Tentative solutions
Games, Competition and Markets
2024/25

Chapter 1

1. (a) It could turn out that marginal costs are 0, or that they are 0.4. In general, when they are c :

$$\pi_1 = (1 - q_1 - q_2 - c)\, q_1$$

so

$$\frac{\partial \pi_1}{\partial q_1} = 1 - 2q_1 - q_2 - c = 0.$$

Imposing symmetry:

$$q = \frac{1-c}{3}.$$

Hence, if it turns out marginal costs are zero, both firms set $q = 1/3$. If they are 0.4, firms set $q = .6/3 = .2$

(b) Expected profits now are:

$$\begin{aligned}\pi_1 &= \frac{1}{2}(1 - q_1 - q_2)\, q_1 + \frac{1}{2}(1 - q_1 - q_2 - 0.4)\, q_1 \\ &= (1 - q_1 - q_2 - 0.2)\, q_1\end{aligned}$$

so the equilibrium has $q = .8/3$ (which in fact, is just the average of the earlier options)

2. The indifferent consumer is now given by

$$v - tz^2 - p_0 = v - t\,(1-z)^2 - p_1$$

which yields

$$z = \frac{1}{2} + \frac{P_1 - P_0}{2t},$$

which is the exact same expression as with linear transport costs. The demand that each firm faces therefore does not change and hence the equilibrium will also be the same as in the lecture notes.

3. We proceed in the same fashion as in the lecture notes. Note however that for $n = 2$ one could also write profits of firm A as a function of prices p_A and p_B and then derive the best-reply function of firm A as a function of p_B.

We now have to take into account that not everyone will buy in equilibrium. In equilibrium, with equal prices, we would have sales of each firm being equal to

$$q = \frac{1}{2} - \frac{1}{2}p^2.$$

Suppose firm A sets a price slightly higher than the other firm. Sales then equal

$$q_A = \frac{1}{2}(1-\Delta)^2 - \frac{1}{2}p^2$$

Again

$$\pi_A = (p_A - c)q_A$$

so

$$\frac{\partial \pi_A}{\partial p_A} = p_A \frac{\partial q_A}{\partial p_A} + q_A = 0$$

Again

$$\frac{\partial q_A}{\partial p_A} = -(1-\Delta)$$

But equilibrium now has

$$q_A = \frac{1}{2} - \frac{1}{2}p^2.$$

So, imposing symmetry, we need

$$p(-1) + \left(\frac{1}{2} - \frac{1}{2}p^2\right) = 0$$

so

$$p^* = \sqrt{2} - 1$$

Chapter 2

1. Now equilibrium profits are $\frac{1}{n}(1-\lambda)$. At price $p \in [\underline{p}, 1]$, firm is expected profits are

$$E(\pi_i(p)) = \frac{1}{n}(1-\lambda)p + (1-F(p))^{n-1}\lambda p.$$

The lower bound of the support can be found by solving:

$$\left[\lambda + \frac{1-\lambda}{n}\right]\underline{p} = \frac{1-\lambda}{n},$$

where the LHS denotes the profits you make when charging the lowest price on the market (you sell to all the informeds and your share of uninformeds) and the RHS denotes the profits you make when setting $p = 1$: then you only sell to your share of uninformeds. So, $\underline{p} = \frac{1-\lambda}{\lambda(n-1)+1}$ Equating the expected profits with $\frac{1}{n}(1-\lambda)$ allows us to solve for the distribution

$$F(p) = 1 - \left(\frac{(1-\lambda)(1-p)}{n\lambda p}\right)^{\frac{1}{n-1}}$$

. You can verify that $F(\underline{p}) = 0, F(1) = 1$.

2. (a) Consider the demand for firm 1. There are two cases to consider. Suppose that $p_2 \leq 2$. There are now 3 obvious choices for the price of firm 1. If it sets a price equal to 2, it only sells to half of all the uninformed consumers, so demand is $1-\lambda$. If it sets a price equal to 3, it only sells to half of the uninformed high types, so demand is $(1-\lambda)/2$. If it slightly undercuts firm 2, it will sell to all the informeds and half of all the uninformeds, so demand is $2\lambda + (1-\lambda) = 1+\lambda$. Profits in these three cases are $2(1-\lambda), 3(1-\lambda)/2$, and $p_2(1+\lambda)$. Obviously, the first option always yields higher profits than the second option. The first option yields higher profits than the third option whenever

$$2(1-\lambda) > p_2(1+\lambda),$$

thus if

$$p_2 < \frac{2(1-\lambda)}{1+\lambda}.$$

Now suppose that $2 < p_2 < 3$. Again, there are three obvious options for firm 1. If it sets a price equal to 2, it sells to all the informeds and half of the uninformeds, so demand is $2\lambda + (1-\lambda) = 1+\lambda$. If it sets a price equal to 3, it only sells to half of the uninformed high types, so demand is $(1-\lambda)/2$. If it slightly undercuts firm 2, it will sell to all the informed high types, and to half of the uninformed high types, so demand is $\lambda + (1-\lambda)/2 = (1+\lambda)/2$. Profits in these three cases are $2(1+\lambda), 3(1-\lambda)/2$, and $p_2(1+\lambda)/2$ respectively. Note that the third option yields higher profits than the first option if

$$p_2(1+\lambda)/2 > 2(1+\lambda),$$

i.e. if $p_2 > 4$. But firm 2 will never set such a price. Thus, the only relevant options in this interval are the first and the second. The first option yields higher profits if

$$2(1+\lambda) > 3(1-\lambda)/2,$$

i.e. if $\lambda > -1/7$, which is always satisfied. Hence, with $2 < p_2 < 3$,[1] the best reply for firm 1 is to always set $p_2 = 2$. This yields the following best reply function

$$p_1 = \begin{cases} 2 & \text{if } p_2 \leq \frac{2(1-\lambda)}{1+\lambda} \\ p_2 - \varepsilon & \text{otherwise} \end{cases}$$

Note that $2(1-\lambda)/(1+\lambda) \leq 2$ for all relevant λ. Hence, no firm will ever find it profitable to set a price higher than 2.

(b) With similar arguments as in the lecture notes, there is no equilibrium in pure strategies; firms have an incentive to undercut prices higher than $2(1-\lambda)/(1+\lambda)$, while for lower prices, the best reply is to set a price equal to 2. Hence we look for an equilibrium in mixed strategies on some interval $[\underline{p}, \bar{p}]$. Note that necessarily $\bar{p} = 2$: no firm will ever set a price higher than 2. If it does, it knows that it has the highest price for sure, but if that is true, it is better off setting $p = 2$. A firm that sets $p = 2$ will sell to half of all the uninformeds, and thus make profits $2(1-\lambda)$. A firm that sets price p will sell $2\lambda + (1-\lambda) = 1 + \lambda$ if this price turns out to be the lowest. Otherwise it will sell $1-\lambda$, which is its share of all the uninformeds. Expected profits then equal

$$F(p)(1-\lambda)p + (1-F(p))(1+\lambda)p = p + p\lambda(1-2F(p)).$$

A mixed strategy necessarily has all prices yielding the same profits of $2(1-\lambda)$. This implies

$$F(p) = \frac{p - 2 + \lambda(p+2)}{2p\lambda}.$$

The lower bound of the interval can be found e.g. by plugging in $F(p) = 0$. This yields

$$\underline{p} = \frac{2(1-\lambda)}{1+\lambda}.$$

3. Consumer behavior does not change. But different types of consumers now have different $\hat{\varepsilon}$. Let's refer to the low search cost people as L and the high search cost people as H. From the analysis in the chapter

[1] Note that if $p_2 = 3$, firm 1 will slightly undercut. But that implies that this will never be set, so for conciseness, we ignore that option in what follows.

$\hat{\varepsilon} = 1 - \sqrt{2s}$ so $\hat{\varepsilon}_L = 1 - 0.4\sqrt{2}$ and $\hat{\varepsilon}_H = 1 - 0.5\sqrt{2}$. A firm that defects now gets (following the slides)

$$\begin{aligned} q_A &= \lambda \left[\frac{1}{2}(1 - \hat{\varepsilon}_H - \Delta)(1 + \hat{\varepsilon}_H) + \frac{1}{2}\hat{\varepsilon}_H^2\right] \\ &+ (1-\lambda)\left[\frac{1}{2}(1 - \hat{\varepsilon}_L - \Delta)(1 + \hat{\varepsilon}_L) + \frac{1}{2}\hat{\varepsilon}_L^2\right]. \end{aligned}$$

So

$$\begin{aligned} \frac{\partial q_A}{\partial P_A} &= \lambda \left[-\frac{1}{2}(1 + \hat{\varepsilon}_H\right] + (1-\lambda)\left[-\frac{1}{2}(1 + \hat{\varepsilon}_L)\right] \\ &= -\frac{1}{2}(1 + \hat{\varepsilon}_A) \end{aligned}$$

with

$$\begin{aligned} \hat{\varepsilon}_A &\equiv \lambda \hat{\varepsilon}_H + (1-\lambda)\varepsilon_L \\ &= 1 - \sqrt{2} \cdot (0.4 + 0.1\lambda) \end{aligned}$$

the weighted average of the two. Equilibrium prices then equal, again following the notes,

$$\begin{aligned} p^* &= \frac{1}{1 + \hat{\varepsilon}_A} \\ &= \frac{1}{2 - \sqrt{2}\,(0.5 - 0.1\lambda)} \end{aligned}$$

4. This is based on Haan and Moraga-González (2011). The first part of the analysis is the same as that leading up to slide 37. However a fraction $a_1/\left(a_1 + a_2\right)$ now visits firm 1 first, rather than 1/2. Hence, demand for firm 1 now equals

$$D_1 = \frac{a_1}{a_1 + a_2}\left(1 - \hat{\varepsilon} - \Delta\right) + \frac{a_2}{a_1 + a_2}\left(1 - \hat{\varepsilon} - \Delta\right)\hat{\varepsilon} + \frac{1}{2}\hat{\varepsilon}^2$$

with again $\Delta \equiv p_1 - p^*$, so

$$\frac{\partial D_1}{\partial p_1} = -\frac{a_1}{a_1 + a_2} - \frac{a_2}{a_1 + a_2}\hat{\varepsilon}.$$

Total profits are

$$\pi_1 = p_1 \cdot D_1 - \frac{1}{4}a_1.$$

First-order conditions are

$$\begin{aligned}
\frac{\partial \pi_1}{\partial p_1} &= D_1 + p_1 \cdot \frac{\partial D_1}{\partial p_1} = 0 \\
\frac{\partial \pi_1}{\partial a_1} &= \frac{a_2}{(a_1+a_2)^2} \cdot (1-\hat{\varepsilon}-\Delta) \cdot p_1 - \frac{a_2}{(a_1+a_2)^2} (1-\hat{\varepsilon}-\Delta)\, \hat{\varepsilon} \cdot p_1 - \frac{1}{4} = 0
\end{aligned}$$

We can now impose symmetry:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial p_1} &= \frac{1}{2} + p^* \cdot \left(-\frac{1}{2} - \frac{1}{2}\hat{\varepsilon}\right) = 0 \\
\frac{\partial \pi_1}{\partial a_1} &= \frac{1}{4a}(1-\hat{\varepsilon})\, p^* - \frac{1}{4a}(1-\hat{\varepsilon})\, \hat{\varepsilon} p^* - \frac{1}{4} = 0
\end{aligned}$$

Hence

$$\begin{aligned}
p^* &= \frac{1}{1+\hat{\varepsilon}} \\
a^* &= (1-\hat{\varepsilon})^2\, p^*
\end{aligned}$$

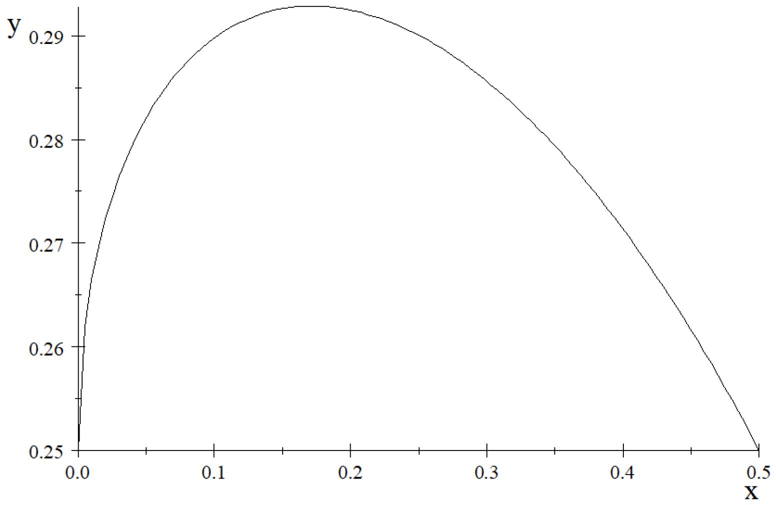
Consumer behavior does not change, so we still have $\hat{\varepsilon} = 1 - \sqrt{2s}$. Note that prices are still the same as in the regular model. This is intuitive: in equilibrium each firm will choose the same level of advertising, so in equilibrium consumers will again visit firms randomly with equal probability. Equilibrium profits are given by

$$\begin{aligned}
\pi^* &= \frac{1}{2}p^* - \frac{1}{4}a^* = \frac{1}{2(1+\hat{\varepsilon})} - \frac{1}{4}(1-\hat{\varepsilon})^2 \cdot \frac{1}{1+\hat{\varepsilon}} - \left(\frac{1}{4}\frac{1+2\varepsilon-\varepsilon^2}{1+\varepsilon}\right) \\
&= \frac{1}{4}\frac{1+2\hat{\varepsilon}-\hat{\varepsilon}^2}{1+\hat{\varepsilon}} = \frac{1}{4}\frac{1+2\left(1-\sqrt{2s}\right)-\left(1-\sqrt{2s}\right)^2}{2-\sqrt{2s}}
\end{aligned}$$

$$\frac{1}{4}\frac{1+2\left(1-\sqrt{2s}\right)-\left(1-\sqrt{2s}\right)^2}{2-\sqrt{2s}}$$

We need that $s \in (0, 1/2)$. Profits are then plotted in the figure above.

That is, they're increasing up to $3 - 2\sqrt{2} \approx 0.17157$ and decreasing afterwards.



Chapter 3

1. (a) The number of consumers that are informed about firm 1 now equal

$$\phi_1 = \Phi_1 + \frac{1}{2}(1 - \Phi_1) = \frac{1}{2}(1 + \Phi_1)$$

Demand is

$$D_1 = \phi_1 \left[(1 - \phi_2) + \phi_2 \left(\frac{1}{2} + \frac{P_2 - P_1}{2t} \right) \right]$$

So

$$\pi_1 = (P_1 - c) D_1 - a (\Phi_1)^2 / 2$$

$$\frac{\partial \pi_1}{\partial P_1} = \phi_1 \left[1 - \phi_2 + \phi_2 \left(\frac{1}{2} + \frac{P_2 - 2P_1}{2t} \right) \right] = 0$$

$$P_1 = \frac{P_2 + t}{2} + \frac{1 - \phi_2}{\phi_2}$$

$$a\Phi_1 = \frac{1}{2} P_1 \left[1 - \frac{1}{2}(1 + \Phi_1) + \frac{1}{2}(1 + \Phi_2) \left(\frac{1}{2} + \frac{P_2 - 2P_1}{2t} \right) \right]$$

(b) Probably: the same intuition still holds. The spillovers in advertisements (in the sense that now ads effectively will reach more people due to word of mouth) will be internalized (i.e., taken into account when setting the advertisement intensity) both by the social planner and in the decentralized equilibrium. Firms will still mix in equilibrium and an ad with the highest price will still capture all the consumer surplus and therefore still be put out at the socially optimal rate. Because of the mixed strategy equilibrium ads with other prices generate the same private benefits as the highest-priced ad and hence will also be put out at the socially optimal rate.

2. The logic is still the same, in that a fraction $\Phi_1 (1 - \Phi_2)$ will definitely buy from firm 1, $(1 - \Phi_1) \Phi_2$ definitely buy from 2, while $\Phi_1 \Phi_2$ are selective.

   In equilibrium, firm 1 again sells to half of all the informeds. That now implies
$$q_1 = \frac{1}{2}\left(1 - (1 - \Phi)^2\right) = \frac{1}{2}\Phi (2 - \Phi).$$
   But now consider a defection both in Φ as well as in price. Demand then equals
$$q_1 = \Phi_1 \left[(1 - \Phi_2) + \frac{1}{2}\Phi_2 \left(1 - (p_1 - p^*)\right)^2\right]$$
   Profits:
$$\pi_1 = (p_1 - c)\, q_1 - \frac{1}{2}a\Phi_1^2$$
   FOCs:
$$\begin{aligned}\frac{\partial \pi_1}{\partial p_1} &= (p_1 - c)\frac{\partial q_1}{\partial p_1} + q_1 = 0 \\ \frac{\partial \pi_1}{\partial \Phi_1} &= \left[(1 - \Phi_2) + \frac{1}{2}\Phi_2 \left(1 - (p_1 - p^*)\right)^2\right](p_1 - c) - a\Phi_1\end{aligned}$$
   Now
$$\frac{\partial q_1}{\partial p_1} = -\Phi_1 \Phi_2 \left(1 - (p_1 - p^*)\right)$$
   Symmetry:
$$\begin{aligned}-(p - c)\,\Phi^2 + q_1 &= 0 \\ (1 - \Phi + \frac{1}{2}\Phi)(p - c) - a\Phi &= 0\end{aligned}$$
   so
$$\begin{aligned}p - c - \frac{2 - \Phi}{2\Phi} &= 0 \\ (1 - \frac{1}{2}\Phi)(p - c) - a\Phi &= 0\end{aligned}$$

This implies

$$
\begin{aligned}
p &= c + \frac{2-\Phi}{2\Phi} \\
4 - 4\Phi + \Phi^2(1-4a) &= 0 \implies \Phi = \frac{4-8\sqrt{a}}{2-8a}.
\end{aligned}
$$

Calculating the Φ that maximizes social welfare is now rather complicated: it involves the expected value of the highest of two match values.

Chapter 4

1. (a) With quality $q_2 = 36$, willingness to pay for the low types is $\sqrt{36} = 6$, that of the high types $\theta\sqrt{36} = 6\theta$. The monopolist can choose to either set a price such that only the high types buy, or a price such that everyone buys. Doing the first entails setting price 6θ and making profits $(6\theta - 2)/2 = 3\theta - 1$. Doing the second entails setting price 6 and making profits $6 - 2 = 4$. The latter yields higher profits iff $\theta < 5/3$. With quality $q_1 = 9$, willingness to pay for the low types is $\sqrt{9} = 3$, that of the high types $\theta\sqrt{9} = 3\theta$. The monopolist can choose to either set a price such that only the high types buy, or a price such that everyone buys. Doing the first entails setting price 3θ and making profits $(3\theta - 2)/2 = \frac{3}{2}\theta - 1$. Doing the second entails setting price 3 and making profits $3 - 2 = 1$. The latter yields higher profits iff $\theta < 4/3$.

   (b) Using the standard result from the lecture notes, if it would offer both qualities, it would set

$$
\begin{aligned}
p_1 &= \sqrt{9} = 3 \\
p_2 &= \theta\left(\sqrt{36} - \sqrt{9}\right) + 3 = 3\theta + 3
\end{aligned}
$$

   so profits would equal

$$\frac{6 + 3\theta}{2} - 2 = \frac{3}{2}\theta + 1.$$

   This is never higher than the profits from only selling the low quality. Only selling the high quality yields $\max\{3\theta - 1, 4\}$ But for every θ, this is higher than what we get from selling both qualities. Hence, it is never profitable to sell both qualities.

   (c) Denote the quality of the low quality product as x^2. When offering both products, the monopolist sets prices

$$
\begin{aligned}
p_1 &= x \\
p_2 &= x + \theta(6 - x),
\end{aligned}
$$

   so profits are

$$\frac{2x + \theta(6 - x)}{2} - 2 = \frac{1}{2}x(2 - \theta) + 3\theta - 2.$$

For $\theta < 2$, this is increasing in x, which suggests the profit-maximizing thing would be to set $x = 6$ (so quality also equals 36). Note that this would boil down to offering only the high quality product and selling it to both, as $p_1 = p_2 = 6$. For $\theta > 2$, this is decreasing in x, so it would be profit-maximizing to set $x = 0$. Note that this boils down to only selling the high-quality product to the high types.

(d)

(e) Suppose the monopolist would sell both cars. His problem is then to

$$
\begin{aligned}
&\max \frac{1}{2}(P_H - 1) + \frac{1}{2}(P_L - c_L) \\
&\text{s.t} \\
&\begin{cases}
3\sqrt{1/2} - P_L \geq 0 & \text{(IR-1)} \\
4\sqrt{1} - P_H \geq 0 & \text{(IR-2)} \\
3\sqrt{1/2} - P_L \geq 3\sqrt{1} - P_H & \text{(IC-1)} \\
4\sqrt{1} - P_H \geq 4\sqrt{1/2} - P_L & \text{(IC-2)}
\end{cases}
\end{aligned}
$$

With the usual arguments, we have that the optimal solution has (IR-1) and (IC-2) binding. Hence

$$
\begin{aligned}
P_L &= 3\sqrt{1/2} \\
4 - P_H &= 3\sqrt{1/2} - P_L
\end{aligned}
$$

or

$$
\begin{aligned}
p_L &= 3\sqrt{1/2} \\
p_H &= 4 - \sqrt{1/2}.
\end{aligned}
$$

Profits then equal

$$
\begin{aligned}
\Pi &= \frac{1}{2}\left(4 - \sqrt{1/2} - 1\right) + \frac{1}{2}\left(3\sqrt{1/2} - c_L\right) \\
&= \frac{1}{2}\sqrt{2} + \frac{3}{2} - \frac{1}{2}c_L
\end{aligned}
$$

Suppose you only sell the high quality car. There are two options: only sell to the high types or sell to both. Only selling to the high types would imply $P = 4$,so $\Pi = \frac{1}{2} \cdot (4 - 1) = \frac{3}{2}$. Selling to both would imply $P = 3$ and $\Pi = 1 \cdot (3 - 1) = 2$. Hence the preferred option would then be to sell to both types. Also supplying the low quality car would be profitable if

$$\frac{1}{2}\sqrt{2} + \frac{3}{2} - \frac{1}{2}c_L > 2, \tag{1}$$

which implies

$$c_L < \sqrt{2} - 1 = 0.41421 \tag{2}$$

(f) Denote the quality (i.e. the expected number of kilometres) of the low-quality car as γ. Suppose that the monopolist would choose to supply both types of car. Its problem then is to

$$\max \frac{1}{2}(P_H - 1) + \frac{1}{2}(P_L - 1) \tag{3}$$

$$\text{s.t} \tag{4}$$

$$\begin{cases} 3\sqrt{\gamma} - P_L \geq 0 & \text{(IR-1)} \\ 4\sqrt{1} - P_H \geq 0 & \text{(IR-2)} \\ 3\sqrt{\gamma} - P_L \geq 3\sqrt{1} - P_H & \text{(IC-1)} \\ 4\sqrt{1} - P_H \geq 4\sqrt{\gamma} - P_L & \text{(IC-2)} \end{cases} \tag{5}$$

With the usual arguments, we have that the optimal solution has (IR-1) and (IC-2) binding. Hence

$$P_L = 3\sqrt{\gamma} \tag{6}$$

$$4 - P_H = 4\sqrt{\gamma} - P_L \tag{7}$$

or

$$p_L = 3\sqrt{\gamma} \tag{8}$$

$$p_H = 4 - \sqrt{\gamma}. \tag{9}$$

Profits then equal

$$\Pi = \frac{1}{2}(4 - \sqrt{\gamma} - 1) + \frac{1}{2}(3\sqrt{\gamma} - 1) \tag{10}$$

$$= \sqrt{\gamma} + 1 \tag{11}$$

which is strictly increasing in γ and thus $\gamma = 1$. Hence, the monopolist is best off by only offering a car of quality 1.

Chapter 5

1. (a) The monopolist faces downward sloping demand curve $p = 1 - q$ (reflecting the demand per period for the services the goods provides). The discount factor is δ and there are no costs of production.

   Renting out the product to consumers; the monopolist retains ownership. After each period, goods are returned to the monopolist but the product returned is broken w.p. half. In the case without breakdown, since there are no costs of production, the monopolist just rents out the monopoly quantity each period and produces this quantity in the first period. In the case with possible breakdown of the product, things are a bit different.

   We assume that the monopolist cannot commit to quantities (although it most likely does not make a difference here). Therefore, we solve the problem by means of backward induction to derive a solution that is in a sense time consistent; the monopolist has no incentive to deviate from his original plan for period two set out at the beginning of period 1 when he arrives at period 2. In period two there are in expectation $Eq_1 = \frac{1}{2}q_1$ units left and the monopolist has to choose how much to produce. Profits from renting out $\frac{1}{2}q_1 + q_2$ (choosing q_2 taking as given Eq_1):

$$\Pi_2(q_2; q_1) = (\frac{1}{2}q_1 + q_2)(1 - \frac{1}{2}q_1 - q_2)$$

   such that $q_2 = \frac{1}{2} - \frac{1}{2}q_1$ (check; if nothing would be left of the previous period quantity produced the monopolist would just produce the monopoly quantity in the second period). Expected profits in the first period taking into account the optimal production in the second period;

$$\begin{aligned} E\Pi_{12}(q_1; q_2(q_1)) &= q_1(1 - q_1) + \delta(\frac{1}{2}q_1 + \frac{1}{2} - \frac{1}{2}q_1)(1 - \frac{1}{2}q_1 - \frac{1}{2} + \frac{1}{2}q_1) \\ &= q_1(1 - q_1) + \frac{1}{4}\delta \end{aligned}$$

   such that in optimum

$$q_1 = \frac{1}{2}, q_2 = \frac{1}{4}$$

   The intuition is that in the first period, the monopolist cannot do better than renting out the monopoly quantity. In the second period, it wants to rent out the monopoly quantity again, but takes into account that there is already production from the first period (of which in expectation half is left) and therefore produces in period two exactly how much is needed to produce such that he can rent out the monopoly quantity in the second period again.

Now consider the case where the monopolist sells the product. Assume consumers are rational to keep things interesting; they will take into account the incentives of the monopolist. The timing remains the same as in the lecture notes:

Stage 1a; The monopolist sets q_1; Stage 1b; Demand determines the price P_1 that will prevail. Stage 2a; The monopolist sets q_2; Stage 2b; Demand determines the price p_2 that will prevail.

In terms of notation, $P_1 = p_1 + \delta p_2^e$ is the price consumers (are willing to) pay if they buy the product and can use it for two periods, and if consumers are rational $p_2 = p^e$. In stage 2b) the expected price is $p_2 = 1 - \frac{1}{2}q_1 - q_2$ and the monopolist maximizes in stage 2a

$$\max_{q_2}(1 - \frac{1}{2}q_1 - q_2)q_2 \implies q_2(q_1) = \frac{2 - q_1}{4}$$

with profits

$$\Pi_2 = \Big(1 - \frac{1}{2}q_1 - \frac{2 - q_1}{4}\Big)\frac{2 - q_1}{4} = \Big(\frac{2 - q_1}{4}\Big)^2$$

. Under rationality then

$$p_2^e = p_2 = 1 - \frac{1}{2}q_1 - \frac{2 - q_1}{4} = \frac{2 - q_1}{4}$$

and the price that prevails (what a consumer can get out of the product if he/she buys it in period 1 and uses it for two periods) is given by

$$\begin{aligned} P_1 = p_1 + \frac{1}{2}\delta p_2 &= 1 - q_1 + \frac{1}{2}\delta\frac{2 - q_1}{4} \\ &= 1 + \frac{1}{4}\delta - q_1(1 + \frac{1}{8}\delta). \end{aligned}$$

since the consumer is w.t.p. p_1 in period one and can resell the product at price δp_2 on the second-hand market, but it takes into account that that only happens w.p. $\frac{1}{2}$. In stage 1a, the monopolist sets q_1 taking into account how this quantity affects profits in the second period. Discounted profits are

$$\Pi = \Pi_1 + \delta\Pi_2 = q_1\Big(1 + \frac{1}{4}\delta - q_1(1 + \frac{1}{8}\delta)\Big) + \delta\Big(\frac{2 - q_1}{4}\Big)^2$$

which is maximized when

$$\begin{aligned} &\Big(1 + \frac{1}{4}\delta - q_1(1 + \frac{1}{8}\delta)\Big) - q_1(1 + \frac{1}{8}\delta) - \frac{1}{2}\delta(1 - \frac{1}{2}q_1) = 0 \\ &q_1 = \frac{4 - \delta}{8} \end{aligned}$$

Chapter 6

1. The setup is the same as in section 6.3; there we start out with a mass of consumers in the first period, and a fraction of the consumers dies after the first period and is replaced by a new mass of consumers that do not have switching costs. Now, we do not have a mass of consumers leaving the market, but after the first period there will be a fraction λ_s of the original mass of consumers that has switching cost and $1-\lambda_s$ that do not. In the standard model, consumers know that with probability λ_s they will survive to the next period, with probability $1-\lambda_s$ they won't. In the model under consideration, with probability $1-\lambda_s$ there choice in period 1 their decision in period 1 becomes immaterial and they don't have to take it into account. Whether consumers exogenously are assigned switching costs at the beginning of period 1 or whether they will be in that group after period one will result in the same proportions of consumers that have switching costs. Hence, following the analysis in the notes (replacing λ_n by $1-\lambda_s$), the equilibrium has first-period prices equal to $p_1 = c + \frac{1}{3}(4-(1-\lambda_s)) = c+1+\frac{1}{3}\lambda_s$ and second-period prices $p_2 = c + 1/(1-\lambda_s)$. The conclusion regarding welfare also does not change; switching costs make firms better off in both periods.

2. We will solve the switching cost model with changing preferences in case consumers are naive. Consumers will draw a new location on the Hotelling line in the second period that is completely unrelated to their location in the first period, and consumers do not take into account in period one that they will change location in period two (and therefore the location of the indifferent consumer in period 1 will not be affected by that prospect, as we will see).

   We solve with backward induction. The analysis for the second period is identical to that in the lecture notes. For completeness; in period 1, a share $\hat{x}^1$ bought from firm A (segment A) and the remaining from firm B (segment B). Given second period prices p_A^2, p_B^2, we can calculate the locations of the indifferent consumers in each segment. The indifferent consumer in segment A again has

   $$\hat{x}_A^2 = \frac{1}{2}\left(1 + p_B^2 - p_A^2 + z\right),$$

   while the indifferent consumer in segment B again has

   $$\hat{x}_B^2 = \frac{1}{2}\left(1 + p_B^2 - p_A^2 - z\right).$$

These expressions are always strictly between 0 and 1 provided that $z < 1$. We assume that to be the case. New consumers will again behave as in the standard Hotelling model, so

$$\hat{x}_n = \frac{1}{2}\left(1 + p_B^2 - p_A^2\right).$$

Firm A's second period demand thus equals

$$\begin{aligned} q_A^2\left(p_A^2, p_B^2\right) &= \frac{1}{2}\lambda_n\left(1 + p_B^2 - p_A^2\right) + \lambda_0\left(\hat{x}_1\hat{x}_A^2 + \left(1 - \hat{x}_1\right)\hat{x}_B^2\right) \\ &= \frac{1}{2}\left(1 + p_B^2 - p_A^2 + \left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)z\right), \end{aligned}$$

where we use that $\hat{x}_A^2 - \hat{x}_B^2 = z$. One can derive a similar expression for firm B. Profits of firm A equal $\pi_A^2 = \left(p_A^2 - c\right)q_A^2$. The first-order condition now becomes

$$\frac{1}{2}\left(1 + p_B^2 - 2p_A^2 + \left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)z + c\right) = 0,$$

so

$$p_A^2 = \frac{1}{2}\left(1 + p_B^2 + c + \left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)z\right).$$

Similarly, for firm B,

$$p_B^2 = \frac{1}{2}\left(1 + p_A^2 + c + \left(1 - 2\hat{x}_1\right)\left(1 - \lambda_n\right)z\right).$$

Plugging these values into each other we can solve for equilibrium prices, which we can then use to evaluate second-period demands and profits. In particular, we have

$$\begin{aligned} p_A^2\left(\hat{x}_1\right) &= 1 + c + \frac{1}{3}\left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)z \\ p_B^2\left(\hat{x}_1\right) &= 1 + c + \frac{1}{3}\left(1 - 2\hat{x}_1\right)\left(1 - \lambda_n\right)z \\ q_A^2\left(\hat{x}_1\right) &= \frac{1}{2}\left(1 + \frac{1}{3}\left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)z\right) \\ \pi_A^2\left(\hat{x}_1\right) &= \frac{1}{2}\left(1 + \frac{1}{3}\left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right)z\right)^2. \end{aligned} \tag{12}$$

Moving back to period 1, total profits for firm A are given by

$$\Pi_A\left(p_A^1, p_B^1\right) = \pi_A^1\left(p_A^1, p_B^1\right) + \delta\pi_A^2\left(\hat{x}_1\right).$$

Taking the first order condition

$$\frac{\partial \Pi_A\left(p_A^1, p_B^1\right)}{\partial p_A^1} = \frac{\partial \pi_A^1\left(p_A^1, p_B^1\right)}{\partial p_A^1} + \delta \frac{\partial \pi_A^2\left(\hat{x}_1\right)}{\partial \hat{x}_1} \cdot \frac{\partial \hat{x}_1}{\partial p_A^1} = 0 \tag{13}$$

With naive consumers, we simply have

$$\hat{x}_1 = \frac{1}{2}\left(1 + p_B^1 - p_A^1\right),$$

so

$$\frac{\partial \hat{x}_1}{\partial p_A^1} = -\frac{1}{2}. \tag{14}$$

First-period profits are

$$\pi_A^1 = \left(p_A^1 - c\right)\hat{x}_1 = \frac{1}{2}\left(1 + p_B^1 - p_A^1\right)\left(p_A^1 - c\right),$$

so

$$\frac{\partial \pi_A^1\left(p_A^1, p_B^1\right)}{\partial p_A^1} = \frac{1}{2}\left(1 + p_B^1 - 2p_A^1 + c\right). \tag{15}$$

From (12),

$$\begin{aligned}
\frac{\partial \pi_A^2\left(\hat{x}_1\right)}{\partial \hat{x}_1} &= \frac{1}{2}\left(1 + \frac{1}{3}\left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right) z\right)^2 \\
&= \frac{2}{3}\left(1 + \frac{1}{3}\left(2\hat{x}_1 - 1\right)\left(1 - \lambda_n\right) z\right) \cdot \left(1 - \lambda_n\right) z. \qquad (16)
\end{aligned}$$

Plugging (14), (15) and (16) into (13) and imposing symmetry yields

$$\frac{1}{2}\left(1 - p^1 + c\right) + \delta \frac{2}{3} \cdot \left(1 - \lambda_n\right) z \cdot \left(-\frac{1}{2}\right) = 0,$$

hence

$$\begin{aligned}
p^1 &= 1 + c - \frac{2}{3} z \delta \left(1 - \lambda_n\right) \\
p^2 &= 1 + c \\
\pi &= \frac{1}{2}\left(p^1 - c\right) + \delta \cdot \frac{1}{2} = (1 + \delta)(1 + c) - \frac{2}{3} z \delta \left(1 - \lambda_n\right)
\end{aligned}$$

Without switching costs, price in each period equals $1 + c$. Hence, second-period profits are unaffected, while first-period profits are definitely lower. Consumers are now better off; many of them will not switch in the second period (note that first-period prices are decreasing in switching costs), so firms are competing fiercer for them in the first period. Yet, different from the case of forward-looking consumers, this does not make consumers less price sensitive in the first period. All this contributes to lower prices.

Chapter 7.

1. Firm would simply set the coupons equal to zero; as they do not have to convince consumers with a taste for their own product to buy from them.

2. We solve with backward induction. In the first period, the markets splits at $\hat{x}_1$: those to the left buy at A, those at the left buy at B. These will be called segment A and segment B respectively. In the second period, firm A sets prices p_{AA}^2 and p_{AB}^2 to consumers in segment A and B, respectively. The indifferent consumer in segment A is denoted $\hat{x}_A$, that in segment B is $\hat{x}_B$. Firm B sets prices p_{BA}^2 and p_{BB}^2 to consumers in segment A and B respectively. Profits then are

$$\begin{aligned}\pi_A &= \left(p_{AA}^2 - c\right)\hat{x}_A + \left(p_{AB}^2 - c\right)(\hat{x}_B - \hat{x}_1) \\ \pi_B &= \left(p_{BA}^2 - c\right)(\hat{x}_1 - \hat{x}_A) + \left(p_{BB}^2 - c\right)(1 - \hat{x}_B)\end{aligned}$$

Segments A and B are completely separated from each other, hence pricing decisions in segment A are independent of those in segment B.

The indifferent consumer in segment A is $\hat{x}_A$. He is given by

$$r - \hat{x}_A - p_{AA}^2 = r - (1 - \hat{x}_A) - p_{BA}^2 - s$$

so

$$\hat{x}_A = \frac{1}{2} + \frac{p_{BA}^2 - p_{AA}^2 + s}{2}.$$

On segment A, firm A maximizes

$$\pi_{AA} = \left(p_{AA}^2 - c\right)\hat{x}_A = \left(p_{AA}^2 - c\right)\left(\frac{1}{2} + \frac{p_{BA}^2 - p_{AA}^2 + s}{2}\right)$$

Firm B maximizes

$$\pi_{BA} = \left(p_{BA}^2 - c\right)(\hat{x}_1 - \hat{x}_A) = \left(p_{BA}^2 - c\right)\left(\hat{x}_1 - \left(\frac{1}{2} + \frac{p_{BA}^2 - p_{AA}^2 + s}{2}\right)\right)$$

Taking the FOCs:

$$\begin{aligned}p_{AA} &= \frac{1}{2}(1 + p_{BA} + c + s) \\ p_{BA} &= \hat{x}_1 + \frac{1}{2}(p_{AA} + c - s - 1)\end{aligned}$$

Solving

$$
\begin{aligned}
p_{AA} &= c+\frac{1}{3}\left(1+2\hat{x}_1+s\right)\\
p_{BA} &= c+\frac{1}{3}\left(4\hat{x}_1-s-1\right)
\end{aligned}
$$

so

$$\hat{x}_A=\frac{1}{6}\left(1+s+2\hat{x}_1\right),$$

hence

$$
\begin{aligned}
\pi_{AA} &= \left(\frac{1}{3}\left(1+2\hat{x}_1+s\right)\right)\left(\frac{1}{6}\left(1+s+2\hat{x}_1\right)\right)=\frac{1}{18}\left(1+2\hat{x}_1+s\right)^2.\\
\pi_{BA} &= \left(\frac{1}{3}\left(4\hat{x}_1-s-1\right)\right)\left(\hat{x}_1-\frac{1}{6}\left(1+s+2\hat{x}_1\right)\right)\\
&= \left(\frac{1}{3}\left(4\hat{x}_1-s-1\right)\right)\left(\frac{1}{6}\left(4\hat{x}_1-s-1\right)\right)=\frac{1}{18}\left(4\hat{x}_1-s-1\right)^2.
\end{aligned}
$$

This implies

$$
\begin{aligned}
\pi_{AB} &= \frac{1}{18}\left(4\left(1-\hat{x}_1\right)-s-1\right)^2=\frac{1}{18}\left(3-4\hat{x}_1-s\right)^2\\
\pi_{BB} &= \frac{1}{18}\left(1+2\left(1-\hat{x}_1\right)+s\right)^2=\frac{1}{18}\left(3+s-2x_1\right)^2
\end{aligned}
$$

Consumers are forward looking. Hence, the indifferent consumer in period 1 foresees that if she buys in A in period 1, she will buy from B in period 2 and vice-versa. Hence, the indifferent consumer follows from

$$r-\hat{x}_1-p_A+\delta\left(r-\left(1-\hat{x}_1\right)-s-p_{BA}\right)=r-\left(1-\hat{x}_1\right)-p_B+\delta\left(r-\hat{x}_1-s-p_{AB}\right)$$

so

$$\hat{x}_1=\frac{1}{2}+\frac{p_B-p_A}{2\left(1-\delta\right)}+\frac{\delta\left(p_{AB}-p_{BA}\right)}{2\left(1-\delta\right)}$$

Now

$$
\begin{aligned}
p_{AB}-p_{BA} &= \left(c+\frac{1}{3}\left(4\left(1-\hat{x}_1\right)-s-1\right)\right)-\left(c+\frac{1}{3}\left(4\hat{x}_1-s-1\right)\right)\\
&= \frac{4}{3}\left(1-2\hat{x}_1\right)
\end{aligned}
$$

so

$$\hat{x}_1=\frac{1}{2}+\frac{p_B-p_A}{2\left(1-\delta\right)}+\frac{2\delta\left(1-2\hat{x}_1\right)}{3\left(1-\delta\right)}.$$

and

$$\hat{x}_1 = \frac{1}{2} + \frac{3\left(p_B - p_A\right)}{2\delta + 6}$$

Profits

$$\begin{aligned}
\pi_A &= \left(p_A - c\right)\hat{x}_1 + \delta\pi_{AA} + \delta\pi_{AB} \\
&= \left(p_A - c\right)\hat{x}_1 + \frac{\delta}{18}\left(1 + 2\hat{x}_1 + s\right)^2 + \frac{\delta}{18}\left(3 - 4\hat{x}_1 - s\right)^2 \\
&= \left(p_A - c\right)\left(\frac{1}{2} + \frac{3\left(p_B - p_A\right)}{2\delta + 6}\right) + \frac{\delta}{18}\left(1 + 2\left(\frac{1}{2} + \frac{3\left(p_B - p_A\right)}{2\delta + 6}\right) + s\right)^2 \\
&\quad + \frac{\delta}{18}\left(3 - 4\left(\frac{1}{2} + \frac{3\left(p_B - p_A\right)}{2\delta + 6}\right) - s\right)^2
\end{aligned}$$

Taking the FOC with respect to p_A and imposing symmetry yields

$$p_A = p_B = 1 + c + \frac{1}{3}\delta\left(1 - 2s\right)$$

In equilibrium $\hat{x}_1 = \frac{1}{2}$, so

$$\begin{aligned}
p_{AA} &= c + \frac{1}{3}\left(2 + s\right) \\
p_{BA} &= c + \frac{1}{3}\left(1 - s\right)
\end{aligned}$$

Chapter 8

1. From Section 8.4, we have that without exclusive contracts, the only credible contracts are those that are a best reply to each other. In the lecture notes, we do this in the context of quantity setting, so the equilibrium contracts are such that the downstream profits are equal to the Cournot profits. The manufacturer can get all these profits by setting the fixed fees equal to the Cournot profits.

   In the context of this exercise, the only credible contracts are those that are a best reply to each other. Hence equilibrium contracts are such that downstream profits equal the equilibrium of the game that is played here: two firms competing in Perloff-Salop fashion with $v = 0$. But this is exactly exercise 3 of chapter 1. From the analysis there, $p^* = \sqrt{2} - 1$, sales of each firm are $q = \frac{1}{2} - \frac{1}{2}p^2 = \frac{1}{2} - \frac{1}{2}\left(\sqrt{2} - 1\right)^2 = \sqrt{2} - 1$, so profits for each firm are $\left(\sqrt{2} - 1\right)\left(\sqrt{2} - 1\right) = 3 - 2\sqrt{2}$. The manufacturer can secure all these profits via its fixed fee so its profits will be $6 - 4\sqrt{2}$

Chapter 9

1. Valuations are now uniformly distributed on [0,1] for product 1, but on $[0, 2]$ for product 2.

   (a) For product 1, the monopolist maximizes $\pi_1 = (1 - p)\, p$. This is maximized by setting $p = 1/2$ which yields $\pi_1 = 1/4$. For product 2, profits are $\pi_1 = \left(1 - \frac{1}{2}p\right) p$ which is maximized by setting $p = 1$ which yields $\pi_2 = 1/2$. In the case of bundling, there are a couple of cases to consider.

   - If $3 \geq P_B \geq 2$, total sales equal

   $$\begin{aligned} & \int_{P_B-2}^{1} \left[\int_{P_B-v_1}^{2} f_2\left(v_2\right) dv_2 \right] f_1\left(v_1\right) dv_1 \\ = & \int_{P_B-2}^{1} \left[1 - F_2\left(P_B - v_1\right)\right] dv_1 \\ = & \int_{P_B-2}^{1} \left(1 - \frac{1}{2}\left(P_B - v_1\right)\right) dv_1 = \frac{1}{4}\left(3 - P_B\right)^2 \end{aligned}$$

     Profits are then given by $\pi = \frac{1}{4}\left(3 - P_B\right)^2 P_B$. Constrained maximization yields, since $\frac{d\Pi}{dp_B} = \frac{1}{4}(3 - P_B)^2 - \frac{2P_B(3-P_B)}{4} < 0$ over the interval $P_B \in [2, 3]$, that the profit-maximizing price in this interval is $P_B = 2$, which yields $\pi = \frac{1}{2}$.
   - If $1 \leq P_B \leq 2$, total sales equal

   $$\begin{aligned} & \int_{0}^{1} \left[\int_{P_B-v_1}^{2} f_2\left(v_2\right) dv_2 \right] f_1\left(v_1\right) dv_1 \\ = & \int_{0}^{1} \left(1 - \frac{1}{2}\left(P_B - v_1\right)\right) dv_1 = \frac{5}{4} - \frac{1}{2}P_B \end{aligned}$$

     Profits are then given by $\pi = \left(\frac{5}{4} - \frac{1}{2}P_B\right) P_B$, which is maximized by setting $P_B = \frac{5}{4}$, which is indeed admissible and yields $\pi = \frac{25}{32} > \frac{3}{4}$.
   - Finally we can also check the case $P_B \leq 1$. Profits then equal

   $$\begin{aligned} & \int_{0}^{P_B} \left[\int_{P_B-v_1}^{2} f_2\left(v_2\right) dv_2 \right] f_1\left(v_1\right) dv_1 + \int_{P_B}^{1} \left[\int_{0}^{2} f_2\left(v_2\right) dv_2 \right] f_1\left(v_1\right) dv_1 \\ = & \int_{0}^{P_B} \left(1 - \frac{1}{2}\left(P_B - v_1\right)\right) dv_1 + \int_{P_B}^{1} dv_1 = 1 - \frac{1}{4}P_B^2 \end{aligned}$$

Profits are then $\pi = \left(1 - \frac{1}{4}P_B^2\right) P_B$, but since $\frac{d}{dp_B}\Pi = 1 - \frac{3}{4}P_B^2 > 0$ in the interval, this is constrained optimized at $P_B = 1$ and yields $\frac{3}{4}$.

(b) First consider the case of no bundling. If the entrant enters with product 1, it can slightly undercut and charge price $1/2 - \varepsilon$ which yields profits 1/4. If it enters with product 1, it can slightly undercut and charge price $1 - \varepsilon$ which yields profits 1/2. So entry is always deterred if $E > 1/2$.

Now consider the case of bundling. Suppose the incumbent again sets $p_B = 5/4$.

- First suppose the entrant enters with product 1. A consumer again buys from the entrant if $v_1 \geq p_E$ and $v_1 + v_2 - p_B < v_1 - p_E$ which implies $v_1 \geq p_E$ and $v_2 < p_B - p_E$. Profits thus equal

$$(1 - p_E) \cdot \frac{1}{2} (p_B - p_E) p_E = (1 - p_E) \cdot \frac{1}{2} \left(\frac{5}{4} - p_E\right) p_E$$

which is maximized by setting $\frac{3}{4} - \frac{1}{12}\sqrt{3}\sqrt{7} \approx 0.368$. This yields profits $(1 - 0.368) \cdot \frac{1}{2} \left(\frac{5}{4} - 0.368\right) 0.368 = 0.10257$.

- Now suppose the entrant enters with product 2. A consumer buys from the entrant if $v_2 \geq p_E$ and $v_1 + v_2 - p_B < v_2 - p_E$ which implies $v_2 \geq p_E$ and $v_1 < p_B - p_E$. Profits thus equal

$$\left(1 - \frac{1}{2}p_E\right) (p_B - p_E) p_E = \left(1 - \frac{1}{2}p_E\right) \left(\frac{5}{4} - p_E\right) p_E.$$

This is maximized by setting $p_E = 1/2$ which yields profits $\frac{9}{32} \approx 0.281\,25$. Hence, entry is always deterred if $E > .28125$. Note that in this case that is not that much higher than in the case without bundling (simply because product 2 is much more valuable on average so bundling it with 1 does not add that much).

Chapter 10

1. (a) When all products are compatible, we know that everyone will consume and we have a standard Salop circle: the total size of the network then equals 1. Thus

$$U_{ij} = (v + \alpha) - td_{ij}.$$

Suppose that a given firm charges price p, whereas all other firms charge price p^*. The indifferent consumer between this firm and its right-hand neighbor is then given by

$$(v+\alpha) - tz - p = (v+\alpha) - t\left(\frac{1}{n} - z\right) - p^*,$$

so

$$z = \frac{1}{2t}\left(\frac{1}{n}t + p^* - p\right) = \frac{1}{2n} + \frac{p^* - p}{2t}$$

The same holds for its left-hand neighbor. Thus, profits of this firm are

$$\pi = (p-c)\left(\frac{1}{t}\left(\frac{1}{n}t + p^* - p\right)\right)$$

Maximizing this wrt to p and imposing symmetry, we have

$$p = c + \frac{t}{n}.$$

Gross profits per firm then equal

$$\pi = \frac{t}{n^2}.$$

Entry will occur until profits are competed away, thus until $\pi = f$, hence

$$n = \sqrt{\frac{t}{f}}.$$

Net profits thus equal zero. A consumer located at $x \in \left[0, \frac{1}{2n}\right]$ has net utility $v + \alpha - tx - \left(c + \frac{t}{n}\right)$. Total consumer surplus then equals

$$\begin{aligned} 2n\int_0^{\frac{1}{2n}} \left(v + \alpha - tx - \left(c + \frac{t}{n}\right)\right) dx &= v + \alpha - c - \frac{5t}{4n} \\ &= v + \alpha - c - \frac{5}{4}\sqrt{tf} \end{aligned}$$

(b) Again assume that a given firm charges price p, whereas all other firms charge price p^*. Given that all the other firms charge p^*, the size of the network of the right-hand neighbor equals $\left(\frac{1}{2n}\right) + \left(\frac{1}{n} - z\right)$: on its right, it has the equilibrium number of consumers of $\frac{1}{2n}$, while on its left, the number equals $1/n - z$. The network of this firm equals $2z$, being the total number of consumers at both the left- and the right-hand side. The indifferent consumer between this firm and its right-hand neighbor is then given by

$$(v + \alpha (2z)) - tz - p = \left(v + \alpha \left(\frac{3}{2n} - z \right) \right) - t \left(\frac{1}{n} - z \right) - p^*,$$

hence

$$z = \frac{1}{2n} + \frac{p^* - p}{(2t - 3\alpha)}.$$

Thus, profits per firm are

$$\begin{aligned} \pi &= (p - c) \, 2z \\ &= 2 (p - c) \left(\frac{1}{2n} + \frac{p^* - p}{(2t - 3\alpha)} \right) \end{aligned}$$

Maximizing wrt p yields

$$p = \frac{2t - 3\alpha + 2p^* n + 2cn}{4n}.$$

Imposing symmetry yields

$$p = \frac{2t - 3\alpha + 2cn}{2n} = c + \frac{2t - 3\alpha}{2n}$$

Gross profits then equal

$$2 \left(\frac{2t - 3\alpha}{2n} \right) \left(\frac{1}{2n} \right) = \frac{1}{2} \frac{2t - 3\alpha}{n^2}.$$

Net profits are driven to zero when this equals 0, thus

$$\frac{1}{2} \frac{2t - 3\alpha}{n^2} = f.$$

This yields

$$n = \sqrt{\frac{2t - 3\alpha}{2f}}$$

Net profits thus equal zero. A consumer located at $x \in \left[0, \frac{1}{2n}\right]$ has net utility $v + \frac{\alpha}{n} - tx - \left(c + \frac{2t - 3\alpha}{2n}\right)$. Total consumer surplus then equals

$$\begin{aligned} 2n \int_0^{\frac{1}{2n}} \left(v + \frac{\alpha}{n} - tx - \left(c + \frac{2t - 3\alpha}{2n} \right) \right) dx &= \frac{1}{4n} (4vn + 10\alpha - 5t - 4cn) \\ &= v - c - \frac{5t - 10\alpha}{4n} \\ &= v - c - \frac{5t - 10\alpha}{4} \sqrt{\frac{2f}{2t - 3\alpha}} \end{aligned}$$

(c) Now consumer welfare in the new case is higher if

$$\frac{5}{4}t\sqrt{\frac{f}{t}}-\alpha>\frac{5t-10\alpha}{4}\sqrt{\frac{2f}{2t-3\alpha}}$$

This is ambiguous. For t large enough, it will not be satisfied.

2. Let me call the firms 0 and 1 rather than A and B, to avoid confusion with Apples and Bananas. Firm 0 is located at $(0,0)$, firm 1 is located at $(1,1)$.

(a) This is just two separate Hotelling lines, so equilibrium price of Apples is $p=t$; that of Bananas $p=2t$.

(b) The indifferent consumer now has

$$2v-tx-2ty-p_0=2v-t(1-x)-2t(1-y)-p_1.$$

The set of indifferent consumers is thus given by the line

$$y=\frac{3-2x}{4}+\frac{p_1-p_0}{4t}.$$

This implies that the indifferent consumers are now on the line in the figure below. Firm 0 sells to the people below and to the left of that line, so we have sales of firm 0 being equal to

$$\left(\frac{1}{4}+\frac{1}{4t}(p_1-p_0)\right)+\frac{1}{4}$$

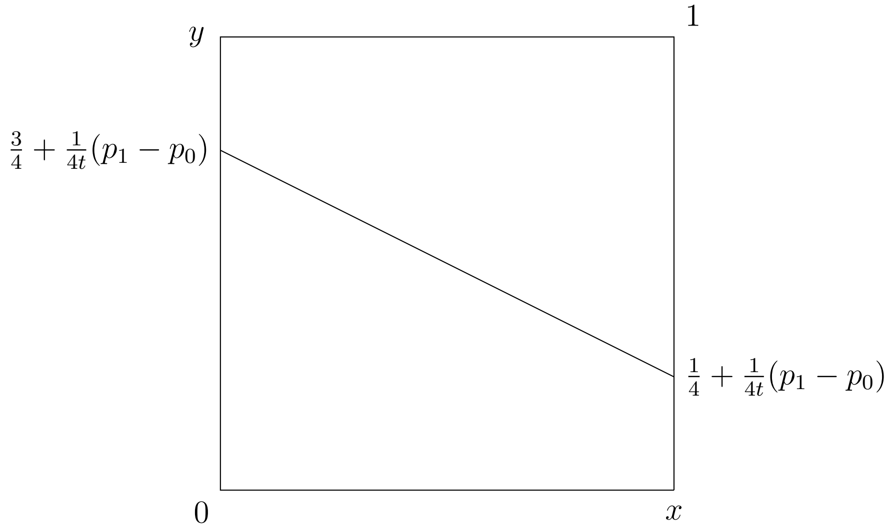
the first term being the lower rectangle, the second term the remaining triangle. Hence profits equal

$$\left(\frac{1}{2}+\frac{1}{4t}(p_1-p_0)\right)p_0$$

Taking the first-order condition with respect to p_0 and imposing symmetry yields $p=2t$.

(c) If one firm bundles then, essentially, both do, since you can either buy everything from one firm or from the other.

(d) No firm will choose to bundle as doing so lowers profits.



Chapter 11

Exercise 1a Profits of the seller side of the market is

$$\pi_S = (p_S \cdot q_B) \cdot q_S = \left(1 - \frac{1}{100} q_S\right) \cdot q_B \cdot q_S$$

Profits of the buyer side of the market equal

$$\pi_B = (p_B \cdot q_S) \cdot q_B = \left(\frac{1}{2} - \frac{1}{200} q_B\right) \cdot q_S \cdot q_B.$$

Taking the FOC of the firm on the seller side:

$$\frac{\partial}{\partial q_S}\left(\left(1 - \frac{1}{100} q_S\right) \cdot q_B \cdot q_S\right) = q_B - \frac{1}{50} q_B q_S = 0,$$

which implies $q_S^* = 50$. Taking the FOC of the firm on the buyer side:

$$\frac{\partial}{\partial q_B}\left(\left(\frac{1}{2} - \frac{1}{200} q_B\right) \cdot q_S \cdot q_B\right) = \frac{1}{2} q_S - \frac{1}{100} q_B q_S = 0,$$

which implies $q_B^* = 50$.

Equilibrium prices are

$$\begin{aligned} p_S^* &= 1 - \frac{1}{100} \cdot 50 = \frac{1}{2} \\ p_B^* &= \frac{1}{2} - \frac{1}{200} \cdot 50 = \frac{1}{4}, \end{aligned}$$

which yields profits

$$\begin{aligned}\pi_S^* &= \frac{1}{2}\cdot 50\cdot 50 = 1250\\ \pi_B^* &= \frac{1}{4}\cdot 50\cdot 50 = 625.\end{aligned}$$

Exercise 1b We now have

$$\begin{aligned}\pi_1 &= p_S\cdot\left(q_B^1+q_B^2\right)\cdot q_S^1 + p_B\cdot\left(q_S^1+q_S^2\right)\cdot q_B^1\\ &= \left(1-\frac{1}{100}\left(q_S^1+q_S^2\right)\right)\cdot\left(q_B^1+q_B^2\right)\cdot q_S^1+\left(\frac{1}{2}-\frac{1}{200}\left(q_B^1+q_B^2\right)\right)\cdot\left(q_S^1+q_S^2\right)\cdot q_B^1.\end{aligned}$$

Taking the derivative w.r.t. q_S^1 and q_B^1 yields the following first-order conditions:

$$\begin{aligned}\left(1-\frac{1}{100}\left(2q_S^1+q_S^2\right)\right)\cdot\left(q_B^1+q_B^2\right)+\left(\frac{1}{2}-\frac{1}{200}\left(q_B^1+q_B^2\right)\right)\cdot q_B^1 &= 0\\ \left(1-\frac{1}{100}\left(q_S^1+q_S^2\right)\right)\cdot q_S^1+\left(\frac{1}{2}-\frac{1}{200}\left(2q_B^1+q_B^2\right)\right)\cdot\left(q_S^1+q_S^2\right) &= 0.\end{aligned}$$

At this point, we can impose symmetry on both sides of the market: $q_S^1 = q_S^2 \equiv q_S$ and $q_B^1 = q_B^2 \equiv q_B$. The conditions above then simplify to

$$\begin{aligned}\left(1-\frac{3q_S}{100}\right)\cdot(2q_B)+\left(\frac{1}{2}-\frac{2q_B}{200}\right)\cdot q_B &= 0.\\ \left(1-\frac{2q_S}{100}\right)\cdot q_S+\left(\frac{1}{2}-\frac{3q_B}{200}\right)\cdot(2q_S) &= 0.\end{aligned}$$

It makes little sense to have equilibria with either $q_S = 0$ or $q_B = 0$, as that would yield zero profits for both firms. Hence (as long as doing so yields strictly positive profits), we can simplify this to

$$\begin{aligned}2\left(1-\frac{3q_S}{100}\right)+\left(\frac{1}{2}-\frac{2q_B}{200}\right) &= 0.\\ \left(1-\frac{2q_S}{100}\right)+2\left(\frac{1}{2}-\frac{3q_B}{200}\right) &= 0.\end{aligned}$$

thus

$$\begin{aligned}250-6q_S-q_B &= 0.\\ 200-2q_S-3q_B &= 0.\end{aligned}$$

This yields $q_S^* = \frac{275}{8} = 34.375$ and $q_B^* = \frac{175}{4} = 43.75$. This yields

$$\begin{aligned} p_S &= \left(1 - \frac{1}{100} \cdot 2 \cdot 34.375\right) = 0.3125 \\ p_B &= \left(\frac{1}{2} - \frac{1}{200} \cdot 2 \cdot 43.75\right) = 0.0625 \end{aligned}$$

Per-firm profits equal

$$\begin{aligned} & p_S \cdot \left(q_B^1 + q_B^2\right) \cdot q_S + p_B \cdot \left(q_S^1 + q_S^2\right) \cdot q_B^1 \\ = \; & 0.3125 \cdot (43.75 + 43.75) \cdot 34.375 + 0.0625 \cdot (34.375 + 34.375) \cdot 43.75 \\ = \; & 1127.90 \end{aligned}$$

Exercise 2 We have

$$\begin{aligned} q_A &= 100 - 100 p_A \\ q_R &= 100 - 200 p_R \end{aligned}$$

$$\begin{aligned} \pi &= p_R \cdot q_R \cdot (100 - q_A) + p_A \cdot q_R \cdot q_A \\ &= p_R \cdot (100 - 200 p_R) \cdot (100 - (100 - 100 p_A)) + p_A \cdot (100 - 200 p_R) \cdot (100 - 100 p_A) \\ &= 10\,000 p_A (1 - 2 p_R)(p_R + 1 - p_A) \end{aligned}$$

Taking first-order conditions:

$$\begin{aligned} \frac{\partial \pi}{\partial p_A} &= 10\,000 \left(1 - p_R - 2 p_A - 2 p_R^2 + 4 p_R p_A\right) = 0 \\ \frac{\partial \pi}{\partial p_R} &= 10\,000 p_A (-4 p_R - 1 + 2 p_A) = 0 \end{aligned}$$

From the second equality, we have

$$p_R = -\frac{1}{4} + \frac{1}{2} p_A.$$

Plugging this into the first equality yields

$$\frac{9}{8} - 3 p_A + \frac{3}{2} p_A^2 = 0.$$

This has two solutions $p_A = \frac{1}{2}$, and $p_A = \frac{3}{2}$. Obviously, the latter is not a feasible solution. We thus find $p_A^* = \frac{1}{2}$ and $p_R^* = 0$.

References

Haan, M. A. and Moraga-González, J. L. (2011). Advertising for attention in a consumer search model. *Economic Journal*, pages 552–579.