**Tentative solutions**
Games, Competition and Markets
2024/25

## Chapter 1

1. (a) It could turn out that marginal costs are 0, or that they are 0.4. In general, when they are $c$ :

$$\pi_1 = (1 - q_1 - q_2 - c)\, q_1$$

so

$$\frac{\partial \pi_1}{\partial q_1} = 1 - 2q_1 - q_2 - c = 0.$$

Imposing symmetry:

$$q = \frac{1-c}{3}.$$

Hence, if it turns out marginal costs are zero, both firms set $q = 1/3$. If they are 0.4, firms set $q = .6/3 = .2$

   (b) Expected profits now are:

$$\begin{aligned} \pi_1 &= \frac{1}{2}(1 - q_1 - q_2)\, q_1 + \frac{1}{2}(1 - q_1 - q_2 - 0.4)\, q_1 \\ &= (1 - q_1 - q_2 - 0.2)\, q_1 \end{aligned}$$

so the equilibrium has $q = .8/3$ (which in fact, is just the average of the earlier options)

2. The indifferent consumer is now given by

$$v - tz^2 - p_0 = v - t(1-z)^2 - p_1$$

which yields

$$z = \frac{1}{2} + \frac{P_1 - P_0}{2t},$$

which is the exact same expression as with linear transport costs. The demand that each firm faces therefore does not change and hence the equilibrium will also be the same as in the lecture notes.

3. We proceed in the same fashion as in the lecture notes. Note however that for $n = 2$ one could also write down profits of firm $A$ as a function of prices $p_A$ and $p_B$ and then derive the best-reply function of firm $A$ as a function of $p_B$.

We now have to take into account that not everyone will buy in equilibrium. In equilibrium, with equal prices, we would have sales of each firm being equal to

$$q = \frac{1}{2} - \frac{1}{2}p^2.$$

Suppose firm $A$ sets a price slightly higher than the other firm. Sales then equal

$$q_A = \frac{1}{2}(1-\Delta)^2 - \frac{1}{2}p^2$$

Again

$$\pi_A = (p_A - c)q_A$$

so

$$\frac{\partial \pi_A}{\partial p_A} = p_A \frac{\partial q_A}{\partial p_A} + q_A = 0$$

Again

$$\frac{\partial q_A}{\partial p_A} = -(1-\Delta)$$

But equilibrium now has

$$q_A = \frac{1}{2} - \frac{1}{2}p^2.$$

So, imposing symmetry, we need

$$p(-1) + \left(\frac{1}{2} - \frac{1}{2}p^2\right) = 0$$

so

$$p^* = \sqrt{2} - 1$$

**Chapter 2**

1. (a) Consider the demand for firm 1. There are two cases to consider. Suppose that $p_2 \leq 2$. There are now 3 obvious choices for the price of firm 1. If it sets a price equal to 2, it only sells to half of all the uninformed consumers, so demand is $1-\lambda$. If it sets a price equal to 3, it only sells to half of the uninformed high types, so demand is $(1-\lambda)/2$. If it slightly undercuts firm 2, it will sell to all the informeds and half of all the uninformeds, so demand is $2\lambda + (1-\lambda) = 1+\lambda$. Profits in these three cases are $2(1-\lambda)$, $3(1-\lambda)/2$, and $p_2(1+\lambda)$. Obviously, the first option

always yields higher profits than the second option. The first option yields higher profits than the third option whenever

$$2(1-\lambda) > p_2(1+\lambda),$$

thus if

$$p_2 < \frac{2(1-\lambda)}{1+\lambda}.$$

Now suppose that $2 < p_2 < 3$. Again, there are three obvious options for firm 1. If it sets a price equal to 2, it sells to all the informeds and half of the uninformeds, so demand is $2\lambda + (1-\lambda) = 1+\lambda$. If it sets a price equal to 3, it only sells to half of the uninformed high types, so demand is $(1-\lambda)/2$. If it slightly undercuts firm 2, it will sell to all the informed high types, and to half of the uninformed high types, so demand is $\lambda + (1-\lambda)/2 = (1+\lambda)/2$. Profits in these three cases are $2(1+\lambda)$, $3(1-\lambda)/2$, and $p_2(1+\lambda)/2$ respectively. Note that the third option yields higher profits than the first option if

$$p_2(1+\lambda)/2 > 2(1+\lambda),$$

i.e. if $p_2 > 4$. But firm 2 will never set such a price. Thus, the only relevant options in this interval are the first and the second. The first option yields higher profits if

$$2(1+\lambda) > 3(1-\lambda)/2,$$

i.e. if $\lambda > -1/7$, which is always satisfied. Hence, with $2 < p_2 < 3$,[1] the best reply for firm 1 is to always set $p_2 = 2$. This yields the following best reply function

$$p_1 = \begin{cases} 2 & \text{if } p_2 \leq \frac{2(1-\lambda)}{1+\lambda} \\ p_2 - \varepsilon & \text{otherwise} \end{cases}$$

Note that $2(1-\lambda)/(1+\lambda) \leq 2$ for all relevant $\lambda$. Hence, no firm will ever find it profitable to set a price higher than 2.

(b) With similar arguments as in the lecture notes, there is no equilibrium in pure strategies; firms have an incentive to undercut prices higher than $2(1-\lambda)/(1+\lambda)$, while for lower prices, the best reply is to set a price equal to 2. Therefore, we will look for an

[1] Note that if $p_2 = 3$, firm 1 will slightly undercut. But that implies that this will never be set, so for conciseness, we ignore that option in what follows.

equilibrium in mixed strategies on some interval $[\underline{p}, \bar{p}]$. Note that necessarily $\bar{p} = 2$: no firm will ever set a price higher than 2. If it does, it knows that it has the highest price for sure, but if that is true, it is better off setting $p = 2$. A firm that sets $p = 2$ will sell to half of all the uninformeds, and thus make profits $2(1-\lambda)$. A firm that sets price $p$ will sell $2\lambda + (1-\lambda) = 1 + \lambda$ if this price turns out to be the lowest. Otherwise it will sell $1 - \lambda$, which is its share of all the uninformeds. Expected profits then equal

$$F(p)(1-\lambda)p + (1 - F(p))(1+\lambda)p = p + p\lambda(1 - 2F(p)).$$

A mixed strategy necessarily has all prices yielding the same profits of $2(1-\lambda)$. This implies

$$F(p) = \frac{p - 2 + \lambda(p+2)}{2p\lambda}.$$

The lower bound of the interval can be found e.g. by plugging in $F(p) = 0$. This yields

$$\underline{p} = \frac{2(1-\lambda)}{1+\lambda}.$$

2. Consumer behavior does not change. But different types of consumers now have different $\hat{\varepsilon}$. Let's refer to the low search cost people as $L$ and the high search cost people as $H$. From the analysis in the chapter $\hat{\varepsilon} = 1 - \sqrt{2s}$ so $\hat{\varepsilon}_L = 1 - 0.04\sqrt{2}$ and $\hat{\varepsilon}_H = 1 - 0.05\sqrt{2}$. A firm that defects now gets (following the slides)

$$\begin{aligned} q_A &= \lambda\left[\frac{1}{2}(1 - \hat{\varepsilon}_H - \Delta)(1 + \hat{\varepsilon}_H) + \frac{1}{2}\hat{\varepsilon}_H^2\right] \\ &+ (1-\lambda)\left[\frac{1}{2}(1 - \hat{\varepsilon}_L - \Delta)(1 + \hat{\varepsilon}_L) + \frac{1}{2}\hat{\varepsilon}_L^2\right]. \end{aligned}$$

So

$$\begin{aligned} \frac{\partial q_A}{\partial P_A} &= \lambda\left[-\frac{1}{2}(1 + \hat{\varepsilon}_H\right] + (1-\lambda)\left[-\frac{1}{2}(1 + \hat{\varepsilon}_L)\right] \\ &= -\frac{1}{2}(1 + \hat{\varepsilon}_A) \end{aligned}$$

with

$$\begin{aligned} \hat{\varepsilon}_A &\equiv \lambda\hat{\varepsilon}_H + (1-\lambda)\varepsilon_L \\ &= 1 - \sqrt{2} \cdot (0.05 - 0.01\lambda) \end{aligned}$$

the weighted average of the two. Equilibrium prices then equal, again following the notes,

$$\begin{aligned} p^* &= \frac{1}{1+\hat{\varepsilon}_A} \\ &= \frac{1}{2-\sqrt{2}\,(0.05-0.01\lambda)} \end{aligned}$$

3. This is based on Haan and Moraga-González (2011). The first part of the analysis is the same as that leading up to slide 37. However a fraction $a_1/\left(a_1+a_2\right)$ now visits firm 1 first, rather than 1/2. Hence, demand for firm 1 now equals

$$D_1 = \frac{a_1}{a_1+a_2}\left(1-\hat{\varepsilon}-\Delta\right) + \frac{a_2}{a_1+a_2}\left(1-\hat{\varepsilon}-\Delta\right)\hat{\varepsilon} + \frac{1}{2}\hat{\varepsilon}^2$$

with again $\Delta \equiv p_1 - p^*$, so

$$\frac{\partial D_1}{\partial p_1} = -\frac{a_1}{a_1+a_2} - \frac{a_2}{a_1+a_2}\hat{\varepsilon}.$$

Total profits are

$$\pi_1 = p_1 \cdot D_1 - \frac{1}{4}a_1.$$

First-order conditions are

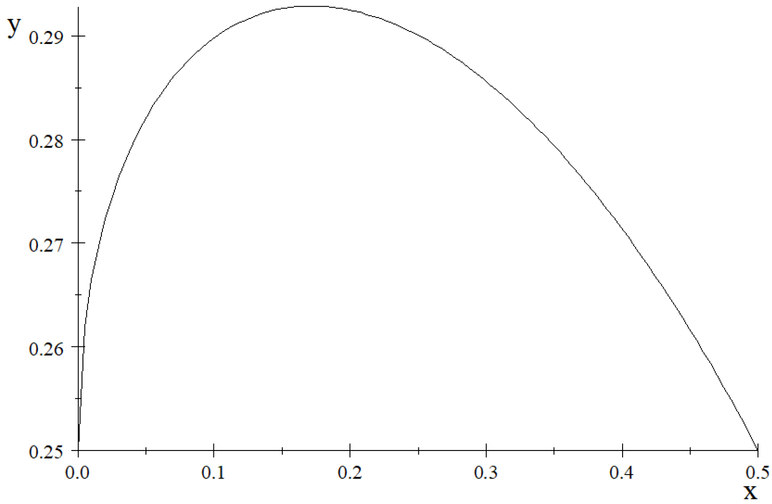
$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} &= D_1 + p_1 \cdot \frac{\partial D_1}{\partial p_1} = 0 \\ \frac{\partial \pi_1}{\partial a_1} &= \frac{a_2}{\left(a_1+a_2\right)^2} \cdot \left(1-\hat{\varepsilon}-\Delta\right) \cdot p_1 - \frac{a_2}{\left(a_1+a_2\right)^2}\left(1-\hat{\varepsilon}-\Delta\right)\hat{\varepsilon} \cdot p_1 - \frac{1}{4} = 0 \end{aligned}$$

We can now impose symmetry:

$$\begin{aligned} \frac{\partial \pi_1}{\partial p_1} &= \frac{1}{2} + p^* \cdot \left(-\frac{1}{2} - \frac{1}{2}\hat{\varepsilon}\right) = 0 \\ \frac{\partial \pi_1}{\partial a_1} &= \frac{1}{4a}\left(1-\hat{\varepsilon}\right)p^* - \frac{1}{4a}\left(1-\hat{\varepsilon}\right)\hat{\varepsilon}p^* - \frac{1}{4} = 0 \end{aligned}$$

Hence

$$\begin{aligned} p^* &= \frac{1}{1+\hat{\varepsilon}} \\ a^* &= \left(1-\hat{\varepsilon}\right)^2 p^* \end{aligned}$$




Consumer behavior does not change, so we still have $\hat{\varepsilon} = 1 - \sqrt{2s}$. Note that prices are still the same as in the regular model. This is intuitive: in equilibrium each firm will choose the same level of advertising, so in equilibrium consumers will again visit firms randomly with equal probability. Equilibrium profits are given by

$$\begin{aligned}
\pi^* &= \frac{1}{2}p^* - \frac{1}{4}a^* = \frac{1}{2(1+\hat{\varepsilon})} - \frac{1}{4}(1-\hat{\varepsilon})^2 \cdot \frac{1}{1+\hat{\varepsilon}} - \left(\frac{1}{4}\frac{1+2\varepsilon-\varepsilon^2}{1+\varepsilon}\right) \\
&= \frac{1}{4}\frac{1+2\hat{\varepsilon}-\hat{\varepsilon}^2}{1+\hat{\varepsilon}} = \frac{1}{4}\frac{1+2\left(1-\sqrt{2s}\right)-\left(1-\sqrt{2s}\right)^2}{2-\sqrt{2s}}
\end{aligned}$$

$$\frac{1}{4}\frac{1+2\left(1-\sqrt{2s}\right)-\left(1-\sqrt{2s}\right)^2}{2-\sqrt{2s}}$$

We need that $s \in (0, 1/2)$. Profits are then plotted in the figure above.

That is, they're increasing up to $3 - 2\sqrt{2} \approx 0.17157$ and decreasing afterwards.

# References


Haan, M. A. and Moraga-González, J. L. (2011). Advertising for attention in a consumer search model. *Economic Journal*, pages 552–579.