Games, Competition and Markets

Lecture Notes

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Chapter 1

Preliminaries

1.1 Introduction

This chapter provides some preliminary material with two main objectives. First, to refresh your knowledge of game theory concepts relevant to this course. Second, to introduce key applications in Industrial Organization, including the use of reaction functions—also in multiperiod settings—and the most commonly used models of competition: Cournot, Bertrand, Hotelling, and Salop. Not all of the content in this chapter will be discussed in class, but it is essential that you study it carefully and ensure you understand it, as it forms the foundation for the rest of the course.

1.2 Game theory

1.2.1 Introduction

This section reviews key game-theoretic concepts that will recur throughout the course. It is not intended as a comprehensive introduction to game theory—there are far better and more detailed treatments available, such as Tadelis (2013), or, at a more advanced level, Fudenberg and Tirole (1991). The aim here is simply to refresh the most relevant concepts and to illustrate how they are applied in the context of Industrial Organization. The focus will be limited to models with complete information.

1.2.2 Nash Equilibrium

In all the applications we consider, we will look for a Nash equilibrium. Such an equilibrium is defined as follows. Suppose that we have a game with n players, with $n > 1$. Player i has to choose a strategy s_i from some strategy space S_i, for $i = 1, \dots, n$. Each player will obtain some utility U_i that is a function of the strategies chosen by all the players in the game. We thus have $U_i \equiv U_i(s_1, s_2, \dots, s_n)$. In the context of Industrial

Organization applications, the utility function will usually be the profit function of a firm. The strategies could entail for example the price or the quantity that a firm chooses to set.

For a Nash equilibrium we need that each player is choosing the best possible strategy given the strategies chosen by all other players. More formally, a Nash equilibrium is a strategy profile $(s_1^*, s_2^*, \ldots s_n^*)$ such that

$$s_i^* \in \arg\max_{s_i} U_i(s_1^*, s_2^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*), \forall i = 1, \ldots, n.$$

Note that we denote the equilibrium values with an asterisk. This is a convention that we will use throughout these notes. Also note that we write "$\in$" rather than "=". This implies that s_i does not have to be the unique maximizer of the function on the right-hand side; it can also simply be just one among many strategies that maximizes U_i.

1.2.3 Reaction functions

In games with a continuous strategy space, it is often useful to derive a *reaction function*, also often referred to as a *best reply* or *best response* function. Such a function simply gives the best action of a particular player for each possible action of the other player(s). Note therefore the difference with the Nash equilibrium: the Nash equilibrium action of a player is the best response to the equilibrium action of the other player(s), whereas a reaction function gives the best response to *any* action of the other player(s).

Example

Consider a Cournot model with two firms and demand function $q = 1 - p$. Firms have constant and equal marginal costs c. The problem for firm 1 is to maximize its profits $\pi_1 = (1 - q_1 - q_2)q_1 - cq_1$. Take q_2 as given. Maximizing π_1 with respect to q_1 yields the first-order condition $1 - 2q_1 - q_2 - c = 0$, which we can write as

$$q_1 = \frac{1}{2}(1 - q_2 - c). \tag{1.1}$$

This is the reaction function of firm 1: it gives the best reply of firm 1 in terms of its quantity q_1, for any possible value of the quantity of the other player, q_2. ■

For a Nash equilibrium, we need that each player chooses their best action given the action of the other player(s). In other words, a Nash equilibrium requires that each firm is on its reaction function. That implies that the Nash equilibrium can be found where the reaction functions of all firms intersect.

1.2.4 Imposing symmetry

A useful trick for finding the Nash equilibrium in many IO problems is the following. Suppose that we have a model with symmetric firms, in the sense that each firm has the same production technology, demand conditions, marginal costs, etcetera. If that is the case, then we effectively have that firms are anonymous, in the sense that the way in which we number, name, or otherwise label the firms, is immaterial. In turn, this implies that we can also expect all firms to choose the same strategy s^* in the equilibrium of the game. Using this knowledge often greatly simplifies finding that equilibrium. After deriving one firm's best-reply function, we can immediately plug in s^* for the strategy of each firm. We can then directly compute that equilibrium value. In the literature, whenever this trick is used, it is almost always accompanied by the words "imposing symmetry".

Example

Consider a Cournot model with 2 firms, and demand function $q = 1 - p$. Firms have constant and equal marginal costs c. Firm 1's best-reply function is given by (1.1). Imposing symmetry, we have that $q_1^* = q_2^* = q^*$, so the best-reply function implies

$$q^* = \frac{1}{2}(1 - q^* - c),$$

which immediately yields that each firm sets $q^* = (1 - c)/3$.

1.2.5 Subgame perfect equilibrium

In the standard Cournot and Bertrand models, firms move simultaneously. That implies that we do not have to worry about the order of moves. In many applications, however, this is different. Many games have an explicit order of moves. In such games, the Nash equilibrium often does not have sufficient bite to yield a unique equilibrium. We then use a refinement of the Nash equilibrium: the subgame perfect Nash equilibrium, or simply subgame perfect equilibrium. This concept was introduced by Selten (1975). It requires that the strategy profile under consideration is not only an equilibrium for the entire game, but also for each subgame of the entire game (see a game theory text for details). In IO models, subgame perfection is a standard requirement. It is so much standard that often authors do not even mention explicitly that they are looking for a subgame perfect equilibrium of their model.

To find the subgame perfect equilibrium of some game requires one to use backward induction. This implies starting at the last stage of the game, solving that stage as a function of all strategies that could possibly have been chosen at the earlier stages of the game, plugging that solution into the penultimate stage of the game, solving for that penultimate stage as a function of all strategies that could possibly have been chosen at earlier stages of the game, etcetera, until the entire game has been solved. This is an important solution concept in IO.

Example: the Stackelberg model

Consider a Cournot model where inverse demand is given by $p = 1 - q$, two firms compete by setting quantities, marginal costs are constant and equal to c, and fixed costs are zero. The timing as as follows:

Stage 1 Firm 1 sets its quantity q_1.

Stage 2 Firm 2 sets its quantity q_2.

Thus, firm 2 only decides on its quantity after firm 1 has done so and *after it has observed the choice that firm 1 has made*. This is crucial: if firm 2 moves after firm 1 but without knowing what firm 1 has chosen, then we are in the exact same case as we are when the two firms move simultaneously.

We solve for the subgame perfect equilibrium. We do so by backward induction. First solve for the last stage of the game. Firm 2 then sets quantity q_2, after having observed q_1. Thus, the q_2 that firm 2 chooses will be a function of q_1. By maximizing its profits, firm 2 will set $q_2^* = (1 - q_1 - c)/2$. Note that this is simply the reaction function we also had in the Cournot model.

We move back one step to solve for the equilibrium of stage 1. When setting its quantity q_1, firm 1 will now take into account the reaction of firm 2. It will thus maximize

$$\begin{aligned}
\pi_1(q_1, q_2) &= (1 - q_1 - q_2^* - c)q_1 \\
&= (1 - q_1 - (1 - q_1 - c)/2 - c)q_1 \\
&= (1 - q_1 - c)q_1/2.
\end{aligned}$$

Maximizing this with respect to q_1, yields $q_1 = (1 - c)/2$, which in turn implies $q_2 = (1 - c)/4$. Note that we could *not* solve this problem by imposing symmetry, simply because the firms are not symmetric from the outset.

1.2.6 Moves of Nature

Many models in IO involve uncertainty. This is often modeled as a “move of nature” in a multistage game. The moment at which the uncertainty is resolved, is referred to as the move of nature. Such games can again be solved using backward induction. It is very important to set up and solve the model in the right manner. Mistakes are often made when solving such models.

Example

Consider a monopolist that faces demand curve $q = 1 - p$. Its marginal cost may either be high or low: $c = 0$ or $c = 1/2$, both with probability $1/2$. The monopolist can decide on the quantity it sets after it has learned the true value of marginal cost. The resulting game can thus be represented as follows:

Stage 1 Nature determines c.

Stage 2 The monopolist sets q.

Using backward induction, we have to consider two possibilities for stage 2: the case in which it turns out that $c = 0$ and the case that it turns out that $c = 1/2$. Profits are given by $\pi(q) = (1 - q - c)q$. Hence profits are maximized by setting $q = (1 - c)/2$, which implies setting $q = 1/2$ if $c = 0$ and $q = 1/4$ if $c = 1/2$. The a priori expected profits are thus $E(\pi) = (1/2) \cdot (1/4) + (1/2) \cdot (1/16) = 5/32$.

Example

Consider a monopolist that faces demand curve $q = 1 - p$. Its marginal cost may either be high or low: $c = 0$ or $c = 1/2$, both with probability $1/2$. When the monopolist decides on the quantity that it sets, it does not know the true value of marginal cost. The resulting game can thus be represented as follows:

Stage 1 The monopolist sets q.

Stage 2 Nature determines c.

Note that this is an entirely different problem from the one in the previous example. When determining its quantity, the monopolist now maximizes

$$\pi(q) = \frac{1}{2} \cdot (1 - q)\, q + \frac{1}{2} \cdot \left(1 - q - \frac{1}{2}\right) q.$$

This is maximized by setting $q = 3/8$. Profits then equal $15/64$ in case $c = 0$ and $3/64$ in case $c = 1/2$. A priori expected profits then equal $E(\pi) = 9/64$. Thus, expected profits in this case are lower than in the previous example. This makes sense: now, the monopolist cannot fully adapt to circumstances.

1.2.7 The Candidate Equilibrium

For ease of exposition, we will often use the concept of a candidate equilibrium. In many models, it is possible to make an educated guess as to what the equilibrium might be. We will refer to such an educated guess as a candidate equilibrium. A candidate equilibrium thus is a strategy profile that may be a (subgame perfect) Nash equilibrium, but for which we still have to check whether that really is the case. This approach for finding an equilibrium is often easier than deriving an equilibrium from scratch. Note however that if it turns out that the candidate equilibrium is a true equilibrium, we still have not established whether that equilibrium is unique.

Example

Consider a Cournot model with three firms. Inverse demand is given by $p = 1 - q$. Marginal costs are constant and equal c for each firm. Fixed costs are zero. Determine the Cournot equilibrium.

In previous examples we already saw that, with this demand and cost function a monopoly (i.e. one firm) will set quantity $(1-c)/2$, whereas each firm in a duopoly sets quantity $(1-c)/3$. One may therefore suspect that, with three firms, the equilibrium may very well be for each firm to set $q_i^* = (1-c)/4$. Hence, this is our candidate equilibrium. To see whether this is a Nash equilibrium, consider firm 1. If our candidate equilibrium is really a Nash equilibrium, then $q_1 = (1-c)/4$ must solve this firm's first-order condition if both other firms set that same quantity. Profits of firm 1 then equal

$$\pi_1 = (1 - q_1 - q_2 - q_3 - c)q_1 = (1 - q_1 - (1-c)/2 - c)q_1.$$

The first-order condition is

$$(1 - 2q_1 - (1+c)/2) = 0.$$

This is indeed solved for $q = (1-c)/4$. As the same analysis holds for the other two firms, this implies that it is indeed a Nash equilibrium for each firm to set $q_i^* = (1-c)/4$.

1.2.8 Mixed strategies

Example

Consider the bimatrix game given in the table below.

		Player 2	
		L	R
Player 1	U	10,0	3,7
	D	4,6	10,0

Table 1.1: A simple game with no pure strategy Nash equilibria.

First of all, note that this game does not have any pure strategy Nash equilibrium. (U, L) is not an equilibrium, as player 2 would defect to R. (U, R) is not an equilibrium, as player 1 would defect to D. (D, R) is not an equilibrium, as player 2 would defect to L. And (L, D) is not an equilibrium, as player 1 would defect to U.

Yet, as Nash (1950) has shown, a Nash equilibrium always exists. In this case, that is a Nash equilibrium in mixed strategies. Suppose that player 1 would play U with probability P_U, and play D with probability $1 - P_U$. For a mixed strategy, we need that $P_U \in (0, 1)$. Also, suppose that player 2 would play L with probability P_L, and play R with probability $1 - P_L$. Again, for a mixed strategy, we need that $P_L \in (0, 1)$.

Consider player 1. Note that, for the above to be an equilibrium, we need that there is no strategy that gives player 1 a strictly higher payoff. But that implies that playing U has to give the same expected payoff as playing D. If the payoff to playing U would be strictly higher, then player 1 would not be willing to mix between U and D: it is strictly better to play U with probability 1. For the same reason, we cannot have that player 1 strictly prefers to play D.

Suppose that player 1 plays U. Given the strategy played by player 2, the expected payoffs of player 1 then equal $10P_L + 3(1 - P_L) = 3 + 7P_L$. If player 1 plays D, his expected payoff is $4P_L + 10(1 - P_L) = 10 - 6P_L$. For an equilibrium, we thus need that $3 + 7P_L = 10 - 6P_L$, or $P_L^* = 7/13$. Note once more that the equilibrium strategy of player 2 is determined by making player 1 indifferent between all the strategies among which player 1 mixes. Player 2 is indifferent between playing L and R if $6(1 - P_U) = 7P_U$, or $P_U^* = 6/13$. Hence, a Nash equilibrium for the game above is for player 1 to play U with probability 6/13 and to play D with probability 7/13, and for player 2 to play L with probability 7/13, and to play R with probability 6/13.

General set-up

When we no longer restrict attention to pure strategies, a strategy for player i is not necessarily some action a_i^* that player i will play in equilibrium, but rather some probability distribution over possible actions. Somewhat more precisely, a mixed strategy Nash equilibrium has each player i drawing its action a_i from a probability distribution $F_i(a_i)$, defined on some domain $\mathcal{A}_i$. Given the strategies played by all other players, each player i is indifferent between the actions among which it mixes. Thus $E(U_i(a_i))$ is constant for all $a_i \in \mathcal{A}_i$, and given the strategies of the other players. Of course, we also need that there is no pure strategy outside $\mathcal{A}_i$ that gives a strictly higher payoff.

We will see many applications in which the probability distributions F_i are continuous, rather than the discrete case we had in the example above. Also note that it is not necessary that a player mixes among *all* possible actions that it can take, as we had in the example above. For example, for both players, we can add a strategy to the game that we analyzed above. Suppose that player 1 can also choose a strategy M that always yields 0 to both players. Similarly, player 2 can also choose a strategy M that yields 0 to both players. This adapted game is the one given in table 1.2.

		Player 2 L	M	R
Player 1	U	10,0	0,0	3,7
	M	0,0	0,0	0,0
	D	4,6	0,0	10,0

Table 1.2: A more complicated game.

The equilibrium in this game is the exact same one as that in the game described in table 1.1: player 1 plays U with probability 6/13 and plays D with probability 7/13. Player 2 plays L with probability 7/13, and plays R with probability 6/13. Neither player plays M in equilibrium.

1.3 Models with differentiated products

In this course, we will use a range of competition models a building blocks to study more advanced issues. In this section, we study those models that you may not have seen yet. These will all be models with horizontal product differentiation, that will be useful for different purposes.

1.3.1 Hotelling competition

Hotelling (1929) formulated the first major model with horizontal product differentiation. With this type of product differentiation, different consumers prefer different products: some like product A better, while others like product B better. In contrast, with vertical product differentiation, all consumers agree which products are most preferable. All consumers prefer, say, product A over product B, yet in equilibrium there are still consumers that buy product B, simply because it is cheaper. One can interpret vertical product differentiation as different products having a different quality: everyone agrees that a product with a higher quality is preferable to a product with a lower quality. However, consumers differ in how much they are willing to pay for the extra quality.

In Hotelling, consumers are uniformly distributed on a line of unit length. The number of consumers is normalized to one. Consumers either buy one unit of the good, or none at all. Each consumer obtains gross utility v from consuming the product. Assume that we have two firms: one is located at 0, the other is located at 1.[1] A consumer located at x thus has to travel a distance x to buy from firm 0, and a distance $1 - x$ to buy from firm 1. Transportation costs equal t per unit of distance. Marginal costs for both firms

[1]The original Hotelling model also has firms choosing locations. For our purposes here, we simply assume that locations are given.

are constant and equal to c. Suppose that firm 0 charges a price P_0, and firm 1 charges a price P_1. A consumer located at x will then buy from firm 0 if and only if

$$v - P_0 - tx > v - P_1 - t(1-x), \tag{1.2}$$

provided that $v - P_0 - tx > 0$. We will assume that the entire market is covered: in equilibrium prices are such that everyone consumes. It can be shown that this requires $v > 2t$. From (1.2), we can define a consumer z who is indifferent between buying from firm 0 and firm 1. This consumer is located at

$$P_0 + tz = P_1 + t(1-z),$$

or

$$z = \frac{1}{2} + \frac{P_1 - P_0}{2t}.$$

Note that every consumer with a location $x < z$ will strictly prefer to buy from firm 0, whereas every consumer with a location $x > z$ will strictly prefer to buy from firm 1. Total sales for firm 0 then equal z, for firm 1 they equal $1 - z$. Also note that, when both firms charge the same price, they share the market, and $z = \frac{1}{2}$. The market share for both firms is decreasing in the price they charge.

Consider the profit maximization problem for firm 0. Its profits are

$$\begin{aligned} \Pi^0 &= (P_0 - c)\, z \\ &= (P_0 - c)\left(\frac{1}{2} + \frac{P_1 - P_0}{2t}\right). \end{aligned}$$

Maximizing this with respect to P_0 yields the reaction function

$$P_0 = R_0(P_1) = \frac{1}{2}\left(c + t + P_1\right).$$

Profits for firm 1 are

$$\begin{aligned} \Pi^1 &= (P_1 - c)\,(1 - z) \\ &= (P_1 - c)\left(\frac{1}{2} + \frac{P_0 - P_1}{2t}\right) \end{aligned}$$

This yields the reaction function

$$P_1 = R_1(P_0) = \frac{1}{2}\left(c + t + P_0\right).$$

Equating the two reaction functions, the market equilibrium thus has

$$P_1^* = P_2^* = c + t,$$

which yields equilibrium profits

$$\Pi^1 = \Pi^2 = \frac{1}{2}t.$$

In many applications of the Hotelling model, location should not be taken literally. Rather, the interpretation is that location reflects a certain characteristic of the product. The possible characteristics of a product are given by a continuum between 0 and 1. A consumer that is located at x has a favorite product that has characteristic x. When this consumer does not consume his favorite product, he suffers a disutility that is increasing in the distance between his favorite product and the product that he ends up consuming. The parameter t then reflects the intensity of his preferences, rather than his preferred product. Formally, this model is identical to the one used above.

1.3.2 The circular city: Salop

Hotelling's (1929) model has one disadvantage: it is very hard to allow for more than two firms. In such a case, firms are asymmetric: those at the endpoint of the line only have one neighbor, whereas other firms have two. As a result, equilibrium prices will differ among firms, which makes the analysis cumbersome. Salop (1979) provides a simple way out of this problem. His model is often used to determine the number of firms in a world with fixed costs of entry. Assuming that there exist a large number of identical potential firms, we will look at the equilibrium number of firms entering the market. Consumers are located uniformly on a circle with a perimeter equal to 1. Density is unitary around this circle. All travel occurs along the circle. Essentially, we thus have a Hotelling line in which the endpoints are connected.

Consumers wish to buy one unit of the good, have a valuation v, and unit transport cost t. Each firm is allowed to locate in only one location. In order to address the issue of the number of firms, we introduce a fixed cost of entry f. Marginal cost are c, so firm i's profit is $(p_i - c)D_i - f$ if it enters, with D_i the demand it faces, and 0 otherwise. In the first stage, potential entrants simultaneously choose whether or not to enter. Let n denote the number of entering firms. Firms are automatically located equidistant from one another on the circle. In the second stage, firms compete in prices given these locations. Omitting the choice of location allows us to study the entry issue in a simple and tractable way.

We solve with backward induction. In stage 2, assume that n firms have entered. We look for a symmetric equilibrium, in which all firm charge the same price p. The strategy to solve this is as follows. Consider the decision of, say, firm 1. Suppose that all other firms charge price p. We can then derive the best reply of firm 1, which we denote $R_1(p)$.

By construction, a Nash equilibrium is a price p^* such that $R_1(p^*) = p^*$. In that case, the price p^* that each firm charges is the profit-maximizing price, given that all other firms also charge p^*.

Suppose that all other firms charge price p. For simplicity, denote the location of firm 1 as 0. Since we have n firms, the location of firm 1's right-hand neighbor, which is firm 2, is given by $1/n$. Consider the consumers located between firm 1 and firm 2. If firm 1 charges price p_1, then the consumer z_{1-2} that is indifferent between visiting firm 1 and firm 2, is given by

$$p_1 + tz_{1-2} = p + t\left(\frac{1}{n} - z_{1-2}\right),$$

which yields

$$z_{1-2} = \frac{1}{2n} + \frac{p - p_1}{2t}.$$

Hence, by charging a price p_1, the total sales to consumers on its right-hand side equal z_{1-2}, provided that firm 1 does not capture sales from firm 3 and beyond. Note that this is necessarily true in the equilibrium, where all firms charge the same price. It can easily be seen that the total sales to consumers on its left-hand side also equal $1/2n + (p - p_1)/2t$. Total profits for firm 1 thus equal

$$\Pi^1(p_1, p) = 2(p_1 - c)\left(\frac{1}{2n} + \frac{p - p_1}{2t}\right).$$

Maximizing this with respect to p_1 yields the reaction function

$$R_1(p) = \frac{1}{2}\left(p + c + \frac{t}{n}\right).$$

Equilibrium requires that $R_1(p^*) = p^*$. This yields

$$p^* = c + \frac{t}{n}.$$

Gross profits per firm (net of entry costs) then equal $\Pi^* = \frac{t}{n^2}$.

Move back to stage 1. Firms will enter the market as long as profits are strictly positive. Hence the number of firms that enters the market is such that

$$\Pi^* - f = \frac{t}{n^2} - f = 0.$$

This yields $n^* = \sqrt{t/f}$, which implies $p^* = c + \sqrt{tf}$. Firms price above marginal cost and yet do not make profits. An increase in f causes a decrease in the number of firms and an increase in the profit margin. An increase in t increases the profit margin and the number of firms. Consumer's average transportation cost is $t/4n^c = \sqrt{tf}/4$. This does not increase as rapidly as t. When the entry cost or fixed production cost converges to zero, the number of entering firms tends to infinity and the market price tends to marginal cost.

The profit margin is only a monetary transfer from consumers to firms. A social planner would therefore choose n in order to minimize the sum of fixed cost and transportation costs: $\min_n(nf + t/4n)$. This yields $n^S = \frac{1}{2}\sqrt{t/f} = \frac{1}{2}n^*$. Hence, the market generates too many firms. Similar results hold for quadratic transportation costs. Entry is socially justified here by the savings in transportation costs. In contrast, the private incentive to enter is linked with stealing the business of other firms while still being able to impose a mark-up. This is sometimes called the trade-diversion effect. The result that in the market equilibrium, too many firms enter, has also been found in other types of models. See e.g. Mankiw and Whinston (1980).

There are three natural extensions that would make the model more realistic: the introduction of a location choice, the possibility that firms do not enter simultaneously, and the possibility that a firm locates at several points in the product space. Economides (1984) considers a three-stage model in which firms choose whether to enter, then choose locations on the circle, and then compete in prices. For quadratic costs there exists a free-entry symmetric equilibrium. A firm may produce several brands and crowd the product space, leaving no room for entry by another firm. An incumbent has a greater incentive to introduce new products than an entrant has. As we will see, this efficiency effect tends to bias the market structure toward multibrand monopoly.

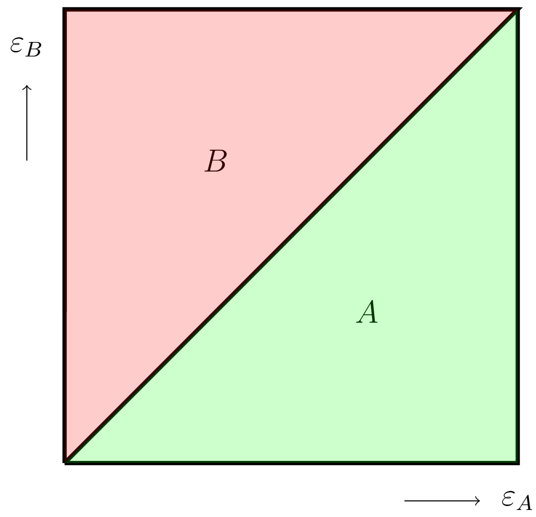
1.3.3 Perloff and Salop

Perloff and Salop (1985) propose another model of product differentiation that will be useful in the following chapters. Suppose there are two firms, A and B, and a unit mass of consumers. Consumer j has a willingness to pay for the product of firm $i \in \{A, B\}$ that is given by $v + \varepsilon_{ij}$, where ε_{ij} is the realization of a random variable with cumulative distribution F on some interval $[0, \bar{\varepsilon}]$, and continuously differentiable density f. We assume that v is high enough such that all consumers buy in equilibrium. All draws are independent from each other. We can interpret ε_{ij} as the *match value* between consumer j and product i; it represents how much consumer j likes the product that firm i sells. Firms cannot observe the realizations of ε_{ij}. For simplicity, assume that firms have constant marginal costs that are constant and equal to c.

Suppose first that both firms charge the same price p^*. Needless to say, this will be what happens in the equilibrium of the model. In that case, a consumer will buy from A whenever $\varepsilon_A > \varepsilon_B$, and buy from B if that is not the case. Figure 1.1 denotes those consumers in $(\varepsilon_A, \varepsilon_B)$-space. Consumers with a $(\varepsilon_A, \varepsilon_B)$ in the green area will buy from firm A, whereas those in the red area will buy from firm B.

To find the equilibrium, we need to analyze what would happen if firm A defects from

Figure 1.1: Perloff-Salop, equal prices.



a candidate equilibrium p^* by setting some different price. Without loss of generality, we may assume that A defects to some $p_A > p^*$.[2] For ease of analysis, we denote $\Delta \equiv p_A - p^*$.

Figure 1.2: Perloff-Salop, $p^A > p^*$.

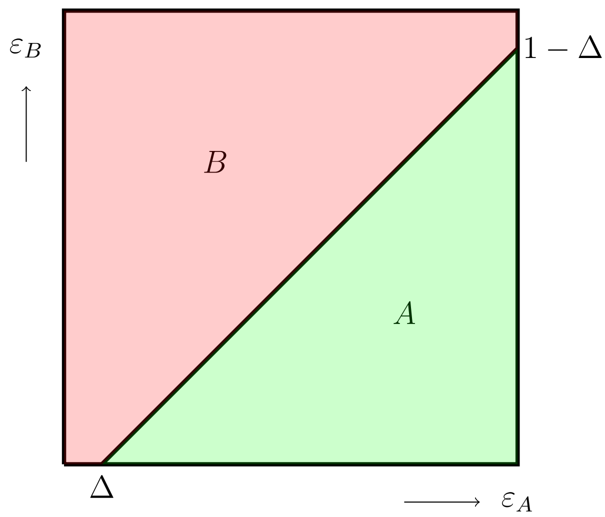


Figure 1.2 gives sales for both firms in this situation. Note that a consumer will now only buy from firm A if $\varepsilon_A - \Delta > \varepsilon_B$, which yields the sales in the Figure. For simplicity, let's assume that the ε's are drawn from a uniform distribution on $[0, 1]$. In that case, from the figure, total sales of firm A from charging a price $p_A = p^* + \Delta$ equal

$$q_A = \frac{1}{2}(1 - \Delta)^2.$$

[2] Technically, as the profits of firm A are twice continuously differentiable in p_A, considering a defection to some $p_A < p^*$ would yield the same outcome.

Profits of firm A can be written

$$\pi_A = (p_A - c) \cdot q_A.$$

Profit maximization thus requires

$$\frac{\partial \pi_A}{\partial p_A} = (p_A - c) \cdot \frac{\partial q_A}{\partial p_A} + q_A = 0. \tag{1.3}$$

Now

$$\frac{\partial q_A}{\partial p_A} = -(1 - \Delta). \tag{1.4}$$

We can now find the equilibrium price by taking the first-order condition (1.3), using (1.4) and imposing symmetry. Note that in equilibrium (thus when imposing symmetry), we have $\Delta = 0$ and $q_A = 1/2$: in equilibrium, at equal prices, each firm will attract exactly half of all consumers, as we can also see in Figure 1.1. Hence, (1.3) simplifies to

$$(p_A - c) \cdot (-1) + \frac{1}{2} = 0.$$

This implies that equilibrium prices equal $p^* = c + \frac{1}{2}$.

In our description of the model, we assumed that v is high enough such that each consumer will buy in equilibrium. We thus need that a consumer with $(\varepsilon_A, \varepsilon_B) = (0, 0)$ would still be willing to buy. In this particular parametrization of the model, with equilibrium prices equal to $p^* = c + 1/2$, this thus requires that $v > c + 1/2$.

Finding the equilibrium price for a general distribution function F is a bit harder. The challenge is to find the proper expression for q_A in that case. From Figure 1.2, we then have

$$\begin{aligned} q_A &= \int_{\Delta}^{1} \left(\int_{0}^{\varepsilon_A - \Delta} f(\varepsilon_B)\, d\varepsilon_B \right) f(\varepsilon_A)\, d\varepsilon_A \\ &= \int_{\Delta}^{1} F(\varepsilon_A - \Delta) f(\varepsilon_A)\, d\varepsilon_A \end{aligned}$$

Using Leibnitz' rule, this implies

$$\frac{\partial q_A}{\partial p_A} = -\int_{\Delta}^{1} f(\varepsilon_A - \Delta) f(\varepsilon_A)\, d\varepsilon_A \tag{1.5}$$

Again, we find the equilibrium price by taking the first-order condition (1.3), using (1.5) and imposing symmetry. In equilibrium, again, $\Delta = 0$ and $q_A = 1/2$. This yields

$$p^* = c + \frac{1}{2\int_0^1 f^2(\varepsilon)\, d\varepsilon}.$$

Note that the Perloff-Salop model is similar in spirit to the Hotelling model, in the sense that each consumer has a certain valuation for each firm's product. The crucial difference is that in the Hotelling model, these valuations are perfectly negatively correlated: a consumer that has a very high valuation for product A, say, necessarily has a very low valuation for product B, and vice-versa. In the Perloff-Salop model, these valuations are independent of each other. A consumer with a high valuation for product A may very well have a high valuation for product B as well. This also implies that the Perloff-Salop model can easily be generalized to more than 2 firms.

Many of the applications that we discuss in this course will build on either the Hotelling model or the Perloff-Salop model. Whichever model is more convenient to use will depend on the particular application.

Exercises

1. Consider the following Cournot model. Two firms set quantities. Demand is given by $q = 1 - p$. Marginal costs are either equal to 0 or to 0.4, both with equal probability. Derive the Cournot equilibrium if
   (a) uncertainty is resolved before firms set their quantities.
   (b) uncertainty is resolved after firms set their quantities.
2. Consider a Hotelling model. Consumers are uniformly distributed on a line of unit length. Consumers either buy one unit of the good or none at all. Each consumer obtains gross utility v from consuming the product. We have two firms: one is located at 0, the other is located at 1. Marginal costs for both firms are constant and equal to c. However, transportation costs are constant: a consumer that has to travel a distance x incurs transport costs tx^2. Derive the equilibrium prices.
3. Consider the Perloff-Salop model where consumers have a valuation that is uniformly distributed on $[0, 1]$. For simplicity, $c = 0$. Assume however that $v = 0$ such that *not* all consumers buy in equilibrium. Derive the equilibrium prices.

Chapter 2

Consumer Search

2.1 Introduction

In this chapter, we will study models of consumer search. Search costs are costs that an individual consumer has to incur to learn the price or the characteristics of a particular firm or product. There are many reasons why it is interesting to do so.

First, standard models of competition (such as Cournot, Bertrand, Hotelling, or almost any other) predict that firms will charge the same price, provided they face the same costs and demand functions. Yet this law of one price often seems violated in the real world. Similar firms often charge wildly different prices for identical products, and even a single firm often charges prices that can fluctuate considerably from day to day. Hence, we need models that can explain such price dispersion. Search models can do exactly that.

Second, when the Internet became an important platform to buy and sell products some time ago, people had high expectations that this would provide a transparent and efficient marketplace where firms would not be able to sell products at any price higher than marginal costs. Somehow, this never materialized. Indeed, studies have found that price dispersion on the Internet is not lower than that in the offline world (see e.g. Morgan et al. (2004)) To understand the implications of innovations that make it easier for consumers to search for products and compare price, we need models that take those search costs explicitly into account.

Third, it is well known that with price competition, identical firms and homogeneous products, the Nash equilibrium has consumers charging prices equal to marginal costs and making zero profits, even if there are just two firms. This is a counterintuitive result; we would expect more competition to lead to lower prices, but not to the extent that two firms is already enough to get perfectly competitive prices. This results is therefore known as the *Bertrand paradox*. Search models provide one way out of this paradox.

Fourth, models of consumer search are interesting in their own right. They provide new insights relative to models that do not take search costs into account. Also, they can serve as building blocks to study other important issues.

2.2 The standard Bertrand Model

To fix thoughts, we start with the standard Bertrand model with perfect substitutes. Although this is one of the easiest models imaginable, we will analyze it and derive the Nash equilibrium in a very formal manner. Being so formal here will be helpful in doing more complicated analyses later.

Consider two firms that produce identical goods. There are no frictions whatsoever, so consumers simply buy from the firm that sets the lowest price. If both firms happen to charge the same price, we assume that both face half the market demand at that price. We also assume that firms can fulfill any demand they happen to face. The demand function is $q = D(p)$. Marginal costs are constant and equal c. We assume that monopoly profits are strictly concave, bounded, and that the monopoly price is given by p^m, i.e.

$$p^m = \arg\max_p (p-c)D(p).$$

Demand for firm i is written $D_i(p_i, p_j)$. Note that we have

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j \\ \frac{1}{2}D(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j. \end{cases}$$

Profits of firm i can now be written

$$\Pi^i(p_i, p_j) = (p_i - c)D_i(p_i, p_j).$$

This profit function is discontinuous. First consider the case in which $p_j > p^m$. In such a case, the best reply of firm i is obviously to set $p_i = p^m$: by doing so, it attracts all consumers, while it charges the monopoly price. Hence, it then earns monopoly profits.

Now suppose that $c < p_j \leq p^m$. Firm i now has three choices. When it charges some price $p_i > p_j$, its profits are zero since it does not attract any customers. When it charges $p_i = p_j$, its sales are $\frac{1}{2}D(p_j)$. Given concavity of the profit function and the fact that $p_j < p^m$, the best is then to set p_i slightly below p_j, say, $p_i = p_j - \varepsilon$, with ε arbitrarily low. The profits of doing so are $D(p_j - \varepsilon) \cdot (p_j - \varepsilon - c)$, which is obviously higher than $\frac{1}{2}D(p_j) \cdot (p_j - c)$. Hence, the best reply in this case is to slightly undercut firm j by setting $p_i = p_j - \varepsilon$.

Consider the case in which $p_j < c$. Undercutting this price now yields negative profits. Hence, the best reply is to set any $p_i > p_j$.

Finally, consider the case in which $p_j = c$. Again, undercutting yields negative profits. But now, firm i is also willing to set $p_i = c$, as that also yields zero profits. The best reply is therefore to set any $p_i \geq c$.

Summarizing, we have

$$R_i(p_j) = \begin{cases} \in (p_j, \infty) & \text{if } p_j < c \\ \in [c, \infty) & \text{if } p_j = c \\ p_j - \varepsilon & \text{if } c < p_j \leq p^m \\ p^m & \text{if } p_j > p^m. \end{cases}$$

We look for a Nash equilibrium, in other words a pair of prices (p_1^*, p_2^*) such that $R_1(p_2^*) = p_1^*$ and $R_2(p_1^*) = p_2^*$. From the reaction functions above, we thus have that the unique Nash equilibrium has both firms charging c. Profits are zero for both firms.

This result is known as the *Bertrand paradox.* It qualifies as a paradox since it is hard to believe that two firms is already enough to restore the competitive outcome in a market. When only two firms are active in a market, one would expect them to make positive profits.

2.3 A model of sales

Consider the following set-up. This is a simplified version of Varian (1980). Again, we have a duopoly that competes in prices. For simplicity, we assume that consumer demand has the simplest form imaginable: there is a mass of consumers that equals 1, where each consumer is willing to pay at most 1 for the product. Each consumer only demands one unit of the product. Hence, formally, demand is given by

$$D(p) = \begin{cases} 1 & \text{if } p \leq 1 \\ 0 & \text{if } p > 1. \end{cases}$$

We assume that marginal costs are zero. In the standard model, this set-up would simply yield $p_1^* = p_2^* = 0$, just like we had in the standard analysis. Both firms then face demand $\frac{1}{2}$, and make profits equal to zero.

The twist in this model is that we now assume that there are two types of consumers. A fraction λ of consumers is informed, which imply that they can observe the prices of both firms, and visit the shop that has the lowest price, just like consumers in the standard Bertrand model. When both firms charge the same price, these consumers split evenly between the two shops. The remaining fraction of consumers $1 - \lambda$, however, is uninformed: they simply pick one firm at random, go there, and buy the product provided its price does not exceed 1. When setting their price, firms thus face a trade-off. They could set a high price, and try to make as much profits as possible from their captive consumers. But alternatively, they could also choose to set a much lower price, to try to capture all the informed consumers as well.

Profits for firm i are given by

$$\Pi^i(p_i,p_j) = \begin{cases} 0 & \text{if } p_i > 1 \\ \frac{1}{2}(1-\lambda)\,p_i & \text{if } 1 \geq p_i > p_j \\ \frac{1}{2}p_i & \text{if } p_i = p_j \leq 1 \\ \left[\lambda + \frac{1}{2}(1-\lambda)\right] p_i & \text{if } p_i < p_j \leq 1 \end{cases}$$

provided $p_i \leq 1$. This can be seen as follows. If firm i has a price higher than that of firm j, then it will only attract its share of the uninformed consumers. With a total of $1-\lambda$ uninformed consumers, it will thus sell to $(1-\lambda)/2$ of them, and thus make profits $(1-\lambda)p_i/2$. If it sets a lower price, however, it will also sell to all the informed consumers, so profits are $\left[\lambda + \frac{1}{2}(1-\lambda)\right] p_i$. If both firms set the same price, they again share the market equally – both the informed as well as the uninformed consumers.

Deriving the best-reply functions is slightly more complicated than in the standard case. First note that, if firm i would want to charge a higher price than firm j, the best price it can charge is simply to set $p_i = 1$: for any $p_i \in (p_j, 1]$, it faces the same demand $(1-\lambda)/2$, and it can make the most of that by charging the highest admissible price. This implies the following. Suppose that $p_j > 0$. The best choice for firm i then is to either slightly undercut p_j which yields profits $\left[\lambda + \frac{1}{2}(1-\lambda)\right] p_j$, or to set $p_i = 1$, which yields profits $(1-\lambda)/2$. Firm i strictly prefers following the first strategy when

$$\lambda p_j + \frac{1}{2}(1-\lambda)\,p_j > \frac{1}{2}(1-\lambda)$$

thus when

$$p_j > \frac{1-\lambda}{1+\lambda}.$$

This yields the following reaction functions:

$$R^i(p_j) = \begin{cases} 1 & \text{if } p_j \leq \frac{1-\lambda}{1+\lambda} \\ p_j - \varepsilon & \text{if } p_j > \frac{1-\lambda}{1+\lambda}. \end{cases}$$

It is clear that we do not have a Nash equilibrium in pure strategies. To understand why this is the case, consider the following thought experiment. Suppose firm 1 sets $p = 1$. Firm 2 will find it profitable to slightly undercut that price. But that situation cannot be a Nash equilibrium either; firm 1 now wants to undercut firm 2's price. This process continues until we reach $p = (1-\lambda)/(1+\lambda)$. At that price, no firm has an incentive to undercut the other firm's price, but each firm does have an incentive to defect by setting its price equal 1, where the whole process starts anew. The reaction functions are discontinuous, which in this case implies that there is no pair of prices (p_1^*, p_2^*) where they intersect. Hence, there is no equilibrium in pure strategies.

We do, however, have an equilibrium in mixed strategies.[1] Suppose that both firms draw their price from some continuous probability distribution function $F(p)$ on the support $[\underline{p}, \bar{p}]$. As we saw in the previous chapter, for this to be an equilibrium, we need that both firms are indifferent between all prices in $[\underline{p}, \bar{p}]$. Thus, given that firm j uses the mixed strategy, any $p_i \in [\underline{p}, \bar{p}]$ must yield the same profit for firm i. Note that the profits that firms make in the mixed strategy equilibrium cannot be lower than $(1-\lambda)/2$: if they were lower, a firm could do better by defecting from the prescribed strategy by simply setting $p = 1$. This implies that we cannot have $\bar{p} > 1$. Such a price would yield zero profits. Hence all prices in $[\underline{p}, \bar{p}]$ would have to yield zero profits, and firms can again do strictly better by setting $p = 1$. Now consider the possibility that $\bar{p} < 1$. A firm charging price $\bar{p}$ is certain that it has the highest price on the market.[2] But, given that that is the case, it can do better by charging $p = 1$ instead. Hence, we must have $\bar{p} = 1$.

We can now pin down the expected profits that firms make in the mixed strategy equilibrium. We necessarily have $\bar{p} = 1$. A firm charging this price has the highest price on the market with certainty, and so makes profits equal to $(1-\lambda)/2$. But that implies that any $p \in [\underline{p}, \bar{p}]$ must yield those same profits. Consider a firm charging $\underline{p}$. With certainty, it charges the lowest price. Its profits then equal $\left[\lambda + \frac{1}{2}(1-\lambda)\right]\underline{p}$. As this has to equal $(1-\lambda)/2$ we thus have that $\underline{p} = (1-\lambda)/(1+\lambda)$.

If firm i charges a price p, the probability that firm j charges a lower price is given by $F(p)$. When setting some p in $[\underline{p}, \bar{p}]$, firm is expected profits are

$$\begin{aligned} E(\pi_i(p)) &= F(p)\left(\frac{1}{2}(1-\lambda)\right)p + (1-F(p))\left(\lambda + \frac{1}{2}(1-\lambda)\right)p \\ &= \frac{1}{2}(1-\lambda)p + (1-F(p))\lambda p. \end{aligned}$$

We thus need

$$\frac{1}{2}(1-\lambda)p + (1-F(p))\lambda p = (1-\lambda)/2, \qquad \forall p \in \left[\frac{1-\lambda}{1+\lambda}, 1\right].$$

Solving for $F(p)$, this implies

$$F(p) = 1 - \frac{(1-\lambda)(1-p)}{2\lambda p}$$

Note that this solution indeed has $F^*\left(\frac{1-\lambda}{1+\lambda}\right) = 0$, and $F^*(1) = 1$. As already shown,

[1] The analysis that follows, is somewhat sloppy. Formally, we also have to prove that the probability density function f that we are deriving, is continuous and does not contain any 'holes' or 'spikes'. See e.g. Varian (1980) for further technical details, albeit in the context of a model that is slightly different, and will be discussed in the next section.

[2] Note that we assume a continuous probability distribution function $F(p)$. This implies that the probability that two firms charge the same price, is equal to 0.

the expected profits of both firms in this equilibrium equal $(1-\lambda)/2$, which is strictly positive. Hence, in that sense, we do not have a Bertrand paradox in this set-up.

Varian (1980) interprets this as a model of sales: in his set-up prices are often high, and every now and then they are low. Note that the model also yields strictly positive profits despite having price competition.

2.4 The Diamond paradox

2.4.1 Model

Consider a slight change of the basic model with homogeneous products and equal marginal costs. This is based on Diamond (1971). We assume that consumers have to incur search costs to find out which price each firm charges. These search costs are s, where s can be arbitrarily small. Visiting the first shop is free, but to visit any additional shop, a consumer has to incur search costs s. Suppose again that there is a unit mass of consumers that all have a willingness to pay for one unit of the product that equals v. Firms have constant marginal costs $c < v$. It is crucial that consumers cannot observe prices before they visit a shop. For simplicity, we assume that there are only two firms.

The timing of the game is as follows. First, firms set prices that are unobservable. Second, consumers visit the shop of their first choice, and observe the price that is set by that shop. Third, consumers either buy from this shop, decide to visit the other shop, or decide not to buy at all. The equilibrium now also depends on what consumers expect. That makes it hard to derive reaction functions in the usual manner; these reaction functions now also depend on the behavior of consumers. We therefore choose a different manner of analysis. We first postulate a candidate equilibrium. Then, we derive whether any firm (or the consumers, for that matter), has an incentive to defect from that equilibrium.

A natural candidate equilibrium seems to be the one that is the equilibrium in the standard Bertrand model, in which both firms charge a price c, consumers split evenly in the choice of their first shop, and consumers also decide to buy at the shop they first visit. It is easy to see, however, that this cannot be an equilibrium. Suppose that firm 1 deviates and increases its price to $c + s/2$. First, such a defection cannot influence the number of consumers that visit this shop, as consumers can only observe the price after they have visited. Hence, all consumers that visit this shop are now unpleasantly surprised. Yet, they still have no reason to choose a different strategy in the subgame that follows. Going to the other shop implies a cost-saving of $s/2$ due to a lower price (note that we consider a deviation from the Nash equilibrium by firm 1, which implies that firm 2 is still charging price c), but an increase in search cost by s. Hence, consumers will not continue search and the deviation is profitable.

But the same argument holds for *any* symmetric candidate equilibrium that has $p < v$. For a price $p \geq v$, consumers also will not switch for a small price increase, but a firm is not willing to defect such a manner, as by definition it would decrease its profits.

Thus, the unique equilibrium has both firms charging the monopoly price, even with infinitesimally small search costs. This is known as the *Diamond paradox*. Also note that this result does not hinge on the number of firms. For any $n \geq 2$, the analysis applies and the unique equilibrium has prices equal to the monopoly price.

2.4.2 What if the first visit is not free?

In this model, the assumption that the first visit is free is somewhat odd. Suppose that a consumer would also have to pay s to visit the first firm. Following the logic above, the unique Nash equilibrium would have all firms charging the monopoly price $p^m = v$. But now suppose that consumers also have to pay s to visit the first firm. This will not change the logic of the analysis; it would still be an equilibrium to have $p = v$. Once the consumer has visited the first firm, the search costs s are already sunk so she could either buy at that firm and pay 1, or go home without the product and still having incurred search costs s.

But a rational consumer will foresee this. Knowing that all firms will charge price v, and knowing that she has to incur search costs s to visit one, she knows that it is not a good idea to enter this market, since doing so so will leave her with a negative surplus of $-s$. Firms would like to commit to charge a lower price of $v - s$, but cannot do so; once a consumer enters their shop they have an incentive to defect to charging a price v anyhow. Hence, the market will break down and cease to exist.

We can get around this by assuming that each consumer has an individual downward sloping demand function $D(p)$. At a monopoly price of p^m, each consumer then demands more than one unit and still has some consumer surplus S. Firms do not have an incentive to defect to a higher price: Again consumers will not switch for a small price increase, but a firm is not willing to defect such a manner, as by definition it would decrease its profits. As long as its consumer surplus S this is bigger than the search cost s, the market would still exist, as the consumer would still be willing to pay the first visit.

2.4.3 Implications

This result, that even infinitessimally small search costs are enough to switch from an equilibrium with marginal cost pricing to one with monopoly pricing, is known as the Diamond paradox. It shows that even the slightest perturbation of the original model is already enough to end up in the other extreme outcome.

Another surprising outcome of the model is that, although we have introduced search costs, people do not actually end up searching in the equilibrium of this model: in equilibrium, they also end up buying from the first firm they encounter.

To find a way out of the Diamond paradox, we can do two things. First, we can assume that not all consumers have positive search costs. If some consumers can observe all prices, firms may still have an incentive to defect from the monopoly price. Second, we may assume that products are differentiated. We will discuss these two approaches in what follows. But before we can do so, we first have to delve somewhat deeper in optimal consumer search.

2.5 Optimal search

In the Diamond paradox, we had a case with a pure strategy equilibrium. Hence, consumers can figure out beforehand that all firms will charge the same price and also what that price will be. This makes the consumers' decision problem very easy; if all firms set the same price and products are homogeneous, it never makes sense to search beyond the first firm.

Things change, however, with a mixed strategy equilibrium, that is, if in equilibrium firms draw their price from some distribution F, as was the case in the Varian model. The models we will consider in the remainder of this chapter will have that form. For now, we assume that $F(p)$ is exogenously given but later we relax that assumption. The optimal search rule now becomes much more complicated. If a consumer faces a price p that is drawn from some cumulative distribution $F(p)$, she has to consider whether it is worthwhile to incur an additional search costs s to visit a new firm and essentially get a new draw from the distribution $F(p)$. The problem becomes even more complicated if there are more firms she can still visit; in that case, she has to take into account the option that there are still more firms left to search after the next.

In this section, we consider that problem. Suppose that firms sell homogeneous products and prices at each firm are drawn from a cumulative distribution $F(p)$, with a different draw for each firm. There are n firms, and the consumer has unit demand. A consumer incurs search costs s per firm that she visits. Then, under what circumstance should she visit another firm?

We will focus on *sequential search.* That implies that after each firm that she visits, the consumer can consider her options and decide whether to continue search. With *non-sequential search,* she has to decide in advance how many firms she will visit. We will also assume *perfect recall:* if at some point in the process the consumer decides to go back to a firm that she has visited before, she can do so for free (i.e. without having to incur another s, or any other costs.) This assumption will greatly simplify the analysis: more on that below.

For simplicity, let's first consider the case in which there are only 2 firms, so $n = 2$. Our consumer has visited one firm, which for simplicity we will denote as firm 1. At that firm, she observed price p_1. Suppose she would visit firm 2. If she observed a price $p_2 > p_1$, she would simply return to firm 1 and buy there. If $p_2 < p_1$, she would buy from firm 2 and her utility would increase by $p_1 - p_2$. Hence, her expected benefit from visiting the second firm are given by

$$b(p_1) = \int_0^{p_1} (p_1 - p_2)\, dF(p_2). \tag{2.1}$$

The expected cost of visiting firm 2 is simply equal to s. Hence, the consumer is willing to visit the last firm whenever $p_1 > \hat{p}$, with $\hat{p}$ implicitly defined by

$$b\left(\hat{p}\right) = s.$$

For existence of $\hat{p}$, note that $b(\bar{p}) = \int_0^{\bar{p}} (\bar{p} - p_2)\, dF(p) = \bar{p} - E(p)$ and $b(0) = 0$. It is obvious that $b(p)$ is strictly increasing in p. Taken together, these observations imply that $\hat{p}$ always exists and is unique, provided that $s < \bar{p} - E(p)$.

Now suppose that there are more firms left to search. One could argue that there is an option value of having more firms left to search and hence, in that case, our consumer will be inclined to continue search more often. That, however, is not the case. It turns out that the $\hat{p}$ we derived above is stationary, in the sense that it does not depend on the number of firms that is left to search. This result is due to Weitzman (1979). It can be seen as follows.

For the sake of argument, first suppose that $n = 3$. At the first firm our consumer again observes some price p_1. Denote $B_1(p)$ as the expected net benefit of doing one more search when the current best price is p, and there is just one firm left to search. Hence $B_1(p) \equiv b(p) - s$. By construction, we have that $B_1(\hat{p}) = 0$.

Similarly, denote $B_2(p)$ as the expected net benefit of doing another search when the current best price is p and there are two firms left to search. Clearly, both B and B_2 are increasing in p. We will now evaluate $B_2(\hat{p})$, with $\hat{p}$ defined as above, so $B(\hat{p}) = 0$. Our consumer thus finds herself at the first of 3 shops, observes $p_1 = \hat{p}$, and considers whether is worthwhile to also visit firm 2.

As always, we use backward induction. First suppose that $p_2 \leq \hat{p}$. In that case, the best price she found after visiting firm 2 now is p_2. As $B(p_2) \leq B(\hat{p}) = 0$, it will not be wortwhile to visit firm 3, hence she will stick to firm 2. Now suppose that $p_2 \geq \hat{p}$. In that case, the best price she found so far still is $p_1 = \hat{p}$. In that case, it will also not be worthwhile to visit firm 3, as by construction $B(\hat{p}) = 0$. But that implies that if the consumer has observed $\hat{p}$ at firm 1, the option to also visit firm 3 is useless, as she will never use that option. In other words, $B_2(\hat{p}) = 0$, so our consumer will use the same reservation value regardless of whether there are 1 or 2 firms left to search.[3]

[3]Of course, this only shows that B and B_2 have the same value in $\hat{p}$, not in any other p.

The argument we used above easily carries over to more firms; given that $B_2(\hat{p}) = 0$, it is now straightforward to show that $B_3(\hat{p}) = 0$, etc. Hence, *the reservation price a consumer uses is independent of the number of firms she has left to search.* This is an important result that highly simplifies the analysis in many models of consumer search. One of its implications is that, to derive the optimal search rule, it is not even necessary for the consumer to *know* how many firms there are left to search.

2.6 Search with homogeneous products

The classic model of consumer search with homogeneous products is due to Stahl (1988) Suppose there are n firms, and a unit mass of consumers. Again, consumers have unit demand and a willingness to pay equal to 1.[4]. A fraction λ of consumers has zero search costs. The remaining fraction $1 - \lambda$ has search costs that equal $s \in (0, 1)$. We will again assume that the first visit is free.

Note that this set-up is very close to that in Varian (1980); consumers with search costs 0 can check all firms for free and hence will become perfectly informed in equilibrium. We will refer to consumers with zero search costs as shoppers, and to those with positive search costs as non-shoppers.

It is easy to see, with the same arguments as in the Varian (1980) model, that an equilibrium in pure strategies does not exist. In any candidate symmetric pure strategy equilibrium, firms either have an incentive to charge a slightly lower price and capture all the shoppers, or to charge a much higher price and make a substantial profit on the non-shoppers that happen to visit.

Solving for the mixed-strategy equilibrium of this model is rather involved. Given the reservation price $\hat{p}$ that non-shoppers use (that is, the price at which they are just willing to visit one more firm), we can derive the price distribution $F(p)$ that firms will use. Given the $F(p)$ that firms will use, we can derive the reservation price $\hat{p}$ that consumers will use. Essentially, solving the model is then a fixed-point problem.

As a first step, suppose that one firm charges a price that is higher that $\hat{p}$. That cannot be part of an equilibrium; if it would, then even a non-shopper will not buy from this firm. Hence, it will not make any sales, as shoppers will definitely be able to find a better deal.[5] Hence *a firm that sets $p > \hat{p}$ will not make any profits, which cannot be part of an equilibrium.*

But this implies that in equilibrium all firms set $p \leq \hat{p}$, which in turn implies that *there will be no search in equilibrium*, in the sense that all non-shoppers buy from the first

[4]The original Stahl paper is more general and considers the case that each consumer has a downward sloping demand function $D(p)$. In that case, it is possible to show that an equilibrium exists, but it is not possible to give an explicit expression. The analysis here is due to Janssen et al. (2005)

[5]Note that it is easy to see that it cannot be an equilibrium for all firms to charge $p > \hat{p}$ either.

firm they encounter; they never have an incentive to continue search, precisely because if they would, the firm they walk away from would not make any profits and hence would be better off lowering its price. *The equilibrium thus necessarily has all shoppers buying from the cheapest firm, and all non-shoppers buying from the first firm they encounter.* Hence, in that sense, also here, there is *no search in equilibrium.*

The full derivation of the equilibrium in this model is beyond the scope of thus course. But essentially, it boils down to the Varian (1980) model, but one where the upper bound of prices is endogenously determined.[6]

[6]If you really want to know, the analysis is as follows. Consider a firm charging some price $p_i \leq \min\{\hat{p}, 1\}$. Given that all other firms use the mixed strategy $F(p)$, the expected profits of doing so equal

$$E(\pi(p_i)) = p_i\left[\frac{1-\lambda}{n} + \lambda(1-F(p_i))^{n-1}\right].$$

It always sells to its share of non-shoppers, which is given by $(1-\lambda)/n$, and if it has the lowest price on the market, which happens with probability $(1-F(p_i))^{n-1}$, it also sells to all λ shoppers.

Again denote the upper bound on the distribution of F as $\bar{p}$. With the same arguments as in the Varian model, it is easy to see that we necessarily have $\bar{p} = \hat{p}$. Profits when charging $\hat{p}$ are then given by $(1-\lambda)\hat{p}/n$. For all prices in the domain, we thus need that

$$p_i\left[\frac{1-\lambda}{n} + \lambda(1-F(p_i))^{n-1}\right] = \frac{(1-\lambda)\hat{p}}{n}$$

which implies

$$F(p) = 1 - \left(\frac{(1-\lambda)(\hat{p}-p)}{n\lambda p}\right)^{\frac{1}{n-1}}. \tag{2.2}$$

Solving for $F(\underline{p}) = 0$ gives the lower bound $\underline{p} = (1-\lambda)\hat{p}/(\lambda n + (1-\lambda))$.

Now consider consumer behavior. Optimal search requires consumers to use a reservation price $\hat{p}$ such that $b(\hat{p}) = s$, with $b(p)$ given by (2.1). This implies

$$\hat{p} - E(p) - s = 0. \tag{2.3}$$

Rewriting (2.2), we have

$$p = \frac{\hat{p}}{1 + n\frac{\lambda}{1-\lambda}(1-F(p))^{n-1}}.$$

This implies

$$E(p) = \int_{\underline{p}}^{\hat{p}} p dF(p) = \int_{\underline{p}}^{\hat{p}} \frac{\hat{p} dF(p)}{1 + n\frac{\lambda}{1-\lambda}(1-F(p))^{n-1}}$$

Changing variables by letting $y \equiv F(p)$, this implies

$$E(p) = \hat{p}\int_0^1 \frac{dy}{1 + n\frac{\lambda}{1-\lambda}(1-y)^{n-1}}.$$

We can use this expression to pin down $\hat{p}$ from (2.3):

$$\hat{p} = \frac{s}{1 - \int_0^1 \frac{dy}{1+n\frac{\lambda}{1-\lambda}(1-y)^{n-1}}},$$

which in turn pins down the equilibrium distribution of prices in (2.2).

As an example, suppose we have two firms and half of consumers are shoppers, so $n = 2$ and $\lambda = 0.5$. In that case $\hat{p} = 2.2188s$, so

$$F(p) = 1 - \frac{2.2188s - p}{2p}.$$

2.7 Search with differentiated products

2.7.1 Introduction

Above, we considered the effect of search cost in models with homogeneous products. We now consider the case of differentiated products. To do so we build on the Perloff/Salop model introduced in the first chapter. Essentially, we add search costs to that. We will also use the analysis of optimal search in the previous chapter. The model we consider is due to Anderson and Renault (1999). An earlier version appeared in Wolinsky (1986).

Suppose there are n firms. For simplicity, their costs are zero. There is a unit mass of consumers that all have search costs s. If a consumer buys from i, the utility she obtains is[8]

$$U_j^i\left(p_i, \varepsilon_i\right) = v + \varepsilon_i - p_i,$$

where v is large enough such that a consumer always buys in equilibrium. We assume that ε_i is the realization of a random variable with cumulative distribution F and continuously differentiable density f. For technical reasons, we assume that F is log-concave. We can interpret ε_{ij} as the *match value* between consumer j and product i; it represents how much consumer j likes the product that firm i sells. To find out, it first has to visit firm i. By assumption, firm i never learns ε_{ij}; otherwise it would be able to price discriminate on the basis of that information. Again, there is sequential search with perfect recall.

Below, we will see that this model does have an equilibrium in pure strategies. For now, assume that all firms charge some price p^*. Suppose that our consumer visits firm i and finds match value ε_{ij}. If she buys from i, she obtains utility $v + \varepsilon_{ij} - p^*$. A visit to some other firm k will give her utility $v + \varepsilon_{kj} - p^*$. This is higher than the utility from buying from firm i if $\varepsilon_{kj} > \varepsilon_{ij}$. The expected benefit from searching once more then equals

$$b\left(\varepsilon_{ij}\right) = \int_{\varepsilon_{ij}}^{\infty} \left(\varepsilon - \varepsilon_{ij}\right) dF\left(\varepsilon\right). \tag{2.4}$$

Note that this is very similar to (2.1). The only difference is that, rather than searching for a price drawn from some distribution F that is low enough, our consumer now searches for a match value drawn from some distribution F that is high enough. Indeed, it is now straightforward to see that this consumer searches until she finds a match value that is at least equal to $\hat{\varepsilon}$, where $b\left(\hat{\varepsilon}\right) = s$. From the analysis earlier, it is also straightforward that such a $\hat{\varepsilon}$ always exists and is unique, provided that s is low enough.

In the more general case, it can be shown that $E(p)$ is increasing in search costs s, but decreasing in the number of shoppers λ. Surprisingly, it is increasing in the number of firms n.[7]

As a final technical note, we still need that s is low enough so consumers will actually search in equilibrium. This requires that their willingness to pay is higher than $\hat{p}$, so $s < 1 - \int_0^1 \frac{dy}{1+n\frac{\lambda}{1-\lambda}(1-y)^{n-1}}$.

[8]Note that, different from the discussion of the Perloff-Salop model in the first chapter, we drop the subscript that refers to the consumer. This simplifies the exposition, Still we do assume that each consumers has their own independent draw at each firm.

2.7.2 The simplest case: 2 firms, uniform distribution

To fix thoughts, let's start with the simple case with only two firms and a uniform distribution of match values. We will first derive the equilibrium behavior of consumers. If all firms charge p^* and the consumer finds ε_i at the first firm, the expected benefit from also visiting the other firm equals

$$b(\varepsilon_i) = \frac{(1-\varepsilon_i)^2}{2}. \tag{2.5}$$

Since visiting one more firm will cost you the search costs s, you will do so whenever $b(\varepsilon_i) > s$, hence if $\varepsilon_i > \hat{\varepsilon}$, with[9]

$$\hat{\varepsilon} = 1 - \sqrt{2s}. \tag{2.6}$$

Note from this expression that if $s = 0$, consumers will continue searching for any value of ε they find at the first firm. The higher s, the lower $\hat{\varepsilon}$, hence the earlier they are going to settle for the product they find at the first firm they visit.

To find the equilibrium price, we proceed as follows. Note that we look for an equilibrium price p^* such that, if the other firm charges this price, it is in the best interest of firm i to charge the exact same price. To find that price, consider what would happen if it would charge some lower price p instead. The consumer expect all firms to charge p^*. Suppose that a consumer visits firm i first and finds price p. If she buys from i, she obtains utility $v + \varepsilon_i - p$. A visit to the other firm will give her utility $v + \varepsilon_j - p^*$. Hence, she will prefer firm j over firm i whenever $\varepsilon_j > \varepsilon_i + \Delta$, with $\Delta \equiv p^* - p$, *If* prices were equal. she would buy from i whenever $\varepsilon_i > \hat{\varepsilon}$. But if firm i now charges a price that is Δ lower, buying from i gives an additional utility of Δ relative to buying from the other firm. Hence she now buys from i whenever $\varepsilon_i + \Delta > \hat{\varepsilon}$, or $\varepsilon_i > \hat{\varepsilon} - \Delta$.

The next step is to find the demand of firm i if it charges p rather than p^*. Essentially, there are 3 ways in which a consumer may end up buying from firm i. First, she may visit firm i first and find a match value there that is high enough, so she buys right away. She visits firms in a random order, so with probability $1/2$ she visits firm i first. If she does, she buys there if $\varepsilon_i > \hat{\varepsilon} - \Delta$, which happens with probability $1 - \hat{\varepsilon} + \Delta$. Hence, the probability that this occurs is

$$\frac{1}{2}\left(1 - \hat{\varepsilon} + \Delta\right). \tag{2.7}$$

Second, our consumer may visit the other firm first, find out that her match value there is too low, then visit firm 1 and find a match value there that is high enough. With probability $1/2$, she visits firm j first. If she does, she finds a match value that is too low with probability $\hat{\varepsilon}$. She then finds a match at firm i that is high enough with probability

[9] Note that the other root is not feasible, as that yields a $\hat{\varepsilon} > 1$.

$1 - \hat{\varepsilon} + \Delta$. Hence, the joint probability that is happens is

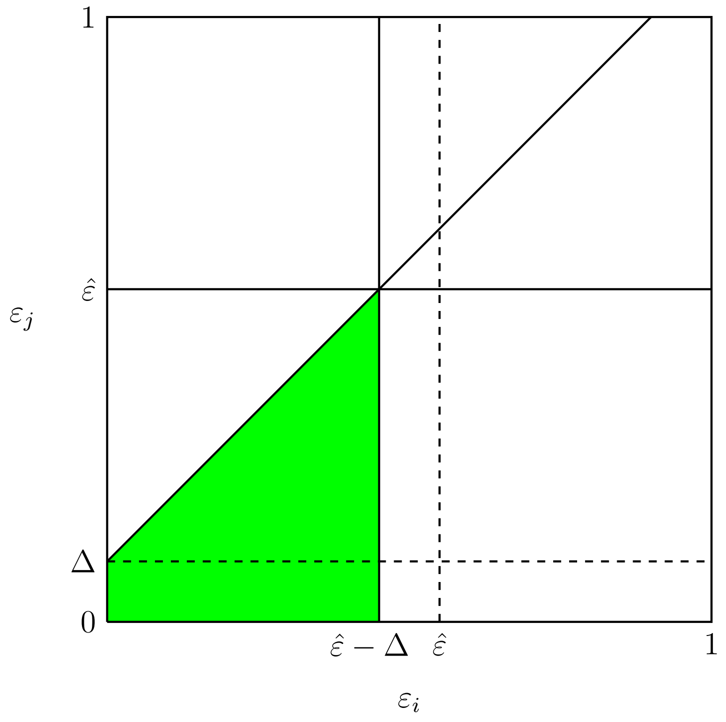
$$\frac{1}{2}\hat{\varepsilon}\left(1 - \hat{\varepsilon} + \Delta\right). \tag{2.8}$$

Third, it could be the case that she visits both firms, but finds a match value at both firms that is too low. Then, she will just go back to the firm that with hindsight offers the best deal (remember that we assume that in equilibrium, the consumer always buys). Note that she buys from firm i in this way whenever all of the following conditions hold:

$$\begin{aligned} \varepsilon_i &< \hat{\varepsilon} - \Delta \\ \varepsilon_j &< \hat{\varepsilon} \\ \varepsilon_i &> \varepsilon_j - \Delta. \end{aligned}$$

The first two conditions together imply that she does not immediately buy from i as well as j: the last condition implies that she prefers i over j and hence ends up buying there. To find out the probability that this happens, it is easiest to make a graph.

Figure 2.1: Visit both, buy from i



In Figure 2.1, we have highlighted the cases in which this applies in $(\varepsilon_i, \varepsilon_j)$-space. The horizontal line in the graph is that for $\varepsilon_j = \hat{\varepsilon}$, the vertical dotted line that for $\varepsilon_i = \hat{\varepsilon}$, the vertical line that for $\varepsilon_i = \hat{\varepsilon} - \Delta$, and the diagonal line that for $\varepsilon_i = \varepsilon - \Delta$. Hence in the green area the inequalities above are all satisfied. The total size of that area is given by the size of the rectangle below $\varepsilon_j = \Delta$ –which is $(\hat{\varepsilon} - \Delta)\Delta$ – plus the size of the remaining triangle – which is $\frac{1}{2}(\hat{\varepsilon} - \Delta)^2$. Hence this case occurs with probability

$$(\hat{\varepsilon}-\Delta)\Delta+\frac{1}{2}(\hat{\varepsilon}-\Delta)^2=\frac{1}{2}\left(\varepsilon^2-\Delta^2\right). \tag{2.9}$$

Therefore, the probability that a consumer buys from firm i is given by the sum of (2.7), (2.8), and (2.9). This yields

$$D_i(p^*,\Delta)=\frac{1}{2}(1-\hat{\varepsilon}+\Delta)+\frac{1}{2}\hat{\varepsilon}(1-\hat{\varepsilon}+\Delta)+\frac{1}{2}\left(\hat{\varepsilon}^2-\Delta^2\right)=\frac{1}{2}+\frac{1}{2}\Delta\left(1+\hat{\varepsilon}-\Delta\right) \tag{2.10}$$

Note that if both firms charge the same price, we have $\Delta=0$, so $D_i=1/2$.

We can now derive equilibrium prices. Total profits of firm i are given by $D_i\cdot(p-c)$. Using the definition of Δ, we thus have

$$D_i(p^*,p)=\frac{1}{2}+\frac{1}{2}(p^*-p)\left(1+\hat{\varepsilon}-p^*+p\right). \tag{2.11}$$

Profits equal $\pi_i = D_i\cdot p$, as we assumed costs to equal zero. Taking the first order condition, we need

$$\frac{\partial D_i}{\partial p}\cdot p+D_i=0.$$

To solve for the equilibrium we impose symmetry. In that case $D_i=1/2$. Moreover,

$$\frac{\partial D_i}{\partial p}=p^*-p-\frac{1}{2}(1+\hat{\varepsilon})$$

With symmetry, this equals $-\frac{1}{2}(1+\varepsilon)$. Plugging this into the first-order condition yields

$$-\frac{1}{2}(1+\hat{\varepsilon})\cdot p^*+\frac{1}{2}=0.$$

Hence the equilibrium price is

$$p^*=\frac{1}{1+\hat{\varepsilon}}.$$

We already derived an expression for $\hat{\epsilon}$ in (2.6). Plugging that into our expression for the equilibrium price yields

$$p^*=\frac{1}{2-\sqrt{2s}}.$$

Hence, when search costs are zero, the price equals 1/2. This is the solution we also found in our analysis of the Perloff-Salop model.[10] As search costs increase, equilibrium prices increase as well. With higher search costs, a consumer that visits a firm is less likely to walk away, hence firms have more market power and hence charge higher prices.

[10]Note that if we would include marginal costs in this model, we would also end up at a price of $p^*=c+1/2$.

2.7.3 Solution of the general model

To find the equilibrium price in the general model, we proceed as follows. Suppose all other firms charge p^* but firm 1 defects to some $p \neq p^*$. In equilibrium, such a defection should not be profitable, and firm 1 should maximize its profits by also charging p^*.

Suppose all others charge p^* but firm 1 charges p. Consumers expect all firms to charge p^*. Suppose a consumer visits firm i first and finds price p. If she buys from i, she obtains $v + \varepsilon_i - p_i$. A visit to some other firm j will give her $v + \varepsilon_j - p^*$. Hence, she will prefer j over i if $\varepsilon_j > \varepsilon_i - \Delta$, with $\Delta \equiv p_i - p^*$. If she does buy from j rather than i, her benefit is $\varepsilon_j - (\varepsilon_i - \Delta)$. Hence, the expected benefit from searching once more now equals

$$b(\varepsilon_i; \Delta) = \int_{\varepsilon_i - \Delta}^{\infty} (\varepsilon - (\varepsilon_i - \Delta)) \, dF(\varepsilon). \tag{2.12}$$

From (2.4) at equal prices, she buys from i whenever $\varepsilon_i > \hat{\varepsilon}$. Comparing (2.4) to (2.12), it is easy to see that she buys from i whenever $\varepsilon_i - \Delta > \hat{\varepsilon}$ or $\varepsilon_i > \hat{\varepsilon} + \Delta$; apart from the regular $\hat{\varepsilon}$, the match value now also has to make up for the price difference.

All consumers visit firms in a random order, so a share $1/n$ visits firm 1 first. They buy there with probability $1 - F(\hat{\varepsilon} + \Delta)$. Another share $1/n$ "plans" to visit firm 1 in second place (in the sense that they only visit firm 1 if they do not sufficiently like the product at the first firm they visit). They first visit some other firm and buy there with probability $1 - F(\hat{\varepsilon})$. Hence the probability that such a consumer buys from firm 1 is $F(\hat{\varepsilon})(1 - F(\hat{\varepsilon} + \Delta))$. Another $1/n$ "plan" to visit firm 1 in third place. They first visit two other firms and end up at firm 1 with probability $F(\hat{\varepsilon})^2$. Hence the probability that such a consumer buys from firm 1 is $F(\hat{\varepsilon})^2(1 - F(\hat{\varepsilon} + \Delta))$. Etcetera.

Some consumers visit all n firms, but find a match value below $\hat{\varepsilon}$ at every one of them. Such a consumer will return to the firm that, with hindsight, offered the highest match value. The total share of consumers that buys from firm 1 through this channel is

$$R_1(p_1, p^*) = \int_{-\infty}^{\hat{\varepsilon} + \Delta} F(\varepsilon - \Delta)^{N-1} \, dF(\varepsilon),$$

where R_1 represents the number of consumers that returns to firm 1. Such a consumer has $\varepsilon_1 < \hat{\varepsilon} + \Delta$, and all other match values smaller than $\varepsilon - \Delta$, hence the integral.

Combining terms, total demand for firm 1 when charging p_1 while others charge p^* is

$$\begin{aligned} D_1(p_1, p^*) &= \frac{1}{n} \sum_{i=1}^{n} F(\hat{\varepsilon})^{i-1} (1 - F(\hat{\varepsilon} + \Delta)) + R_1(p_1, p^*) \\ &= \frac{1}{n} \left[\frac{1 - F(\hat{\varepsilon})^n}{1 - F(\hat{\varepsilon})} \right] (1 - F(\hat{\varepsilon} + \Delta)) + \int_{-\infty}^{\hat{\varepsilon} + \Delta} F(\varepsilon - \Delta)^{n-1} \, dF(\varepsilon) \end{aligned}$$

Profits of firm 1 are given by $\pi_1(p_1, p^*) = p_1 D_1(p_1, p^*)$. Take the first order condition and set $\partial\pi_1(p_1, p^*)/\partial p_1 = 0$. Equilibrium requires that this is satisfied for $p_1 = p^*$. Note

$$\frac{\partial\pi_1(p_1, p^*)}{\partial p_1} = D_1(p_1, p^*) + p_1\frac{\partial D_1(p_1, p^*)}{\partial p_1} = 0. \tag{2.13}$$

Note that

$$\begin{aligned}\frac{\partial D_1(p_1, p^*)}{\partial p_1} &= -\frac{1}{n}\left[\frac{1-F(\hat{\varepsilon})^n}{1-F(\hat{\varepsilon})}\right] f(\hat{\varepsilon}+\Delta) + F(\hat{\varepsilon})^{n-1} f(\hat{\varepsilon}+\Delta) \\ &- (n-1)\int_{-\infty}^{\hat{\varepsilon}+\Delta} F(\varepsilon-\Delta)^{n-2} f^2(\varepsilon)\, d\varepsilon.\end{aligned}$$

Imposing symmetry, we have

$$\begin{aligned}\frac{\partial D_1(p^*, p^*)}{\partial p_1} &= -\frac{1}{n}\left[\frac{1-F(\hat{\varepsilon})^n}{1-F(\hat{\varepsilon})}\right] f(\hat{\varepsilon}) \\ &+ F(\hat{\varepsilon})^{n-1} f(\hat{\varepsilon}) - (n-1)\int_{-\infty}^{\hat{\varepsilon}} F(\varepsilon)^{n-2} f^2(\varepsilon)\, d\varepsilon.\end{aligned}$$

Taking the last two terms together, this implies[11]

$$\frac{\partial D_1(p^*, p^*)}{\partial p_1} = -\frac{1}{n}\left[\frac{1-F(\hat{\varepsilon})^n}{1-F(\hat{\varepsilon})}\right] f(\hat{\varepsilon}) + \int_{-\infty}^{\hat{\varepsilon}} f'(\varepsilon) F(\varepsilon)^{n-1}\, d\varepsilon.$$

Note that firms are symmetric, and in equilibrium all consumers buy. That implies that necessarily $D_1(p^*, p^*) = 1/n$. From (2.13), we then have (again imposing symmetry)

$$p^* = -\frac{D_1(p^*, p^*)}{\partial D_1(p^*, p^*)/\partial p_1} = \frac{1}{\frac{1-F(\hat{\varepsilon})^n}{1-F(\hat{\varepsilon})} f(\hat{\varepsilon}) - n\int_{-\infty}^{\hat{\varepsilon}} f'(\varepsilon) F(\varepsilon)^{n-1}\, d\varepsilon}.$$

For simplicity, let's assume that F is uniform on $[0,1]$. In that case

$$D_1(p_1, p^*) = \frac{1}{n}\left[\frac{1-\hat{\varepsilon}^n}{1-\hat{\varepsilon}}\right](1-\hat{\varepsilon}-\Delta) + \int_{-\infty}^{\hat{\varepsilon}+\Delta} (\varepsilon-\Delta)^{n-1}\, d\varepsilon,$$

so

$$\frac{\partial D_1(p^*, p^*)}{\partial p_1} = -\frac{1}{n}\left[\frac{1-\hat{\varepsilon}^n}{1-\hat{\varepsilon}}\right].$$

Note that this greatly simplifies the analysis, as the derivative of the number of returning

[11] Note that, using integration by parts,

$$\begin{aligned}\int_{-\infty}^{\hat{\varepsilon}} f'(\varepsilon) F(\varepsilon)^{n-1}\, d\varepsilon &= f(\varepsilon) F(\varepsilon)^{n-1} - \int f(\varepsilon)\cdot(n-1) F(\varepsilon)^{n-2} f(\varepsilon)\, d\varepsilon \\ &= f(\varepsilon) F(\varepsilon)^{n-1} - (n-1)\int F(\varepsilon)^{n-2} f^2(\varepsilon)\, d\varepsilon\end{aligned}$$

which implies the result.

consumers with respect to p_1, that is, $\partial R_1(p_1, p^*)/\partial p_1$, is equal to zero. The equilibrium price now equals

$$p^* = \frac{1-\hat{\varepsilon}}{1-\hat{\varepsilon}^n}.$$

From (2.4),

$$b(\hat{\varepsilon}) = \int_{\hat{\varepsilon}}^{1} (\varepsilon - \hat{\varepsilon})\, d\varepsilon = \frac{1}{2}(1-\hat{\varepsilon})^2$$

hence

$$\hat{\varepsilon} = 1 - \sqrt{2s},$$

which implies

$$p^* = \frac{\sqrt{2s}}{1-\left(1-\sqrt{2s}\right)^n}.$$

Using this specification, it is straightforward to do comparative statics. First suppose the number of firms increases. From Weitzman (1979), we know that this does not affect $\hat{\varepsilon}$. Hence

$$\frac{\partial p^*}{\partial n} = \frac{\partial}{\partial n}\left(\frac{1-\hat{\varepsilon}}{1-\hat{\varepsilon}^n}\right) = \hat{\varepsilon}^n (\ln \hat{\varepsilon}) \frac{1-\hat{\varepsilon}}{(1-\hat{\varepsilon}^n)^2} < 0,$$

as $\ln \hat{\varepsilon} < 0$. Hence, *having more firms leads to lower prices.* We can also evaluate the effect of an increase in search costs on the equilibrium price. First note that $\hat{\varepsilon}$ is defined by $b(\hat{\varepsilon}) = s$. With $b(\varepsilon)$ decreasing in ε, we unambiguously have that an increase in search costs s leads to a lower $\hat{\varepsilon}$. This is intuitive: if search costs are higher, a consumer is willing to settle for a lower match value. Now consider

$$\frac{\partial p^*}{\partial \hat{\varepsilon}} = \frac{\partial}{\partial \hat{\varepsilon}}\left(\frac{1-\hat{\varepsilon}}{1-\hat{\varepsilon}^n}\right) = \frac{(1-\hat{\varepsilon})\, n\hat{\varepsilon}^{n-1} - (1-\hat{\varepsilon}^n)}{(1-\hat{\varepsilon}^n)^2} = \frac{\hat{\varepsilon}^{n-1}\left(n(1-\hat{\varepsilon}) + \hat{\varepsilon}\right) - 1}{(1-\hat{\varepsilon}^n)^2} < 0.$$

To see the inequality note that the numerator is increasing in $\hat{\varepsilon}$ as

$$\frac{\partial\left(\hat{\varepsilon}^{n-1}\left(n(1-\hat{\varepsilon}) + \hat{\varepsilon}\right)\right)}{\partial \hat{\varepsilon}} = n\hat{\varepsilon}^{n-2}(1-\hat{\varepsilon})(n-1),$$

hence it is maximized for $\hat{\varepsilon} = 1$, when it equals $\hat{\varepsilon}^n - 1 < 0$.

Hence, *equilibrium prices are increasing in search costs*; as search costs increase, firms have more market power over the consumers that do visit them, which implies that they can set higher prices.[12]

Exercises

1. Consider Varian's model of sales as described in Section 2.3. Suppose the number of firms is $n > 2$. Derive the mixed strategy equilibrium.

[12] It can be shown that the comparative statics with respect to n and s also hold in the general model, see Anderson and Renault (1999) for details.

2. Consider the following variation of Varian (1980)'s model of sales. A duopoly competes in prices. Costs of production are zero. There is a mass 1 of consumers that is willing to pay at most 2, and a mass 1 of consumers that is willing to pay at most 3. For both groups there is a fraction λ that is informed and a fraction $1-\lambda$ that is uninformed.

   (a) Derive the best-reply function of each firm.

   (b) Derive the Nash equilibrium in prices.

3. Consider a model of search with differentiated products as analyzed in this chapter. There are two firms that each have zero costs. Match values are uniformly distributed on $[0,1]$. A share λ of consumers have search costs $s = 0.25$. The other $1-\lambda$ have search costs $s = 0.16$. Derive the equilibrium in prices.

4. * In the model of search with differentiated products described in these notes, we assumed that consumers pick a firm at random to visit first. Let us now assume that that is no longer the case, and can advertise to attract consumers. More precisely, if firm 1 puts out a_1 ads and firm 2 puts out a_2, then the probability that firm 1 will be visited first is given by $a_1/(a_1+a_2)$. The cost of each ad are $1/4$,

   Suppose that match values are uniformly distributed on [0,1], and that firms set prices and advertising levels simultaneously. Derive equilibrium profits as a function of search costs s.

   (WARNING: this is tricky, so here is a hint. Proceed as follows

   (a) Write demand in this case along the lines of (2.10).

   (b) Take derivative of profits with respect to p_i and a_i.

   (c) Impose symmetry.

   (d) Solve for equilibrium values of p and a.

   (e) Plug back into profits.

   (f) Plot the resulting profits as a function of s using your favourite software.

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Chapter 3

Advertising

3.1 Introduction

Whenever a consumer is interested in buying a particular product but does not know where to find it and at what price, it has a problem. Yet, the consumer can solve that problem by actively searching for a firm that offers that product, as we analyzed in the previous chapters. But, of course, if a firm sells a particular product but consumers are not aware of that, the firm faces a problem as well. It can solve that problem by advertising its product. It is such informative advertising that we analyze in this chapter.

This chapter thus focuses on *informative advertising*: advertising with the sole function to inform consumers that a product is available and, possibly, at what price. Advertising can have many other roles as well. For example, *persuasive advertising* tries to convince a consumer of the usefulness of a particular product, and hence tries to increase their willingness-to-pay for that product. An extensive survey of the IO literature on advertising can be found in Bagwell (2007).

3.2 The Butters Model

3.2.1 Set-up

This model is due to Butters (1977) Firms produce a homogeneous product at unit cost c. There is free entry of firms and there are N consumers that can only learn a firm's existence and price by receiving an ad from that firm. Each firm decides how many ads to send, and which price to put in those ads. Ads are randomly distributed among consumers at a cost of k per ad. Consumers have unit demand an are willing to pay at most R. We assume that $R > c + k$. we are interested in, first, the equilibrium of this model in terms of prices and advertising levels. But, perhaps even more importantly, we are interested in whether the market outcome provides too much or too less advertising from a welfare perspective.

3.2.2 Analysis

Suppose that A ads are sent. There will be three kinds of consumers. Some consumers receive no ads and are uninformed. Other consumers only receive ads from 1 firm. They are captive. Still other consumers receive ads from at least 2 firms. These consumers are selective. These consumers buy from the firm that offers the lowest price, provided it is smaller than or equal to R.

Let Φ denote the probability that a consumer receives at least one ad. The probability that a consumer is uninformed then equals $1 - \Phi = (1 - 1/N)^A$. If N is large enough, this is approximately equal to $e^{-A/N}$. Thus, if a proportion Φ of consumers are to receive at least one ad, then the number of ads that has to be sent equals

$$A(\Phi) = N \cdot \ln \left[\frac{1}{1 - \Phi} \right]. \tag{3.1}$$

A pure strategy equilibrium in prices does not exist. This can be seen as follows. If all other firms charge some $p \in (k + c, R]$, then this firm has an incentive to slightly undercut this price. If all other firms charge $p = k + c$ however, then this firm has an incentive to charge $p = R$ and sell to its captive consumers. The equilibrium has firms mixing on $[k + c, R]$. For any price below $k + c$, a firm would be better off not sending any ads.

There is free entry of firms, which implies that equilibrium profits are zero, as in our analysis of the Salop circle in the first chapter. Denote by $x(P)$ the probability that an ad with price P will be accepted by the consumer receiving it. Then $x(P)$ is the probability that a consumer does not receive an ad with a lower price. This is decreasing in P. Equilibrium requires that, for each $P \in [k + c, R]$, we have

$$(P - c)\, x(P) - k = 0.$$

This implies $x(c + k) = 1$ and $x(R) = k/(R - c)$. But in equilibrium we also need that the probability that a consumer will accept price R exactly equals the probability that he does not receive any other ad, so $x(R) = 1 - \Phi$. Equilibrium thus requires:

$$\Phi^* = 1 - k/(R - c).$$

For the purposes of the analysis it is not necessary to explicitly derive what the equilibrium price distribution looks like: it involves a mass point. What we have derived so far is already sufficient to derive the required result on social welfare.

3.2.3 Social optimum

Now consider the amount of advertising that a social planner would choose. Prices are just transfers between consumers and firms, so they will not affect total welfare. When one additional consumer that was initially uninformed learns about the existence of some firm, the social benefit is $R - c$. Thus, social benefits are $N\Phi(R - c)$. However, there is also a cost of reaching these consumers, which is given by (3.1). The social planner thus maximizes

$$\max_{\Phi} \left\{ \Phi N \left(R - c\right) - kN \cdot \ln \left[\frac{1}{1 - \Phi} \right] \right\}.$$

Taking the FOC:

$$N(R - c) - \frac{kN}{1 - \Phi} = 0$$

Solving for Φ yields

$$\Phi^S = 1 - k/(R - c).$$

But this implies that we have $\Phi^S = \Phi^*$. Thus, the market provides the socially optimal level of advertising.

This can be understood as follows. Consider the private benefit to a firm of sending an ad at the price R. This benefit equals $(R - c)$ times the probability that the consumer receives no other ad. But this is also the social benefit of sending an ad, since the ad increases social surplus by $R - c$ but only if no other ads are received by the consumer. Thus, the highest-priced firm appropriates all consumer surplus and steals no business from rivals. Therefore, it advertises at the socially optimal rate. Now consider an ad at a price lower than R. The private benefits to a firm of sending such an ad are equal to those of sending an ad at price $P = R$, as expected profits are always zero. This implies that social benefits also equal private benefits at any price below R.

3.3 Informative advertising, differentiated products

Grossman and Shapiro (1984) extend the Butters model by allowing for differentiated products. In the original paper, firms are distributed on a circle. For simplicity, we consider a Hotelling line. This discussion follows that in Tirole (1988).

Consumers are uniformly distributed on a line of unit length. Each has unit demand and is willing to pay at most R, but also faces transportation costs of t per unit of distance. Two firms are located at 0 and 1 respectively. As in Butters, ads are sent randomly. The cost of reaching a fraction Φ_i is denoted $A(\Phi_i)$. Grossman and Shapiro allow for general advertising technologies, of which Butters is a special case. For simplicity, suppose that $A(\Phi_i) = a(\Phi_i)^2/2$, with $a > t/2$.

Again, there are three kinds of consumers. Suppose that firms 1 and 2 inform fractions Φ_1 and Φ_2 of consumers respectively. A fraction $[1 - \Phi_1]\,[1 - \Phi_2]$ of consumers are uninformed. A fraction $\Phi_1 [1 - \Phi_2]$ receive only firm 1's ads and are captive to firm 1. A fraction $[1 - \Phi_1] \Phi_2$ are captive to firm 2. A fraction $\Phi_1 \Phi_2$ is selective. Suppose that the market is always covered, in the sense that a consumer who has received at least one ad will always buy. Also assume that the number of selective consumers is sufficiently large such that firms are willing to compete for them. This is true if advertising is not too costly. Firm 1's demand function can be written

$$D_1(P_1, P_2, \Phi_1, \Phi_2) = \Phi_1 \left[(1 - \Phi_2) + \Phi_2 \left(P_2 - P_1 + t\right)/2t\right].$$

Consider a game in which firms simultaneously choose price and advertising levels. Profits of firm 1 equal

$$\pi_1 = (P_1 - c)\, D_1 - A(\Phi_1).$$

or

$$\pi_1 = (P_1 - c)\, \Phi_1 \left[(1 - \Phi_2) + \Phi_2 \frac{P_2 - P_1 + t}{2t}\right] - a(\Phi_1)^2/2.$$

Taking the derivative with respect to P_1 yields

$$\frac{\partial \pi_1}{\partial P_1} = \Phi_1 \left[(1 - \Phi_2) + \Phi_2 \frac{P_2 - 2P_1 + c + t}{2t}\right] = 0,$$

which yields reaction function

$$P_1 = \frac{P_2 + t + c}{2} + \frac{1 - \Phi_2}{\Phi_2} t. \tag{3.2}$$

Note that the first term is exactly the reaction function in a regular Hotelling model. The second term is the additional markup that firm 1 is able to set due to the fact that not every consumer is informed about firm 2. Note that this term is decreasing in Φ_2 : the more consumers are informed about firm 2, the lower the price that firm 1 will charge in equilibrium. Taking the derivative with respect to Φ_1 and putting it equal to zero yields

$$a\Phi_1 = (P_1 - c) \left[(1 - \Phi_2) + \Phi_2 \frac{(P_2 - P_1 + t)}{2t}\right]. \tag{3.3}$$

The left-hand side gives the marginal costs of having more informed consumers. The right-hand side is the marginal benefit of having more informed consumers: the price-cost margin times the probability of a sale. The equilibrium can be found by imposing symmetry and solving (3.2) and (3.3) simultaneously. From (3.2), we have

$$P^* = c + t \frac{2 - \Phi^*}{\Phi^*}.$$

From (3.3), we then have

$$a\Phi = t\frac{2-\Phi}{\Phi}\left((1-\Phi)+\frac{\Phi}{2}\right).$$

or

$$a\Phi^2 = 2t\left(1-\frac{1}{2}\Phi\right)^2$$

which yields

$$\sqrt{a}\Phi = \sqrt{2t}\left(1-\frac{1}{2}\Phi\right)$$

hence

$$\Phi^* = \frac{2}{1+\sqrt{2a/t}},$$

which implies

$$P^* = c + \sqrt{2at}.$$

Equilibrium profits are

$$\Pi^* = \frac{2a}{\left(1+\sqrt{2a/t}\right)^2}.$$

First of all, note that the equilibrium price is higher than in the case without advertising. The equilibrium advertising level is higher when advertising is less costly, and when products are more differentiated (that is, when t is higher). Most surprisingly, equilibrium profits are increasing in the cost of advertising. When a increases, this implies an increase in costs, but will also lower the equilibrium level of advertising. In this model, the net effect is positive.

Note that, from a welfare point of view, prices are again just transfers from consumers to firms. Consumers that are informed about the existence of both firms will face transportation costs $t/4$ on average. Those that only know the existence of one firm will face transportation costs $t/2$ on average. Hence, social welfare is given by

$$SW(\Phi) = \Phi^2\left(v-c-t/4\right) + 2\Phi(1-\Phi)\left(v-c-t/2\right) - a\Phi^2.$$

Maximizing this with respect to Φ yields

$$\Phi^S = \frac{2\left(v-c\right)-t}{2\left(v-c\right)+2a-3t/2}.$$

This implies that the market equilibrium can either have too much or too little advertising, depending on the exact parameters of the model.

Exercises

1. (a) Consider the Grossman/Shapiro model as described in the lecture notes, but with $t = 1$ and $c = 0$. However, different from the model in the notes, half of consumers that are not informed by the ads of firm 1, still learn about product 1 through word-of-mouth. Something similar holds for firm 2. Write down the profit functions for this case, and derive the system of equations that pin down equilibrium prices and equilibrium numbers of informed consumers (hence **do not** explicitly solve for those values; that is too much hassle. But do write equilibrium prices in terms of equilibrium fractions of informed consumers and vice versa)

   (b) Suppose we would do the same in a Butters model (that is, assume that half of consumers that are not informed by the ads of firm 1, still learn about product 1 through word-of-mouth). Argue whether the result that the amount of ads are socially optimal would still hold (hence **do not** derive, only give an intuitive argument).

2. Solve the Grossman-Shapiro model in the case that preferences of consumers are given by the Perloff-Salop model described in the first chapter, rather than the Hotelling model. (Do **not** solve for the welfare optimum: that becomes too complicated.)

Chapter 4

Menu Pricing

4.1 Introduction

We now move to an entirely different topic. So far, we have assumed that a firm only charges a single price for its product. Yet, examples abound in which the same economic good is sold at different prices to different consumers by the same firm. Such examples may be viewed as attempts by the producer to capture a higher fraction of consumer surplus than he would if he charged a uniform price. The producer *price discriminates* when two units of the same physical good are sold at different prices, either to the same consumer or to different consumers. There is no price discrimination if differences in prices between consumers exactly reflect differences in the costs of serving these consumers.

Classic examples of price discrimination include railroad pricing, student discounts, and senior discounts. Railways price discriminate in many dimensions; prices are lower for holders of a discount card, and prices also differ between first and second class carriages. Admittedly, part of that price difference may be due to a difference in costs, but it is hard to believe that that is the entire story. Another example in railways are the lower prices that are paid by senior citizens. But also in many technology markets, the use of price discrimination is rampant. In such markets, marginal costs are often close to zero, which gives firms an incentive to sell their product to as many consumers as possible, yet, without losing the revenues from consumers that have a high willingness-to-pay. Software firms often use intricate pricing schemes to sell their software. The price a consumer has to pay often depends on how he uses the product, for what purpose, etc. The same is true for mobile telephony; many mobile phone operators offer a menu of possible calling plans, which differ in subscription costs, the marginal price of a phone call, the exact times when peak and off-peak rates are valid, etc. On the internet, a lot of information is sold using price discrimination. Here, the more recent information is, the higher the price one has to pay. For example, stock quotes are often freely available with a 15 minute delay, but only available in real-time at a non-zero price. Airlines also often use price discrimination, usually referring to it as yield management.

The analysis of price discrimination dates back to Dupuit (1849). Following Pigou (1920) we distinguish between three types of price discrimination. With *first-degree* or perfect price discrimination, the monopolist captures the entire consumer surplus. Of course, this will often be just a theoretical construct; in practice, a monopolist will never be able to charge consumers the exact price they are willing to pay for a product, simply because it does not have this information, and consumers have no incentive to reveal it. With *second-degree* price discrimination, the monopolist offers different bundles, and lets the consumers self-select. The idea is that bundles are designed in such a manner that each group of consumers voluntarily choose the bundle that is designed with that group in mind. An example is the use of first class and second class in trains. Although the railroad company is not able to determine who has a high, and who has a low willingness to pay, it can let the consumers self-select by offering two bundles; one bundle consisting of a first-class seat, the other consisting of a second-class seat. With *third-degree* price discrimination, the monopolist can directly distinguish between different submarkets, and can set different prices on each of them. For example, a railroad can charge senior citizens a different price from non-senior citizens. The fact that one is a senior citizen is a characteristic that is observable and verifiable.

We will restrict attention to second degree price discrimination, nowadays also known as menu pricing. Section 4.2 is primarily based on citetsalanie97. Section 4.3 follows Deneckere and McAfee (1996).

4.2 Menu pricing

With menu pricing, a monopolist wants to price discriminate, but there is no exogenous signal of each consumer's demand function. In other words: the monopolist is not able to divide the consumers into groups purely on the basis of some exogenous variable such as age. Some consumers have a higher elasticity of demand than others, but the monopolist is not able to tell which is which. What the monopolist can now do is to offer a menu of bundles where consumers can choose from. One bundle is designed especially for high-demand consumers, a different bundle is designed especially for low-demand consumers. The monopolist must however take into account the possibility of personal arbitrage. In other words, a high-demand consumer should not prefer the bundle designed for the low-demand consumer over the bundle designed for the high-demand consumer. This introduces ”self-selection” or ”incentive-compatibility” constraints, as we will see below.

4.2.1 The Model

Assume that consumers have a surplus of consuming quality q of a good that equals $U(\theta, q)$, where θ is a parameter that differs per consumer. The cost of providing quality q is $c(q)$. We make the following assumptions:

1. $U(\theta, 0) = 0$. Hence, if you don't consumer anything, you have utility 0.

2. $\frac{\partial U}{\partial q} > 0$. Hence, all consumers obtain a higher utility if they consume more.

3. $\frac{\partial^2 U}{\partial q^2} < 0$. Hence, although utility increases in θ, it does so at a decreasing rate.

4. $c' \geq 0$. Hence costs are non-decreasing in quality.

5. $c'' \geq 0$. Hence marginal costs are non-decreasing in quality.

We assume that there are two types of consumers. They have taste parameters θ_1, θ_2 with $\theta_1 < \theta_2$. The proportion of type-1 consumers is λ. Hence, the proportion of type-2 consumers is $1 - \lambda$. The costs of providing one unit of a product with quality q is given by $c(q)$, with $c' > 0$ and $c'' \geq 0$.

Suppose the monopolist offers two packages. One is denoted (p_1, q_1): this package has quality q_1, and the monopolist sells it at a price p_1. The other package has quality q_2 and sells for p_2. Hence, we denote it as (p_2, q_2). We assume that the following single-crossing condition is satisfied:

$$U(\theta_2, q_2) - U(\theta_2, q_1) \geq U(\theta_1, q_2) - U(\theta_1, q_1)$$

if $q_2 > q_1$. Note that this implies that a type 2 consumer is always willing to pay more to get quality q_2 rather than quality q_1 than a type 1 consumer is. In other words, a high type is always willing to pay more for any upgrade than a low type is.

The monopolist now designs the packages such that a type-1 consumer prefers to consume (p_1, q_1), and the type-2 prefers to consume (p_2, q_2). Given these constraints, the monopolist designs these packages such that its profits are maximized. Its problem is thus the following

$$\max_{p_1, q_1, p_2, q_2} \lambda (p_1 - c(q_1)) + (1 - \lambda)(p_2 - c(q_2)) \tag{4.1}$$

$$\text{s.t.} \quad \begin{cases} U(\theta_1, q_1) - p_1 \geq U(\theta_1, q_2) - p_2 & \text{(IC-1)} \\ U(\theta_2, q_2) - p_2 \geq U(\theta_2, q_1) - p_1 & \text{(IC-2)} \\ U(\theta_1, q_1) - p_1 \geq 0 & \text{(IR-1)} \\ U(\theta_2, q_2) - p_2 \geq 0 & \text{(IR-2)} \end{cases}$$

Condition (IC-1) requires that the net utility that a type 1 obtains from consuming (p_1, q_1) is at least as high as the net utility that he obtains from consuming (p_2, q_2). In other words, consumer 1 prefers the package designed for him, over the package designed for a type-2 consumer. Similarly, (IC-2) requires that a type-2 consumer prefers (p_2, q_2) over (p_1, q_1). These conditions are referred to as *incentive constraints*. Condition (IR-1) is the *individual rationality constraint* for a type 1. Often, it also referred to as his *participation constraint*. It requires that consuming package (T_1, q_1) yields non-negative utility to a type 1. If this condition were not satisfied, a type 1 would simply refrain from buying this product. The individual rationality constraint of a type 2 is given by (IR-2).

4.2.2 Benchmark: complete information

First consider the case in which the monopolist knows the types of the consumers he faces. Hence, he does not have to worry about a type 1 choosing bundle 2 and vice-versa, simply because a type 1 consumer only has the possibility to choose a type 1 bundle, and a type 2 only can choose bundle 2. Technically speaking, this implies that the incentive constraints do not have to be satisfied.

The monopolist will now design bundle i to maximize

$$\begin{aligned}
&\max_{p_i, q_i} p_i - c(q_i) \\
&\text{s.t. } U(\theta_i, q_i) - p_i \geq 0
\end{aligned}$$

It is obvious that, in the profit-maximizing solution, the constraint will be binding. Otherwise, the monopolist can simply increase p_i, which increases his profits. The problem then becomes to

$$\max_{q_i} U(\theta_i, q_i) - c(q_i), \tag{4.2}$$

which implies that the optimal solution is implicitly given by

$$\begin{aligned}
U_2(\theta_i, q_i) &= c'(q_i) \\
p_i &= U(\theta_i, q_i).
\end{aligned}$$

Note that this implies that the consumer is indifferent between buying the bundle and not buying it: his surplus is zero.

The situation is depicted in figure 4.2.2. We have drawn utility of both types of consumers, U_1 and U_2, as a function of the quantity consumed q. We have also drawn the total cost function of the monopolist $C(q) = cq$, thus assuming that marginal costs are constant. For a given bundle q, the monopolist always maximizes its profit by charging an amount that equals a consumer's willingness to pay for that bundle, so $p_i = U_i(q) \equiv U(\theta_i, q_i)$. The profit-maximizing bundle to offer to type i consumers then is the one where

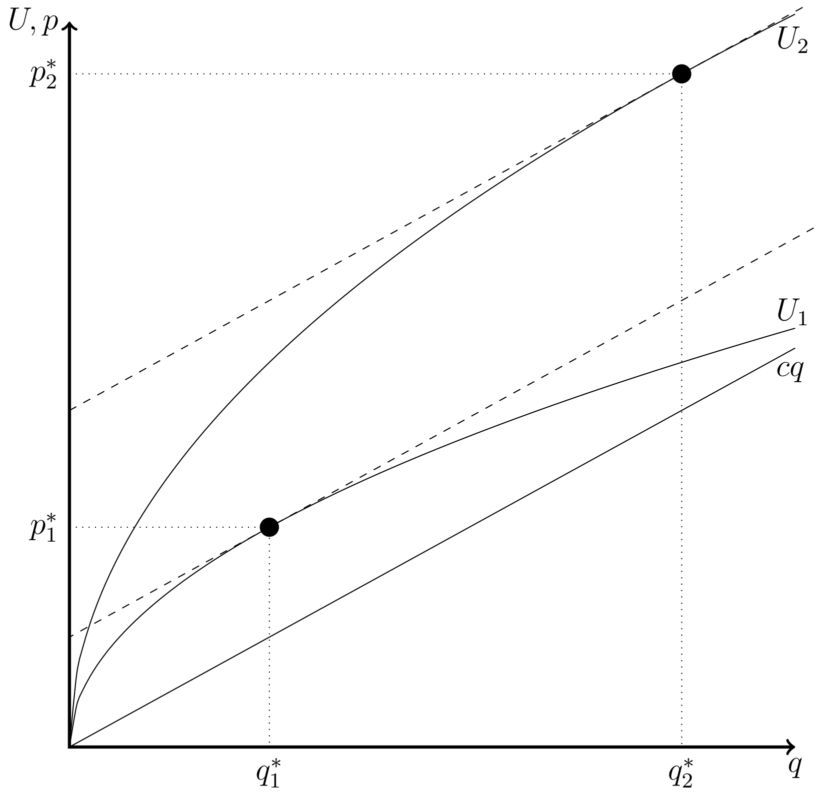

Figure 4.1: Second-degree price discrimination, complete information

the difference between total revenue and total costs from a particular consumer is the highest, i.e. where the slope of the consumer's utility function equals the slope of the firm's total cost function, which is the case in q_1^* and q_2^*.

4.2.3 Social optimum

Now consider social welfare. As always, social welfare equals the firm's profits plus the consumer surplus. For a consumer of type i, this implies

$$SW_i = (p - c(q_i)) + (U(\theta_i, q_i) - p) = U(\theta_i, q_i) - c(q_i).$$

But this is exactly (4.2). Hence, the monopoly will maximize social welfare. The intuition is the same as that for first-degree price discrimination. By setting an "entrance fee" p, the monopoly is able to capture the entire surplus, which induces him to choose the socially optimal output.

4.2.4 Solving the model with incomplete information

We now solve for (4.1), the model with incomplete information. Above, we assumed that the monopolist can observe the type of a consumer, and can offer that consumer only one package; the package that maximizes the monopolist's profits from that consumer. Now we assume that the monopolist cannot observe the type of a consumer. The only thing it can now do is offer two bundles to each consumer, and hope that each consumer will pick the 'right' bundle.

Note that we have

$$U(\theta_2, q_2) - p_2 \geq U(\theta_2, q_1) - p_1 > U(\theta_1, q_1) - p_1. \tag{4.3}$$

The first inequality is simply (IC-2), the second inequality then follows from $\theta_2 > \theta_1$. This implies that if $U(\theta_1, q_1) - p_1 > 0$, we also have $U(\theta_2, q_2) - p_2 > 0$. Now suppose that (IR-1) is not binding. Then the chain of inequalities above implies that (IR-2) is also not binding. But if that is the case, the monopolist can increase both p_1 and p_2 by some small enough ε, without violating either (IR-1) or (IR-2). It is easy to see that this would not affect (IC-1) or (IC-2) either. Since increasing p_1 and p_2 does increase profits, we have established that this situation cannot be profit-maximizing. Hence we must have

Lemma 1 *At the profit-maximizing solution, (IR-1) is binding.*

Given that (IR-1) is binding, we have from (4.3) that $U(\theta_2, q_2) - p_2 > 0$. Thus

Lemma 2 *If (IR-1) is binding, then (IR-2) is strictly satisfied.*

Combined with Lemma 1, this implies that we can delete (IR-2) from our list of restrictions.

Next, for the sake of argument, suppose that (IC-2) is not binding. We then have

$$U(\theta_2, q_2) - p_2 > U(\theta_2, q_1) - p_1 > U(\theta_1, q_1) - p_1 = 0, \tag{4.4}$$

where the second inequality again follows from $\theta_2 > \theta_1$, and the equality from Lemma 1. Note that this implies that increasing p_2 by some small ε does not violate (IR-2). Obviously, it does not violate the other three conditions either. Hence, this situation cannot be profit-maximizing.

Lemma 3 *At the profit-maximizing solution, (IC-2) is binding.*

For the next step, add (IC-1) and (IC-2) to find

$$U(\theta_1, q_1) + U(\theta_2, q_2) \geq U(\theta_1, q_2) + U(\theta_2, q_1)$$

or

$$U(\theta_2, q_2) - U(\theta_2, q_1) \geq U(\theta_1, q_2) - U(\theta_1, q_1)$$

This is exactly the single-crossing condition, provided that $q_2 \geq q_1$. Thus

Lemma 4 *At the profit-maximizing solution, $q_2 \geq q_1$.*

Hence, the optimal package designed for type 2 has at least as many products as the one designed for type 1. Next, note that Lemma 3 implies

$$p_2 - p_1 = U(\theta_2, q_2) - U(\theta_2, q_1) > U(\theta_1, q_2) - U(\theta_1, q_1),$$

using $\theta_2 > \theta_1$, Lemma 4 and the single-crossing condition. This implies

$$U(\theta_1, q_2) - p_2 < U(\theta_1, q_1) - p_1.$$

Hence

Lemma 5 *If (IC-2) is binding, then (IC-1) is strictly satisfied.*

Combined with Lemma 3, this implies that we can delete (IC-1) from our list of restrictions. Combining all results, we have that we can write the monopolist's problem as

$$\max_{q_1, q_2} \lambda (p_1 - c(q_1)) + (1 - \lambda)(p_2 - c(q_2))$$
$$\text{s.t.} \quad \begin{cases} p_2 = U(\theta_2, q_2) - U(\theta_2, q_1) + p_1 \\ p_1 = U(\theta_1, q_1) \end{cases}$$

or

$$\begin{aligned} \max_{q_1, q_2} \Pi(q_1, q_2) &= \lambda (U(\theta_1, q_1) - c(q_1)) + \\ &\quad (1 - \lambda)(U(\theta_2, q_2) - U(\theta_2, q_1) + U(\theta_1, q_1) - c(q_2)). \end{aligned}$$

Taking first-order conditions yields

$$\begin{aligned} \frac{\partial \Pi}{\partial q_1} &= U_2(\theta_1, q_1) - (1 - \lambda) U_2(\theta_2, q_1) - \lambda c'(q_1) = 0, \\ \frac{\partial \Pi}{\partial q_2} &= (1 - \lambda)(U_2(\theta_2, q_2) - c'(q_2)) = 0. \end{aligned}$$

or

$$U_2(\theta_1, q_1) = \lambda c'(q_1) + (1 - \lambda) U_2(\theta_2, q_1) \tag{4.5}$$
$$U_2(\theta_2, q_2) = c'(q_2). \tag{4.6}$$

Note that $q_2 > q_1$.Hence, as $U_{22}(\theta_2, q) < 0$, we have $U_2(\theta_2, q_1) > U_2(\theta_2, q_2) = c'(q_2) \geq c'(q_1)$. This implies that $\lambda c'(q_1) + (1 - \lambda) U_2(\theta_2, q_1) > c'(q_1)$, so the optimal solution has $U_2(\theta_1, q_1) > c'(q_1)$.

There are several things to note here:

1. Note that (4.6) implies that the type 2 consumes exactly the amount that he would also choose to consume would he face a price c. Hence, the type 2 consumes the amount that is socially optimal. This is a general result in these types of models: there is *no distortion at the top.*

2. Also note from Lemma 2 that the type 2 gets a strictly positive surplus: with $U(\theta_2, q_2) > p_2$, a type 2 pays less for package 2 than he would be willing to pay. The difference is type 2's *informational rent*: if the monopolist would know his type, he would not get any surplus. Hence, the surplus is due to type 2 having private information regarding his type.

3. From Lemma 1, we have that a type 1 pays exactly the amount he is willing to pay for q_1 units of the product. Hence, the lowest type does not obtain any utility from consuming the good: there is *no surplus at the bottom.*

4. Under the assumption that, in equilibrium, both types are served (which was made throughout the entire analysis), we have that $U_2(\theta_1, q_1) > c'(q_1)$. With $U_{22}(\theta_1, q_1) < 0$, this implies that type 1 consumes *less* than his socially optimal amount. In fact, from Lemma 3, we have that the package (p_1, q_1) is designed such that a type 2 is exactly indifferent between consuming (p_1, q_1) and (p_2, q_2). This is also a general result: in a model with more types, we always have that a type is indifferent between consuming his own package and the package designed for the next-highest type. In a sense, this also holds for the lowest type: he is indifferent between consuming the package designed for him, and consuming nothing.

5. Note that the rhs of (4.5) is decreasing in λ, again conditional on both types being served. This implies that as λ, the fraction of types 1 in the population, increases, the package designed for type 1 moves in the direction of his socially optimal package. Also, as θ_2 and θ_1 move closer together, the same is true.

6. Finally, note that throughout the analysis we assume that the monopolist finds it profitable to offer two different packages to the two different types of consumers. We solved the model under that restriction. However, that may not always be the case. Depending on the parameters, the monopolist may find it more profitable to only offer one package. For example, if there are relative few low types, then a monopolist may find it more profitable to only offer a package aimed at the high type, and not selling anything to the low types. In that case, it can charge a higher price to the high types, but has to give up any profits it was making on the low types.

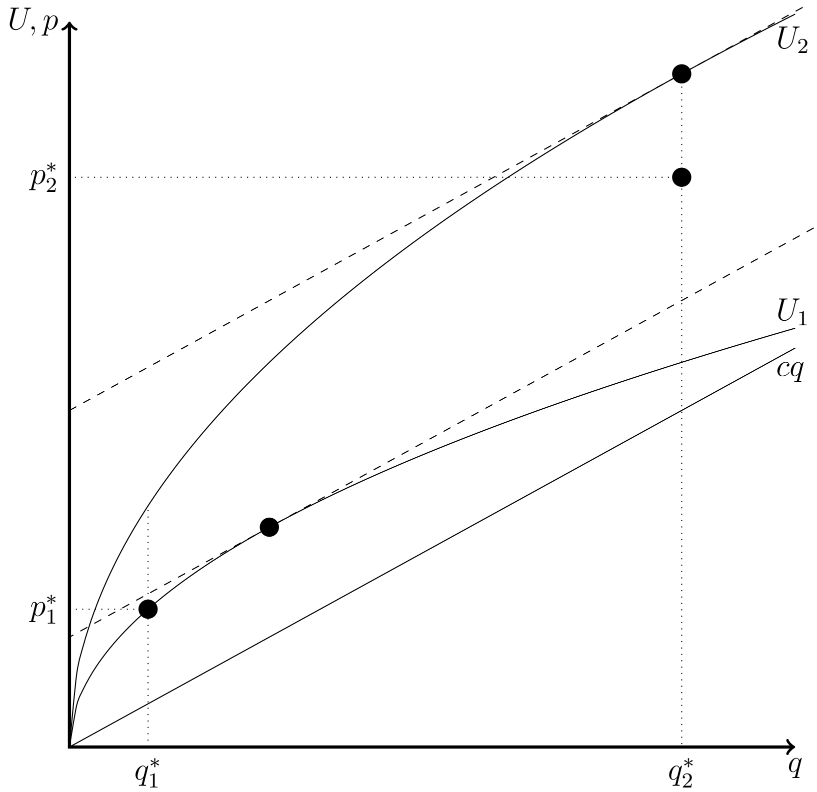

Figure 4.2: Second-degree price discrimination, incomplete information

The equilibrium we derived is depicted in figure 4.2. In the figure, we have also depicted the complete information solution. If we would implement that in the incomplete information case, then obviously the incentive compatibility constraint of the high type would not be satisfied: he would prefer consuming bundle q_1 and obtain utility $U(\theta_2, q_1) - p_1 > 0$ rather than consuming bundle q_2 and obtain utility $U(\theta_2, q_2) - p_2 = 0$.

To increase its profits, the monopolist will do two things: it will lower the price that it charges to a type 2, and it will decrease the quantity in the bundle that it offers to type 1. Both actions make it less attractive for a type 2 to choose the bundle of a type 1. In the new optimum, the low type again pays exactly what he is willing to pay, whereas the high type is indifferent between consuming his own bundle and that of a low type, i.e. $U_2(q_2) - P_2 = U_2(q_1) - P_1$.

Note that the fact that the low type get a quality that is lower than the social optimum is *exactly* the intuition already offered by Dupuit (1849) in explaining the difference between second and third class in trains: "*What the company is trying to do is prevent the passengers who can pay the second-class fare from traveling third-class; it hits the poor, not because it wants to hurt them, but to frighten the rich*". (quoted e.g. by Tirole, p. 150).

4.3 Damaged Goods

This application follows Deneckere and McAfee (1996), who observe that there are many instances where a monopolist sells two versions of a product: one regular product, and one version that has been deliberately made worse, often even at a cost. For example, in the early 1990s, IBM sold the first laser printers at a price of some \$2400. This printer was able to print at a speed of some 10 pages per minute. At some point, however, IBM introduced a cheaper version. For only \$1500, it of course had lower quality: in fact it was able to print only 5 pages per minute. As it turned out, however, the two printers were almost identical. The only difference was that the cheaper version had an additional chip whose sole function was to slow down the printer. A slightly adapted version of the model set out above suggests why this could be profitable. There are many other examples in which firms deliberately "damage" their product only to sell it at a lower price.

Let's formalize the story above. Suppose that a monopolist offers a good with quality q_2 (the high-end printer). The monopolist, however can also "damage" the good he produces (adding a chip to the printer that slows it down). By doing so, it produces a quality $q_1 < q_2$. Marginal costs for the damaged goods are $c_1 > c_2$: these costs are higher since, after all, the monopolist has to provide it with additional chips.

We again assume two types of consumers, with valuations $\theta_i V(q)$, where $\theta_2 > \theta_1$. Also, we again assume that a fraction λ of the population has type 1. We make the following additional assumptions:

1. When the monopolist only sells the high-end product, it would find it profitable to serve only the type 2 consumers.

2. When the monopolist would only sell the damaged good, it would sell find it most profitable to sell that to both types of consumers.

In other words, if it only sells the high-end product, the monopolist would set a price that only the high types would be willing to pay. But if it only sells the low-end product, it would do so at a price that both types of consumers would be willing to pay.

Let's analyze what the assumptions above imply for the model parameters. Firts consider assumption 1. When it chooses to only serve the high types, the profit-maximizing price would be $p_2 = \theta_2 V(q_2)$. Profits of doing so equal

$$\Pi_H = (1 - \lambda)\left(\theta_2 V(q_2) - c_2\right),$$

where the subscript H refers to the situation in which the monopolist only sells the high quality product. Suppose the monopolist would serve both consumers. He would then set $p_2 = \theta_1 V(q_2)$. His profits would then equal

$$\Pi = \theta_1 V(q_2) - c_2.$$

The assumption that he would only serve the high-type thus boils down to assuming that

$$(1-\lambda)(\theta_2 V(q_2) - c_2) > \theta_1 V(q_2) - c_2$$

or

$$(\theta_1 - (1-\lambda)\theta_2) V(q_2) < \lambda c_2.$$

Now consider the case that the monopolist only produces the damaged good. When only serving high types, the profit-maximizing price would be $p_1 = \theta_2 V(q_1)$. Profits then equal

$$\Pi_D = (1-\lambda)(\theta_2 V(q_1) - c_1),$$

where subscript D refers to the monopolist only selling the damaged product. Suppose it would serve both consumers. It would then set $p_1 = \theta_1 V(q_1)$, yielding profits

$$\Pi = \theta_1 V(q_1) - c_1.$$

The assumption that he would serve both types thus boils down to assuming that

$$\theta_1 V(q_1) - c_1 > (1-\lambda)(\theta_2 V(q_1) - c_1) \tag{4.7}$$

We are now able to tackle the problem of the monopolist that produces both goods. It then has to set T_1 and T_2 as to maximize

$$\Pi_{HD} = \lambda(p_1 - c_1) + (1-\lambda)(p_2 - c_2).$$

The subscript HD refers to the fact that the monopolist produces both the high-quality and the damaged good. Note that the difference between this problem and the one discussed earlier, is that the qualities q_1 and q_2 are exogenously given. With the same arguments as before, we can show that in equilibrium the individual rationality constraint of type 1, and the incentive compatibility constraint of type 2 have to bind:

$$\begin{aligned} p_1 &= \theta_1 V(q_1) \\ p_2 &= \theta_2 (V(q_2) - V(q_1)) + p_1. \end{aligned}$$

The monopolist's profits are thus given by

$$\begin{aligned} \Pi_{HD} &= \lambda(\theta_1 V(q_1) - c_1) + (1-\lambda)(\theta_2 (V(q_2) - V(q_1)) + \theta_1 V(q_1) - c_2) \\ &= (1-\lambda)(\theta_2 V(q_2) - c_2) + \theta_1 V(q_1) - \theta_2 V(q_1) + \lambda\theta_2 V(q_1) - \lambda c_1 \\ &= \Pi_H + \theta_1 V(q_1) - \theta_2 V(q_1) + \lambda\theta_2 V(q_1) - \lambda c_1 \\ &= \Pi_H + (\theta_1 V(q_1) - c_1) - (1-\lambda)(\theta_2 V(q_1) - c_1). \end{aligned}$$

Using (4.7), this implies

$$\Pi_{HD} > \Pi_H.$$

Hence, offering both the high-quality and the damaged goods strictly increases the monopolist's profits. The types 2 will also be strictly better off with the introduction of the damaged good: in the case without damaged goods, they were kept at zero surplus. Now, they earn a strictly positive surplus, for the usual reasons. In this simplified specification, the types 1 are indifferent between whether or not the damaged good is introduced: in both cases, their net surplus is zero. In Deneckere and McAfee (1996), the low types are strictly better off. This is due to the fact that they assume that each individual has a downward sloping demand curve for both qualities. In such a case, by setting a single price for each quality, the monopolist is never able to capture the entire consumer surplus. Therefore, with two products, the types 1 are left with some consumer surplus as well. Note that this result would disappear if the monopolist would simply sell packages of the high-quality and the damaged good.

Exercises

1. There are two types of consumers. High types have a willingness to pay $\theta\sqrt{q}$ for a product of quality q, with $\theta > 1$, low types have willingness to pay $\sqrt{q}$. Both types occur in equal numbers. A monopolist is able to produce two qualities $q_1 = 9$ and $q_2 = 36$. Production costs are 2 per unit, regardless of the quality produced.

   (a) Suppose the monopolist can only produce the high quality product. Determine the price that it will charge and the profits it will make (note: this will depend on θ). Do the same in case the monopolist can only produce the low quality product.

   (b) Suppose the monopolist can produce both qualities. Determine the profit-maximizing prices, and resulting profits. For what values of θ will the monopolist choose to produce both qualities?

   (c) Now suppose that the monopolist can choose the quality of the low quality product, whereas the quality of the high-quality product is still fixed at 36. Determine what low quality he is going to set. Explain.

2. A monopolist sells cars. She faces two types of consumers. The utility a consumer obtains from the car is solely determined by the expected amount of kilometres that the car is able to drive. Denote this expected kilometrage as $E(k)$, denoted in units of 100.000 kilometer. Type 1 consumers have a utility of owning the car that equals $3\sqrt{E(k)}$. Type 2 consumers have a utility of owning the car that equals

$4\sqrt{E(k)}$. The fraction of type 1 consumers in the population is 1/2. The fraction of type 2 consumers is also 1/2. Initially, the monopolist produces cars that run 100.000 kilometers for sure, so $E(k) = 1$. The costs of producing such a car are 1. But the monopolist realizes that she is also able to produce a car that is able to run for 100.000 kilometers with probability 1/2, and will not run at all (so 0 kilometers) with probability 1/2 as well. Consumers can perfectly observe this. The costs of producing such a low-quality car are denoted by c_L.

(a) Determine for which values of c_L it is profitable for the monopolist to also supply the low-quality car, given that she can charge prices for both types of cars in a profit-maximizing manner.

(b) Now suppose that $c_L = 1$ and the monopolist can choose the expected kilometrage of the cheap car to be any value below 100.000. Which value will he choose? Again, of course, we assume that after he has chosen this value, he can charge prices for both types of cars in a profit-maximizing manner.

Chapter 5

Durable Goods

5.1 Introduction

Normally we (implicitly) assume that goods are perishable: the monopolist makes one pricing decision, consumers react by deciding how much to purchase at that price, transactions take place, and the world ends. When goods are durable, however, the picture sketched changes entirely. In that case, consumers can use a product for several periods and the monopolist is still able to change its price after some consumers have already made their purchase. Those consumers will probably be those that have the highest willingness-to-pay for the product. But that gives the monopolist an incentive to lower its price, to also capture the consumers that have a slightly lower willingness-to-pay, and that have not bought the product yet. In turn, this may give the initial consumers with a high willingness-to-pay an incentive to postpone their purchase, since they know that the monopolist will drop its price in the future. As first noted by Coase (1972) this may imply that the monopolist loses all of its market power.

This chapter is largely based on Bulow (1982) who couched the verbal argument of Coase (1972) in a two-stage framework, and on the discussion in Tirole (1988). Please do note that there are some confusing typos and mistakes in the original Bulow article.[1] The discussion in section 5.7 is largely based on Tirole.

5.2 The model

Consider the following set-up, which is a simplified version of Bulow (1982). A monopolist produces a product that is durable, in that it lasts for two periods. After one period, the product is still in perfect working condition, and is a perfect substitute for a new

[1] On pg. 320, end of penultimate paragraph: "To maximize profits in that situation the firm will choose to produce $q_{2s} = (\alpha - \beta\bar{q}_{1S})/2$". This should read "...the firm will choose to produce $q_{2s} = (\alpha - \beta\bar{q}_{1S})/2\beta$". Also, equation 6 on pg. 321 is confusing. It suggests that q_{1S} is some fraction times q_{2S}. This is not the case. The first fraction is simply the equilibrium value for q_{1S}, while the second one is that for q_{2S}.

product. After two periods, the world ends. Consumers are heterogeneous. For ease of exposition, we assume that each consumer have unit demand: they consume either 0 or 1 units of the product. As the willingness to pay differs among consumers, this yields a downward sloping demand function. For simplicity, we assume that this demand curve is given by

$$p = 1 - q. \tag{5.1}$$

Note that this demand function does *not* reflect the demand for the product itself, but rather the demand per period for the services that the product provides. In other words, it reflects the demand for ownership during one period. For example, if the product is a car, then the demand function reflects the demand for the use of the car for one period. Obviously, consumers are willing to pay a higher price if they can use the product for two periods. For simplicity, we assume for now that production is costless. The discount factor is given by δ. Again for ease of exposition, we will assume that the monopoly will decide on the quantity q that it sells, and that the price is then determined on the market, through the demand function (5.1).

5.3 Renting

First suppose that the monopolist does not sell its product but, rather, *rents* it out to consumers (or, in the context of cars, *leases* out the car). This implies that it retains ownership over the products that it produces. After each period, the goods are returned to the monopolist. We assume that the quality of the product that consumers return to the monopolist is identical to the quality of the good that they obtain. In other words, there is no depreciation, and a product that is returned to the monopolist after one period of use is a perfect substitute for a brand new product. In every period, the monopolist thus faces the same problem as a regular perishable-good monopolist: it has to decide which price to charge consumers for the use of its product during one period. Per period profits of the monopolist equal

$$\pi = q\left(1 - q\right).$$

These are maximized by setting

$$q^* = \frac{1}{2}, \tag{5.2}$$

so per-period profits are

$$\pi^* = \frac{1}{4}$$

in each period. Total discounted profits are

$$\Pi^* = \frac{1}{4}\left(1 + \delta\right). \tag{5.3}$$

Note that *production* in period 2 will be zero. In period 1, the monopolist will produce the entire $q^* = 1/2$. This is rented out to consumers, who return their product at the end of period 1. In period 2, the same $q^* = 1/2$ are then rented out.

5.4 Selling to naive consumers

Now suppose that the monopolists sells its product. In period 1, it produces and sells a quantity q_1. In period 2, it sells a quantity q_2. For the sake of argument, we first consider what happens if consumers in period 1 are naive, and do not take the incentives of the monopolist into account. Hence, these consumers assume that the price in the second period will be the same as that in the first: $p_2^e = p_1$, where p_2^e denotes the second-period price that consumers expect in period 1. Since consumers can use the product for two periods, the equilibrium price that will prevail in period 1 is given by

$$P_1 = p_1 + \delta p_2^e = (1 - q_1) + \delta(1 - q_1) = (1 + \delta)(1 - q_1).$$

First period profits of the monopolist are then given by

$$\pi_1 = (1 + \delta)(1 - q_1) q_1,$$

which are again maximized by setting $q_1^* = 1/2$. The price on the market then equals $P_1^* = \frac{1}{2}(1 + \delta)$.

But now consider what happens in period 2. There are still q_1^* units of the product on the market. All consumers with a relatively high willingness-to-pay already own the product, keep using it and hence do not buy a new product in the second period. That implies that the monopolist still faces a residual demand curve $p_2 = 1 - q_1^* - q_2$. Second period profits are thus given by

$$\begin{aligned} \pi_2 &= (1 - q_1^* - q_2) q_2 \\ &= (1/2 - q_2) q_2 \end{aligned}$$

Maximizing this yields $q_2^* = \frac{1}{4}$. This implies a second-period price of $p_2^* = \frac{1}{4}$.

The consumers who buy the product in period 1 now pay an effective per-period price of $P_1^* / (1 + \delta) = \frac{1}{2}$. Consumers who buy the product in period 2 pay a price per period of $p_2^* = \frac{1}{4}$. This is lower. It implies that, if consumers are rational, some of them will postpone their purchase. Knowing that the monopolist has an incentive to effectively lower its price in period 2, they have have an incentive not to buy in period 1, but rather to wait until period 2.

To show that this is indeed the case, consider a consumer with a per-period willingness-to-pay of $WTP = \frac{1}{2} + \varepsilon$, with ε some small number. We will analyze when this consumer is better off; when purchasing the durable good in period 1, or when purchasing it in period 2. In the naive analysis above, we assumed that this consumer would buy in period 1. By doing so, she has a net surplus of

$$(1+\delta)\, WTP - P_1^* = (1+\delta)\,\varepsilon > 0.$$

But now assume that this consumer rationally foresees what the monopolist will do in period 2. She then knows that $p_2^* = \frac{1}{4}$. Hence, by postponing her purchase, this consumer's net surplus is

$$\delta\,(WTP - p_2^*) = \delta\left(\frac{1}{4} + \varepsilon\right).$$

The consumer is thus better off by postponing her purchase if $\delta\left(\frac{1}{4}+\varepsilon\right) > (1+\delta)\,\varepsilon$, or if $\varepsilon < \frac{1}{4}\delta$. Rational consumers that have a willingness-to-pay between 0.5 and $0.5 + 0.25\delta$ will thus postpone their purchase, even though buying the product now does give them a positive surplus. Hence, if we assume that consumers are rational, the analysis carried out above cannot be an equilibrium.

5.5 Selling to rational consumers

We now assume that consumers are rational, and hence take the incentives of the monopolist into account when making their first-period purchase decision. We solve the model using backward induction. The timing is as follows:

Stage 1

a The monopolist sets q_1.

b Demand determines the price P_1 that will prevail.

Stage 2

a The monopolist sets q_2.

b Demand determines the price p_2 that will prevail.

Note again that we denote as p_1 the price consumers (implicitly) pay for use of the good during period 1, and as P_1 the price they pay if they buy the good in period 1, and can use it in both period 1 and 2. Denoting the expected price as p_2^e we thus again have $P_1 = p_1 + \delta p_2^e$. The difference with the analysis above is that we now assume that consumers are rational, so we require that $p_2^e = p_2$. Effectively, we assume that there is a second-hand market, on which the product can be traded after stage 2a. Consumers that buy in stage 1 know in advance that the price for which they can sell their product on that second-hand market equals p_2^e (of course, they can also just keep it themselves). Hence, they are willing to pay P_1 in period 1.

To solve the model, we start with stage 2b. Trivially, $p_2 = 1 - q_1 - q_2$. Now move back to stage 2a. The monopolist's problem is then to

$$\max_{q_2} (1 - q_1 - q_2)\, q_2,$$

which yields

$$q_2^* = \frac{1 - q_1}{2}.$$

The monopolist's second-period profits are then given by

$$\pi_2 = \left(\frac{1 - q_1}{2}\right)^2$$

Consider stage 1b. Taking into account the incentives the firm has in period 2, and given the quantity q_1 that the monopoly sets in period 1, consumers know that the second-period price will equal

$$p_2^e = p_2 = 1 - q_1 - \frac{1 - q_1}{2} = \frac{1 - q_1}{2}.$$

Therefore, the price that will prevail in period 1 is given by

$$P_1 = p_1 + \delta p_2 = (1 - q_1) + \delta \left(\frac{1 - q_1}{2}\right) = \left(1 + \frac{\delta}{2}\right)(1 - q_1)\,.$$

We move back to stage 1a. The monopolist will set q_1 to maximize total discounted profits, taking into account how the choice of q_1 will affect what happens in the second period.

Total discounted profits of the monopolist are given by

$$\begin{aligned} \Pi &= \pi_1 + \delta \pi_2 = P_1 q_1 + \delta \left(\frac{1 - q_1}{2}\right)^2 \\ &= \left(1 + \frac{\delta}{2}\right)(1 - q_1)\, q_1 + \delta \left(\frac{1 - q_1}{2}\right)^2 . \end{aligned}$$

Taking the first-order condition with respect to q_1:

$$\frac{\partial \Pi}{\partial q_1} = 1 - \frac{1}{2} q_1 (4 + \delta) = 0$$

The monopolist thus sets

$$q_1^* = \frac{2}{4 + \delta}.$$

This implies that the second-period quantity will be

$$q_2^* = \frac{1}{2} \cdot \frac{2+\delta}{4+\delta}.$$

Total profits are

$$\Pi^* = \frac{1}{4} \cdot \frac{(2+\delta)^2}{4+\delta}$$

Note that these profits are always lower than the profits in the case of renting. This can be seen as follows. Selling profits are higher than renting profits if and only if

$$\frac{1}{4}(1+\delta) > \frac{1}{4}\frac{(2+\delta)^2}{4+\delta}$$

$$\begin{aligned} &\Rightarrow \quad (1+\delta)(4+\delta) > (2+\delta)^2 \\ &\Rightarrow \quad 4+5\delta+\delta^2 > 4+4\delta+\delta^2 \\ &\Rightarrow \quad \delta > 0. \end{aligned}$$

Hence, the monopolist is always better off renting than selling. Also note that as consumers (and the firm) become very impatient and δ goes to 0, profits are equal to the case of renting. This makes sense: if players are extremely impatient, they will only take into account what happens in period 1 when taking their decisions, so the analysis collapses to one of perishable goods.

The intuition for this result is as follows. When selling its product, the monopolist has a commitment problem. It wants to convince consumers that it will not flood the market in the future, but it is very hard to make such a claim, simply because it is not credible one. Once the future comes, the monopolist has an incentive to renege on its promise and to increase output anyhow.

Another thing worth noting is that our monopolist engages in *intertemporal price discrimination*, in that it charges a different price in each period. But in this case, that is not to take optimal advantage of the fact that there are different types of consumers out there, but rather an inevitable outcome from the fact that it is not able to commit.

5.6 The Case of Commitment

To see that this really is a commitment problem, suppose that the monopolist *could* commit to a quantity beforehand. Its total profits are

$$\begin{aligned} \pi(q_1, q_2) &= q_1(p_1 + \delta p_2^e) + \delta q_2 p_2 \\ &= q_1(1-q_1+\delta(1-q_1-q_2)) + \delta q_2(1-q_1-q_2). \end{aligned}$$

Maximizing with respect to q_1 and q_2 yields first-order conditions

$$\begin{aligned}\frac{\partial \pi}{\partial q_1} &= 1 - 2q_1 + \delta\left(1 - 2q_1 - q_2\right) - \delta q_2 = 0 \\ \frac{\partial \pi}{\partial q_2} &= -\delta q_1 + \delta\left(1 - q_1 - 2q_2\right) = 0.\end{aligned}$$

Solving for q_2 from the second condition yields

$$q_2 = \frac{1}{2} - q_1.$$

Plugging this back into the first condition yields indeed $q_1 = 1/2$, so $q_2 = 0$; the same solution we found in the rental case.

5.7 Getting around the problem

We thus have that the firm is worse off when it sells its product rather than renting it. The problem of the monopolist is that it cannot commit itself to restrict output in period 2. Consumers know this to be the case, and take it into account when making their purchase decisions. Effectively, the monopolist is competing against its future self.

So far we have only considered a very simple two-stage model. When the number of periods increases to infinity, it can be shown that the monopolist loses all its monopoly power, and produces a quantity in period 1 that is equal to what a competitive market would produce (see e.g. Stokey (1981).

Obviously, a monopoly would like to have a way to convince consumers that it will not increase production after period 1. It can do so in several ways.

1. By leasing or renting the good, as we showed above. This may however be a problem when *moral hazard* is involved, i.e. when the monopolist cannot observe perfectly how consumers treat its product. If that is the case, then a product that has once been rented to consumers is no longer a perfect substitute for a new product, since on average its quality will be lower.

2. By committing itself not to produce (too much) in period 2. It can do so in several ways

    (a) By contracting with a third party who will monitor the behavior of this firm.

    (b) By establishing a reputation for not flooding the market. The classic example here is the diamond monopoly DeBeers.

    (c) By destroying the factory. This may sound far-off, but is actually a common practice in the arts: lithographs, for example, are always produced in limited amounts. Every single lithograph is numbered and also informs the buyer of the total number of lithographs that have been (and will be) produced. This is done exactly to get around this durable-good problem.

(d) The monopolist may also give a *price guarantee*, also known as a most-favored customer clause. With such a clause, the monopolist commits itself to refund the customer whenever it lowers prices in the future. With such a clause, a monopolist has much less of an incentive to flood the market in the future, since that involves having to refund past customers as well. Note therefore that although such a practice may sound like a good deal for consumers, actually the opposite is true. Such a guarantee only provides the monopolist with a way to commit not to lower prices in the future, which ultimately implies that prices today are much higher. This is analyzed in Butz (1990).

(e) By having opportunity costs of staying in the market. If a monopolist has e.g. high fixed costs of staying in a market, it can credibly commit not to produce anything in period 2, as the profits of doing so would simply be too low.

It can also be shown that the durable-goods problem may also be less of a problem when consumers do not know the exact value of marginal cost, or when there is a constant inflow of new customers.

5.8 Planned Obsolescence

Let's now consider a case in which the monopolist can affect the durability of its product. This section is loosely based on Bulow (1986). For simplicity, we again assume a two-period model and let ρ denote the fraction of products produced in period 1 that "survive" up to the second period. Thus, in period 2 there are still ρq_1 left of the products that were produced in the first period. We again assume that demand in each period for the services provided by the product are given by $p = 1 - q$. For this section, we do have to assume that the monopolist also faces marginal costs c for each unit that it produces in either period. For simplicity, we set the discount factor equal to 1: $\delta = 1$. In the first period, the monopolist now also has to set the durability ρ of its product. We assume that there are costs $C(\rho)$ involved in doing so: a higher durability requires more upfront investment. A higher durability requires a higher upfront costs, so we have $C'(\rho) > 0$.

Let's first assume again that the monopolist can rent out its product in each period. Its total profits then equal

$$\pi(q_1, q_2, \rho) = q_1(1 - q_1) + (\rho q_1 + q_2)(1 - \rho q_1 - q_2) - cq_1 - cq_2 - C(\rho). \qquad (5.4)$$

The first term reflects total revenue in period 1; the second term are the revenues in period 2. The remaining terms reflect the costs of production and durability.

The monopolist will choose its decision variables to maximize these profits. Taking first-order conditions we obtain

$$\frac{\partial \pi}{\partial q_1} = 1 - 2q_1 + \rho(1 - \rho q_1 - q_2) - \rho(\rho q_1 + q_2) - c = 0. \tag{5.5}$$

$$\frac{\partial \pi}{\partial q_2} = (1 - \rho q_1 - q_2) - (\rho q_1 + q_2) - c = 0 \tag{5.6}$$

$$\frac{\partial \pi}{\partial \rho} = -(\rho q_1 + q_2)q_1 + q_1(1 - \rho q_1 - q_2) - C' = 0. \tag{5.7}$$

Combining the last two yields

$$cq_1 = C'.$$

That implies that, given the production decisions of the monopolist, the investment in durability is socially efficient: the monopolist invests up to the point where the marginal cost of durability C'/q to the marginal benefit: for every unit of additional durability it now has to produce q_1 less in the second period which saves cq_1.

Now suppose the monopolist has to sell its product. Given q_1, in the second period it will produce q_2 to maximize

$$\pi_2 = (1 - \rho q_1 - q_2 - c)\, q_2.$$

Hence

$$q_2 = \frac{1}{2}(1 - \rho q_1 - c)$$

while second-period profits are

$$\pi_2 = \frac{1}{4}(1 - \rho q_1 - c)^2.$$

This implies that the second-period price will equal

$$p_2^e = p_2 = 1 - \rho q_1 - \frac{1}{2}(1 - \rho q_1 - c) = \frac{1}{2}(1 - \rho q_1 + c),$$

hence in the first period consumers are willing to pay

$$P_1 = p_1 + p_2 = (1 - q_1) + \frac{1}{2}(1 - \rho q_1 + c) = \frac{1}{2}(3 - (2 + \rho)\, q_1 + c).$$

Profits then equal

$$\pi(q_1, \rho) = \frac{1}{2}(3 - (2 + \rho)\, q_1 - c)\, q_1 + \frac{1}{4}(1 - \rho q_1 - c)^2 - C(\rho). \tag{5.8}$$

Maximizing

$$\begin{aligned}\frac{\partial \pi\left(q_1, \rho\right)}{\partial q_1} &= \frac{1}{2}\left(3-2\left(2+\rho\right) q_1-c\right)-\frac{1}{2} \rho\left(1-\rho q_1-c\right)=0 \\ \frac{\partial \pi\left(q_1, \rho\right)}{\partial \rho} &= -\frac{1}{2} q_1^2-\frac{1}{2}\left(1-\rho q_1-c\right)-C^{\prime}=0\end{aligned}$$

But the second condition implies that at the optimal solution we would now have $C'(\rho) < 0$. Given the conditions we imposed, this is not feasible. Hence, we have that the durable goods monopolist will choose a corner solution and set $\rho = 0$. Effectively, this turns the durable good into a non-durable good. By lowering the lifetime of its product, the monopolist circumvents the durable goods problem.

In the literature, this is known as *planned obsolescence*: more generally, this implies that a monopolist chooses a lifetime for its product that is lower than the social optimum. There are many real-world examples of this, the most notorious being the Phoebus cartel, a conspiracy of lightbuld manufacturers that in the 1920s agreed to restrict the lifetime of lightbulbs to 1000 hours. Publisher of textbooks often have new editions every one or two years, in an attempt to kill off the second-hand market. Also, books often appear first in a hardcover edition, and only then in paperback, even though the two editions cost virtually the same to produce.

Exercise

Consider a durable good monopolist that sells a product that breaks down after one period with probability 1/2. Both the monopolist and the consumers are fully aware of this. There are two periods. Derive the equilibrium in terms of quantities, prices and profits if the monopolist rents, and if it sells its product.

Chapter 6

Switching Costs

6.1 Introduction

This chapter considers models of switching costs. Switching costs are the costs that a consumer incurs as a result of changing suppliers. These costs can be monetary, but also in terms of effort. Such switching costs are relevant in, for example, banking. To switch from one bank to another can imply a huge amount of costs: all your financial contacts have to be informed about the new bank account. Another example is telecom: it takes a lot of effort and costs to switch from one mobile phone company to another, especially when that involves having to change your phone number. But it is also an issue in software (as it takes time and effort to learn a new program).

To model switching costs, we usually consider a two-period model — as we do need to give consumers the possibility to switch. In the first period, consumers choose their supplier, in the second period, they do so again but have to incur additional costs when changing their supplier.

Our base model of switching costs will be as follows. Two firms compete on a Hotelling line of unit length. They are located on opposite ends of the line. Once consumers have chosen one of the firms, they face costs of switching to the other suppliers. There will be two types of consumers in period 2: some consumers will be new, and have not consumed in period 1, while others are old and have already consumed in period 1.

We will first consider a model in which consumer tastes do not change from one period to the next. For ease of exposition, we will first in Section 6.2 consider a case in which consumers are completely naive and do not foresee that they will face switching costs in period 2. In Section 6.3 we study consumers that are forward-looking. Section 6.4 studies a model in which consumer tastes change from one period to the next.

The analyses in this chapter are largely based on Klemperer (1987) and follow the discussion in Belleflamme and Peitz (2015).

6.2 Naive consumers

Two firms are located at the endpoint of a Hotelling line; firm A is located at 0, while firm B is located at 1. For simplicity, we set transportation costs equal to $t = 1$. There are two periods, and as usual, we assume that the market is entirely covered. In period 2, a share λ_n of consumers are replaced by a new bunch of consumers. Hence, these new consumers make their first decision choice in period 2 and thus do not face switching costs. The other $\lambda_o = 1 - \lambda_n$ are "old" consumers that also consumed in period 1 and hence face switching costs when switching suppliers. A consumer that bought from A in the first period, has to incur additional switching costs z when buying from B in the second period, and vice versa. For now, we assume that each consumer has the same preference (i.e. the same location on the Hotelling line) in each period.

As always, we solve using backward induction. Suppose that a share $\hat{x}^1$ of consumers bought from firm A in the first period. Hence, the remaining $1 - \hat{x}^1$ bought from firm B. We will refer to those that bought from A in period 1 as segment A, and to those that bought from B as segment B.

Consider a consumer in segment A. Given second-period prices p_A^2 and p_B^2, she will buy again from A in period 2 if her location x is such that

$$v - x - p_A^2 \geq v - (1 - x) - p_B^2 - z$$

This implies that the indifferent consumer in segment A in period 2 is given by

$$\hat{x}_A = \frac{1}{2}\left(1 + p_B^2 - p_A^2 + z\right).$$

Similarly, the indifferent consumer in segment B in period 2 is given by

$$\hat{x}_B = \frac{1}{2}\left(1 + p_B^2 - p_A^2 - z\right).$$

Note that there will be no switching if $\hat{x}_A \geq \hat{x}_1$ and $\hat{x}_B \leq \hat{x}_1$. This is true if

$$z \geq \left(p_A^2 - p_B^2\right) + 2\hat{x}_1 - 1.$$

We will show below that this is satisfied. Hence, in equilibrium, consumers in segment A will also buy from A in the second period, while consumers in segment B will also buy from B in the second period.

Now consider the consumers that are new in period 2. They will just behave as regular consumers in a one-period Hotelling model. Hence, the indifference new consumer will be located at

$$\hat{x}_n = \frac{1}{2}\left(1 + p_B^2 - p_A^2\right).$$

Second-period demand for firm A is thus given by

$$q_A^2 = \lambda_o \hat{x}_1 + \frac{1}{2}\lambda_n \left(1 + p_B^2 - p_A^2\right).$$

Second-period profits equal $\pi_A^2 = \left(p_A^2 - c\right) q_A^2$. Taking the first-order condition:

$$\frac{\partial \pi_A^2}{\partial p_A^2} = \lambda_o \hat{x}_1 + \frac{1}{2}\lambda_n \left(1 + p_B^2 - 2p_A^2 + c\right),$$

which yields reaction function

$$p_A^2 = \frac{\lambda_o}{\lambda_n}\hat{x}_1 + \frac{1}{2}\left(1 + p_B^2 + c\right).$$

For firm B, we have

$$q_B^2 = \lambda_o \left(1 - \hat{x}_1\right) + \frac{1}{2}\lambda_n \left(1 + p_A^2 - p_B^2\right)$$

which yields reaction function

$$p_B^2 = \frac{\lambda_o}{\lambda_n}\left(1 - \hat{x}_1\right) + \frac{1}{2}\left(1 + p_A^2 + c\right).$$

The equilibrium thus has

$$p_A^2(\hat{x}_1) = c + \frac{1}{\lambda_n}\left(1 + \frac{1}{3}\left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)\right) \tag{6.1}$$

$$q_A^2(\hat{x}_1) = \frac{1}{2}\left(1 + \frac{1}{3}\left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)\right)$$

$$\pi_A^2(\hat{x}_1) = \frac{1}{2\lambda_n}\left(1 + \frac{1}{3}\left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)\right)^2, \tag{6.2}$$

with similar expressions for firm B.[1]

Now move back to period 1. If consumers are naive, total profits for firm A are

$$\Pi_A\left(p_A^1, p_B^1\right) = \pi_A^1\left(p_A^1, p_B^1\right) + \delta \pi_A^2\left(\hat{x}_1\right). \tag{6.3}$$

Taking the first order condition

$$\frac{\partial \Pi_A\left(p_A^1, p_B^1\right)}{\partial p_A^1} = \frac{\partial \pi_A^1\left(p_A^1, p_B^1\right)}{\partial p_A^1} + \delta \frac{\partial \pi_A^2\left(\hat{x}_1\right)}{\partial \hat{x}_1} \cdot \frac{\partial \hat{x}_1}{\partial p_A^1} = 0 \tag{6.4}$$

With naive consumers, we simply have

$$\hat{x}_1 = \frac{1}{2}\left(1 + p_B^1 - p_A^1\right),$$

[1] Wherever there is an $\hat{x}_1$ in the expression for firm A, we get a $1 - \hat{x}_1$ in the expression for firm B.

so $\partial \hat{x}_1 / \partial p_A^1 = -\frac{1}{2} < 0$. From (6.2), we immediately have $\partial \pi_A^2 (\hat{x}_1) / \partial \hat{x}_1 > 0$. Taken together, this implies from (6.4) that the equilibrium now has $\partial \pi_A^1 (p_A^1, p_B^1) / \partial p_A^1 > 0$. Without switching costs, firms would simply maximize profits in each period, so we would have $\partial \pi_A^1 (p_A^1, p_B^1) / \partial p_A^1 = 0$. That implies[2] that in this context switching costs necessarily lead to lower first-period prices. To solve for the equilibrium, first note that

$$\pi_A^1 = \left(p_A^1 - c\right) \hat{x}_1 = \frac{1}{2} \left(1 + p_B^1 - p_A^1\right) \left(p_A^1 - c\right),$$

so

$$\frac{\partial \pi_A^1 (p_A^1, p_B^1)}{\partial p_A^1} = \frac{1}{2} \left(1 + p_B^1 - 2p_A^1 + c\right). \tag{6.5}$$

Moreover

$$\frac{\partial \pi_A^2 (\hat{x}_1)}{\partial \hat{x}_1} = \frac{1}{\lambda_n} \left(1 + \frac{1}{3} (2\hat{x}_1 - 1)(1 - \lambda_n)\right) \left(\frac{2}{3} (1 - \lambda_n)\right). \tag{6.6}$$

Imposing symmetry (which in this case implies that $\hat{x}_1 = 1/2$), we thus have from (6.4),

$$\frac{\partial \Pi_A (p_A^1, p_B^1)}{\partial p_A^1} = \frac{1}{2} \left(1 - p_A^1 + c\right) - \delta \left(\frac{1}{3\lambda_n} (1 - \lambda_n)\right) = 0.$$

Hence

$$p_A^{1*} = 1 + c - \delta \frac{2(1 - \lambda_n)}{3\lambda_n}.$$

Without switching costs, we would have a standard Hotelling model, so profits in each period would equal $p = 1 + c$. Hence, first-period prices in a world with switching costs are *lower* than prices in a world without switching costs. For the second period, we have

$$p_A^{2*} = c + \frac{1}{\lambda_n}.$$

With $\lambda_n < 1$, this implies that these prices are *higher* than they would be in a world without switching costs. With switching costs, firms have more market power, which allows them to set a higher price in the second period. But, as firms can make substantial profits on any given consumer in the second period, they are also much more eager to attract such a consumer in the first period. that implies that competition will then be much fiercer - so prices will be lower.

To evaluate the total effect, consider the total discounted price that consumers end up paying:

$$\begin{aligned} P = p_1 + \delta p_2 &= 1 + c - \delta \frac{2(1 - \lambda_n)}{3\lambda_n} + \delta \left(c + \frac{1}{\lambda_n}\right) \\ &= 1 + c + \frac{1}{3} \delta \left(\frac{1}{\lambda_n} + 2 + 3c\right) \end{aligned}$$

[2]given that at the optimal solution profits are necessarily strictly concave.

Without switching costs we have $P = (1+c)(1+\delta)$. If we take the difference between total discounted price with and without switching costs, we obtain

$$1 + c + \frac{1}{3}\delta\left(\frac{1}{\lambda_n} + 2 + 3c\right) - (1+c)(1+\delta) = \frac{1}{3}\frac{\delta}{\lambda_n}(1-\lambda_n).$$

This implies that consumers that live for two periods end up paying a higher price if there are switching costs: switching costs make them worse off. The advantage of having a lower price in the first period is outweighed by having a higher price in the second.

6.3 Forward-looking consumers

Now suppose that consumers are forward looking: n the first period they realize that there will be switching costs, and also that current behavior may affect future prices.

If consumer x buys from firm A in period 1, she knows she has a probability λ_o of staying in the market in period 2 and to buy again from A. Otherwise, she enters the market and does not earn any surplus. For simplicity, in the analysis that follows we will set $\delta = 1$, otherwise expressions become very messy. Her total expected surplus when buying from A in period 1 equals

$$u_A = v - x - p_A^1 + \lambda_o\left(v - x - p_A^2(\hat{x}_1)\right),$$

where the term before the plus sign gives first-period utility from buying A, while the term after the plus sign gives second-period utility, given that second-period price will be $-p_A^2(\hat{x}_1)$, and that there will be no switching in equilibrium. Buying from B gives her expected surplus

$$u_B = v - (1-x) - p_B^1 + \lambda_o\left(v - (1-x) - p_B^2(\hat{x}_1)\right).$$

Hence, equating u_A and u_B, the indifferent consumer is located at

$$\hat{x}_1 = \frac{1}{2} + \frac{p_B - p_A + \lambda_o\left(p_B^2(\hat{x}_1) - p_A^2(\hat{x}_1)\right)}{2(1+\lambda_o)}.$$

Note that the second-period analysis will be exactly the same from that in the previous section; given the decisions that consumers have made in the first period, that are reflected by $\hat{x}_1$, the incentives for firms in the second period are not affected. From (6.1), we have

$$\begin{aligned} p_B^2(\hat{x}_1) - p_A^2(\hat{x}_1) &= \frac{1}{\lambda_n}\left(\left(\frac{1}{3}(2(1-\hat{x}_1)-1)(1-\lambda_n)\right) - \left(\frac{1}{3}(2\hat{x}_1 - 1)(1-\lambda_n)\right)\right) \\ &= \frac{2\lambda_o}{3\lambda_n}(1-2\hat{x}_1). \end{aligned}$$

We can plug that back into our expression for $\hat{x}_1$ to find

$$\hat{x}_1 = \frac{1}{2} + \frac{p_B - p_A + \lambda_o \left(\frac{2\lambda_o}{3\lambda_n}(1-2\hat{x}_1)\right)}{2(1+\lambda_o)}.$$

Solving for $\hat{x}_1$ now implies

$$\hat{x}_1 = \frac{1}{2}\left(1 + \frac{3\lambda_n (p_B - p_A)}{2 + 2\lambda_n - \lambda_n^2}\right). \tag{6.7}$$

Note that the term $3\lambda_n / (2 + 2\lambda_n - \lambda_n^2) < 1$ for all $\lambda_n < 1$. That means that first period demand is now less responsive to price than the standard model without switching costs. That suggests that first-period prices will be higher. This can be seen as follows. For firms, lowering prices always yields a trade-off: it will increase sales but it will also yield a lower profit margin. When sales are less responsive to price, it becomes less tempting to lower one's price for any price that the competitor charges. Hence, reaction functions shift upwards, which also implies that equilibrium prices will be higher.

Total profits for firm A can again be written

$$\Pi_A\left(p_A^1, p_B^1\right) = \pi_A^1\left(p_A^1, p_B^1\right) + \delta\pi_A^2(\hat{x}_1),$$

while taking the first order condition again yields

$$\frac{\partial \Pi_A\left(p_A^1, p_B^1\right)}{\partial p_A^1} = \frac{\partial \pi_A^1\left(p_A^1, p_B^1\right)}{\partial p_A^1} + \delta\frac{\partial \pi_A^2(\hat{x}_1)}{\partial \hat{x}_1} \cdot \frac{\partial \hat{x}_1}{\partial p_A^1} = 0 \tag{6.8}$$

In this case, that simplifies to

$$\frac{\partial \Pi_A\left(p_A^1, p_B^1\right)}{\partial p_A^1} = \left(p_A^1 - c + \frac{\partial \pi_A^2(\hat{x}_1)}{\partial \hat{x}_1}\right)\frac{\partial \hat{x}_1}{\partial p_A^1} + x_1 = 0 \tag{6.9}$$

Here, $\partial \pi_A^2(\hat{x}_1)/\partial \hat{x}_1$ is again given by (6.6), From (6.7) and imposing symmetry, this implies

$$\left(p_A^1 - c + \frac{1}{\lambda_n}\left(\frac{2}{3}(1-\lambda_n)\right)\right)\left(-\frac{1}{2}\cdot\frac{3\lambda_n}{2+2\lambda_n-\lambda_n^2}\right) + \frac{1}{2} = 0$$

hence

$$\begin{aligned} p_A^{1*} &= 1 + c + \frac{1}{3}(1-\lambda_n) \\ p_A^{2*} &= c + \frac{1}{\lambda_n} \end{aligned}$$

Also note here that second-period equilibrium prices are the same as in the case with naive consumers. This should not come as a surprise; the second period is the same as in a model with naive consumers, hence given that in equilibrium we will have $\hat{x} = 1/2$, second-period prices will not be affected.

But the price in the second period is now higher. It is easy to see that it is also higher than in the case when there are no switching costs. That holds for the second period as well, since λ_n <1. Hence, switching costs now make consumers worse off in each period – which necessarily implies that they make firms better off in each period. Interestingly, consumers would be better off if they could commit to being naive. The reason is that a rational forward-looking consumer is less inclined to be tempted by a lower price. She knows that a firm charging a lower price today will have higher sales today, but also that that implies that it will charge a higher price tomorrow. With consumers being less reactive to prices in the first period, this will also give firms less of an incentive to compete fiercely in that period.

6.4 Changing tastes

In the analysis in the previous section, we assumed that consumers have the same taste in each period. We now consider the other extreme, and assume that consumers draw a new location in the second period that is completely unrelated to their location in the first. We thus assume that tastes change, and for simplicity we consider the simplest case in which tastes in period 1 are completely uninformative about tastes in period 2. Still, the analysis now becomes rather cumbersome. Using backward induction, we again start with the second period. The indifferent consumer in segment A again has

$$\hat{x}_A^2 = \frac{1}{2}\left(1 + p_B^2 - p_A^2 + z\right),$$

while the indifferent consumer in segment B again has

$$\hat{x}_B^2 = \frac{1}{2}\left(1 + p_B^2 - p_A^2 - z\right).$$

These expressions are always strictly between 0 and 1 provided that $z < 1$. We assume that to be the case. Hence, there will now always be some consumers that switch, different from the model with fixed preferences.

New consumers will again behave as in the standard Hotelling model, so

$$\hat{x}_n = \frac{1}{2}\left(1 + p_B^2 - p_A^2\right).$$

Firm A's second period demand thus equals

$$\begin{aligned} q_A^2\left(p_A^2, p_B^2\right) &= \frac{1}{2}\lambda_n\left(1 + p_B^2 - p_A^2\right) + \lambda_0\left(\hat{x}_1 \hat{x}_A^2 + \left(1 - \hat{x}_1\right)\hat{x}_B^2\right) \\ &= \frac{1}{2}\left(1 + p_B^2 - p_A^2 + \left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right) z\right), \end{aligned}$$

with a similar expression for firm B. Profits of firm A equal $\pi_A^2 = (p_A^2 - c)\, q_A^2$. The first-order condition now becomes

$$\frac{1}{2}\left(1 + p_B^2 - 2p_A^2 + (2\hat{x}_1 - 1)(1 - \lambda_n) z + c\right) = 0,$$

so

$$p_A^2 = \frac{1}{2}\left(1 + p_B^2 + c + (2\hat{x}_1 - 1)(1 - \lambda_n) z\right).$$

Similarly, for firm B,

$$p_B^2 = \frac{1}{2}\left(1 + p_A^2 + c + (1 - 2\hat{x}_1)(1 - \lambda_n) z\right).$$

Plugging these values into each other we can solve for equilibrium prices, which we can then use to evaluate second-period demands and profits. In particular, we have

$$p_A^2(\hat{x}_1) = 1 + c + \frac{1}{3}(2\hat{x}_1 - 1)(1 - \lambda_n) z \tag{6.10}$$

$$p_B^2(\hat{x}_1) = 1 + c + \frac{1}{3}(1 - 2\hat{x}_1)(1 - \lambda_n) z \tag{6.11}$$

$$q_A^2(\hat{x}_1) = \frac{1}{2}\left(1 + \frac{1}{3}(2\hat{x}_1 - 1)(1 - \lambda_n) z\right) \tag{6.12}$$

$$\pi_A^2(\hat{x}_1) = \frac{1}{2}\left(1 + \frac{1}{3}(2\hat{x}_1 - 1)(1 - \lambda_n) z\right)^2. \tag{6.13}$$

Moving to the first period, assume that consumers are forward looking. A consumer at location x_1 that buys from A in the first period and survives to the second, knows that in the second period she will buy from firm A if her new location $x \leq \hat{x}_A^2$, and from B otherwise. Hence, her expected total surplus from buying A in the first period is, again setting $\delta = 1$:

$$\begin{aligned} u_A &= v - x_1 - p_A^1 \\ &+ \lambda_o \left[\int_0^{\hat{x}_A^2} \left(v - x - p_A^2(\hat{x}_1)\right) dx + \int_{\hat{x}_A^2}^1 \left(v - (1 - x) - p_B^2(\hat{x}_1) - z\right) dx\right]. \end{aligned}$$

When buying from B :

$$\begin{aligned} u_B &= v - (1 - x_1) - p_B^1 \\ &+ \lambda_o \left[\int_0^{\hat{x}_B^2} \left(v - x - p_A^2(\hat{x}_1) - z\right) dx + \int_{\hat{x}_B^2}^1 \left(v - (1 - x) - p_B^2(\hat{x}_1)\right) dx\right]. \end{aligned}$$

Taking the difference between the two yields

$$\Delta u = u_A - u_B = 1 - 2x_1 + p_B^1 - p_A^1 + z\lambda_o\left(p_B^2(\hat{x}_1) - p_A^2(\hat{x}_1)\right)$$

Plugging in the equilibrium values of $p_A^2(\hat{x}_1)$ and $p_B^2(\hat{x}_1)$ from (6.10) and (6.11), this yields

$$\Delta u = 1 - 2x_1 + p_B^1 - p_A^1 + \frac{2}{3}z^2\lambda_o^2(1 - 2x_1)$$

The indifferent consumer thus has

$$1 - 2\hat{x}_1 + p_B^1 - p_A^1 + \frac{2}{3}(1 - 2\hat{x}_1)\lambda_o^2 z^2 = 0.$$

hence

$$\hat{x}_1 = \frac{1}{2} + \frac{3}{6 + 4z^2\lambda_o^2}(p_B - p_A).$$

Hence, consumers are less responsive to price changes than in the model without switching costs, provided $\lambda_o > 0$.

As always, total profits for firm A can again be written

$$\Pi_A\left(p_A^1, p_B^1\right) = \pi_A^1\left(p_A^1, p_B^1\right) + \pi_A^2(\hat{x}_1),$$

while taking the first order condition again yields

$$\frac{\partial \Pi_A\left(p_A^1, p_B^1\right)}{\partial p_A^1} = \frac{\partial \pi_A^1\left(p_A^1, p_B^1\right)}{\partial p_A^1} + \frac{\partial \pi_A^2(\hat{x}_1)}{\partial \hat{x}_1} \cdot \frac{\partial \hat{x}_1}{\partial p_A^1} = 0 \tag{6.14}$$

From the analysis above, we have

$$\pi_A^2 = \frac{1}{2}\left(1 + \frac{1}{3}(2\hat{x}_1 - 1)(1 - \lambda_n)z\right)^2$$

hence

$$\frac{\partial \Pi_A}{\partial p_A^1} = \hat{x}_1 + \left(p_A^1 - c\right)\frac{\partial \hat{x}_1}{\partial p_A^1} + \left(1 + \frac{1}{3}(2\hat{x}_1 - 1)(1 - \lambda_n)z\right) \cdot \frac{2}{3}(1 - \lambda_n)z\frac{\partial \hat{x}_1}{\partial p_A^1} = 0.$$

Imposing symmetry and using $\partial \hat{x}_1/\partial p_A^1 = -3/\left(6 + 4z^2\lambda_o^2\right)$, this implies

$$\frac{\partial \Pi_A}{\partial p_A^1} = \frac{1}{2} - \left(p_A^1 - c + \frac{2}{3}(1 - \lambda_n)z\right)\left(\frac{3}{6 + 4z^2\lambda_o^2}\right) = 0.$$

hence

$$\begin{aligned} p_A^1 &= 1 + c - \frac{2}{3}z\lambda_0(1 - z\lambda_o) \\ p_A^2 &= 1 + c \end{aligned}$$

Hence, the first period price is lower than that in a standard Hotelling model, while the second-period price is equal. Hence, in this environment, switching costs make consumers better off, and firms worse off. Firms are more eager to attract consumers in the first period, as more demand in the first period will *ceteris paribus* also lead to more demand in the second period. However, in equilibrium consumers divide evenly in the first period, so second period demand will not be affected.

Exercises

1. Consider the standard model of switching costs as described in the book. Consumers are uniformly distributed on a Hotelling line, where firms A and B are located at 0 and 1, respectively. Transportation costs are normalized to 1, costs are zero. There are two periods. Consumers are a priori identical. However, after period 1, a fraction λ_s of consumers will have switching costs $s > 0$, whereas the remaining fraction $1 - \lambda_s$ will have no switching costs. Consumers know this in advance, but they do not know in which group they will be. No consumers die, no new consumers are born. Consumers are forward-looking, and have the same taste in both periods. Derive equilibrium prices in periods 1 and 2. What are the welfare effects of having switching costs in this case? (HINT: be smart)

2. In this chapter, we studied the model of switching costs with fixed preferences both for the case of naive consumers, as well as for the case of rational, forward-looking consumers. We also studied the switching cost model with changing preferences, but only for the case of rational, forward-looking consumers. Derive the equilibrium (in terms of first-period prices, second-period prices, and profits) in case consumers have changing preferences but are naive. Compare your outcome to the model without switching costs. Are consumers better off or worse of with switching costs. Try to give an intuition?

Chapter 7

Behavior Based Price Discrimination

7.1 Introduction

This chapter builds on the models we studied in the previous chapter, to discuss a related but slightly different issue; that of behavior-based price discrimination. With behavior-based price discrimination, firms can base the price they charge to a consumer on her past buying behavior. For example, firms may give consumers that buy in the first period a **coupon** that entitles them to a lower price in the second period. We study this issue in Section 7.2. Alternatively, a firm may try to tempt consumers that bought from their competitor in the previous period (and hence have revealed that they prefer the product of that firm) with a lower price in the second period. In the literature, that is known as **poaching**. We discuss it in Section 7.3.

7.2 Coupons

The framework we developed in the previous chapter also allows us to study the use of coupons. Suppose consumers do not face switching costs, but instead firms may issue coupons in the first period. A coupon gives consumers a discount on their next purchase. This is just one example of this type of loyalty pricing that is particularly prevalent in the US. We can also think of many other loyalty schemes such as frequent flier miles, card that give you a free coffee after you have consumed 10, etc. To keep things interesting, we do the analysis in the model of varying tastes that we discussed in the previous chapter.

Suppose that in period 1, firm A issues a coupon that entitles the consumer to a discount of γ_A in the next period, while firm B issues a coupon with a discount of γ_B. Consumers can already observe this in the first period before they make their purchase decision. In the second period, the indifferent consumer in segment A now has

$$\hat{x}_A^2 = \frac{1}{2}\left(1 + p_B^2 - p_A^2 + \gamma_A\right).$$

Hence, the discount has the exact same effect as a switching costs; the costs now are not the actual switching costs z, but rather the fact that one has to forego the discount. However, potentially, the switching costs when switching from A to B may be different from switching in the other direction while the switching costs will now also directly affect the profits of both firms, as we will see below.

The indifferent consumer in segment B has

$$\hat{x}_B^2 = \frac{1}{2}\left(1 + p_B^2 - p_A^2 - \gamma_B\right).$$

For ease of exposition, we assume that $\lambda_n = 0$. As firm A has to give a discount in the second period to his loyal consumers, his second profit now is

$$\begin{aligned}
\pi_A^2\left(p_A^2, p_B^2\right) &= \hat{x}_1 \hat{x}_A^2 \left(p_A - \gamma_A - c\right) + \left(1 - \hat{x}_1\right) \hat{x}_B^2 \left(p_A - c\right) \\
&= \frac{1}{2}\left(1 + p_B^2 - p_A^2 + \gamma_A\right) \hat{x}_1 \left(p_A^2 - \gamma_A - c\right) \\
&\quad + \frac{1}{2}\left(1 + p_B^2 - p_A^2 - \gamma_B\right)\left(1 - \hat{x}_1\right)\left(p_A^2 - c\right)
\end{aligned}$$

with a similar expression for firm B. The first-order condition of firm As profit maximization problem now becomes

$$\begin{aligned}
&\frac{1}{2} x_1 \left(1 + c + 2\gamma_A - 2p_A^2 + p_B^2 + 1\right) + \frac{1}{2}\left(1 + p_B^2 - 2p_A^2 - \gamma_B + c\right)\left(1 - \hat{x}_1\right) \\
= &\frac{1}{2}\left(1 + c - 2p_A^2 + p_B^2 + 2\gamma_A \hat{x}_1 - \gamma_B\left(1 - \hat{x}_1\right)\right) = 0
\end{aligned}$$

so

$$p_A^2 = \frac{1}{2}\left(1 + c + p_B^2 + 2\gamma_A \hat{x}_1 - \gamma_B\left(1 - \hat{x}_1\right)\right),$$

and

$$p_B^2 = \frac{1}{2}\left(1 + c + p_A^2 + 2\gamma_B\left(1 - \hat{x}_1\right) - \gamma_A \hat{x}_1\right).$$

Plugging these into each other yields

$$\begin{aligned}
p_A^2\left(\hat{x}_1\right) &= 1 + c + \gamma_A \hat{x}_1 \\
\pi_A^2\left(\hat{x}_1\right) &= \frac{1}{2} - \frac{1}{2}\left(1 - \gamma_A \hat{x}_1\left(1 - \hat{x}_1\right)\left(\gamma_A + \gamma_B\right)\right)
\end{aligned}$$

In the first period, a consumer at location x_1 that buys from A in the first period, again knows that in the second period she will buy from firm A if her new location $x \leq \hat{x}_A^2$, and from B otherwise. Hence, her expected total surplus from buying A in the first period is

$$\begin{aligned} u_A &= v - x_1 - p_A^1 + \\ &\lambda_o \left[\int_0^{\hat{x}_A^2} \left(v - x - \left(p_A^2(\hat{x}_1) - \gamma_A \right) \right) dx + \int_{\hat{x}_A^2}^1 \left(v - (1-x) - p_B^2(\hat{x}_1) \right) dx \right] \end{aligned}$$

When buying from B :

$$\begin{aligned} u_B &= v - (1 - x_1) - p_B^1 + \\ &\lambda_o \left[\int_0^{\hat{x}_B^2} \left(v - x - p_A^2(\hat{x}_1) \right) dx + \int_{\hat{x}_B^2}^1 \left(v - (1-x) - \left(p_B^2(\hat{x}_1) - \gamma_B \right) \right) dx \right] \end{aligned}$$

Taking the difference between the two yields, plugging in the equilibrium values of $p_A^2(\hat{x}_1)$ and $p_B^2(\hat{x}_1)$,

$$\Delta u = 1 - 2x_1 + p_B^1 - p_A^1 + \frac{1}{4}\left((\gamma_A + \gamma_B)^2 + 2(\gamma_A - \gamma_B) \right) - \frac{1}{2}(\gamma_A + \gamma_B)^2 \hat{x}_1.$$

The indifferent consumer has $\Delta u = 0$, hence

$$\hat{x}_1 = \frac{4(1 + p_B^1 - p_A^1) + (\gamma_A + \gamma_B)^2 + 2(\gamma_A - \gamma_B)}{2\left(4 + (\gamma_A + \gamma_B)^2\right)}.$$

Firm A's total profits equal

$$\Pi_A\left(p_A^1, p_A^2, \gamma_A^1, \gamma_A^2\right) = \left(p_1^A - c\right)\hat{x}_1 + \frac{1}{2} - \frac{1}{2}\left(1 - \gamma_A \hat{x}_1 (1 - \hat{x}_1)(\gamma_A + \gamma_B)\right)$$

To find the equilibrium, we have to take the first-order conditions with respect to p_A^1 and γ_A, impose symmetry, and solve the resulting system to find

$$\begin{aligned} \gamma_A &= 2/3 \\ p_A^1 &= c + \frac{13}{9} \\ p_A^2 &= c + \frac{4}{3} \\ \Pi_A &= \frac{10}{9}. \end{aligned}$$

Hence, profits are higher than in the case without coupons (when profits would be 1/2 in each period). Loyal customers pay a lower net price in period 2, yet the total price they pay is $2c + \frac{19}{9}$, which is higher than in the case without coupons.

7.3 Poaching

This model is based on Fudenberg and Tirole (2000). Consider a model of poaching. The set-up will be very similar to that in the previous chapter; two firms are located at the endpoints of a Hotelling line. There are two periods. Yet, in this case there are no switching costs, but firms can base their second-period price on the purchase history of consumers. That is, in period 2, firm A can charge different prices to consumers that bought from A in period 1, and consumers that bought from B. in period 1. Conceptually, this is close to the coupons model in the previous chapter. The crucial difference, however, is that firms cannot commit to the discount in period 1, but can only do so in period 2. Another difference is that in the coupons model, a firm can only commit to a discount to consumers that also bought from A in the previous period. It cannot choose to give a discount to consumers that bought from B in the previous period. This will make a huge difference.

Again, we solve with backward induction. In period 1, a share $\hat{x}_1$ of consumers buys from firm A. We will refer to these consumers as segment A. The remaining $1 - \hat{x}_1$ buy from firm B. We refer to them as segment B. In period 2, consider a consumer in segment A, who has thus bought from firm A in period 1. Denote the price charged by firm A to these consumers in period 2 as p^2_{AA}. The price charged to them by firm B is p^2_{BA}.

Second period In equilibrium at least some type consumers in segment A will be tempted by the poaching price of firm B.[1] The second period will then have some $\hat{x}^2_A < \hat{x}_1$ again choosing for firm A, while the remaining $\hat{x}_1 - \hat{x}^2_A$ switch to B. Something similar holds for consumers in segment B. The indifferent consumers on segments A and B are given by

$$\hat{x}^2_A = \tfrac{1}{2}\left(1 + p^2_{BA} - p^2_{AA}\right); \qquad \hat{x}^2_B = \tfrac{1}{2}\left(1 + p^2_{BB} - p^2_{AB}\right), \tag{7.1}$$

provided that these expressions are strictly between 0 and $\hat{x}_1$. We will show that this holds in equilibrium.

Second-period profits for firm A are given by

$$\Pi^2_A = \Pi^2_{AA} + \Pi^2_{AB} \equiv (p^2_{AA} - c)\hat{x}^2_A + (p^2_{AB} - c)(\hat{x}^2_B - \hat{x}_1), \tag{7.2}$$

where Π^2_{AA} (the first term) reflects total profits from loyal consumers, and Π^2_{AB} (the second term) total profits from consumers that are poached. Similarly, firm B's profits

[1] To show that that is indeed the case, we have to derive the equilibrium like we do in the main text, and then show that from the resulting equilibrium no firm has an incentive to defect to a price such that no consumers switch. But by construction, that is the case.

are given by

$$\Pi_B^2 = \Pi_{BB}^2 + \Pi_{BA}^2 \equiv (p_{BA}^2 - c)(\hat{x}_1 - \hat{x}_A^2) + (p_{BB}^2 - c)(1 - \hat{x}_B^2). \tag{7.3}$$

Plugging in the expressions from (7.1) into the second line of (7.2), we have

$$\Pi_{AA}^2 = \frac{1}{2}(p_{AA}^2 - c)\left(1 + p_{BA}^2 - p_{AA}^2\right). \tag{7.4}$$

Similarly, for firm B, from (7.3), and (7.1),

$$\Pi_{BA}^2 = (p_{BA}^2 - c)\left(\hat{x}^1 - \frac{1}{2}\left(1 + p_{BA}^2 - p_{AA}^2\right)\right), \tag{7.5}$$

Maximizing (7.4) with respect to p_{AA}^2 and (7.5) with respect to p_{BA}^2 yields the following reaction functions:

$$p_{AA}^2 = \tfrac{1}{2}\left(1 + p_{BA}^2 + c\right); \qquad p_{BA}^2 = \tfrac{1}{2}\left(2\hat{x}^1 - 1 + p_{AA}^2 + c\right).$$

Solving the system gives:

$$p_{AA}^2 = c + \tfrac{1}{3}(1 + 2\hat{x}^1); \qquad p_{BA}^2 = c + \tfrac{1}{3}(4\hat{x}^1 - 1). \tag{7.6}$$

We then immediately have

$$\hat{x}_A^2 = \frac{1}{6}\left(1 + 2\hat{x}^1\right). \tag{7.7}$$

and

$$\Pi_{AA}^2 = \tfrac{1}{18}(1 + 2\hat{x}^1)^2; \qquad \Pi_{BA}^2 = \tfrac{1}{18}(4\hat{x}^1 - 1)^2. \tag{7.8}$$

On segment B, we can do a similar analysis. Here

$$\begin{aligned} \Pi_{BB}^2 &= \frac{1}{2}(p_{BB}^2 - c)\left[1 + p_{AB}^2 - p_{BB}^2\right], \\ \Pi_{AB}^2 &= \frac{1}{2}(p_{AB}^2 - c)\left[1 - \hat{x}^1 - \frac{1}{2}\left(1 + p_{AB}^2 - p_{BB}^2\right)\right]. \end{aligned}$$

Hence

$$p_{AB}^2 = c + \tfrac{1}{3}(3 - 4\hat{x}^1); \qquad \Pi_{AB}^2 = \tfrac{1}{18}(3 - 4\hat{x}^1)^2. \tag{7.9}$$

First period We now solve for the first period. Consumers are forward-looking and rationally take into account the events that will unfold in the second period. A consumer that is indifferent between A and B in period 1 thus anticipates that, whatever she chooses, she will switch in period 2. Denoting the discount factor by δ, the indifferent type i located at $\hat{x}_1$ has

$$\begin{aligned} v - \hat{x}_1 - p_A^1 &+ \delta(v - (1 - \hat{x}_1) - p_{BA}^2) \\ &= v - (1 - \hat{x}_1) - p_B^1 + \delta(v - \hat{x}_1 - p_{AB}^2), \end{aligned} \tag{7.10}$$

where the left-hand side gives her total lifetime utility if she chooses A in period 1, while the right-hand side gives that of choosing B in period 1.[2] Solving (7.10) gives

$$\hat{x}^1 = \frac{1 + p_B^1 - p_A^1 - \delta(1 + p_{BA}^2 - p_{AB}^2)}{2(1-\delta)}. \tag{7.11}$$

Substituting second-period equilibrium prices from (7.6) and (7.9) and solving for $\hat{x}^1$ yields

$$\hat{x}^1 = \frac{1}{2} + \frac{3(p_B^1 - p_A^1)}{6 + 2\delta}. \tag{7.12}$$

In the first period, firm A sets p_A^1 as to maximize total discounted profits

$$\begin{aligned} \Pi_A &= (p_A^1 - c)\hat{x}_1 + \delta\Pi_{AA}^2 + \delta\Pi_{AB}^2 \\ &= (p_A^1 - c)\hat{x}_1 + \frac{\delta}{18}(1 + 2\hat{x}^1)^2 + \frac{\delta}{18}(3 - 4\hat{x}^1)^2. \end{aligned} \tag{7.13}$$

Taking the derivative with respect to p_A^1 :

$$\frac{\partial \Pi_A}{\partial p_A^1} = (p_A^1 - c)\frac{\partial \hat{x}^1}{\partial p_A^1} + \hat{x}^1 + \frac{2\delta}{9}\left(1 + 2\hat{x}^1\right)\frac{\partial \hat{x}^1}{\partial p_A^1} - \frac{4\delta}{9}\left(3 - 4\hat{x}^1\right)\frac{\partial \hat{x}^1}{\partial p_A^1}.$$

A symmetric equilibrium requires $p_A^1 = p_B^1$ hence $\hat{x} = \frac{1}{2}$. From (7.12), we have $\frac{\partial \hat{x}^1}{\partial p_A^1} = \frac{-3}{6+2\delta}$. Hence, the FOC becomes

$$\frac{1}{2} - \frac{3}{6 + 2\delta}\left(p_A^1 - c\right) = 0.$$

This yields equilibrium prices

$$\begin{aligned} p_A^1 &= c + 1 + \frac{\delta}{3}. \\ p_{AA}^2 &= c + \frac{2}{3} \\ p_{AB}^2 &= c + \frac{1}{3} \\ \Pi_A &= \frac{1}{2} + \frac{4}{9}\delta \end{aligned}$$

In a model with standard Hotelling competition, without poaching we would have $p = c + 1$ in each period, so $P = (c+1)(1+\delta)$. In this case loyal consumers pay

[2] Note that in this model, it is crucial to have a discount factor $\delta < 1$. With $\delta = 1$, $\hat{x}^1$ would drop out of this equality; the indifferent consumer knows that she will always consume one product in this period and the other product in the next. Without discounting, she would not care which one to consume first.

$P_{\text{loyal}} \equiv p_A^1 + \delta p_{AA}^2 = \left(c + 1 + \frac{\delta}{3}\right) + \delta\left(c + \frac{2}{3}\right) = (1+c)(1+\delta)$, so they are unaffected. Consumers that are poached end up paying $P_{\text{poach}} = p_A^1 + \delta p_{AB}^2 = c + 1 + \frac{\delta}{3} + \delta\left(c + \frac{1}{3}\right) = 1 + \frac{2\delta}{3} + c(1+\delta) < (1+c)(1+\delta)$, the difference being $-\delta/3$. Hence they are better off. Note that consumers that are poached pay a lower price than loyal consumers. Firms are worse off; their profits decrease.

Doing a full welfare analysis is harder than in the models with switching costs; we have to take into account that some consumers that are located close to the middle no longer consume their preferred product in period 2. More specifically, if we focus on segment A, consumers located between $\hat{x}_A^2$ and $1/2$ now face transportation costs $1-x$ which in their case is higher than x, which are the transportation costs they face if they would not switch. That means that total welfare goes down because of poaching; again, prices are just transfers between firms and consumers and do not affect welfare.

To evaluate consumer welfare of consumers in segment A, first note from 7.7 that $\hat{x}_A^2 = 1/3$. The total additional discounted disutility from traveling for these consumers thus equals

$$\int_{\hat{x}_A^2}^{1/2} \delta(1-x-x)\,dx = \int_{1/3}^{1/2} (1-x-x)\,dx = \frac{\delta}{36}.$$

The total savings in price for these consumers equal

$$\int_{1/3}^{1/2} \frac{1}{3}\delta dx = \frac{\delta}{18}.$$

Hence, they are better off as a result. The analysis for consumers in segment B is of course identical.

Exercises

1. When discussing the model of coupons, I argued that this model only makes sense and yields interesting results if we assume that consumer tastes change. Explain why that is the case (i.e. what would happen with coupons in a model where tastes do not change?).

2. Consider the model of poaching. Derive the equilibrium prices in both periods in a case in which all consumers also have switching costs s that are relatively small. Assume again that consumers are forward looking.

Chapter 8

Vertical Control

8.1 Introduction and acknowledgement

So far, we have assumed that the monopolist sells its product directly to final consumers. Often, this is not the case. Manufacturers usually sell their product to retailers, which in turn sell their product to the final consumer. This may give rise to a *double marginalization problem* as we will explain in this chapter. In the literature, the manufacturer producing the product is often referred to as the *upstream firm*, whereas the retailer that sells the product to the final consumer is referred to as the *downstream firm*.

What follows is based on Tirole, chapter 4. Section 8.4 closely follows Whinston (2006)

8.2 The Problem

Consider the following set-up. A monopolist producer sells its product to a monopolist retailer. The retailer then sells the product to the final consumer. The marginal costs of the monopolist are c. It sets a per-unit price p_w. We thus assume a *linear price*: a contract specifying only a payment $T(q) = p_w q$ from retailer to manufacturer. For simplicity, the retailer has no retailing costs: the only costs it faces are those of purchasing the product from the manufacturer. Demand from final consumers is given by $D(p) = 1 - p$. Note that we have a two-stage problem:

Stage 1 The monopolist manufacturer sets p_w.

Stage 2 The monopolist retailer sets p.

We solve the model using backward induction. Given the price p_w the manufacturer sets, the retailer's problem is to set p to

$$\max_p \left[(p - p_w)(1 - p) \right],$$

so

$$p^* = \frac{1+p_w}{2}. \tag{8.1}$$

Demand for the final good, and hence for the intermediate good, is then given by

$$q = \frac{1-p_w}{2}.$$

The retailer's profit thus is

$$\Pi_r = \left(\frac{1-p_w}{2}\right)^2.$$

The manufacturer's problem is then to

$$\begin{aligned}\max_{p_w} \Pi_m &= (p_w - c)(1-p) \\ &= (p_w - c)\left(\frac{1-p_w}{2}\right).\end{aligned}$$

Hence

$$p_w^* = \frac{1+c}{2}.$$

The final price thus equals

$$p^* = \frac{1+\frac{1+c}{2}}{2} = \frac{3+c}{4}.$$

Total profits of the non-integrated structure are given by

$$\Pi^{ni} = \Pi_m + \Pi_r = \frac{(1-c)^2}{8} + \frac{(1-c)^2}{16} = \frac{3}{16}(1-c)^2.$$

Now consider the integrated industry or, equivalently, the case in which the manufacturer sells directly to the public. Total profits are then given by

$$\Pi^i = (p-c)(1-p),$$

which are maximized by setting

$$p_m^* = \frac{1+c}{2},$$

so total profits are

$$\Pi^i = \frac{(1-c)^2}{4} > \Pi^{ni}.$$

The model is sketched in the figure below. Demand from final consumers is given by demand curve D. The manufacturer's marginal costs are given by c. It will then set price p_w^*, that maximizes its profits.[1] The retailer has to buy its product from the monopolist

[1]Note that in the figure and the accompanying discussion, it is as if the manufacturer sets its price without taking into account that the retailer will subsequently set a price as well. With a linear demand curve, this is immaterial: the manufacturer sets the same price $p = (1+c)/2$ that he would set if he

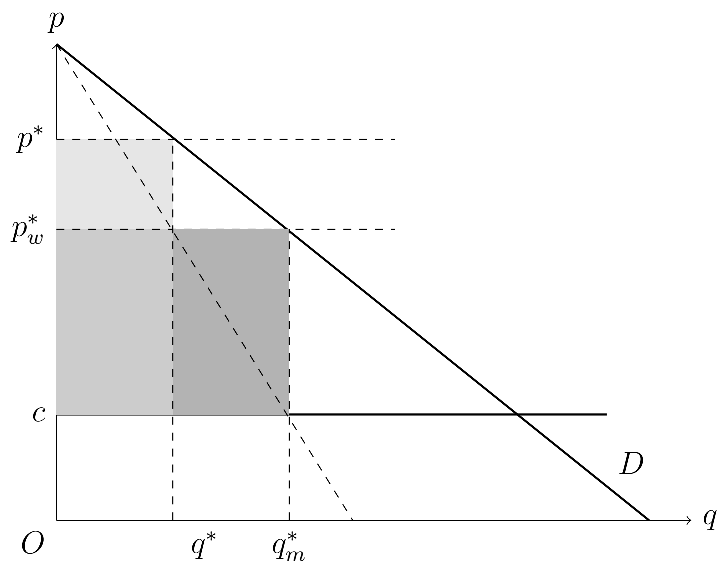

Figure 8.1: Double Marginalization

and then sells it to the final consumer. For each unit, it has to pay a price p_w^*. Hence, p_w^* are the marginal costs of the retailer. Given those marginal costs, it will set price to maximize profits, i.e. it sets a final price to consumers that equals p^*.

Total sales now equal q^*. Profits of the retailer are given by the light-shaded square in the figure. Profits of the manufacturer are given by the slightly darker-shaded rectangle. A manufacturer selling directly to the final consumer would set a price p_w^* and have sales q_m^*. Its total profits would then equal the two darkest-shaded rectangles in the figure. This would also be the case with vertical integration, i.e. when the upstream manufacturer would merge with the downstream retailer.

We thus have that the integrated industry makes more profit than the non-integrated industry. In the non-integrated case, there are two firms with market power. Both firms distort prices, thereby hurting each other. Both firms, when setting their price, do not take into account the negative external effect their price has on the other monopolist. Spengler (1950) coined this the *double marginalization problem*: the problem is caused by both firms taking a positive profit margin. Indeed, there is no double marginalization problem if one of the two firms is competitive.

Note that the double marginalization problem is a problem not only for the two monopolists, but also for consumers: with an integrated monopoly, they would pay a lower price than with a non-integrated monopoly. Hence, a move toward integration would be a Pareto improvement: profits of the integrated firm would increase, but consumer prices would also come down.

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would be a monopolist selling directly to consumers. With a non-linear demand function however, this is no longer true.

8.3 Solving the problem

There are a number of ways in which the monopolist manufacturer can solve the double marginalization problem. Note that in the discussion below, we will always assume that the manufacturer has all the bargaining power, and can offer a contract to the retailer that the retailer is only able to either accept or reject.

The first solution is **vertical integration.** This is the most obvious solution, already suggested above.

Another solution for the manufacturer is to offer a different type of contract. In particular. it can set a **franchise fee**. The retailer then has to pay a fixed fee A to sell the manufacturer's product in the first place, plus a per unit price p_f. The manufacturer thus uses a *two-part tariff*: $T(q) = A + p_f q$, for $q > 0$. The game is then as follows:

Stage 1 The manufacturer sets $T(q)$.

Stage 2 The retailer sets p.

Again, we solve using backward induction. Given the tariff $T(q)$ the manufacturer sets, the retailer's problem is to set p to

$$\max_p \left[(p - p_f)(1 - p) - A\right],$$

which again yields

$$p^* = \frac{1 + p_f}{2}.$$

The retailer's profit thus is

$$\Pi_r = \left(\frac{1 - p_f}{2}\right)^2 - A.$$

Moving back to stage 1, the manufacturer's problem is to

$$\begin{aligned} \max_{A,p_f} \Pi_m &= (p_f - c)(1 - p) + A \\ \text{s.t. } \Pi_r &\geq 0 \end{aligned}$$

hence

$$\begin{aligned} \max_{A,p_f} \Pi_m &= (p_f - c)\left(\frac{1 - p_f}{2}\right) + A \\ \text{s.t. } \left(\frac{1 - p_f}{2}\right)^2 - A &\geq 0 \end{aligned}$$

This problem is easy to solve. First note that the manufacturer will set A such that the constraint holds with equality: it can be easily seen from the maximand that a higher

A unambiguously increases the manufacturer's profit, hence it wants to set it as high as possible. Plugging the resulting A back into the profit function, we obtain the following:

$$\begin{aligned} A^* &= \left(\frac{1-p_f}{2}\right)^2, \\ p_f^* &= \arg\max_{p_f} \Pi_m = (p_f - c)\left(\frac{1-p_f}{2}\right) + \left(\frac{1-p_f}{2}\right)^2 \end{aligned}$$

We now have

$$\frac{\partial \Pi_m}{\partial p_f} = \frac{1}{2}(c - p_f),$$

so

$$p_f^* = c; \qquad A^* = (1-c)^2/2.$$

The manufacturer thus sets its price equal to marginal cost, hence avoiding the double marginalization problem. However, the manufacturer captures the entire profits of the downstream retailer through the franchise fee. The market outcome is the same as in the case of an integrated monopoly. Effectively, this solution amounts to selling the vertical structure at price A to the downstream monopolist.

There are some drawbacks to a franchise fee. First, when the retailer is risk-averse, and there is uncertainty in the model, the retailer bears too much risk. Second, the retailer may have private information. Third, this solution does not work with multiple retailers.

A third solution to the double marginalization problem is the use of **resale price maintenance**. Suppose the monopolist is only willing to supply the product to the retailer when the latter charges a price p_{rpm} to the consumer. Formally, we have $T(q) = p_r q$ if and only if $p = p_{rpm}$. Obviously, the manufacturer will charge $p_r^* = p_m^*$ and impose $p_{rpm}^* = p_m^*$, with p_m^* the price that would be set by an integrated monopolist. Note that the double marginalization problem is now solved by the retailer rather than the manufacturer setting price equal to marginal cost.

A fourth solution is **price fixing**. In that case, the manufacturer specifies the amount q to be bought by the retailer. In the context of this simple model, this boils down to the same solution as resale price maintenance. With price fixing, we have $T(q) = p_{pf} q$ if and only if $q = q_{pf}$. Obviously, the manufacturer will then choose to set $p_{pf}^* = p_m^*$ and $q_{pf}^* = q_m^*$.

8.4 Exclusive contracts

In some cases, a manufacturer commits itself to deal with only one retailer. This is the case of *exclusive contracts.* At first sight, such contracts may be hard to understand: more downstream competition implies lower prices for the final consumer, which implies higher sales for the manufacturer. Hart and Tirole (1990) provide a model that can explain such exclusive dealing. The discussion in this section is based on Whinston (2006) who uses a more general model. There is a single upstream manufacturer (M) and two downstream retailers (R_A and R_B). M has constant unit cost c_M, the retailers have constant unit cost c_R, over and above the costs of obtaining the product from M.

Without exclusive contracts, the game is as follows. First, M makes simultaneous private offers to each R_j in the form (q_j, t_j), with q the quantity that the manufacturer offers and t the total payment that is required. Second, retailers simultaneously announce whether or not they accept the offer. Third, retail competition occurs. In the final stage, each retailer offers all the units he has purchased and the market clears. In the most general case, prices are $p_j(q_A, q_B)$, so profits are given by $\pi_j(q_A, q_B) = [p_j(q_A, q_B) - c_R]q_j - t_j$ and $\pi_M(q_A, q_B) = t_A + t_B - c_M(q_A, q_B)$. When retailers are active in distinct local markets, the price that one retailer receives will not be affected by the output of the other retailer, so $p_j(q_A, q_B) = P_j(q_j)$. In the case of one common market and homogeneous products, both retailers receive the same price for their product, so $p_j(q_A, q_B) = P(q_A, q_B)$. The sales levels (q_A^{**}, q_B^{**}) that maximize joint profits are given by

$$(q_A^{**}, q_B^{**}) = \arg\max_{q_A, q_B} \sum_{j=A,B} [p_j(q_A, q_B) - (c_M + c_R)]\, q_j.$$

This is exactly the problem of a monopolist selling two differentiated products, each with marginal costs $c_M + c_R$. For undifferentiated retailers, any (q_A, q_B) that sum to the joint profit-maximizing Q^{**} would solve this problem. Define as Π^{**} the resulting profit level. It is also useful to consider the joint profit-maximizing sales level if only one of the two products is being sold. For product j, we denote this sales level as x_j^e. Thus, x_j^e solves

$$x_j^e = \max_{x_j} [p_j(q_j, 0) - (c_M + c_R)]\, q_j.$$

This yield joint profits of $\Pi_j^e \leq \Pi^{**}$.

When a retailer receives an offer from the manufacturer, he must form some conjecture about the offer that the other retailer has received. With *passive beliefs* each retailer R_j has a fixed conjecture about the offer that is received by the other retailer R_{-j}. In equilibrium, each fixed conjecture must be correct. Hence, any manufacturer-retailer pair must then agree to a contract that maximizes their joint payoff taking as given the contract being signed between M and R_{-j}. Given the contract offered to firm B, the best that the manufacturer can do is to offer A a contract with a quantity q_A that is such that As profits are maximized, and then to charge an amount t_A that equals those profits.

Hence (q_A^*, q_B^*) must satisfy the following conditions:

$$\begin{aligned} q_A^* &= \arg\max_{x_A} \left[p_A(q_A, q_B^*) - (c_M + c_R)\right] q_A \\ q_B^* &= \arg\max_{x_B} \left[p_B(q_A^*, q_B) - (c_M + c_R)\right] q_B \end{aligned}$$

But these are exactly the conditions that would hold if R_A and R_B competed as duopolists, each with marginal costs $c_M + c_R$. In distinct local markets, this outcome coincides with the joint monopoly outcome. Hence, when the contract that M signs with A does not have an effect on B (hence when contracting externalities are absent), bilateral contracting achieves the joint profit-maximizing outcome. But now suppose that products are homogeneous and A and B operate on the same market. The equilibrium contracts then have

$$\begin{aligned} q_A^* &= \arg\max_{x_A} \left[P(q_A + q_B^*) - (c_M + c_R)\right] q_A \\ q_B^* &= \arg\max_{x_B} \left[P(q_A^* + x_B) - (c_M + c_R)\right] q_B \end{aligned}$$

This is exactly the Cournot outcome. This immediately implies that the manufacturer M can also not earn more than the total Cournot profit. The manufacturer is hampered by a commitment problem: it would like to offer each retailer half of the monopoly output at a price that equals half of the monopoly profit. But it cannot credibly commit to do so: if one retailer has signed such a contract, the manufacturer has an incentive to offer a higher amount to the other retailer. In equilibrium, the manufacturer should not have an incentive to renege on its contracts. This is only the case if it offers Cournot outputs. Note that this logic is very similar to that of the durable goods monopolist, where the monopolist is also hurt by its inability to commit.

Now introduce the possibility that M may offer an exclusive contract to a retailer. A retailer that receives an offer from the manufacturer then knows that the other retailer has not received (and will not receive) any offer. We still assume that without an exclusive contract, a retailer believes that M has offered the other retailer his equilibrium nonexclusive quantity. The equilibrium then involves exclusive contracts for j whenever $\Pi_j^e > \max\left\{\Pi_{-j}^e, \hat{\Pi}\right\}$, with $\hat{\Pi}$ the profits of M in the nonexclusive equilibrium. If $\hat{\Pi} > \max\left\{\Pi_A^e, \Pi_B^e\right\}$, the equilibrium thus has nonexclusive contracts. With retailers selling in distinct local markets, we never get exclusives as $\hat{\Pi} = \Pi_A^e + \Pi_B^e = \Pi^{**} > \max\{\Pi_A^e, \Pi_B^e\}$. With undifferentiated retailers, $\Pi_A^e = \Pi_B^e = \Pi^{**} > \hat{\Pi}$, so we always get exclusive contracts.

Exercise

A monopolist manufacturer sells its products to two downstream firms, using a contract of the form (q_j, t_j) with q_j the quantity produced and t the total payment that is required. For simplicity, all marginal costs are zero. Consumers' tastes differ for the two retailers. Their preferences can be described by a Perloff-Salop model as described in exercise 3 of chapter 1: for each of the two retailers, consumers have a preference that is uniformly distributed on $[0, 1]$, but $v = 0$ so not all consumers buy in equilibrium.

1. Derive the manufacturer's profits if it sells to both downstream firms
2. Derive the manufacturer's profits if it uses an exclusive contract.

Chapter 9

Bundling

9.1 Introduction

In this chapter we return to the case of a monopolist firm. We will now assume, however, that the monopolist sells two products. It can then choose to either sell the two products separately, or to sell them as a bundle. Selling the products as a bundle may increase its profits, and may also serve as an mechanism to deter entry.

To see that this may be the case, consider the following simple example. A monopolist sells two products. Costs are normalized to zero. There is a mass of consumers that equals 1. Consumers want to purchase at most one unit of each product. Half of all consumers have a willingness to pay for product 1 that is equal to 9, while they are willing to pay 3 for product 2. For the other half of the consumers, things are exactly the other way round: they are willing to pay 9 for product 2, but only 3 for product 1.

First consider the case in which the monopolist sells the two products separately. His profit-maximizing price is then to charge 9 for each product. If he does so, each consumer will only sell one product, the monopolist sells each product to half of all consumers, and his profit on each product is 9/2, giving 9 in total profits. To sell all products to all consumers, he would have to set each price equal to 3, which would yield total profits of only 6.

Now suppose that the monopolist does no longer sell the two products separately, but only sells them as a bundle containing one unit of product 1, and one unit of product 2. Using the same valuations as above, each consumer is willing to pay 12 for such a bundle. Hence, the monopolist will set price for the bundle equal to 12. This yields profits of 12, which is higher than his profits if he were to sell the two products separately.

This example suggests that bundling is a profitable strategy whenever the valuations of consumers for the two products are negatively correlated. This, however, is not a necessary condition. In section 9.2, we will show that in the case of independent valuations, the monopolist can still gain by bundling. In section 9.3, we derive the optimal bundling strategies in the case that consumers' valuations are independently and uniformly distributed. Section 9.5 considers the role of bundling as an entry barrier.

9.2 Bundling with independent valuations

Consider a generalized version of the example in the introduction. A monopolist sells two products. There is a mass of consumers with size 1. Valuations for product 1 are given by the probability distribution F_1. Thus, the number of consumers with a valuation for product 1 that is lower than or equal to v is given by $F_1(v)$. We will denote the valuation of a consumer for product 1 as v_1, and that of product 2 as v_2. Similarly, valuations for product 2 are given by the probability distribution F_2. We will denote the valuation of a consumer for product 1 as v_1, and that of product 2 as v_2. Note that we implicitly assume that for each consumer, her valuation for product 1 is independent of her valuation for product 2. Also note that the set-up in terms of consumer preferences is essentially identical to that in the Perloff/Salop model we discussed in the first chapter. Costs of production are normalized to zero.

Denote as p_1^* and p_2^* the prices that the monopolist would set if he would only sell the products separately. The monopolist has three options. First, he could indeed price each bundle separately. Second, he could offer the two products *only* as a bundle. Following Adams and Yellen (1976) we refer to this case as one of *pure bundling.* Third, the monopolist can also choose to sell the items both separately and as a bundle, with a price for the bundle that is different from the sum of the single-good prices. This is a case of *mixed bundling.*

First if all, it is easy to see that firms are always at least as well off with mixed bundling as they are with pure bundling; it has always the option to set the individual prices prohibitively high. But we can also show the following:

Theorem 6 *With independent valuations, mixed bundling always dominates nonbundling.*

Proof. Denote the profit-maximizing nonbundling prices as p_1^* and p_2^*. Now suppose that the monopolist instead offers a bundle price $p_B^* = p_1^* + p_2^*$, and separate prices $p_1 = p_1^*$ and $p_2 = p_2^* + \varepsilon$. Consider a consumer. His net utility from buying product 1 is $v_1 - p_1^*$, that from buying product 2 is $v_2 - p_2^* - \varepsilon$, and that from buying the bundle $v_1 + v_2 - p_1^* - p_2^*$. That implies that he will only buy product 1 if

$$\begin{aligned} v_1 - p_1^* &\geq 0 \\ v_2 - p_2^* - \varepsilon &< 0 \\ v_1 - p_1^* &> v_1 + v_2 - p_1^* - p_2^* \end{aligned}$$

which is true if $v_1 \geq p_1^*$ and $v_2 < p_2^*$ (see the figure). The consumer will choose product

2 if

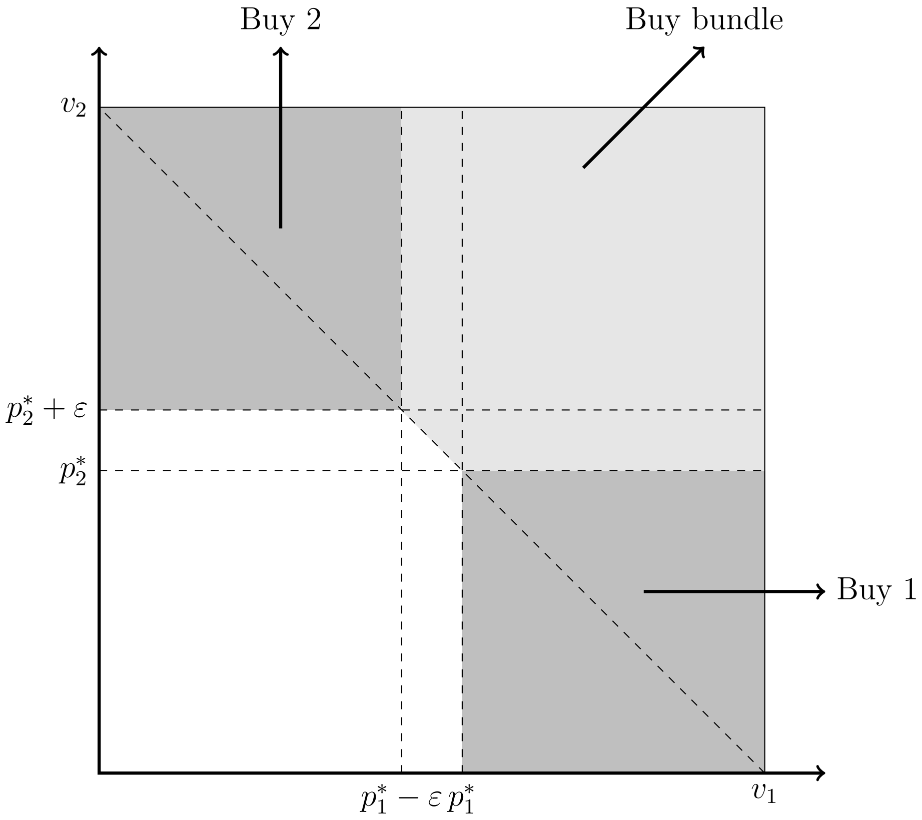

Figure 9.1: Bundling is profitable.

$$\begin{aligned} v_1 - p_1^* &< 0 \\ v_2 - p_2^* - \varepsilon &\geq 0 \\ v_2 - p_2^* - \varepsilon &> v_1 + v_2 - p_1^* - p_2^* \end{aligned}$$

which is true if $v_1 < p_1^* - \varepsilon$ and $v_2 \geq p_2^* + \varepsilon$. The consumer will choose to consume the bundle if

$$\begin{aligned} v_1 - p_1^* &\leq v_1 + v_2 - p_1^* - p_2^* \\ v_2 - p_2^* - \varepsilon &\leq v_1 + v_2 - p_1^* - p_2^* \\ v_1 + v_2 - p_1^* - p_2^* &\geq 0 \end{aligned}$$

which implies $v_2 \geq p_2^*$, $v_1 \geq p_1^* - \varepsilon$, and $v_1 + v_2 \leq p_1^* + p_2^*$. In all other cases, the consumer will buy nothing.

Now consider the case without bundling. In that case, we simply have that all consumers with $v_1 \geq p_1^*$ buy product 1, while all consumers with $v_2 \geq p_2^*$ buy product 2.

Let's consider a change from the case without mixed bundling to the one with mixed bundling. First of all note that those with $v_1 \geq p_1$ are unaffected; those that only bought product 1 before also do that now, with the price still at p_1^*. Those who bought both products before buy the bundle now and, by construction, pay the same price.

So we only have to compare the profit $\tilde{\Pi}$ made on consumers with $v_1 < p_1^*$. First of all, these include profits from consumers that buy product 2. The consumers that choose to do so have $v_1 < p_1^* - \varepsilon$ and $v_2 < p_2^* + \varepsilon$, so there mass is $[1 - F_2(p_2^* + \varepsilon)]\,[F_1(p_1^* - \varepsilon)]$. Consumers that buy the bundle and are in the relevant interval are those with $v_1 < p_1^*$ and $v_2 > p_1^* + p_2^* - v_1$ (see figure). The mass of these consumers is $\int_{p_1^*-\varepsilon}^{p_1^*} f_1(s)\,[1 - F_2(p_1^* + p_2^* - s)]\,ds$. The relevant profits thus are

$$
\begin{aligned}
\tilde{\Pi} &= (p_2^* + \varepsilon)\,[1 - F_2(p_2^* + \varepsilon)]\,[F_1(p_1^* - \varepsilon)] \\
&\quad + (p_1^* + p_2^*) \int_{p_1^*-\varepsilon}^{p_1^*} f_1(s)\,[1 - F_2(p_1^* + p_2^* - s)]\,ds
\end{aligned}
$$

Take the derivative of these with respect to ε :

$$
\begin{aligned}
\frac{\partial \tilde{\Pi}}{\partial \varepsilon} &= [1 - F_2(p_2^* + \varepsilon)]\,[F_1(p_1^* - \varepsilon)] \\
&\quad + (p_2^* + \varepsilon)\,[-f_2(p_2^* + \varepsilon)F_1(p_1^* - \varepsilon) - f_1(p_1^* - \varepsilon)\,(1 - F_2(p_2^* + \varepsilon))] \\
&\quad + (p_1^* + p_2^*)\,f_1(p_1^* - \varepsilon)\,[1 - F_2(p_2^* + \varepsilon)]
\end{aligned}
$$

Evaluate this in $\varepsilon = 0$ to obtain

$$
\begin{aligned}
\left.\frac{\partial \Pi}{\partial \varepsilon}\right|_{\varepsilon=0} &= [1 - F_2(p_2^*)]\,[F_1(p_1^*)] + p_2^*\,[-f_2(p_2^*)F_1(p_1^*) - f_1(p_1^*)\,(1 - F_2(p_2^*))] \\
&\quad + (p_1^* + p_2^*)\,f_1(p_1^*)\,[1 - F_2(p_2^*)] \\
&= [1 - F_2(p_2^*) - p_2^* f_2(p_2^*)]\,F_1(p_1^*) + p_1 f_1(p_1^*)\,[1 - F_2(p_2^*)]
\end{aligned}
$$

Optimality of p_2^* requires

$$
\frac{\partial}{\partial p_2}\left((1 - F_2(p_2))\,p_2\right) = 1 - F_2(p_2) - f_2(p_2)p_2 = 0,
$$

so we have

$$
\left.\frac{\partial \Pi}{\partial \varepsilon}\right|_{\varepsilon=0} = p_1 f_1(p_1^*)\,[1 - F_2(p_2^*)] > 0
$$

as $f_1 > 0$. Hence, by starting from the unbundled monopoly solution and using the strategy described above, the monopolist can always strictly increase profits by choosing ε sufficiently small. ■

McAfee et al. (1989) analyze the case in which valuations for product 1 and product 2 are *not* independently distributed. They derive general conditions under which bundling increases profits. These conditions boil down to the requirement that valuations are not too closely correlated, confirming the intuition from the numerical example in the introduction.

9.3 Bundling with uniform distributions

In the previous section, we showed that with independent valuations, a monopolist that sells two products can always strictly increase profits by also offering a bundle. In this section, we derive the optimal prices in the case that valuations are uniformly distributed on $[0,1]$, and that costs are zero. For simplicity, we restrict attention to the case in which the monopolist uses pure bundling. It can be shown that, in this particular case, the monopolist cannot do strictly better by using mixed bundling.

First, consider the case in which the monopolist would only sell separate products. He would then set $p_1 = p_2 = 1/2$, and profits would equal $\pi = 1/2$. Now consider the case in which he would only offer a pure bundle. For our analysis, we have to distinguish between two cases: one in which the monopolist offers a bundle price $p_B < 1$, and one in which he offers a bundle price $p_B \geq 1$. In the latter case, his profits equal

$$\Pi = \frac{1}{2}(2 - p_B)^2 p_B.$$

As this is strictly decreasing for $p_B \in [1,2]$, the optimal solution in this interval would be to set $p_B^* = 1$, which yields profits $\pi = 1/2$. Now consider the case that $p_B < 1$. In that case, profits equal

$$\Pi = \left(1 - \frac{1}{2}p_B^2\right) p_B$$

Maximizing this yields $p_B = \frac{1}{3}\sqrt{6}$, which implies profits of $\frac{2}{9}\sqrt{6} \approx 0.544\,33$. Hence, this is the profit-maximizing solution.

To evaluate the welfare effects of bundling in this example, we first have to calculate net consumer surplus. In the case of independent pricing, this equals

$$CS = 2\int_{1/2}^{1}\left(v - \frac{1}{2}\right) dv = \frac{1}{4}.$$

In the case of bundling, we have

$$\begin{aligned} CS &= \int_0^{\frac{1}{3}\sqrt{6}} \int_{\frac{1}{3}\sqrt{6}-v_1}^{1} \left(v_1 + v_2 - \frac{1}{3}\sqrt{6}\right) dv_2 dv_1 + \int_{\frac{1}{3}\sqrt{6}}^{1} \int_0^1 \left(v_1 + v_2 - \frac{1}{3}\sqrt{6}\right) dv_2 dv_1 \\ &= \frac{5}{3} - \frac{31}{54}\sqrt{6} \approx 0.26048. \end{aligned}$$

Thus, in this particular example, both the monopolist and consumers gain from bundling.

9.4 Competition in bundling

Suppose that there are two firms A and B, that sell two separate products, 1 and 2. For each product, consumers are distributed on a Hotelling line of unit length, while firms A and B are located at 0 and 1, respectively. For simplicity, transportation costs are equal for both products and given by t. Also, willingness to pay for each product is v for each consumer. Essentially, we thus have that consumers are uniformly distributed on a unit square, with firm A located at $(0,0)$ and firm B at $(1,1)$, where the horizontal axis represents preferences for product 1, and the vertical axis those for product 2. Note that the set-up as well as the analysis are identical to that in the Matutes and Regibeau (1988) model of compatibility: essentially, if both firms sell bundles, this is equivalent to them selling incompatible products. If they don't, then their products are compatible.

Suppose that the two products are sold separately. Essentially, we then have two separate Hotelling analyses. Hence equilibrium prices for both products will be equal to $p^* = t$ and total profits for each firm will be $\pi^* = t$.

Now suppose that both firms decide to bundle their product. For simplicity, we assume that they do not have the option of mixed bundling. A consumer now has to decide to either buy both products from firm A, or to buy both from B. Denote bundle prices of firms A and B as p_A and p_B. Denoting a consumers location as (x,y), the indifferent consumer thus has

$$2v - t(x+y) - p_A = 2v - t(2-x-y) - p_B.$$

We thus have a set of indifferent consumers, given by the line

$$y = 1 + \frac{p_B - p_A}{2t} - x.$$

When $p_A \geq p_B$, the total mass of consumers opting for A is the triangle below the line that crosses both the horizontal and the vertical axis at $x = 1 + (p_B - p_A)/2t$. The total demand for firm A is thus given by

$$Q_A = \frac{1}{2}\left[1 + \frac{p_B - p_A}{2t}\right]^2,$$

the remaining consumers go to firm B. Profits are thus given by

$$\begin{aligned} \Pi_A &= \frac{p_A}{2}\left[1 + \frac{p_B - p_A}{2t}\right]^2 \\ \Pi_B &= p_B\left[1 - \frac{1}{2}\left(1 + \frac{p_B - p_A}{2t}\right)^2\right]. \end{aligned}$$

Maximizing profits and imposing symmetry yields $p_A^* = p_B^* = t$, and $\Pi_A^* = \Pi_B^* = t/2$. Consumer surplus is

$$CS = 2\int_0^1 \int_0^{1-x} \left[2v - t\left(x+y\right) - p^*\right] dydx = 2v - 5t/3.$$

When the two products are sold separately by both suppliers, consumer surplus is

$$CS = 4\int_0^{1/2} \int_0^{1/2} \left[v - t\left(x+y\right) - 2t\right] dxdy = 2v - 5t/2.$$

Consumers are thus better off with bundling, but profits are lower. Social welfare with separate products is higher ($2v - \frac{2}{3}t$ vs. $v - \frac{1}{2}t$).

Intuitively, when firms consume in bundles, competition becomes fiercer as losing a consumer now implies losing two sales rather than one.

Now consider the case that firm A only sells a bundle at price p_A while firm B sells both products separately at prices p_{B1} and p_{B2}. This does not substantially change the analysis above; given that firm A only sells bundles, and given that we assume that the market is covered, this implies that any consumer still only has the choice to either sell both products from A or both products from B. Total prices will thus be the same as in the analysis of competing bundles. In turn, this implies that in this set-up, the only equilibrium has both firms selling separate products rather than bundles.

9.5 Bundling as an entry barrier

Nalebuff (2004) argues that the role of bundling as an entry deterrent may be much more important than its role as a means of price discrimination, which we had above. He makes his point in a model with a similar set-up to the one we discussed previously: valuations for each product are uniformly distributed on $[0, 1]$, and the costs of the monopolist are normalized to zero.

Assume however that this monopolist faces an entry threat. The potential entrant can sell a product that is a perfect substitute for either product 1 or product 2. The monopolist does not know which product the potential entrant is able to produce. Both are equally likely. Assume that the monopolist has to commit to prices before the entrant enters.

First consider the case of independent pricing. If the monopolist sets the unrestricted monopoly price, $p_1 = p_2 = 1/2$. This implies that the entrant will enter with a price $p_i = 1/2 - \varepsilon$ whenever fixed entry costs are smaller than $1/4$. To deter entry of an entrant with entry costs E requires the incumbent to set prices p such that its profits are not higher than $2E$. By slightly undercutting the incumbent's price of one product, the entrant can then exactly earn E. The incumbent's choice is thus either to accept entry and make profits of $1/4$, or to deter it and make $2E$. He will thus choose to deter if $E > 1/8$.

Now assume that the incumbent uses pure bundling. If he would sell the bundle at price $1/2$, it would still sell to half of the market and make profits $1/2$. But entry now becomes much more difficult for the entrant. Suppose that he enters the market with product 2, and sells it at a price slightly below $1/2$. It will then sell to only those consumers with $v_2 \geq 1/2$, and $v_1 + v_2 - 1 \leq v_2 - 1/2$, i.e. $v_1 \leq 1/2$. Hence, the potential entrant will make profits of only $1/8$. Nalebuff (2004) refers to this as the *pure bundling effect.* However, there is also a second effect, which is the *bundle discount effect*: a monopolist that does not face the threat of entry, has an incentive to price the bundle at $\frac{1}{3}\sqrt{6}$, which is lower than 1, the sum of separate monopoly prices. This lower price makes it even harder for the entrant to profitably enter.

Suppose that the incumbent sells the bundle at price p_B. Now the entrant enters with price p_E. For ease of exposition, we assume that the entrant enters with product 2. A consumer will buy from the entrant whenever

$$\begin{aligned} v_2 &\geq p_E \\ v_1 + v_2 - p_B &< v_2 - p_E, \end{aligned}$$

which implies

$$\begin{aligned} v_2 &\geq p_E \\ v_1 &< p_B - p_E \end{aligned}$$

Its profits thus equal

$$(1 - p_E)(p_B - p_E)\, p_E.$$

Suppose that entry is just not profitable in the case that the incumbent sets the unconstrained profit-maximizing bundle price of $\frac{1}{3}\sqrt{6}$. At such a price, the potential entrant would maximize profits upon entry by setting p_E to maximize

$$(1 - p_E)\left(\frac{1}{3}\sqrt{6} - p_E\right) p_E,$$

which yields $p_E \approx 0.29815$. Hence, entry is just deterred at this price if entry costs are

$$E = (1 - 0.29815)\left(\frac{1}{3}\sqrt{6} - 0.29815\right) 0.29815 \approx 0.10847.$$

Compare this to a case without bundling. With $E = 0.108$, the monopolist will choose not to deter entry. Hence, his profits equal $1/4$. His profits when he does bundle are 0.54433. Hence, the monopolist stands to gain much more from bundling when facing a threat of bundling, than when he does when he does not face such a threat.

Exercise

1. Suppose there are two products, 1 and 2. For product 1, valuations are uniformly distributed on $[0, 1]$; for product 2 they are uniformly distributed on $[0, 2]$. Each consumer's valuation for product 1 is independent of her valuation for product 2.

   (a) Suppose that a firm has a monopoly on both products. Derive profit-maximizing prices and resulting profits if it sells both products separately, and if it sells them as a bundle.

   (b) Now suppose the monopolist faces a threat of entry. A potential product can sell a product that is a perfect substitute for either product 1 or product 2, with both being equally likely. The monopolist has to commit to prices before the entrant enters (this is the same set-up as in Section 9.5). For which value of entry costs E will entry *always* be deterred if the monopolist does not bundle? And if she bundles?

Chapter 10

Network Externalities and Compatibility

10.1 Introduction

This chapter discuss **network externalities** and **compatibility**. For many products, such as social networks, the utility a consumer obtains from consuming the product is increasing in the total number of consumers of that product. These are positive network externalities. For example, a phone is more useful if more people are using a phone. Also, if more people use the same software, that software becomes more useful as a larger user base implies that there are also more people that can help you out if you get stuck either in real life or on the internet. These are examples of **direct network externalities**. There can also be **indirect network externalities**: in that case, your utility of using the product also increases with the number of users, but not since you like to have more users around, but rather since having more users also increases the usefulness of the product. One example is Google Search: as more people use this search engine, its algorithm will improve so it will deliver better search results.

Competition on markets with network externalities is often fierce: exactly because the product becomes more valuable as more people are using it, firms have every incentive to compete very intensively to get such a critical mass of consumers. It also implies that there are often multiple equilibria in such markets, as we will see in the next section.

On a related note, we will also study to what extent producers of two products have an incentive to make their products compatible with each other. This is an important issue in e.g. social networks (if you're on WhatsApp you cannot directly communicate with someone on Instagram) and telecommunication (telecom networks of different providers are compatible: you can easily make a phone call to someone using a different provider). We will study that issue in Section 10.3.1. A related issue is the extent to which you can mix and match components. For example, Apple has long used its own charger that it incompatible with chargers for other phones. This issue is discussed in Section 10.3.2.

10.2 A monopolist with network externalities

In this section, we study the classic model of a monopolist with positive network externalities due to Rohlfs (1974). The discussion is based on Shy (2001), chapter 5.

Consider a group of a η continuum of telecommunication customers uniformly indexed by x on $[0,1]$. Those with low x have a high willingness to pay. Utility of x is $(1-x)q^e - p$ iff she subscribes, with q^e the expected number of subscribers. This captures the network externality: for any given consumer, the higher the total number of consumers, the higher the willingness-to-pay for any given consumer. For a connection fee p, the indifferent consumer is given by $\hat{x} = (q^e - p)/q^e$. All $x > \hat{x}$ will not subscribe. The actual number of consumers is $q = \eta\hat{x}$. In equilibrium, the expectations of all consumers will be fulfilled, so we have $q^e = q = \eta\hat{x}$. Substuting this yields the inverse demand function

$$p = (1-\hat{x})\eta\hat{x}.$$

Thus, demand is upward sloping at small demand levels and downward sloping at high demand levels. The network effect dominates the price effect at a small network size. Once network size reaches half the population, the negative price effect dominates. A connection fee p_0 intersects twice the inverse demand curve, at $\hat{x}_0^L$ and $\hat{x}_0^H > \hat{x}_0^L$. Only $\hat{x}_0^H$ is a stable equilibrium. At p_0 the critical mass is $\eta\hat{x}_0^L$. (In fact, there is also an equilibrium at $\hat{x} = 0$). Assume connection costs are zero. The monopoly maximizes profits

$$\pi(\hat{x}) = p\eta\hat{x} = (1-\hat{x})\left(\eta\hat{x}\right)^2.$$

Maximizing this, we have $\hat{x} = 2/3$: the number of customers exceeds half of the population but is less than the entire population. Plugging this equilibrium back into earlier expressions, we have

$$\begin{aligned} p &= \frac{2\eta}{9}, \\ \pi &= \frac{4\eta^2}{27}. \end{aligned}$$

For the consumers who buy (i.e. those with $x < 2/3$), utility is given by

$$U = \frac{2}{3}(1-x)\eta - \frac{2\eta}{9} = \frac{2\eta(2-3x)}{9}.$$

Now consider an increase in η. Prices will go up. The fraction of connected consumers remains constant, thus the absolute number increases. Profits of the firm also increase.

Yet, utility of the connected consumers increases as well. The monopoly cannot capture the entire surplus.

Social welfare is given by

$$\begin{aligned}
& (1-\hat{x})(\eta\hat{x})^2 + \eta\int_0^{\hat{x}} (\hat{x}(1-x)\eta - (1-\hat{x})\eta\hat{x})\,dx \\
= & (1-\hat{x})(\eta\hat{x})^2 + \eta\int_0^{\hat{x}} (\hat{x}\eta(\hat{x}-x))\,dx \\
= & (1-\hat{x})(\eta\hat{x})^2 + \frac{1}{2}\hat{x}^3\eta^2
\end{aligned}$$

This is maximized by setting

$$\hat{x} = \frac{4}{3},$$

which, of course, is infeasible. This implies that the social optimum simply has the corner solution $\hat{x} = 1$.

10.3 Compatibility

We now study how network effects may affect competition between two firms. We do so in the simplest model: that of Hotelling competition. When we study competition, it also becomes important to what extent the networks of the two competitors are compatible. We study this issue in Section 10.3.1.

Yet, compatibility may also be an issue in environments without network externalities: it then mainly concerns the compatibility of different components of a single system. We will study this issue in Section 10.3.2.

10.3.1 Compatibility with network externalities

Suppose that we add network effects to the standard Hotelling model. The net utility of a consumer located at x that buys from firm 0 is given by

$$U = v + \alpha q_0 - tx - p_0,$$

with q_0 the number of users from which consumer x_i enjoys network externalities. Again, we assume that production costs are zero. First consider the case in which the goods that the two firms produce are not compatible. In that case, q_0 simply equals the mass of consumers that chooses to consume firm 0's product. When z is the indifferent consumer, we thus have that

$$v + \alpha z - tz - P_0 = v + \alpha(1-z) - t(1-z) - P_1,$$

or

$$P_0 + (t-\alpha)\,z = P_1 + (t-\alpha)\,(1-z),$$

hence

$$z = \frac{1}{2} + \frac{P_1 - P_0}{2\,(t-\alpha)}.$$

Note that, compared with the standard Hotelling model, the addition of network externalities has the same effect as a decrease in transportation costs. Going through the same analysis as in the standard Hotelling model, we thus have that the market equilibrium has

$$P_1^* = P_2^* = t - \alpha.$$

The net utility for a consumer located at some $x < 1/2$ thus equals

$$\begin{aligned} U &= v + \frac{1}{2}\alpha - tx - (t-\alpha) \\ &= v - t\,(1+x) + \frac{3}{2}\alpha. \end{aligned}$$

Equilibrium profits are

$$\Pi^1 = \Pi^2 = \frac{1}{2}\,(t-\alpha)\,.$$

Now consider the case in which the two products are compatible. Hence, the products of firms 0 and 1 are able to freely "communicate" with each other. This implies that the relevant size of the network is simply the entire mass of consumers. Since we assume that the entire market is always covered, this implies that $q_0 = q_1 = 1$. The indifferent consumer is thus given by

$$v + \alpha - tz - P_0 = v + \alpha - t\,(1-z) - P_1,$$

or

$$P_0 + tz = P_1 + t(1-z),$$

hence

$$z = \frac{1}{2} + \frac{P_1 - P_0}{2t}.$$

This is the standard Hotelling case. Equilibrium prices and profits are thus given by

$$\begin{aligned} P_0 &= P_1 = t, \\ \Pi_0 &= \Pi_1 = \frac{1}{2}t. \end{aligned}$$

The net utility for a consumer located at some $x < 1/2$ thus equals

$$\begin{aligned} U &= v + \alpha - tx - t \\ &= v - t(1+x) + \alpha. \end{aligned}$$

Consumer utility is thus higher in the case of incompatibility than it is in the case of compatibility! With incompatibility, competition will be fiercer, which implies that prices will be lower. Firms are better off with compatibility. Consumer surplus with incompatibility equals.

$$2\int_0^{1/2} \left(v - t(1+x) + \frac{3}{2}\alpha\right) dx = v - \frac{5}{4}t + \frac{3}{2}\alpha.$$

With compatibility, we have

$$2\int_0^{1/2} (v - t(1+x) + \alpha)\, dx = v - \frac{5}{4}t + \alpha.$$

Social welfare with incompatibility thus equals

$$W = \left(v - \frac{5}{4}t + \frac{3}{2}\alpha\right) + (t - \alpha) = v - \frac{1}{4}t + \frac{1}{2}\alpha,$$

and with compatibility

$$W = \left(v - \frac{5}{4}t + \alpha\right) + t = v - \frac{1}{4}t + \alpha.$$

Thus, social welfare is higher with compatibility.

10.3.2 A 'mix-and-match' model

This is due to Matutes and Regibeau (1988). Assume that we have a duopoly. A full system consists of two components. Both firms produce both components. Marginal costs are zero. If the two components are compatible, only two systems are available for consumers: X_{AA} and X_{BB}. If the components are compatible, consumers have the additional options of X_{BA} and X_{AB}. Consumers are uniformly distributed on the unit square. Firm A is located at the origin, while B is located at (1,1). A consumer located at (x_1^o, x_2^o) has a preferred component that is x_1^o away from firm As first component and a preferred second component that is x_2^o away from firm As second component. The distances from firm Bs components are $1 - x_1^o$ and $1 - x_2^o$. A consumer buying one unit of system X_{ij} has a surplus of

$$v - \lambda(d_{1i} + d_{2j}) - P_{ij},$$

with v the valuation for the entire system, d_{ij} the distance between the consumer's preferred specification of the ith component and the specification of the ith component sold by firm j, P_{ij} is the price of the system X_{ij} and $\lambda > 0$ measures the degree of horizontal product differentiation.

Suppose the firms sell incompatible components. Assume that the whole market is covered. For the indifferent consumer, we have

$$P_{AA} + \lambda(z_1 + z_2) = P_{BB} + \lambda(2 - z_1 - z_2).$$

We thus have a set of consumers, given by the line

$$z_2 = 1 + \frac{P_{BB} - P_{AA}}{2\lambda} - z_1.$$

When $P_{AA} \geq P_{BB}$, the total mass of consumers opting for system 1 is the triangle below the line that crosses both the horizontal and the vertical axis at $z_1 = 1 + (P_{BB} - P_{AA})/2\lambda$. The total demand for firm A is thus given by

$$Q_{AA} = \frac{1}{2}\left[1 + \frac{P_{BB} - P_{AA}}{2\lambda}\right]^2,$$

the remaining consumers go to firm B. Profits are thus given by

$$\begin{aligned} \Pi_A &= \frac{P_{AA}}{2}\left[1 + \frac{P_{BB} - P_{AA}}{2\lambda}\right]^2 \\ \Pi_B &= P_{BB}\left[1 - (1/2)\left(1 + \frac{P_{BB} - P_{AA}}{2\lambda}\right)^2\right]. \end{aligned}$$

Maximizing profits and imposing symmetry yields $P_{AA}^* = P_{BB}^* = \lambda$, and $\Pi_A^* = \Pi_B^* = \lambda/2$. Consumer surplus is

$$\text{CS} = 2\int_0^1 \int_0^{1-z_1} \left[v - \lambda\left(z_1 + z_2\right) - P^*\right] dz_2 dz_1 = v - 5\lambda/3.$$

The whole market is covered with $v > 2\lambda$.

Now suppose the two components are compatible. Then, the two decisions are independent: consumers simply buy the 1-component they prefer and the 2-component they prefer. Equilibrium prices for both components are thus given by λ, and consumer surplus is

$$\text{CS} = 4\int_0^{1/2} \int_0^{1/2} \left[C - 2\lambda - \lambda z_1 - \lambda z_2\right] dz_1 dz_2 = v - 5\lambda/2.$$

Also in this model, consumer surplus is thus higher with incompatibility than it is with compatibility. Yet, profits are higher with compatibility. Social welfare with compatibility is higher ($v - \frac{2}{3}\lambda$ vs. $v - \frac{1}{2}\lambda$).

Exercises

1. Consider a Salop circle with unit length. Firms sell products that exhibit network externalities. The utility of a consumer i located at x_i that consumes the product produced by firm j is given by

$$U_{ij} = v - td_{ij} + \alpha q_j,$$

   with v the stand-alone valuation of i for its most preferred product, t the transportation costs (or rather, the disutility per unit of 'distance' between the most preferred product and the product of firm j), d_{ij} the distance between consumer i and firm j, α the strength of the network effect, and q_j the total mass of consumers that consume a product that is compatible with the one that firm j is producing. For firms, marginal costs of production are given by c, and fixed costs of entry F. Firms are located equidistant from each other.

   (a) Suppose all products are compatible. Calculate the equilibrium number of firms, firm profits, and consumer surplus.

   (b) Suppose all products are incompatible. Calculate the equilibrium number of firms, firm profits, and consumer surplus.

   (c) Compare total welfare in the two scenarios.

2. Consider the following set-up. Two firms compete. They both supply Apples and Bananas. Consumers wish to buy one unit of each good, i.e. 1 Apple and 1 Banana. In the market for Apples, firms are horizontally differentiated: firm 1 is located at one endpoint of a Hotelling line with unit length, firm 2 is located at the other endpoint. Consumers are uniformly distributed on the line. In the market for Bananas, we also have horizontal differentiation. Again, firm 1 is located at one endpoint of a Hotelling line with unit length, firm 2 is located at the other endpoint. Consumers are uniformly distributed on the line. Yet a consumer's location on the 'Apple-line' is completely unrelated to its location on the 'Banana-line'. Thus, 'location' should be interpreted as taste here, rather than as physical location. Hence, consumers can be viewed as being uniformly distributed on a unit square: a consumer that is located at (x, y) has location x on the 'Apple-line' and location y on the 'Banana-line'. Transportation costs for consumers are t per unit of distance for Apples, but $2t$ per unit of distance for Bananas. For simplicity, marginal costs are zero for each firm and for each product. You may assume that the market is always entirely covered.

(a) Suppose that both firms sell Apples and Bananas separately. Determine the equilibrium price for Apples, and the equilibrium price for Bananas.

(b) Firm 1 decides no longer to sell Apples and Bananas separately, but only as a package containing 1 Apple and 1 Banana. Firm 2 does the same thing. Determine the equilibrium price of a package containing one Apple and one Banana.

(c) Suppose that firm 1 decides to sell only packages, but firm 2 decides to only sell separate Apples and Bananas. Determine equilibrium prices.

(d) Now consider a model in which in stage 1 each firm decides what to offer (i.e. either a package or the separate fruits), and in stage 2 firms set prices. Determine the equilibrium.

Chapter 11

Platform Competition

11.1 Introduction

Monopolies are often involved in the production of complicated products. Increasingly, especially in the context of technological markets, monopolists do not just sell their product, but primarily serve as an intermediary between suppliers and consumers. In the market for video game consoles, for example, the monopolist provides the game console, and hence serves as an intermediary between suppliers of video games and their consumers.[1] Commercial television stations serve as an intermediary between the suppliers of advertisements, and their consumers. The same holds for newspapers. Credit card suppliers serve as an intermediary between merchants and consumers. Real estate agents serve as an intermediary between sellers and buyers of houses. Other examples abound.

What these examples have in common is that there are always two sides of the market that are served by the monopolist. Therefore, the literature refers to these cases as *two-sided markets*. Alternatively, one often uses the term *platform competition*. Crucially, in these models, the utility obtained by each side of the market is affected by the size of the other side of the market. For example, suppliers of video games on a particular console are better off if more consumers use that console, and vice-versa. Advertisers on a TV station are better off if that station attracts more viewers – but viewers are worse off if that station attracts more advertisers. Sellers of houses are better off if more potential buyers use the same real estate agent, and vice-versa. Two-sided markets are currently one of the hottest research topics in Industrial Organization.

In this chapter, we can only give a small sample of that literature. In Section 11.2, we study the classic model of two-sided markets, due to Rochet and Tirole (2003). Section 11.3 we study a slightly different business model, one in which the platform acts as an intermediary between firms selling homogeneous products. Section 11.4 looks at a model in which a platform also makes money from advertisements, and we study the choice of

[1] Although note that this market no longer qualifies as a monopoly, as there are now three suppliers of game consoles: Nintendo, Sony, and Microsoft.

platform content in that context.

11.2 Two-sided markets

11.2.1 Model

We consider the following model, based on Rochet and Tirole (2003) On a two-sided market, buyers want to interact with sellers. The monopolist provides the platform that allows such interaction. For each interaction that a buyer has on the platform, the monopolist will charge it some price p_B. Think of p_B as e.g. the fee that a bank charges a consumer each time that the consumer uses the bank's credit-card for making a purchase at some merchant. For each interaction that a seller has on the platform, the monopolist will charge it a price p_S. Think of p_S as e.g. the fee that a bank charges a merchant each time the merchant accepts the bank's credit card from a consumer making a purchase. For simplicity, we assume that each buyer interacts exactly once with each seller, and vice-versa. This is harmless: our results will continue to hold as long as the number of interactions is proportional to the size of the other side of the market.

Buyer i will obtain some benefit b_B^i from each interaction that it has on the platform. Seller j will obtain some benefit b_S^j from each interaction that it has on the platform. The number of buyers that wants to use the platform if the price equals p^B, is the number of buyers for whom $b_B > p_B$. The number of sellers that wants to use the platform if the price equals p_S, is the number of sellers for whom $b_S > p_S$. These numbers are represented by downward sloping demand curves: $D_B(p_B)$ for buyers, and $D_S(p_S)$ for sellers. The marginal costs for the monopolist owner are c per transaction. The total revenues that the monopolist will earn from sellers equals $p_S \cdot D_S(p_S) \cdot D_B(p_B)$. The total revenues that it will earn from buyers equals $p_B \cdot D_S(p_S) \cdot D_B(p_B)$. Total profits thus equal

$$\pi = (p_B + p_S - c)D_S(p_S)D_B(p_B).$$

This profit function reflects that this monopolist faces a trade-off. Setting a very low price for buyers, for example, will cause it to make little money on buyers (as the revenues per buyer will be very low), but a lot of money on sellers (as the number of buyers will be high, which is attractive for sellers). Taking the first-order conditions of the profit function yields:

$$D_S D_B + (p_B + p_S - c)D_S D_B' = 0,$$
$$D_S D_B + (p_B + p_S - c)D_S' D_B = 0.$$

Denote the elasticity of demand for the buyers as ε_B and that for sellers as ε_S. Note that, strictly speaking, the interpretation of these elasticities is slightly different from the interpretation of the regular demand elasticities that we had earlier in this chapter. The main reason for that is that the demand functions are also slightly different from normal. It is not so much total demand for the monopolist's product as such, but rather the demand for the platform services given that each interaction on that platform has price p_B or p_S. That is the reason that Rochet and Tirole refer to D_B and D_S as *quasi-demand functions*.

Since we have that $\varepsilon_i = -p_i D_i'/D_i$, for $i \in B, S$, we can rewrite the first-order conditions as

$$p_B + p_S - c = \frac{p_B}{\varepsilon_B} = \frac{p_S}{\varepsilon_S}$$

We can also rewrite these as

$$p_i = \varepsilon_i (p_B + p_S - c), \tag{11.1}$$

for $i \in B, S$. Summing these expressions for p_B and p_S yields

$$p_S + p_B = (\varepsilon_S + \varepsilon_B)(p_B + p_S - c),$$

or

$$\frac{p_B + p_S - c}{p_B + p_S} = \frac{1}{\varepsilon_B + \varepsilon_S}.$$

If we define ε as the sum of the two demand elasticities ($\varepsilon \equiv \varepsilon_S + \varepsilon_B$) and p as the sum of the two prices ($p \equiv p_B + p_S$), then this equation simplifies to the familiar Lerner condition:

$$\frac{p - c}{p} = \frac{1}{\varepsilon}. \tag{11.2}$$

Thus, in this sense, the problem is identical to the standard monopoly problem, albeit that some variables are defined in a somewhat different manner. But now consider the way in which the total price is allocated between the two sides of the market. First of all, note that from (11.2) we have that

$$p = \frac{\varepsilon c}{\varepsilon - 1}. \tag{11.3}$$

From (11.1),

$$p_B = \varepsilon_B (p - c)$$

Using (11.3), this yields

$$p_B = \left(\frac{\varepsilon_B}{\varepsilon_B + \varepsilon_S - 1} \right) c$$

Rewriting yields

$$(\varepsilon_S + \varepsilon_B - 1)\, p_B = \varepsilon_B c,$$

or

$$\varepsilon_B \cdot (p_B - c) = (1 - \varepsilon_S)\, p_B,$$

hence

$$\frac{p_B - c}{p_B} = \frac{1 - \varepsilon_S}{\varepsilon_B}.$$

Similarly, we have

$$\frac{p_S - c}{p_S} = \frac{1 - \varepsilon_B}{\varepsilon_S}.$$

These equations look like the standard Lerner conditions, although the interpretation of the variables is different. In particular, c is the marginal cost of the entire transaction, i.e. that of serving both the seller and the buyer. Hence, the terms $p_i - c$ cannot be interpreted strictly as mark-ups. The most striking result is that the price that is charged to buyers is also affected by the elasticity of demand for sellers, and vice-versa.

Suppose that both sides of the market have a demand function that exhibits constant elasticity of demand. Consider the price charged to buyers. This price will depend on the elasticity of demand for buyers, in the usual manner. But it will also depend on the elasticity of demand for sellers. If the elasticity of demand for sellers is very high, then the price charged to buyers will be very low. This can be understood as follows. To make profits on sellers, the monopolist can do two things. It can either charge a high price to sellers, or it can make sure that there are many buyers, which makes the platform more attractive to sellers. When the elasticity of demand for sellers is high, then setting a high price to sellers will become less attractive, as it will scare away many sellers. In order to lure sellers to the platform, it is then more profitable for the monopolist to try to attract more buyers, by lowering the price charged to them.

This is what almost happens in these kinds of markets: one side of the market is subsidized in order to get the other side ”on board”. One side of the market serves as a loss leader, in order to make profits on the other side of the market. For example, game consoles are often sold below cost, in order to make profits from software developers. Viewers pay a zero price to watch commercial television, such that advertisers are willing to pay a higher price per advertisement. Newspapers are also sold below cost. In some nightclubs, men have to pay an entrance fee, where women don’t. This result also implies that one should be very careful with antitrust policy on such markets.

In a simple example with linear demand functions, all this will become much clearer. Note however that in that case demand functions do not exhibit constant elasticity, so the intuition given above does not carry through directly.

11.2.2 Example

Consider an internet site that organizes auctions. There are 100 potential buyers and 100 potential sellers. To use the site, one needs a subscription. Each potential seller is willing to pay on the basis of the number of buyers that the site will attract. Each potential buyer is willing to pay on the basis of the number of sellers that the site will attract. Demand from sellers is such that if a seller has to pay p_S for each buyer that subscribes to the site, the total number of sellers that is willing to subscribe equals

$$q_S = 100 - 100p_S.$$

Demand from buyers is such that if a buyer has to pay p_B for each seller that subscribes to the site, the total number of buyers that is willing to subscribe equals

$$q_B = 100 - 200p_B.$$

Note that, as a whole, sellers are more eager to join the website than buyers are, in the sense that for any given price per interaction, more sellers than buyers are willing to subscribe. For simplicity, we assume that marginal costs are zero.

First consider a benchmark in which both sides of the market are independent, in the sense that the willingness to pay for sellers does not depend on the number of buyers, and vice versa. In that case, maximizing profits given the two demand functions, the monopolist would set $p_S = 1/2$ and $p_B = 1/4$.

Now consider the two-sided market model. Profits of the monopolist are given by

$$\begin{aligned}\pi &= (p_B + p_S) \cdot q_B \cdot q_S \\ &= (p_B + p_S)(100 - 200p_B)(100 - 100p_S).\end{aligned}$$

Taking the first-order conditions yields

$$\begin{aligned}p_S &= (1 - p_B)/2 \\ p_B &= (1 - 2p_S)/4.\end{aligned}$$

Solving this system yields $p_S = 1/2$ and $p_B = 0$. Hence, buyers can enter the website for free, and the monopolist will not make any direct profits on them. Yet, by attracting many buyers in this manner, the monopolist is able to charge much more to sellers. The total subscription price charged to sellers will equal $q_B \cdot p_S = 100 \cdot 1/2 = 50$.

11.3 Intermediaries

Next, we consider the role of a platform as an intermediary in a market where firms sell homogeneous products. This is largely based on Baye and Morgan (2000). What follows is only a sketch of their model. Further details can be found in the original paper.

A platform allows firms to post prices and consumers to save on search costs. The underlying question is: do we get rid of price dispersion if we have price comparison sites on the internet? There are two local markets. On each, there is a single firm and a unit mass of consumers. Costs are zero. Each consumer has demand $q(p) = 2 - p$. Initially, consumers only have access to their own local market. Note that $\pi(p) = p(2-p)$, so $p^m = 1$ and $\pi^m = 1$. It costs a consumer $z < 1/2$ to visit a local store. Consumer surplus at price 1 is exactly 1/2.

Now suppose an intermediary opens a price comparison site on the internet. It charges an access fee M_s to firms, and M_b to consumers. For each M_s and M_b, we look for an equilibrium in terms of probabilities that consumers subscribe to the site, probabilities that firms do, and a distribution of prices. The platform then sets M_s and M_b to maximize profits. In equilibrium, nonsubscribing consumers shop at their local store. Subscribing consumers visit the website and purchase at the lowest price there. Thus: if you only observe the other firm's price, you're not going to check out your local store. The argument is similar to Stahl (1988).

11.3.1 Firm behavior

We first derive the equilibrium behavior of firms. Suppose that on each local market, a fraction λ of consumers subscribe to the platform. Each firm posts on the platform with probability α. Suppose moreover that the prices that they set are drawn from some distibution $F(p)$. Expected profits of a non-posting firm are given by: $E\pi_i^N = (1-\alpha) + \alpha(1-\lambda)$. Expected profits of a posting firm are:

$$\begin{aligned} E\pi_i^P &= (1-\lambda)\pi(p) + 2\lambda\left[(1-\alpha) + \alpha(1-F(p))\right]\pi(p) - M_s \\ &= (1+\lambda)\pi(p) - 2\alpha\lambda F(p)\pi(p) - M_S \end{aligned}$$

In equilibrium, firms must be indifferent between the two options. This pins down the distribution $F(p)$. We necessarily have

$$F(p) = \frac{(1+\lambda)p(2-p) + \alpha\lambda - 1 - M_s}{2\alpha\lambda p(2-p)},$$

with lower bound

$$p_0 = 1 - \sqrt{\frac{\lambda - M_s + \alpha\lambda}{1+\lambda}}.$$

Moreover, we need

$$E\pi_i^P = (1+\lambda)\pi(p) - 2\alpha\lambda F(p)\pi(p) - M_S$$

The highest price charged is 1. That yields $E\pi_i^P = 1 + \lambda - 2\alpha\lambda - M_S$. This should equal $1 - \alpha\lambda$, hence $\alpha^* = \max\{0, 1 - M_s/\lambda\}$. Hence, we have price dispersion in equilibrium!

11.3.2 Consumers

We now derive equilibrium behavior of consumers. Denote by $v(p)$ the consumer surplus of a consumer at price p. Denote by $h_2(p)$ the lowest-order statistic of two draws from F, so $h_2(p) = 2(1 - F(p)) f(p)$ Then the value to consumers of subscribing to the platform is given by

$$\begin{aligned} V^I &= \alpha^2 \int_{p_0}^{1} v(p) h_2(p) \, dp + (1-\alpha)^2 \left(\frac{1}{2} - z \right) \\ &\quad + 2\alpha (1-\alpha) \int_{p_0}^{1} v(p) f(p) \, dp - M_b. \end{aligned}$$

A non-subscriber has

$$V^U = \alpha \int_{p_0}^{1} v(p) f(p) dp + (1-\alpha) \frac{1}{2} - z$$

Denote by $\beta\left(M_s, \lambda^I\right)$ the value of M_b that equates $V^I = V^U$.

11.3.3 Equilibrium

Given M_s and M_b, we can have two types of equilibria:

1. No participation.
2. Some firms participate, some consumers do.
3. Some firms participate, all consumers do. (hence $\alpha^* = 1 - M_s$).

The intermediary sets fee to maximize

$$E\Pi = 2\alpha M_s + 2\lambda M_b - K$$

It turns out that it is most profitable to have full consumer participation. Hence, in equilibrium, all consumers will go on the platform, and firms will mix. As a result, firms will also use a mixed strategy in prices. As a result, we will have price dispersion, despite the internet search engine.

11.4 Platform content

11.4.1 Introduction

This section is based on (and the text is taken from) a recent paper by Haan et al. (2024).

Big techs started out as very distinct firms. Google used to be a simple search engine. Amazon started as an online book seller, Facebook as a social network, and Apple as a computer manufacturer. But increasingly these firms are developing full digital ecosystems that compete head-on for consumers, as also noted in a recent leader in the Economist.[2] Almost all now offer competing cloud computing services, home assistants, and media distribution platforms. Amazon is considered a major threat to Google's targeted ad services, using its detailed consumer data to help others find their audiences.[3] Facebook recently launched "Facebook Shops" that will use its social network data to direct consumers to online stores, challenging for example Amazon and Ebay.[4] Google's Youtube explores selling products through its videos.[5]

From a competition perspective it is puzzling why these firms choose to compete head-on for consumers rather than just enjoying a monopoly within their original niche. In this Section, we explore this question. We argue that platforms can choose on which side of the market to compete. By choosing strategies leading to head-on competition on the consumer market, platforms can induce consumers to *singlehome* rather than to spend time on many different platforms. This may increase competition for these consumers – and reduce prices on that side of the market – but it also gives the platform more market power on the other side of the market where they sell consumer attention or data to advertisers.[6] When consumers singlehome on a platform, firms that want to reach those consumers have no choice but to go through that platform. The platform then acts as a gatekeeper, and can charge monopoly access fees to advertisers (Armstrong, 2006; Armstrong and Wright, 2007). This ability to monopolize the advertiser side may outweigh the loss in market power on the consumer side. Platforms can thus choose their battles: they can choose where to compete fiercely in order to gain market power on the other side of the market.

In our general framework, two firms compete in prices in a Perloff and Salop (1985) type model. They face a non-negative price constraint (as in e.g. Choi and Jeon, 2021). Patterns of substitutability may vary. At one extreme, products may be completely independent, so the choice to buy one product is independent of whether the other product

[2]As the Economist describes,"First, the companies are increasingly selling the same products or services. Second, they are providing similar products and services on the back of different business models, for example giving away things that a rival charges for (or vice versa, charging for a service that a competitor offers in exchange for user data sold to advertisers). Third, they are eyeing the same nascent markets, such as artificial intelligence (ai) or self-driving cars." (from: The new rules of competition in the technology industry, The Economist, Feb 27th 2021).

[3]see e.g. "Amazon Knows What You Buy. And It's Building a Big Ad Business From It." by Karen Weise in the New York Times, Jan. 20 2019.

[4]"Facebook takes on Amazon with online shopping venture", Financial Times, May 2020

[5]"YouTube explores selling products to take on Amazon", competitionpolicyinternational.com, Oct 2020.

[6]More generally, we are thinking about advertising slots, product referrals, targeting data, etc. We will refer to this as advertising, with the implicit understanding that we have in mind broader applications than merely showing an ad on one's website.

is bought. At the other extreme, products may be perfect substitutes, so consuming multiple products has no advantages to a consumer whatsoever over and above consuming just one. Take newspapers for example; a consumer may be perfectly happy to buy a sports paper and a general interest newspaper, but is unlikely to consume two sports papers as these will largely carry the same news. Head-on competition (say, both firms producing a sports paper) reduces each platform's consumer base, but also leads to less sharing of consumers with rivals. A platform will have higher advertising revenues from every consumer that single- rather than multihomes at its platform. We will refer to these additional advertising revenues as the *singlehoming rent*. These rents may outweigh the loss of revenues from fiercer competition on the consumer-side of the market.

11.4.2 The General Framework

We consider a two-stage model. In stage 1, firms make strategic choices on their product portfolios. In stage 2, they compete on the product market. Firms make money not only from selling their products to consumers, but also from gaining access to these consumers' eyeballs or data. For example, they may show advertisements to their users. They may also sell consumer data to potential advertisers for targeting purposes, or use this data to improve their recommendation systems. Either way, having access to consumer data and eyeballs is valuable for a firm. For ease of exposition, in the remainder we simply refer to this as advertising. The value of this advertising depends on whether consumers only buy from one firm (singlehoming) or buy from multiple firms (multihoming). In the case of singlehoming, a firm is a monopoly gatekeeper to its consumer's eyeballs. With multihoming, some of the gatekeeping rents are dissipated.

Consumers We model consumers in Perloff and Salop (1985) fashion. A unit mass of consumers differ in their valuation for each product $i = 1, \ldots, n$. Consider one representative consumer. Her stand-alone utility from consuming i is denoted v_i, and is a random draw from some distribution F on $[0, 1]$. For simplicity, we assume that valuations for all products are independent draws from the same distribution.

The price of product i is denoted P_i. We assume that firms can only charge non-negative prices, as abuse by consumers of negative prices would be hard to police. Hence, $P_i \geq 0$. Consumers have unit demand for each individual product. However, a consumer may consume more than one product. If she does, her total utility may differ from the sum of the individual utilities of all products she buys: products may be (partial) substitutes. When reading two newspapers for example, there will be some overlap in the news that they cover. Also, when subscribing to a second video streaming service, most people will not double their time spent watching TV – nor their enjoyment in doing so. In other words, the total utility a consumer derives from consuming i and j will be lower than the sum of the willingnesses-to-pay for these two products when consumed in isolation. Following Gentzkow (2007), we simply assume that total utility will shift downwards by some constant σ_{ij} when i and j are both consumed: $\sigma_{ij} \in [0, 1]$ represents the degree of substitutability between i and j. Suppose that $\sigma_{ij} = 0$. Then the total utility of consuming i and j simply equals the sum of their individual utilities. The two products are completely independent of each other: consuming one does not compromise the utility obtained from consuming the other. However, if $\sigma_{ij} = 1$, then consuming both products does not add value to the utility of consuming just the better one of the two.[7] In that sense, i and j are now perfect substitutes.[8] This formulation generalizes Gentzkow (2007), who assumes that each two products have the same σ.

Advertising If a consumer uses the products of only one firm, then that firm has monopoly power over access to that consumer's data or eyeballs. If the consumer buys products from multiple firms, then those firms compete in selling access to the consumer. Such competition can be modelled in numerous ways. We simply assume that a firm can earn an amount π_s in advertisement revenues from each of its singlehoming consumers. A consumer that multihomes generates advertising revenues π_m for that firm, with $\pi_m \leq \pi_s$, reflecting that there is a premium to having a monopoly on access to a consumer.

For simplicity, we assume that there is no disutility to consumers from watching advertisements: allowing for that would not affect our qualitative results.

Firms Marginal costs of production for each product are constant and equal to c. Every sale a firm makes also yields an advertising revenue of (at least) π_m. Define R as the monopoly premium a firm earns from controlling unique access to a consumer's eyeballs: $R \equiv \pi_s - \pi_m$. We refer to R as the *singlehoming premium*. It captures the incremental value highlighted in Anderson et al. (2019). For ease of exposition, we will work with the net marginal cost defined as $C \equiv c - \pi_m$. We may very well have $C < 0$, if (multihoming) advertising benefits outweigh production costs. A firm that sells product i to a multihoming consumer thus has net marginal costs C and revenues P_i. If it sells i to a singlehoming consumer, it has net marginal costs C and revenues $P_i + R$.

Firms play a two-stage game. First, they make strategic decisions regarding their product portfolio. Second, they compete in prices. In the price-setting stage, firms maximize profits that consist of the margin $P_i - C$ on each product sold, plus the premium R for each consumer that singlehomes at that firm. We are particularly interested in the competitive strategies firms choose in stage 1.

[7] Consuming only i, for example, would yield $u = v_i - P_i$, while consuming both i and j would yield $u = v_i - P_i + v_j - P_j - \sigma_{ij}$. With $\sigma_{ij} = 1$, $v_j \leq 1$ and non-negative prices, this is always weakly lower.

[8] At least, they are so for the consumers on aggregate. Of course, each individual consumer does have different valuations for the two products.

11.4.3 The choice of platform content

As a first application, we consider competition between two single-product firms that first position their product by choosing the content of their platform, and then compete in prices. Product positioning is captured by our measure of product substitutability σ_{12}. For simplicity, we refer to this as σ in the remainder of this application. For example, consider two news sites that have to decide what type of news to run. They may each choose to run general news. In that case σ is close to 1. Alternatively, one may run general news while the other focuses on sports. In that case σ is close to zero.

The timing is as follows:

1. Firm 1 enters the market and chooses its content.

2. Firm 2 enters and chooses its content relative to firm 1, as measured by σ.

3. After having observed σ, firms simultaneously and noncooperatively set non-negative prices P_1 and P_2.

4. Consumers make their purchasing decisions and ad revenues are realized.

We first study the subgames with $\sigma = 0$ (independent products) and $\sigma = 1$ (perfect substitutes). We relegate the analysis of intermediate choices of σ to an appendix. We focus on symmetric equilibria in the subsequent pricing games. The equilibrium positioning choice will then simply be the choice that maximizes total profits, making the assumption that it is firm 2 that chooses σ inconsequential.[9]

Analysis With $\sigma = 1$ platforms are perfect substitutes and it makes no sense to multihome: doing so always lowers net utility. A consumer prefers platform 1 whenever

$$v_1 - P_1 \geq v_2 - P_2$$
$$v_1 - P_1 \geq 0$$

This is area 1 in Figure 11.1 – which we have drawn for the case that $P_1 > P_2$. In the Figure, consumers in area 2 buy from firm 2, while consumers in area $\emptyset$ refrain from consumption.

Consumer singlehoming implies that platforms are effectively monopolists on the market for advertisements: as there is no multihoming they simply earn the singlehoming premium R in ad revenues for each consumer that they attract. To derive equilibrium

[9] Alternatively, we could consider simultaneous content choices, resulting in a coordination game, with the same outcome for the profit maximizing equilibrium, but leaving the question how firms would coordinate on this equilibrium.

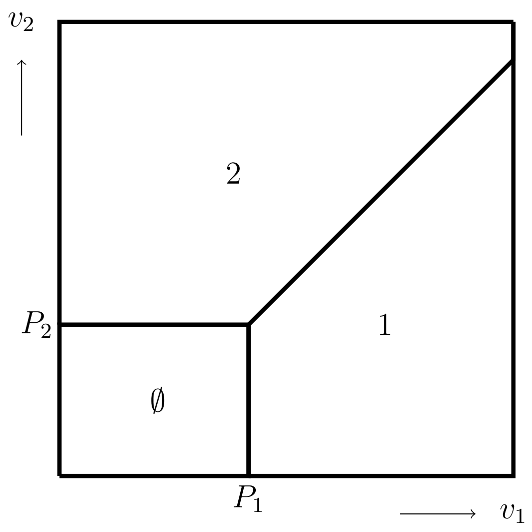

Figure 11.1: $\sigma = 1$, perfect substitutes

prices, assume without loss of generality $P_1 \geq P_2$. From Figure 11.1, total profits of firm 1 are then given by

$$\pi_1 = (P_1 - C + R) \int_{P_1}^{1} F(v_1 + P_2 - P_1) dF(v_1) \tag{11.4}$$

In a symmetric equilibrium, each firm's profits are

$$\pi^* = (P^* - C + R) \frac{1}{2} \left(1 - F^2(P^*)\right).$$

This is intuitive: firms simply share the part of the market that is covered (i.e. all consumers except for the "zerohoming" consumers in area $\emptyset$ in Figure 11.1, that has size $F^2(P^*)$). P^* is determined by the first-order conditions with respect to P_1 and imposing symmetry:

$$P^* - C + R = \frac{1 - F^2(P^*)}{2f(P^*)F(P^*) + 2\int_{P^*}^{1} f(v) dF(v)},$$

where the left-hand side equals the average mark-up. If this yields negative prices, we have a corner solution with $P^* = 0$.

Next, consider independent products: $\sigma = 0$. In this scenario, platforms are effectively monopolists on the consumer side of the market; their demand does not depend on the price charged by the other firm. Of course, they do now compete on the ad market for advertisements to consumers that multihome. Consumers with $v_1 \geq P_1$ will use platform 1, those with $v_2 \geq P_2$ will use platform 2, those for whom both conditions are satisfied will multihome (see Figure 11.2).

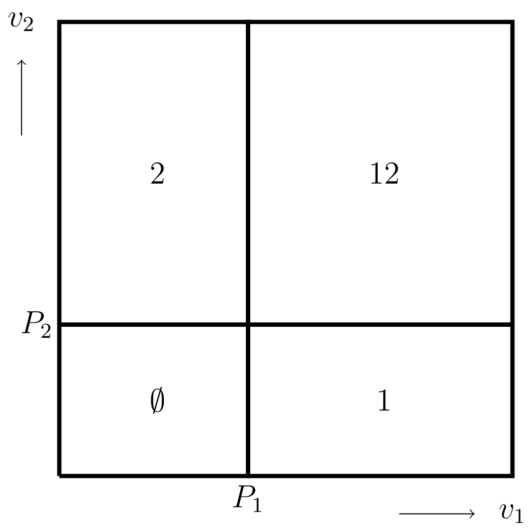

Figure 11.2: $\sigma = 0$, independent products

Profits of platform 1 are given by

$$\pi_1 = (P_1 - C)(1 - F(P_1))(1 - F(P_2)) + (P_1 - C + R) F(P_2)(1 - F(P_1)),$$

where the first term represents the multihomers (area 12 in Figure 11.2), while the second represents the singlehomers (area 1 in Figure 11.2). The firm only earns the advertising monopoly premium R for the latter group. In a symmetric equilibrium, profits are

$$\pi^* = (P^* - C)(1 - F(P^*)) + RF(P^*)(1 - F(P^*)).$$

where prices P^* satisfy first-order conditions for the symmetric equilibrium,

$$P^* - C + RF(P^*) = \frac{1 - F(P^*)}{f(P^*)}.$$

The left-hand side is the average mark-up, which includes the singlehoming premium R for the fraction $F(P^*)$ of consumers that do not buy the rival's product. The right-hand side is the hazard rate for the distribution of v. As long as net marginal costs C are non-negative (in other words, marginal costs c exceed the advertising revenue made on multihoming consumers π_m), prices will never go to zero: at $P^* = 0$, all consumers would multihome and platforms would make non-positive profits.

Example For the uniform distribution, $F(v) = v$, and with $C = 0$, the first-order conditions for $\sigma = 1$ can be solved to

$$P^* = \sqrt{2}\sqrt{1 - R} - 1.$$

Note that this solution is only feasible with $R \leq 1/2$: for higher values of R, optimal prices would be negative. As we do not allow for that, we get a corner solution at $P^* = 0$.

Equilibrium profits if $\sigma = 1$ and $R \leq 1/2$ equal

$$\pi^* = (1 - R)\left(3 - R - 2\sqrt{2}\sqrt{1 - R}\right).$$

It is easy to see that this is increasing in R. With $R > 1/2$, equilibrium prices are zero, so profits become

$$\pi^* = \frac{1}{2}R.$$

If, instead, firms choose to provide independent products $\sigma = 0$, we find

$$P^* = \frac{1}{2 + R}.$$

In this case of independent products, although prices decrease in R (platforms are more eager to attract consumers as their ad revenues from a consumers increase), they never go down to zero. Total profits equal

$$\pi_1 = \left(\frac{1 + R}{2 + R}\right)^2,$$

which is also increasing in R.

We now turn to the first stage of the game, the choice of strategy. Comparing profits in the two scenarios, we can now show that the following result holds not only for the uniform distribution, but for any distribution function:

Proposition 7 *For general distribution $F(v)$, if singlehoming premium R is sufficiently small, platforms maximize profits by differentiating their content as much as possible: $\sigma^* = 0$. For R sufficiently large, profits are maximized with perfect substitutes: $\sigma^* = 1$.*

Proof See Haan et al. (2024), Appendix.

With $R = 0$ we have standard profit maximization: firms seek a high margin on a high volume of consumers. They can achieve this by differentiating their product as much as possible from their competitor's. By doing so they establish a monopoly which allows them to charge their monopoly prices without having to worry about their rivals' actions. Consumers with a high willingness to pay for both products end up buying both.

When $R > 0$, however, a platform cannot extract singlehoming advertising rents from consumers that also buy the competitor's product. The higher R, the more platforms start focusing on obtaining a high volume of consumers *that are not also served by their rival.* This can be achieved by offering a product that is virtually identical, creating fierce competition for consumers. For large R, the gain in singlehoming advertising rents from offering a perfect substitute, more than offsets the decrease in profits due to fierce competition on the consumer side. The non–negative-pricing constraint is important in this argument: if firms could charge negative prices, higher advertising rents would mostly be competed away by charging ever lower prices to consumers.

Hence, by their choice of σ, platforms can effectively choose to compete head-on for consumers (by producing perfect substitutes and setting $\sigma = 1$) or to compete head-on for advertisers (by producing independent products and setting $\sigma = 0$). If advertising rents from singlehoming are not so important, R is low, and platforms prefer to monopolize the consumer market, while competing vigorously for advertisers. As advertising singlehoming rents become more important however, at some point it becomes more profitable to turn all consumers into singlehomers: by doing so platforms monopolize the advertising market, competing fiercely for consumers instead.

Exercise

1. Consider the model used in the example in Section 11.2. An internet site organizes auctions. There are 100 potential buyers and 100 potential sellers. To use the site, one needs a subscription. Each potential seller is willing to pay on the basis of the number of buyers that the site will attract. Each potential buyer is willing to pay on the basis of the number of sellers that the site will attract. Demand from sellers is downward sloping. If q_S subscriptions are sold to sellers, then the price that results equals

$$p_S = 1 - \frac{1}{100} q_S. \tag{11.5}$$

Note again that this is the price that each seller is willing to pay *per buyer that will ultimately subscribe*. The total amount that each seller pays thus equals $p_S \cdot q_B$, with q_B the number of subscriptions sold to buyers. Demand from buyers is downward sloping. If q_B subscriptions are sold to sellers, then the price that results equals

$$p_B = \frac{1}{2} - \frac{1}{200} q_B. \tag{11.6}$$

Note that this is the price that each buyer is willing to pay *per seller that will ultimately subscribe*. The total amount that each buyer pays thus equals $p_B \cdot q_S$. For simplicity, marginal costs are always zero.

   (a) Suppose that the subscriptions to buyers and the subscriptions to sellers are sold by two separate firms that both maximize their own profits. Determine the equilibrium values q_S^* and q_B^*, and the equilibrium prices and profits that will result.

(b) Now consider the case of a Cournot duopoly. Each of the two firms sell subscriptions to both sides of the markets. Again, the price each seller is willing to pay for an interaction with a buyer is given by (11.5). When the total number of buyers is q_B, then the total revenues that firm 1 receives from sellers is thus given by $p_S \cdot q_S^1 \cdot q_B$, with q_S^1 the total number of subscriptions sold by firm 1 to sellers. Similar expressions hold for firm 2, and for the revenues received from buyers. Determine the equilibrium quantity of subscriptions that each firm sells to buyers, and the equilibrium quantity of subscriptions that each firm sells to sellers. Also determine equilibrium prices and profits.

2. Consider a monopolist newspaper that wants to attract readers and advertisers. There are 100 potential readers and 100 potential advertisers. Each potential advertiser is willing to pay on the basis of the number of readers that the newspaper will attract. Demand from advertisers is downward sloping. At a price p_A, the number of advertisements sold to sellers equals

$$q_A = 100 - 100p_A. \tag{11.7}$$

Note that p_A is the price that each advertiser has to pay *per reader that will ultimately subscribe*, just like we had in the lecture notes. The total amount that each advertiser pays thus equals $p_A \cdot q_R$, with q_R the number of subscriptions sold to readers.

Demand from readers is downward sloping. At a price p_R, the number of subscriptions sold to readers equals

$$q_R = 100 - 200p_R. \tag{11.8}$$

Readers are willing to pay more if there are *less* advertisements in the newspaper. One convenient way to model this, is to assume that the reader is willing to pay for the *lack* of advertisements. Potentially, there could be 100 advertisements in the newspaper. We assume that for every advertisement less than that, readers have to pay the amount p_R. The total amount that each reader pays thus equals $p_R \cdot (100 - q_A)$. For simplicity, marginal costs are zero.

Determine the profit-maximizing prices p_A and p_R.

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