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8: Bundling

Games, Competition and Markets 2023/24

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Overview

1. **Where We Stand**

Where We Stand

Topics

1. Preliminaries

Introductory lecture. Review of game-theoretic concepts. Some basic models of competition.

2. Consumer Search

What if consumers have to engage in costly search to find out about products and/or prices?

3. Advertising

What if producers have to inform consumers about their products and/or prices?

4. Menu Pricing

What if firms design different products and different prices for different consumers?

5. Durable Goods

What if a monopolist sells a durable good and cannot commit to future quantities?

6. Switching Costs

What if consumers have to pay extra if they switch suppliers?

7. Behavior-Based Price Discrimination

What if firms can base their prices on a consumer's past behavior?

8. Vertical control

What if firms sell products to retailers who then sell it to final consumers?

9. Bundling

What if firms can sell bundles of products?

10. Network externalities and compatibility

What if products exhibit network effects: they becomes more (or sometimes less) useful if more consumers use it. Also: when do firms want to make their products compatible with that of their competitor?

11. Platform competition

What if online platforms bring buyers and sellers together? Or consumers and advertisers?

Overview

Bundling

A firm sells two separate products as a bundle.

What are the implications?

1. A simple example
2. Monopoly, independent valuations
3. Monopoly, uniform distributions
4. Duopoly, uniform distributions
5. Entry deterrence

A simple example

- Suppose a monopolist sells two products, at zero costs.
- Unit mass of consumers with unit demand for each product.
- Half of consumers has willingness-to-pay 3 for product 1, and 9 for product 2.
- Other half: other way round.
- Classic example: cable.
- When selling both products separately, you can make at most 9.
- But if you bundle, you can make 12.

Some terminology

- **Pure bundling:** only sell the products as a bundle.
- **Mixed bundling:** sell both the bundle, and the two products separately.
- Trivially, with mixed bundling you can at least do as well as with pure bundling.

A general model

- A monopolist sells two products.
- There is a mass of consumers with size 1.
- Valuations for product 1 are given by the pdf F_1.
- Valuations for product 2 are given by the pdf F_2.
- Denote the valuation of a consumer for product 1 as v_1, that of product 2 as v_2.
- For each consumer, valuations are independent.
- Under separate selling, monopoly prices are p_1^* and p_2^*.
- Can it do better by offering mixed bundling?
- This follows Adams and Yellen (1976).

- Suppose the monopolist offers bundle price $p_B^* = p_1^* + p_2^*$ and separate prices $p_1 = p_1^*$ and $p_2 = p_2^* + \varepsilon$. How does that affect profits?
- A consumer will only buy product 1 if

$$\begin{aligned} v_1 - p_1^* &\geq 0 \\ v_2 - p_2^* - \varepsilon &< 0 \\ v_1 - p_1^* &> v_1 + v_2 - p_1^* - p_2^* \end{aligned}$$

which is true if $v_1 \geq p_1^*$ and $v_2 < p_2^*$

- She will choose product 2 if

$$\begin{aligned} v_1 - p_1^* &< 0 \\ v_2 - p_2^* - \varepsilon &\geq 0 \\ v_2 - p_2^* - \varepsilon &> v_1 + v_2 - p_1^* - p_2^* \end{aligned}$$

which is true if $v_1 < p_1^* - \varepsilon$ and $v_2 \geq p_2^* + \varepsilon$.

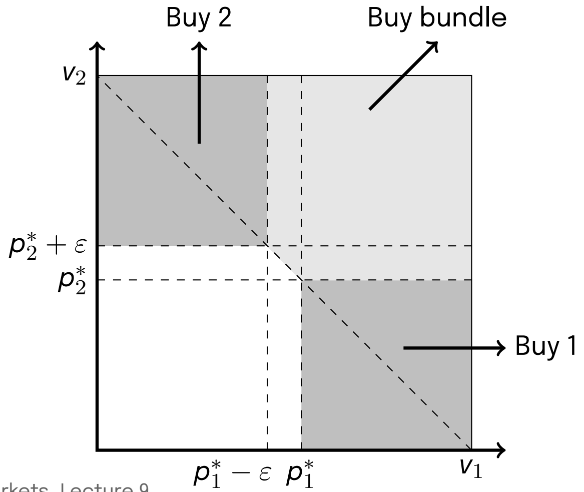
Analysis

- She will choose the bundle if

$$\begin{aligned} v_1 - p_1^* &\leq v_1 + v_2 - p_1^* - p_2^* \\ v_2 - p_2^* - \varepsilon &\leq v_1 + v_2 - p_1^* - p_2^* \\ v_1 + v_2 - p_1^* - p_2^* &\geq 0 \end{aligned}$$

  which implies $v_2 \geq p_2^*$, $v_1 \geq p_1^* - \varepsilon$, and $v_1 + v_2 \leq p_1^* + p_2^*$.
- In all other cases, the consumer will buy nothing.
- Without bundling. all consumers with $v_1 \geq p_1^*$ buy product 1, all with $v_2 \geq p_2^*$ buy product 2.

Consumer behavior



How are profits affected?

- Those with $v_1 \geq p_1$ are unaffected.
- We only have to compare profits made on consumers with $v_1 < p_1^*$.
- Those that only buy 2 have mass $[1 - F_2(p_2^* + \varepsilon)]\,[F_1(p_1^* - \varepsilon)]$.
- Those that buy the bundle: $\int_{p_1^*-\varepsilon}^{p_1^*} f_1(s)\,[1 - F_2(p_1^* + p_2^* - s)]\,ds$.
- Relevant profits

$$
\begin{aligned}
\tilde{\Pi} \quad &= \quad (p_2^* + \varepsilon)\,[1 - F_2(p_2^* + \varepsilon)]\,[F_1(p_1^* - \varepsilon)] \\
&\quad + (p_1^* + p_2^*) \int_{p_1^*-\varepsilon}^{p_1^*} f_1(s)\,[1 - F_2(p_1^* + p_2^* - s)]\,ds
\end{aligned}
$$

How are profits affected?

$$\tilde{\Pi} = (p_2^* + \varepsilon)\left[1 - F_2(p_2^* + \varepsilon)\right]\left[F_1(p_1^* - \varepsilon)\right] + (p_1^* + p_2^*)\int_{p_1^* - \varepsilon}^{p_1^*} f_1(s)\left[1 - F_2(p_1^* + p_2^* - s)\right] ds$$

Take the derivative with respect to ε, evaluate in $\varepsilon = 0$:

$$\left[1 - F_2(p_2^*) - p_2^* f_2(p_2^*)\right] F_1(p_1^*) + p_1 f_1(p_1^*)\left[1 - F_2(p_2^*)\right]$$

Optimality of p_2^* requires

$$\frac{\partial}{\partial p_2}\left((1 - F_2(p_2))\, p_2\right) = 1 - F_2(p_2) - f_2(p_2)p_2 = 0,$$

so we have

$$\left.\frac{\partial \Pi}{\partial \varepsilon}\right|_{\varepsilon=0} = p_1 f_1(p_1^*)\left[1 - F_2(p_2^*)\right] > 0$$

Unbundled monopoly profits can be increased by choosing ε sufficiently small.

Uniform distribution

- $p_1^* = p_2^* = 1/2$.
- With $p_B \geq 1$, we have $\Pi = \frac{1}{2}(2 - p_B)^2 p_B$.
- Restricted maximization then yields $p^B = 1$.
- With $p_B < 1$, we have $\Pi = \left(1 - \frac{1}{2}p_B^2\right) p_B$.
- Maximizing this yields $p_B = \frac{1}{3}\sqrt{6}$, which implies profits of $\frac{2}{9}\sqrt{6} \approx 0.544\,33$.
- Hence, this is the profit-maximizing solution.
- With independent pricing $CS = 2\int_{1/2}^{1}\left(v - \frac{1}{2}\right) dv = \frac{1}{4}$.
- Bundling
  $CS = \int_0^{\frac{1}{3}\sqrt{6}} \int_{\frac{1}{3}\sqrt{6}-v_1}^{1} \left(v_1 + v_2 - \frac{1}{3}\sqrt{6}\right) dv_2 dv_1 + \int_{\frac{1}{3}\sqrt{6}}^{1} \int_0^1 \left(v_1 + v_2 - \frac{1}{3}\sqrt{6}\right) dv_2 dv_1 = \frac{5}{3} - \frac{31}{54}\sqrt{6} \approx 0.26048.$

Competition in bundling

- Two firms A and B, sell two separate products, 1 and 2.
- For each product, consumers are distributed on a Hotelling line.
- A and B are located at 0 and 1.
- Transportation costs are t for both products.
- Willingness to pay for each product is v.
- No bundling: Hotelling twice. $p^* = t$ and $\pi^* = t$.
- Bundling: for simplicity, only pure. Prices: p_A and p_B.
- The indifferent consumer has

$$2v - t\left(x + y\right) - p_A = 2v - t\left(2 - x - y\right) - p_B.$$

- We thus have a set of indifferent consumers, given by the line

$$y = 1 + \frac{p_B - p_A}{2t} - x.$$

- Suppose $p_A \geq p_B$. Profits for A

$$\Pi_A = \frac{p_A}{2}\left[1 + \frac{p_B - p_A}{2t}\right]^2$$

- Maximizing and imposing symmetry yields $p^* = t$, and $\Pi^* = t/2$.
- Consumer surplus is

$$CS = 2\int_0^1 \int_0^{1-x} \left[2v - t\left(x+y\right) - p^*\right] dydx = 2v - 5t/3.$$

- Without bundling

$$CS = 4\int_0^{1/2} \int_0^{1/2} \left[v - t\left(x+y\right) - 2t\right] dxdy = 2v - 5t/2.$$

- Consumers are better off with bundling, but profits are lower.
- Social welfare with separate products is higher.
- This is just our compatibility model!

Bundling as an entry barrier

- Nalebuff (2004).
- Model with uniform distributions on $[0, 1]$.
- Monopolist faces an entry threat, either with product 1 or 2.
- With $p_1 = p_2 = 1/2$, an entrant will set price $1/2 - \varepsilon$ and is profitable if fixed entry costs $E < 1/4$.
- If the entrant deters entry, it makes $2E$.
- Hence it will deter entry if $E > 1/8$.

With bundling

- Suppose $p^B = 1$. In that case, it still makes 1/2.
- Suppose again the entrant enters, say with product 2, and charges $p_2 = 1/2$.
- It will sell to those with $v_2 \geq 1/2$ and $v_1 + v_2 - 1 \leq v_2 - 1/2$, hence $v_1 < 1/2$.
- Profit now is only 1/8.
- *Pure bundling effect*.
- There is also a *bundle discount effect*: without entry threat, optimally $p_B = \frac{1}{3}\sqrt{6} < 1$. This lower price makes it even harder to profitably enter.

- Incumbent sets bundle price p_B. Entrant sets p_E.
- Buy from entrant when $v_2 \geq p_E$ and $v_1 + v_2 - p_B < v_2 - p_E$,
- which implies $v_2 \geq p_E$ and $v_1 < p_B - p_E$.
- Its profits thus equal $(1 - p_E)(p_B - p_E)\, p_E$.
- Suppose entry is just not profitable if $p_B = \frac{1}{3}\sqrt{6}$.
- The entrant would then maximize profits upon entry by setting p_E to maximize $(1 - p_E)\left(\frac{1}{3}\sqrt{6} - p_E\right) p_E$, which yields $p_E \approx 0.29815$.
- Hence, entry is just deterred at this price if entry costs are

$$E = (1 - 0.29815)\left(\frac{1}{3}\sqrt{6} - 0.29815\right) 0.29815 \approx 0.10847.$$

- Without bundling, if $E = 0.108$, the monopolist will not deter entry. Hence, profits equal 1/4. Profits with bundles are 0.54433.
- Hence, the monopolist stands to gain much more from bundling when facing a threat of entry, than when he does not.

Thank you for your attention

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