2: Consumer Search

Games, Competition and Markets 2024/25

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Overview

1. Where We Stand
2. Search with homogeneous products
3. Mixed strategy equilibria
4. Bertrand
5. Varian's model of sales
6. Diamond paradox
7. Optimal search
8. Stahl's model of search
9. Anderson and Renault (1999)

Where We Stand

Topics

1. Preliminaries

Introductory lecture. Review of game-theoretic concepts. Some basic models of competition.

2. Consumer Search

What if consumers have to engage in costly search to find out about products and/or prices?

3. Advertising

What if producers have to inform consumers about their products and/or prices?

4. Menu Pricing

What if firms design different products and different prices for different consumers?

5. Durable Goods

What if a monopolist sells a durable good and cannot commit to future quantities?

6. Switching Costs

What if consumers have to pay extra if they switch suppliers?

7. Behavior-Based Price Discrimination

What if firms can base their prices on a consumer’s past behavior?

8. Vertical control

What if firms sell products to retailers who then sell it to final consumers?

9. Bundling

What if firms can sell bundles of products?

10. Network externalities and compatibility

What if products exhibit network effects: they becomes more (or sometimes less) useful if more consumers use it. Also: when do firms want to make their products compatible with that of their competitor?

11. Platform competition

What if online platforms bring buyers and sellers together? Or consumers and advertisers?

Search with homogeneous products

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Why?

- Price dispersion
- Realism
- Bertrand paradox
- Consumers are not fully informed
- As a vehicle to understand other issues
- Many online, platform issues

Basic Premise

1. Standard model: consumers are informed about all prices.
2. We now drop that assumption.
3. Consumers have to make an effort to learn prices. This effort is costly (at least for some).
4. Next week, firms will make an effort to inform consumers about prices...

Program

1. Bertrand: competition with homogeneous products.
2. Varian (1980): shoppers and non-shoppers.
3. Diamond (1971): search costs.
4. Stahl (1989): shoppers and non-shoppers, search costs.
5. Anderson and Renault (1999): differentiated products.

- This is an interesting, but rather technical literature.
- I do not have time to go into all technical details, and will give a broad overview of the (technical) issues involved.)

Mixed strategy equilibria

		Player 2	
		L	R
Player 1	U	10,0	3,7
	D	4,6	10,0

- A Nash equilibrium is for player 1 to play *U* with probability 6/13 and for player 2 to play *L* with probability 7/13.
- More generally: every player is indifferent among the strategies she mixes in equilibrium.

Bertrand

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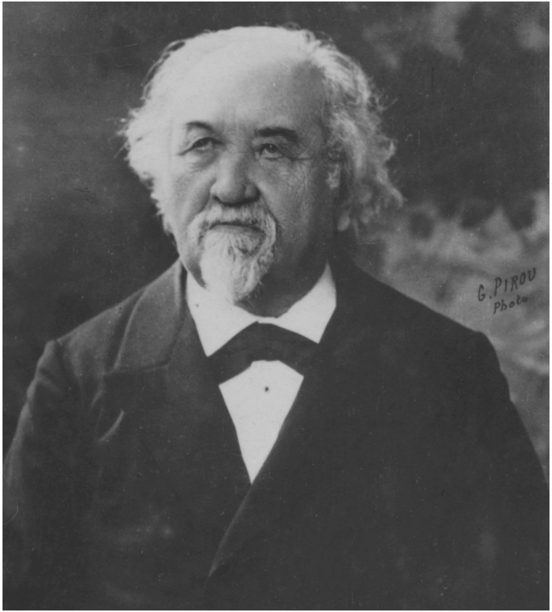

G. PIROU
Photo

Bertrand

- Homogenous products. Two firms.
- Marginal costs are constant and equal to c.
- Demand function $D(p)$.
- Define the monopoly price $p^m \equiv \arg\max_p (p-c)D(p)$.
- Demand firm i:

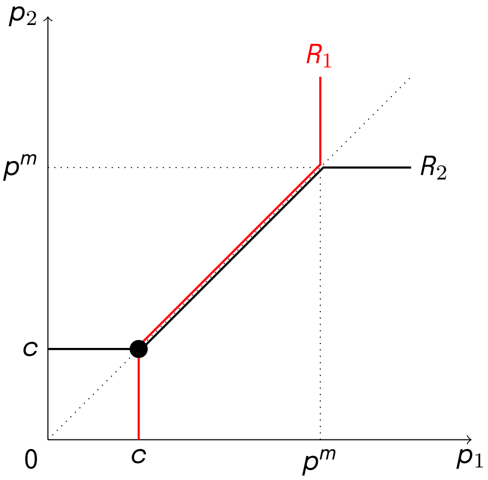
$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j \\ \frac{1}{2}D(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j. \end{cases}$$

- Profits firm i: $\Pi^i(p_i, p_j) = (p_i - c)D_i(p_i, p_j)$.
- Discontinuous!

Bertrand (ctd)

- Reaction function:

$$R_i(p_j) = \begin{cases} \in (p_j, \infty) & \text{if } p_j < c \\ \in [c, \infty) & \text{if } p_j = c \\ p_j - \varepsilon & \text{if } c < p_j \leq p^m \\ p^m & \text{if } p_j > p^m. \end{cases}$$


p_2
R_1
p^m
R_2
c
0
c
p^m
p_1

Bertrand (ctd)

- Reaction function:

$$R_i(p_j) = \begin{cases} \in (p_j, \infty) & \text{if } p_j < c \\ \in [c, \infty) & \text{if } p_j = c \\ p_j - \varepsilon & \text{if } c < p_j \leq p^m \\ p^m & \text{if } p_j > p^m. \end{cases}$$

- So the unique Nash equilibrium has both firms charging price equal to marginal costs and making zero profits.
- Bertrand paradox.

Varian's model of sales

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Varian (1980)

- Two firms.
- Costs are (normalized to) zero.
- Mass 1 of consumers, unit demand, willing to pay at most 1.
- A fraction λ of consumers is informed, and knows all prices.
- The remaining $1-\lambda$ simply pick a firm at random and buy there, provided $p \leq 1$.
- Profits for firm i :

$$\Pi^i(p_i, p_j) = \begin{cases} \left[\lambda + \frac{1}{2}(1-\lambda)\right] p_i & \text{if } p_i < p_j \leq 1 \\ \frac{1}{2} p_i & \text{if } p_i = p_j \leq 1 \\ \frac{1}{2}(1-\lambda) p_i & \text{if } 1 \geq p_i > p_j \\ 0 & \text{if } p_i > 1 \end{cases}$$

Equilibrium

- Derive the reaction function of firm i.
- Two obvious strategies: slightly undercut p_j, or set $p_i = 1$.
- Profits are $\left[\lambda + \frac{1}{2}(1-\lambda)\right] p_j$ and $(1-\lambda)/2$,respectively.
- The first yields higher profits if

$$\lambda p_j + \frac{1}{2}(1-\lambda) p_j > \frac{1}{2}(1-\lambda)$$

or

$$p_j > \frac{1-\lambda}{1+\lambda}.$$

- Reaction function:

$$R^i(p_j) = \begin{cases} 1 & \text{if } p_j \leq \frac{1-\lambda}{1+\lambda} \\ p_j - \varepsilon & \text{if } p_j > \frac{1-\lambda}{1+\lambda} \end{cases}$$

Equilibrium (ctd.)

- No equilibrium in pure strategies. Look for a mixed strategy equilibrium.
- Firm i draws its price from some $F(p)$ on $[\underline{p}, \bar{p}]$.
- Necessarily, all prices yield the same expected profits.
- Necessarily, $\bar{p} = 1$.
- Necessarily, these profits equal $\frac{1}{2}(1-\lambda)$.
- At price $p \in [\underline{p}, 1]$, firm is expected profits are

$$E(\pi_i(p)) = \frac{1}{2}(1-\lambda)p + (1-F(p))\lambda p.$$

- $\underline{p} = \frac{1-\lambda}{1+\lambda}$.

$$F(p) = 1 - \frac{(1-\lambda)(1-p)}{2\lambda p}$$

- Varian interprets this as a model of sales.

n firms

- Straightforward extension.
- Now equilibrium profits are $\frac{1}{n}(1-\lambda)$.
- At price $p \in [\underline{p}, 1]$, firm is expected profits are

$$E(\pi_i(p)) = \frac{1}{n}(1-\lambda)\,p + (1 - F(p))^{n-1}\,\lambda p.$$

- $\underline{p} = \frac{1-\lambda}{1+\lambda}$.

$$F(p) = 1 - \left(\frac{(1-\lambda)(1-p)}{n\lambda p}\right)^{\frac{1}{n-1}}$$

Diamond paradox

Diamond (1971)

- n firms.
- Costs are (normalized to) zero.
- Each consumer has unit demand and willingness to pay v (for now).
- Observing the price of a single firm costs s.
- The first observation is free (for now).
- With $s = 0$: Bertrand pricing.
- $s > 0$: monopoly pricing!
- Diamond paradox.

Diamond: what if the first visit is not free?

- In the current set-up: the equilibrium does not change.
- Once a consumer has visited, again any price $p < v$ cannot be an equilibrium.
- Search costs s are then sunk.
- This implies consumers will not visit in the first place!
- Market breaks down.
- Way out: assume each consumer has individual downward sloping demand function $D(p)$
- Still an equilibrium to set the monopoly price.
- It is now worthwhile to pay the first visit - as long as consumer surplus is high enough to compensate for s.

Optimal search

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Optimal search

- Intermezzo.
- Suppose prices are drawn from some distribution $F(p)$ (as in Varian).
- Your costs for each firm that you visit are s (as in Diamond).
- You have unit demand.
- When should you continue search!?
- We look at *sequential search* and *perfect recall*.

Optimal search - 2 firms

- For now, assume there are 2 firms.
- You go to firm 1 and observe price p_1.
- If you also go to firm 2, you may find a lower price: $p_2 < p_1$.
- If you do, your benefit is $p_1 - p_2$.
- Expected benefit of visiting firm 2: $b(p_1) = \int_0^{p_1} (p_1 - p_2)\, dF(p_2)$.
- Your costs of doing so are s.
- There is a unique $\hat{p}$ such that $b(\hat{p}) = s$.
- Hence, you will visit firm 2 whenever $p_1 > \hat{p}$.

Optimal search - $n > 2$ firms

- But now suppose there are more than 2 firms.
- Let's start with 3.
- Backward induction.
- Denote as $B_1(p)$ the expected *net* benefit of continuing search if there is just 1 firm left to search: $B_1(p) = b(p) - s$.
- By construction, $B_1(\hat{p}) = 0$.
- Denote as $B_2(p)$ the expected *net* benefit of continuing search if there are 2 firms left to search.
- Hence, at firm 1, you should stop searching if you find a p_1 with $B_2(p_1) \leq 0$.

Optimal search - $n > 2$ firms, ctd

- For the sake of argument, suppose $p_1 = \hat{p}$.
- If you continue search, two options:
- You may find $p_2 > p_1$. In that case, your best price is still p_1 and by construction, you will be indifferent between continuing search and not doing so.
- You may find $p_2 < p_1$. Then, you prefer not to continue search.
- Hence, with $p_1 = \hat{p}$, *you never exercise your option to visit the last firm.*
- Hence, *the situation you are facing is exactly the same as if there is just one firm left to search.*
- $B_2(\hat{p}) = B_1(\hat{p}) = 0$.

Optimal search - $n > 2$ firms, ctd

- By induction, the same argument applies if there are 3, 4, 5, …firms left to search.
- Hence, the reserve price $\hat{p}$ is *stationary*; it does not matter how many firms we have left to search.
- We don't even have to know how many firms there are left to search.
- Note: we need to have perfect recall for this.

Stahl's model of search

Games, Competition and Markets. Lecture 2

Stahl (1989)

- n firms, unit mass of consumers.
- Costs are (normalized to) zero.
- Each consumer has willingness to pay 1, unit demand.
- A fraction λ of consumers has search costs 0. Hence, they know all prices. Shoppers.
- The remaining $1 - \lambda$ have search costs s. Non-shoppers.
- Also here: no equilibrium in pure strategies.
- Hence, we look for a price distribution $F(p)$.
- Given $F(p)$, we can determine $\hat{p}$.
- Given $\hat{p}$, we can determine $F(p)$.

Firm behavior

- Suppose a firm charges a price $p > \hat{p}$.
- Can this be part of an equilibrium?
- Note: even non-shoppers continue search if they encounter a $p > \hat{p}$.
- Hence: you will not sell anything.
- Not a good idea.
- This implies that *in equilibrium, non-shoppers only visit one firm*.
- Hence the model collapses into Varian (1980), but with an endogenous upper bound.
- Equilibrium can be derived, if you really want to.

If you really want to know...

- Expected profits: $E\left(\pi\left(p_i\right)\right) = p_i\left[\frac{1-\lambda}{n} + \lambda\left(1 - F\left(p_i\right)\right)^{n-1}\right]$
- When setting $\hat{p}$, profits are $(1-\lambda)\hat{p}/n$.
- Hence $F\left(p\right) = 1 - \left(\frac{(1-\lambda)(\hat{p}-p)}{n\lambda p}\right)^{\frac{1}{n-1}}$.
- Hence $p = \frac{\hat{p}}{1+n\frac{\lambda}{1-\lambda}(1-F(p))^{n-1}}$.
- So
$$E(p) = \int_{\underline{p}}^{\hat{p}} p dF(p) = \int_{\underline{p}}^{\hat{p}} \frac{\hat{p} dF(p)}{1 + n\frac{\lambda}{1-\lambda}\left(1 - F\left(p\right)\right)^{n-1}}$$
- Change of variables $y \equiv F(p)$, this implies
$$E(p) = \hat{p}\int_0^1 \frac{dy}{1 + n\frac{\lambda}{1-\lambda}\left(1 - y\right)^{n-1}}.$$

If you really want to know...

- $$E(p) = \hat{p} \int_0^1 \frac{dy}{1 + n\frac{\lambda}{1-\lambda}(1-y)^{n-1}}.$$
- Note $s = \int_0^{\hat{p}} (\hat{p} - p_2)\, dF(p_2) = \int_0^{\hat{p}} \hat{p} dF(p_2) - \int_0^{\hat{p}} p_2 dF(p_2) = \hat{p} - E(p)$
- This allows us to pin down $\hat{p}$:
$$\hat{p} = \frac{s}{1 - \int_0^1 \frac{dy}{1+n\frac{\lambda}{1-\lambda}(1-y)^{n-1}}},$$
- Solved!

Some notes

- Stahl shows existence of equilibrium for the general case (i.e. consumers with downward sloping demand functions).
- Only shoppers: Bertrand. Only non-shoppers: Diamond.
- With lower entry costs, prices go up!
- The average price increases with the number of stores.

Anderson and Renault (1999)

Model

- n firms. Unit mass of consumers
- For a consumer the utility from buying from firm $i : u(p_i) = v - p_i + \varepsilon_i$.
- ε_i is a *match value*. Firms can never observe it. Consumers only observe it upon visiting a firm.
- $\varepsilon_i \sim F$, with $1 - F$ log-concave.
- The consumer has search costs s.
- Sequential search, perfect recall.
- v is high enough such that the consumer will always buy in equilibrium.
- We do get a pure strategy equilibrium in prices.
- Consumers search for a match value that is high enough.

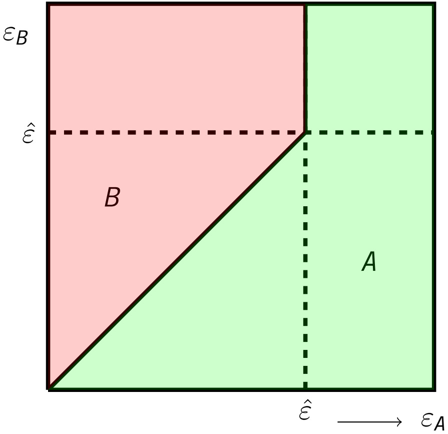
Consumer behavior

- For the sake of argument, first suppose that all firms charge the same price p^*.
- Hence consumers only search for a match value that is high enough.
- Suppose the current best offer gives match value ε_i
- Expected benefit of one more search:

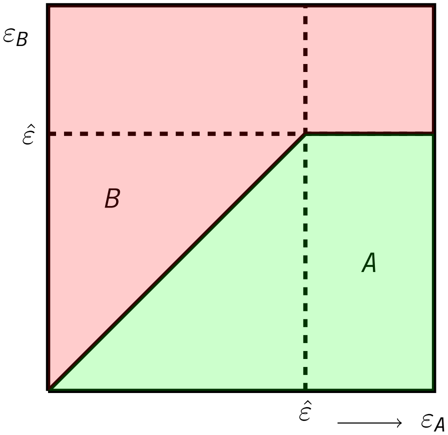
$$g(x) = \int_{\varepsilon_i}^{\infty} (\varepsilon - \varepsilon_i) f(\varepsilon) d\varepsilon.$$

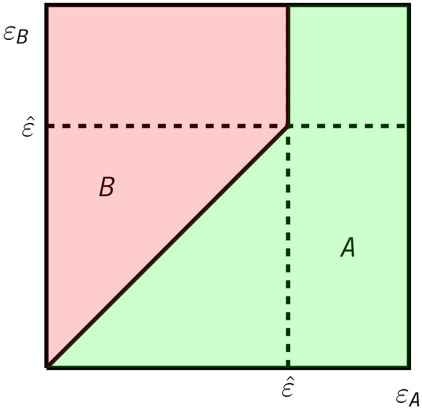
- Define $\hat{\varepsilon}$ such that $g(\hat{\varepsilon}) = s$.
- Then the consumer will continue searching iff $\varepsilon < \hat{\varepsilon}$.
- Let's start with the simple uniform 2-firm case.

Consumers that visit A first...

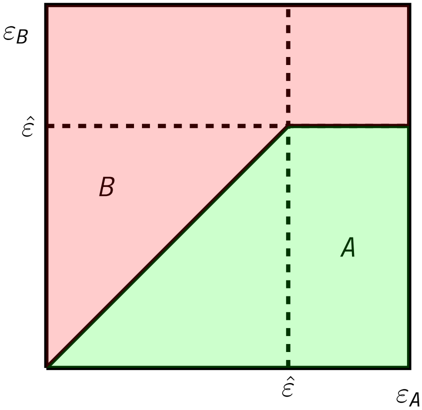


Consumers that visit B first...



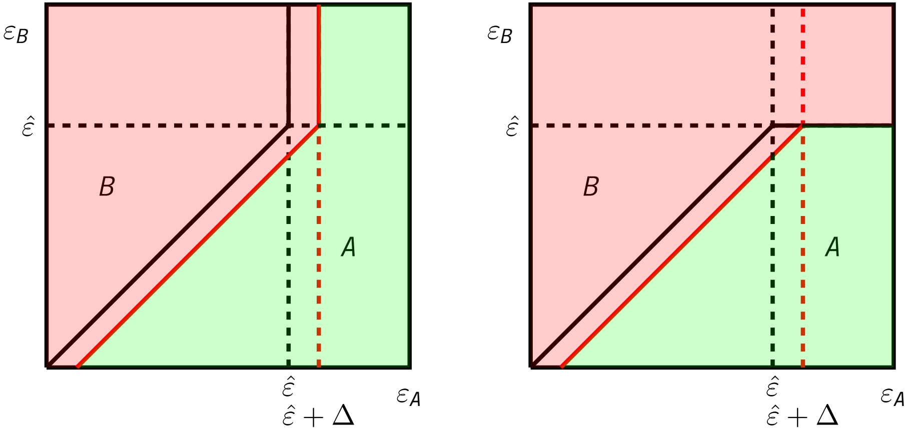


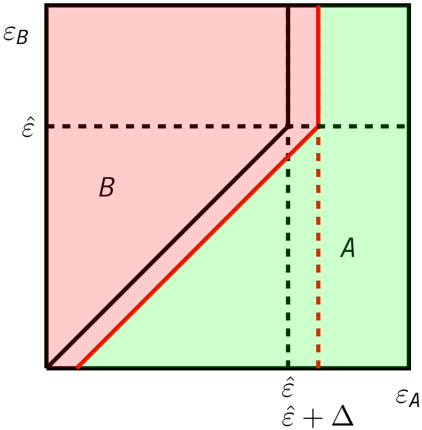
ε_B
$\hat{\varepsilon}$
B
A
$\hat{\varepsilon}$
ε_A



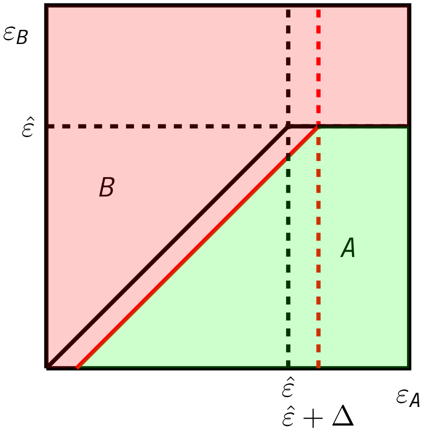
ε_B
$\hat{\varepsilon}$
B
A
$\hat{\varepsilon}$
ε_A

What if A defects to a higher price?





$$(1 - \hat{\varepsilon} - \Delta) + \frac{1}{2}\hat{\varepsilon}^2$$



$$(1 - \hat{\varepsilon} - \Delta)\hat{\varepsilon} + \frac{1}{2}\hat{\varepsilon}^2$$

So

$$D_A = \frac{1}{2}(1 - \hat{\varepsilon} - \Delta)(1 + \hat{\varepsilon}) + \frac{1}{2}\hat{\varepsilon}^2$$

Now

$$\pi_A = p_A \cdot D_A$$

so

$$\frac{\partial \pi_A}{\partial p_A} = D_A + p_A \cdot \frac{\partial D_A}{\partial p_A} = 0$$

Note:

$$\frac{\partial D_A}{\partial p_A} = -\frac{1}{2}(1 + \hat{\varepsilon})$$

Symmetry:

$$\frac{1}{2} + p \cdot -\frac{1}{2}(1 + \hat{\varepsilon}) = 0$$

Hence

$$p^* = \frac{1}{1 + \hat{\varepsilon}}$$

We still have to figure out $\hat{\varepsilon}$

$$b(\varepsilon_i) = \int_{\varepsilon_i} (\varepsilon - \varepsilon_i) f(\varepsilon) d\varepsilon_i = \frac{(1-\varepsilon_i)^2}{2}$$

So

$$\hat{\varepsilon} = 1 - \sqrt{2s}$$

Hence

$$p^* = \frac{1}{1+\hat{\varepsilon}} = \frac{1}{2-\sqrt{2s}}.$$

General model: Consumer behavior

- For the sake of argument, first suppose that all firms charge the same price p^*.
- Hence consumers only search for a match value that is high enough.
- Suppose the current best offer gives match value ε_i
- Expected benefit of one more search:

$$g(x) = \int_{\varepsilon_i}^{\infty} (\varepsilon - \varepsilon_i) f(\varepsilon) d\varepsilon.$$

- Define $\hat{\varepsilon}$ such that $g(\hat{\varepsilon}) = s$.
- Then the consumer will continue searching iff $\varepsilon < \hat{\varepsilon}$.

Firm sales

- What are the expected sales of a firm in this case?
- Suppose you are firm 1.
- For the sake of argument, suppose the consumer starts out by making a list of the random order in which she plans to visit firms.
- The probability that you are 1st on that list is $1/n$. If you are, the probability she buys from you is $1 - F(\hat{\varepsilon})$.
- The probability you are 2nd on the list is $1/n$. If you are, the probability she buys from you is $F(\hat{\varepsilon})\,(1 - F(\hat{\varepsilon}))$.
- The probability you are 3rd on the list is $1/n$. If you are, the probability she buys from you is $F(\hat{\varepsilon})^2\,(1 - F(\hat{\varepsilon}))$.
- etc.

Firm sales

- Hence, probability she buys from you:

$$\frac{1}{n}\sum_{j}F(\hat{\varepsilon})^{j-1}\left(1-F(\hat{\varepsilon})\right)=\frac{1}{n}\left[\frac{1-F(\hat{\varepsilon})^{n}}{1-F(\hat{\varepsilon})}\right]\cdot\left(1-F(\hat{\varepsilon})\right).$$

- But there is also a probability that she visits all firms, and after the last finds out that you had the best deal after all:

$$\int_{-\infty}^{\hat{\varepsilon}}F(\varepsilon)^{n-1}f(\varepsilon)d\varepsilon$$

- Your demand D_1 is the sum of these two terms.
- In equilibrium, it should equal $1/n$.

Equilibrium

- But what price will firms charge in equilibrium?
- We look for an equilibrium such that if all other firms charge p^*, it is a best reply for j to do the same.
- So suppose all other firms charge p^*, but firm 1 defects to some p_1.
- Define $\Delta \equiv p_1 - p^*$.
- Suppose firm 1 is the first firm that a consumer happens to visit.
- Her optimal strategy is still to continue search as long as the net utility she obtains at firm 1 is smaller than the $\hat{\varepsilon}$ defined above.
- Thus she buys from you with probability $1 - F(\hat{x} + \Delta)$.
- If she (plans to) visit you second, you sell with probability $F(\hat{\varepsilon})(1 - F(\hat{\varepsilon} + \Delta))$.
- etc.

Equilibrium (ctd)

- Your expected demand now equals $D_1(p_1, p^*) =$

$$\frac{1}{n}\left[\frac{1-F(\hat{\varepsilon})^n}{1-F(\hat{\varepsilon})}\right]\left[1-F\left(\hat{\varepsilon}+\Delta\right)\right]+\int_{-\infty}^{\hat{\varepsilon}+\Delta}F(\varepsilon-\Delta)^{n-1}f(\varepsilon)d\varepsilon.$$

- Profits: $p_1 D_1$. Take FOC, impose symmetry: $p_1 D_1' + D_1 = 0$. In equilibrium, $D_1 = 1/n$, hence

$$p^* = \frac{1}{\frac{1-F(\hat{\varepsilon})^n}{1-F(\hat{\varepsilon})}f(\hat{\varepsilon}) - n\int_{-\infty}^{\hat{\varepsilon}} f'(\varepsilon)F(\varepsilon)^{n-1}d\varepsilon}$$

Example

- Match values uniform on $[0, 1]$.

$$D_1\left(p_1, p^*\right) = \frac{1}{n}\left[\frac{1-\hat{\varepsilon}^n}{1-\hat{\varepsilon}}\right]\left(1-\hat{\varepsilon}-\Delta\right) + \int_{-\infty}^{\hat{\varepsilon}+\Delta}\left(\varepsilon-\Delta\right)^{n-1} d\varepsilon,$$

$$\frac{\partial D_1\left(p^*, p^*\right)}{\partial p_1} = -\frac{1}{n}\left[\frac{1-\hat{\varepsilon}^n}{1-\hat{\varepsilon}}\right].$$

$$b\left(\hat{\varepsilon}\right) = \int_{\hat{\varepsilon}}^{1}\left(\varepsilon-\hat{\varepsilon}\right) d\varepsilon = \frac{1}{2}\left(1-\hat{\varepsilon}\right)^2$$

$$\hat{\varepsilon} = 1-\sqrt{2s},$$

$$p^* = \frac{\sqrt{2s}}{1-\left(1-\sqrt{2s}\right)^n}.$$

Uniform match values

$$p^* = \frac{\sqrt{2s}}{1 - \left(1 - \sqrt{2s}\right)^n}.$$

$$\frac{\partial p^*}{\partial n} = \frac{\partial}{\partial n}\left(\frac{1-\hat{\varepsilon}}{1-\hat{\varepsilon}^n}\right) = \hat{\varepsilon}^n \left(\ln \hat{\varepsilon}\right) \frac{1-\hat{\varepsilon}}{\left(1-\hat{\varepsilon}^n\right)^2} < 0,$$

- Having more firms leads to lower prices
- An increase in search costs s leads to a lower $\hat{\varepsilon}$.

$$\frac{\partial p^*}{\partial \hat{\varepsilon}} = \frac{\partial}{\partial \hat{\varepsilon}}\left(\frac{1-\hat{\varepsilon}}{1-\hat{\varepsilon}^n}\right) = \frac{\hat{\varepsilon}^{n-1}\left(n\left(1-\hat{\varepsilon}\right)+\hat{\varepsilon}\right)-1}{\left(1-\hat{\varepsilon}^n\right)^2} < 0.$$

- Higher search costs, higher prices.

Thus...

- In equilibrium, consumers do actually search...
- They may even return to a shop they visited before.
- Prices increase in search costs s, decrease in number of firms n.
- This can also be shown for a general distribution function.
- Equilibrium does **not** exhibit price dispersion.

Applications include

- Armstrong, Vickers and Zhou (2009)
- Haan and Morága-Gonzalez (2011)
- Zhou (2011)

Thank you for your attention

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