Tentative Solution Mock Midterm
Games, Competition and Markets
2024/2025

1. A consumer will buy from firm 0 iff

$$v_0 - p_0 - tx > v_1 - p_1 - t(1-x)$$

Hence the indifferent consumer is given by

$$z = \frac{1}{2} + \frac{p_1 - p_0 + v_0 - v_1}{2t}$$

Profits of firm 0 are

$$\pi_0 = (p_0 - c)\, z$$

Taking the FOC and deriving the best reply function:

$$p_0 = \frac{1}{2}(c + t + v_0 - v_1 + p_1)$$

Profits of firm 1 are

$$\pi_1 = (p_1 - c)(1 - z)$$

Taking the FOC and deriving the best reply function:

$$p_1 = \frac{1}{2}(c + t + v_1 - v_0 + p_0)$$

Plugging this into the reaction function of firm 0 and solving:

$$p_0 = \frac{1}{2}\left(c + t + v_0 - v_1 + \left(\frac{1}{2}(c + t + v_1 - v_0 + p_0)\right)\right)$$

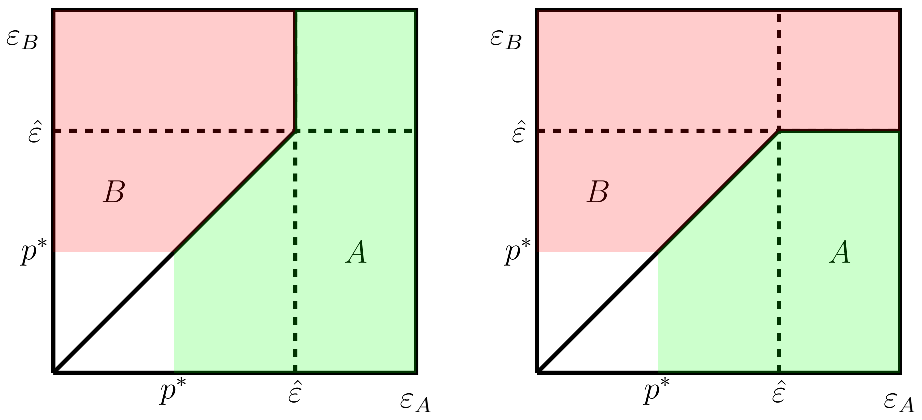
hence

$$p_0 = c + t + \frac{1}{3}(v_0 - v_1),$$

which implies

$$p_1 = c + t - \frac{1}{3}(v_0 - v_1),$$

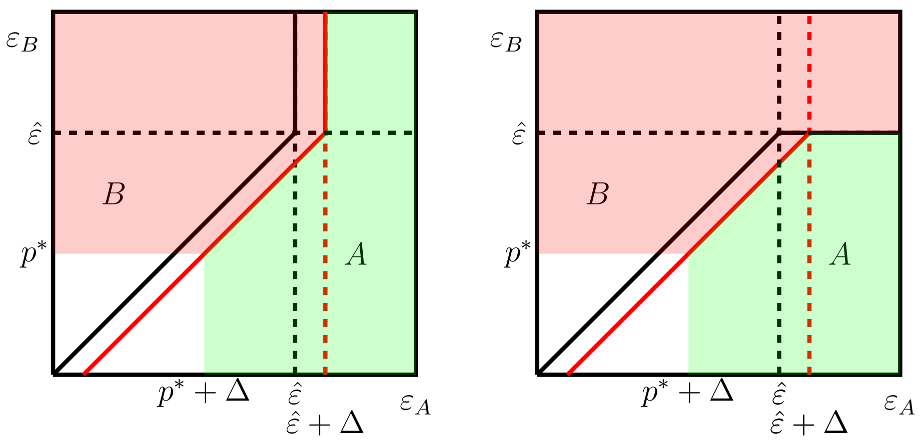
2. Solve the Anderson-Renault model with 2 firms and a uniform distribution of match values for the case that $v = 0$. Write equilibrium prices in terms of search costs s.



It is convenient to use figures similar to those in the lecture. Again, the green people are those that buy from A, while the red buy from B, in case prices are equal:

where the left-hand side reflects those who visit firm A first, and the right-hand side those that visit B first.

Now suppose firm A defects to a price that is Δ higher. The resulting sales are given by



It is straightforward to verify that the number of consumers visiting A in the left panel is $(1 - \hat{\varepsilon} - \Delta) + \frac{1}{2}\hat{\varepsilon}^2 - \frac{1}{2}p^{*2}$ and in the right panel $(1 - \hat{\varepsilon} - \Delta)\hat{\varepsilon} + \frac{1}{2}\hat{\varepsilon}^2 - \frac{1}{2}p^{*2}$. Total sales thus equal

$$D_A = \frac{1}{2}(1 - \hat{\varepsilon} - \Delta)(1 + \hat{\varepsilon}) + \frac{1}{2}\hat{\varepsilon}^2 - \frac{1}{2}p^{*2}$$

Now

$$\pi_A = p_A \cdot D_A$$

so

$$\frac{\partial \pi_A}{\partial p_A} = D_A + p_A \cdot \frac{\partial D_A}{\partial p_A} = 0$$

Note:

$$\frac{\partial D_A}{\partial p_A} = -\frac{1}{2}(1+\hat{\varepsilon})$$

Impose symmetry:

$$\frac{1}{2} - \frac{1}{2}p^2 + p \cdot -\frac{1}{2}(1+\hat{\varepsilon}) = 0$$

Hence

$$p^* = \frac{1}{4}\sqrt{(1+\hat{\varepsilon})^2+8} - \frac{1}{4}(1+\hat{\varepsilon})$$

The consumer's decision problem does not change. Hence again $\hat{\varepsilon} = 1-\sqrt{2s}$. Plugging that into p^* yields

$$p^* = \frac{1}{4}\sqrt{2}\left(\sqrt{s+4}+\sqrt{s}\right) - \frac{1}{2}.$$

3. Just like in the solution to exercise 1 of Chapter 3, we can write the reaction function of firm 1 as

$$P_1 = \frac{1}{2} + \frac{P_2}{2} + \frac{1-\phi_2}{\phi_2},$$

with ϕ_2 the total fraction of consumers that ends up being informed about firm 2. Of course, the challenge is to derive this expression.

- A fraction $\Phi_1\Phi_2$ of consumers are informed about both firms (hence also about firm 2).
- A fraction $\Phi_2(1-\Phi_1)$ only receive an ad from firm 2. They are informed about firm 2 as well.
- A fraction $\Phi_1(1-\Phi_2)$ only receive an ad from 1. But half of these also become informed about 2.
- Hence $\phi_2 = \Phi_1\Phi_2 + \Phi_2(1-\Phi_1) + \frac{1}{2}\Phi_1(1-\Phi_2) = \frac{1}{2}\Phi_1(1-\Phi_2) + \Phi_2$. Plugging this into the reaction function above gives the final answer:

$$P_1 = \frac{1}{2} + \frac{P_2}{2} + \frac{\frac{1}{2}(1-\Phi_2)(2-\Phi_1)}{\frac{1}{2}\Phi_1(1-\Phi_2)+\Phi_2}.$$

4. (a) In equilibrium we need that there is no surplus at the bottom, i.e. that the low type exactly pays what she is willing to pay, so

$$P_{75} = 75q_{75} - q_{75}^2.$$

Moreover, we need that the high type is indifferent between buying her own bundle and that designed for the low type. Hence we need:

$$80q_{80} - q_{80}^2 - P_{80} = 80q_{75} - q_{75}^2 - P_{75}.$$

Using the expression we derived for P_{75}, this implies

$$P_{80} = 80q_{80} - q_{80}^2 - 5q_{75}$$

(b) The firm maximizes

$$\begin{aligned}\pi &= \lambda\left(P_L - 25q_L\right) + (1-\lambda)\left(P_H - 25q_H\right) \\ &= \lambda\left(50q_L - q_L^2\right) + (1-\lambda)\left(55q_H - q_H^2 - 5q_L\right)\end{aligned}$$

Now

$$\begin{aligned}\frac{\partial\pi}{\partial q_{75}} &= \frac{\partial}{\partial q_{75}}\left(\lambda\left(50q_{75} - q_{75}^2\right) + (1-\lambda)\left(55q_{80} - q_{80}^2 - 5q_{75}\right)\right) \\ &= 55\lambda - 2\lambda q_{75} - 5 = 0\end{aligned}$$

so

$$q_{75} = \frac{55\lambda - 5}{2\lambda}$$

Also

$$\begin{aligned}\frac{\partial\pi}{\partial q_{80}} &= \frac{\partial}{\partial q_{80}}\left(\lambda\left(50q_{75} - q_{75}^2\right) + (1-\lambda)\left(55q_{80} - q_{80}^2 - 5q_{75}\right)\right) \\ &= (1-\lambda)\left(55 - 2q_{80}\right)\end{aligned}$$

so

$$q_{80} = \frac{55}{2}.$$

(Note: this is also the quantity that maximizes the monopolist's profits with perfect information).