consumers have to pay extra if they switch suppliers?

7. Behavior-Based Price Discrimination

What if firms can base their prices on a consumer's past behavior?

8. Vertical control

What if firms sell products to retailers who then sell it to final consumers?

9. Bundling

What if firms can sell bundles of products?

10. Network externalities and compatibility

What if products exhibit network effects: they becomes more (or sometimes less) useful if more consumers use it. Also: when do firms want to make their products compatible with that of their competitor?

11. Platform competition

What if online platforms bring buyers and sellers together? Or consumers and advertisers?

Outline

- 10 lectures, 4 tutorials.
- 7 weeks, one lecture-free week.
- One midterm – 30%.
- One exam – 70%.
- One resit – 100%.

Today

Game Theory

Games, Competition and Markets. Lecture 1

Game Theory

1. Nash equilibrium
2. Reaction functions
3. Imposing symmetry
4. Subgame perfect equilibrium
5. Moves of nature
6. Candidate equilibrium
7. Mixed strategies

Nash Equilibrium

$$s_i^* \in \arg\max_{s_i} U_i(s_1^*, s_2^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*), \forall i = 1, \ldots, n.$$

Reaction functions

Your best action *as a function* of the action of the other player(s).

Hence: Nash equilibrium is where reaction functions intersect.

Imposing symmetry

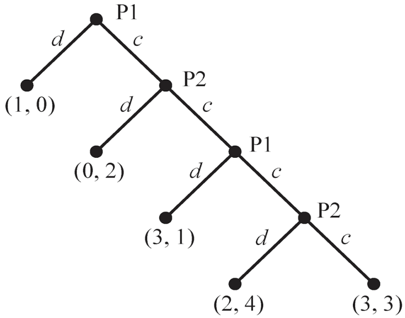
If all players are a priori identical, we may assume that in equilibrium they all take the same action.

Example

If the reaction function is $q_1 = \frac{1}{2}(1 - q_2 - c)$, we can impose symmetry to find $q^* = \frac{1}{2}(1 - q^* - c)$, hence $q^* = \frac{1}{3}(1 - c)$.

Subgame perfection

- Subgame perfection is a *refinement* of Nash equilibrium that requires that we also have an equilibrium at every subgame.
- Solve with backward induction.



Moves of Nature

Convenient way to model uncertainty.

A monopolist that sets a quantity before uncertainty concerning marginal costs is resolved will make a different decision than if uncertainty is resolved afterwards.

Candidate equilibrium

If you need to find a Nash equilibrium just make an (educated) guess and check whether that is indeed an equilibrium.

Mixed strategies

- Not all games have an equilibrium *in pure strategies*.

		Player 2 L	Player 2 R
Player 1	U	10,0	3,7
	D	4,6	10,0

- Equilibrium: $P_U = \frac{6}{13}, P_L = \frac{7}{13}$.
- Player 1's strategy *makes player 2 indifferent,*
- Player 2's strategy *makes player 1 indifferent,*

Some useful models

Hotelling

- A unit mass of consumers are uniformly distributed on a line of unit length.
- Two firms are located on the endpoints of that line: one at 0, the other at 1.
- Consumers have unit demand, and willingness-to-pay v.
- Consumers face *transportation costs*: t per unit of distance.
- Suppose firms set price P_0 and P_1, respectively.
- The indifferent consumer z is then located at

$$v - P_0 - tz = v - P_1 - t(1 - z).$$

- This yields

$$z = \frac{1}{2} + \frac{P_1 - P_0}{2t}.$$

Hotelling (ctd)

- Note: if z is indifferent, then everyone located left of z will strictly prefer 0.
- Those to the right of z will strictly prefer 1.

$$\Pi^0 = (P_0 - c)z = (P_0 - c)\left(\frac{1}{2} + \frac{P_1 - P_0}{2t}\right).$$

- Maximizing yields $P_0 = (c + t + P_1)/2$.
- Symmetry: $P^* = c + t$.
- Alternative interpretation: taste.
- More product differentiation, more market power, higher prices.

The circular city: Salop

- Hotelling doesn't easily generalize to more than 2 firms.
- Alternative: consider consumers that are located on a *circle* of unit length.
- Again, willingness-to-pay v, unit demand, linear transportation costs.
- Suppose there are fixed costs of entry f.
- Firms will be located at equal distances along the circle.
- How many firms will enter?

Salop (ctd)

- Suppose n firms have entered.
- Suppose all other firms $2, \ldots, n$ charge the same price p.
- If firm 1 then maximizes profits by also charging p, we have an equilibrium.
- Denote the location of firm 1 as 0.
- If firm 1 charges price p_1, the consumer indifferent between 1 and 2 is given by

$$p_1 + tz_{1-2} = p + t\left(\frac{1}{n} - z_{1-2}\right),$$

$$z_{1-2} = \frac{1}{2n} + \frac{p - p_1}{2t}.$$

$$z_{1-2} = \frac{1}{2n} + \frac{p - p_1}{2t}.$$

Profits of firm 1:

$$\Pi^1(p_1, p) = 2\,(p_1 - c)\left(\frac{1}{2n} + \frac{p - p_1}{2t}\right).$$

This yields reaction function:

$$R_1(p) = \frac{1}{2}\left(p + c + \frac{t}{n}\right).$$

The equilibrium then yields

$$p^* = c + \frac{t}{n}.$$

Net profits per firm:

$$\Pi^* - f = \frac{t}{n^2} - f.$$

Salop circle (ctd.)

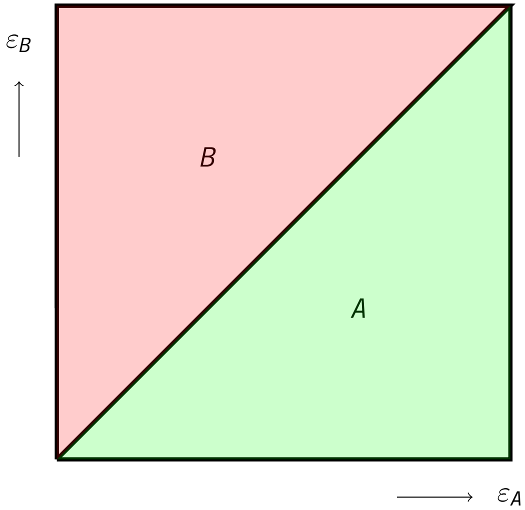
Net profits per firm:

$$\Pi^* - f = \frac{t}{n^2} - f.$$

- Equilibrium number of firms: $n^* = \sqrt{t/f}$.
- Equilibrium prices: $p^* = c + \sqrt{tf}$.
- Note: a social planner would set n to $\min_n(nf + t/4n)$.
- This yields $n^S = \frac{1}{2}\sqrt{t/f} = \frac{1}{2}n^*$.
- Hence, the market has too many firms.

Perloff and Salop (1985)

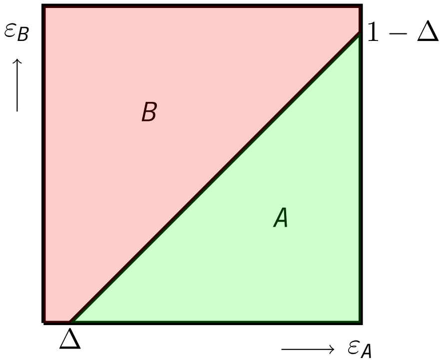
- Unit mass of consumers with unit demand.
- Consumer j has a willingness to pay for firm i that is given by $v + \varepsilon_{ij}$.
- $\varepsilon_{ij} \sim F$.
- For now: market is covered, two firms, A and B (but easily extendable).
- Note the difference with Hotelling.
- What if both firms charge the same price?



ε_B
B
A
ε_A

How to find the equilibrium?

- Again, take the price(s) of the other firm(s) as given. Say p.
- Suppose firm A charges a slightly higher price, say $p_A = p^* + \Delta$.
- What are the sales of firm A in that case?
- Indifferent consumer: $\varepsilon_A - p^* - \Delta = \varepsilon_B - p^*$.
- $\varepsilon_B = \varepsilon_A - \Delta$.



- Uniform: $q_A = \frac{1}{2}(1-\Delta)^2$.
- More generally:

$$q_A = \int_{\Delta}^{1} \left(\int_{0}^{\varepsilon_A - \Delta} f(\varepsilon_B)\, d\varepsilon_B \right) f(\varepsilon_A)\, d\varepsilon_A = \int_{\Delta}^{1} F(\varepsilon_A - \Delta)\, f(\varepsilon_A)\, d\varepsilon_A.$$

Now $\pi_A = (p_A - c) \cdot q_A$. Profit maximization thus requires

$$\frac{\partial \pi_A}{\partial p_A} = (p_A - c) \cdot \frac{\partial q_A}{\partial p_A} + q_A = 0.$$

With uniform $q_A = \frac{1}{2}(1 - \Delta)^2$ so

$$\frac{\partial q_A}{\partial p_A} = -(1 - \Delta).$$

In equilibrium, impose symmetry and find

$$(p_A - c) \cdot (-1) + \frac{1}{2} = 0,$$

so $p^* = c + \frac{1}{2}$.

$$\frac{\partial \pi_A}{\partial p_A} = (p_A - c) \cdot \frac{\partial q_A}{\partial p_A} + q_A = 0.$$

In general

$$q_A = \int_{\Delta}^{1} F(\varepsilon_A - \Delta) f(\varepsilon_A) \, d\varepsilon_A.$$

so

$$\frac{\partial q_A}{\partial p_A} = -\int_{\Delta}^{1} f(\varepsilon_A - \Delta) f(\varepsilon_A) \, d\varepsilon_A.$$

In equilibrium, impose symmetry to find

$$p^* = c + \frac{1}{2\int_0^1 f^2(\varepsilon) d\varepsilon}.$$

Thank you for your attention

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