Chapter 5: Comparing one or two means

> **Learning objectives of this chapter:**
>
> - Hypothesis testing by hand using the z- and t-score
> - Generating and interpreting z-scores and t-scores
> - Independent two-sample t-test by hand and in R
> - Dependent two-sample t-test in R

Assignment 5.1: Hypothesis testing by hand using the z- and t-score

The maximum allowed amount of PFAS chemicals in soil is 0.9 microgram per kg of dry soil. In a regular soil check, researchers take 49 soil **samples** in a town near a plastic factory and find a **mean** PFAS level of 0.918 microgram/kg with a **standard deviation** of 0.071. They want to show with 95% confidence that the PFAS levels in the town are significantly above the norm.

5.1 a) Write down the relevant information from this case.

> **Answer 5.1a:**
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5.1 b) Write down the **null hypothesis** H_0 and the **alternative hypothesis** H_1 for the researchers' test of the **mean** PFAS level against the norm.

> **Answer 5.1b:**
>
> H_0: _____ H_1: _____

The **critical z-value** for a one-sided 95% confidence interval is equal to 1.645 (see also Table 1 on page 95)

5.1 c) Calculate the **lower bound** of the one-sided **confidence interval** for the PFAS level **population mean** with 95% confidence.

Answer 5.1c:

Lower bound: _____

Calculation:

_____

5.1 d) Draw the conclusion for the researchers. Include the following four elements:

☐ Show how μ_0 relates to the **confidence interval.**
☐ Discuss whether H_0 is rejected or not.
☐ Describe what this tells us about μ and μ_0.
☐ Describe what type of error is relevant *(type-I or type-II)*.

Answer 5.1d:

_____

_____

_____

_____

_____

5.1 e) Calculate the **z-score** for this situation.

💡 *Hint 5.1: You can find the formula for the **z-score** in the formula sheet. Use μ_0 in this formula.*

Answer 5.1c:

z-score: _____

Calculation:

_____

5.1 f)   Compare the calculated **z-score** with the **critical z-value** . What does this tell you about the **p-value** ?

Hint 5.2: Use the absolute value of the calculated **z-score** .

Answer 5.1f:

5.1 g)   Draw the conclusion again, but now using the **z-score** , is it the same? Include the following elements:

☐ Show how the calculated **z-score** relates to the **critical z-score** .
☐ Discuss whether H_0 is rejected or not.
☐ Describe what this tells us about μ and μ_0.
☐ Describe what type of error is relevant *(type-I or type-II)*.

Answer 5.1g:

The researchers want to quickly check another town near the plastic factory, but have less time available. They take 16 **samples** in the second town and find a much higher **mean** PFAS level of 0.930 microgram/kg with a comparable **standard deviation** of 0.070.

5.1 h)   Calculate the **t-score** for this situation.

Hint 5.3: You can find the formula for the **t-score** *in the formula sheet. Use μ_0 in this formula.*

Answer 5.1h:

t-score: _____

Calculation:

The **critical t-value** for a one-sided 95% **confidence interval** with 15 **degrees of freedom** is equal to 1.753.

5.1 i)   Draw the conclusion for the researchers using the (absolute) t-score.   Include the following elements:

- ☐ Show how the calculated **t-score** relates to the **critical t-score** .
- ☐ Discuss whether H_0 is rejected or not.
- ☐ Describe what this tells us about μ and μ_0.
- ☐ Describe what type of error is relevant *(type-I or type-II)*.

Answer 5.1i:

5.1 j)   Also calculate the **lower bound** of the one-sided **confidence interval** for the PFAS level **population mean** with 95% confidence.

Answer 5.1j:

Lower bound: _____

Calculation:

5.1 k)   If you would draw the conclusion based on this **lower bound** would you get the same result?
Explain why.

Answer 3.4g:

The conclusion **would** / **would not** be the same.

Explanation:

5.1 l)   Why is it that even though the second town shows much higher PFAS level in the **sample**
(0.930 vs 0.918) H_0 cannot be rejected?

Answer 5.1l:

Assignment 5.2: Generating and interpreting z-scores and t-scores

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In assignment 5.1 you have seen that to calculate a 95% one-sided **confidence interval** for a **sample** with $n \geq 30$ you need to use the correct **z-value**. Now you are going to find out how to get that number from R using the **qnorm()** function. The **qnorm()** function returns the **x**-value for a certain probability for a specified **normal distribution**, if you do not specify the **mean** (**mean**) and **standard deviation** (**sd**) arguments it returns the **z-value** for the **standard normal distribution**.

Run the following code in R:

```
qnorm(p = 0.95, mean = 0, sd = 1, lower.tail = TRUE)
# Or because the standard normal distribution is the default simply use:
qnorm(0.95)
```

5.2 a)   Round the number to 3 decimals. Do you recognise this **z-value** ?

Answer 5.2a:

     Result: _____

In assignment 5.1 you have also seen that for the 95% two-sided **confidence interval** for a **sample** with $n \geq 30$ you can use z = 1.960. Now you are going to learn to reproduce that number in R.

Run the following code in R:

```
qnorm(p = 0.975)
```

5.2 b)   Why do you have to use the 97.5 percentile to get the z-value for the 95% two-sided **confidence interval** ?

Answer 5.2b:

Run the following code in R:

```
pnorm(q = 1.645, mean = 0, sd = 1)
pnorm(1.960)
```

5.2 c)   Based on the results of this R code explain what the **pnorm()** function does.

Answer 5.2c:

In assignment 5.1 it was stated that the **critical t-value** for a one-sided 95% **confidence interval** with 15 **degrees of freedom** is equal to 1.753.

Run the following code in R:

```
qt(p = 0.95, df = 15)
```

5.2 d)   Based on the results of this R code explain what the **qt()** function does.
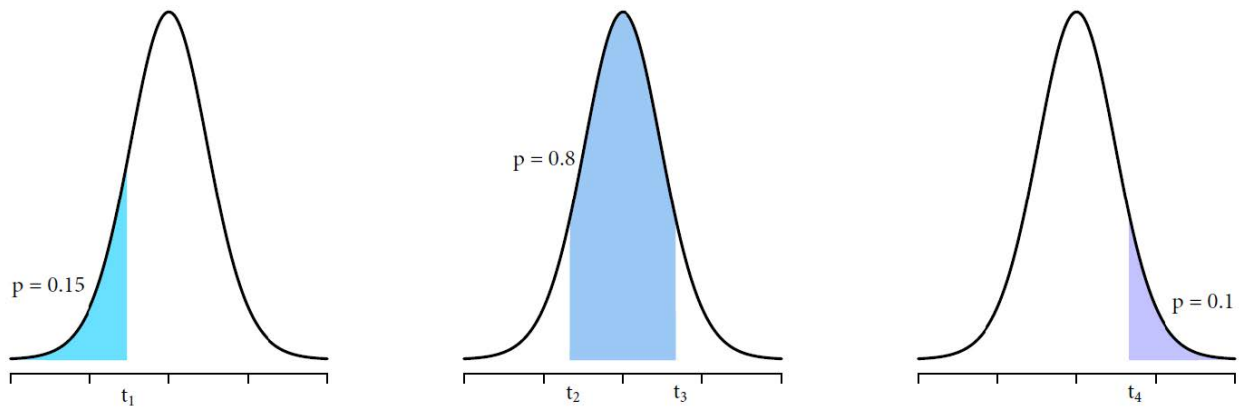
Answer 5.2d:

Run the following code in R:

```
pt(q = 1.753, df = 15)
```

5.2 e)   Based on the results of this R code explain what the **pt()** function does.

Answer 5.2e:

5.2 f) For the situation where there are 19 degrees of freedom, use the `qt()` function to get the **t-values** (t_1, t_2, t_3, t_4) for the three situations shown in the picture above.

💡 *Hint 5.4: For the picture on the right keep in mind that the `qt()` function by default returns the value for the left tail of the distribution.*
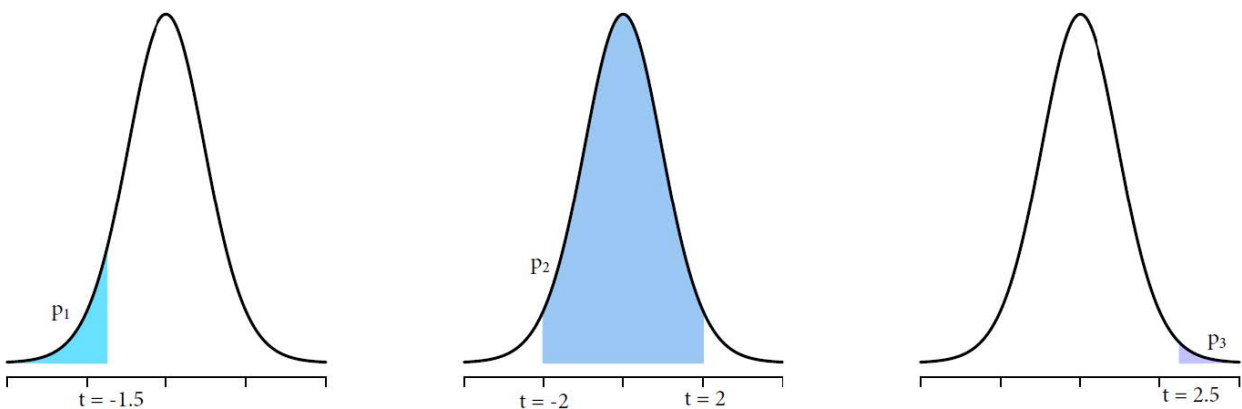

R code 5.2f:

Answer 5.2f:

t_1: _____ t_2: _____

t_3: _____ t_4: _____

5.2 g) For the situation where there are 11 **degrees of freedom** , use the **pt()** function to get the probabilities p_1, p_2, and p_3 (area under the curve) for the three situations shown in the figures above.

Hint 5.5: For the picture in the middle and the right keep in mind that the **pt()** *function by default uses the left tail and the total area under the curve is 1.*

R code 5.2g:

Answer 5.2g:

   p_1:   ——— p_2:   ——— p_3:   ———

When you test a value for a **population mean** , there exist three types of tests:

Two-tailed inequality test: One-tailed right sided test: One-tailed left sided test:

$H_0 : \mu = \mu_0$ $H_0 : \mu \leq \mu_0$ $H_0 : \mu \geq \mu_0$
$H_1 : \mu \neq \mu_0$ $H_1 : \mu > \mu_0$ $H_1 : \mu < \mu_0$

5.2 h) Give a critical **t-value** for the three tests described above using a confidence of 98% and 28 **degrees of freedom** .

Hint 5.6: Use the **qt()** *function in R*

.

R code 5.2h:

Answer 5.2h:

   Two-tailed inequality test: ———————

   One-tailed right sided test: ———————

   One-tailed left sided test: ———————

Three $n = 29$ **samples** were done on three different populations and the **t-score** based on these **samples** was calculated:

Two-tailed inequality test: One-tailed right sided test: One-tailed left sided test:

$H_0 : \mu = 50$ $H_0 : \mu \leq 0$ $H_0 : \mu \geq 0$
$H_1 : \mu \neq 0$ $H_1 : \mu > 0$ $H_1 : \mu < 0$

$t = 2.6$ $t = -2.3$ $t = 1.6$

5.2 i)   Using a confidence of 98%, find out whether the **null hypothesis** H_0 is rejected for each of these three cases.

💡 *Hint 5.7 : Use the **critical t-values** from the previous assignment and compare these with the sample **t-scores** to draw your conclusion for each of these outcomes.*

Answer 5.2i:

Two-tailed inequality test: H_0 **rejected** / H_0 **not rejected**

Explanation: _____

One-tailed right sided test: H_0 **rejected** / H_0 **not rejected**

Explanation: _____

One-tailed left sided test: H_0 **rejected** / H_0 **not rejected**

Explanation: _____

Assignment 5.3: Independent two-sample t-test by hand and in R

To test the effect of caffeine on the respiratory exchange ratio (RER), 9 men get caffeine and 9 different men get a placebo, their RER is measured while doing sports[1]:

RER measurement	n	Mean ($\bar{x}$)	Standard deviation (s_x)
Caffeine	9	94.22	5.61
Placebo	9	100.56	7.70

Researchers want to use this experiment to show with 95% confidence that caffeine reduces the RER in men. They are going to evaluate the results by comparing the **mean** RER using a **two-sample t-test** .

5.3 a)   Why is this test also called an **independent samples t-test** ?

> **Answer 5.3a:**
>
>
>
>

5.3 a)   Write down the **null hypothesis** H_0 and the **alternative hypothesis** H_1 for a one-sided test where the researchers want to show that the placebo **mean** RER is higher than the caffeine **mean** RER.

> **Answer 5.3b:**
>
> H_0: _____ H_1: _____

In the formula sheet on page 93 you can see that for **independent samples t-tests** :

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} \qquad s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

5.3 c)   Using the formula provided calculate the **pooled standard deviation** s_p.

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[1]These data are taken from http://learntech.uwe.ac.uk/da/Default.aspx?pageid=1438

Answer 5.3c:

s_p : _____

Calculation:

_____

5.3 d) Using the formula provided calculate the **t-score** .

Answer 5.3c:

t-score: _____

Calculation:

_____

The **critical t-value** for 16 **degrees of freedom** (n_1+n_2-2) for a one-sided test is 1.746.

5.3 e) Draw the conclusion for this test. Include the following elements:

☐ Show how the calculated **t-score** relates to the **critical t-score** .
☐ Discuss whether H_0 is rejected or not.
☐ Describe what this tells us about μ and μ_0.
☐ Describe what type of error is relevant *(type-I or type-II)*.

Answer 5.3e:

_____

_____

_____

_____

_____

_____

Now you will learn how to do the same independent t-test in R.

Run the following code in R:

```r
# These are the values for the RER test data set
placebo <- c(105, 119, 100, 97, 96, 101, 94, 95, 98)
caffeine <- c(96, 99, 94, 89, 96, 93, 88, 105, 88)
```

5.3 f)   Calculate the **mean** and **standard deviation** for the **placebo** and **caffeine** variables, confirm that your results match with the previous assignment.

R code 5.3f:

Answer 5.3f:

placebo   **caffeine**

Mean: _____   Mean: _____

Standard deviation: _____   Standard deviation: _____

Run the following code in R:

```r
t.test(x = placebo, y = caffeine, alternative = 'greater', mu = 0,
       paired = FALSE, var.equal = TRUE, conf.level = 0.95)
```

5.3 g)   Interpret the R code above and check if the results match your conclusion for assignment 5.3e.

Answer 5.3g:

_____

_____

The **t-test** assumes equal **variances** . In this case the **standard deviations** are actually a bit different.

5.3 h) Change the `var.equal` argument in the R code above to do a **Welch t-test** to account for unequal **variances** . Check whether the results differ.

R code 5.3h:

Answer 5.3h:

5.3 i) How can you check if you need to do a **Welch t-test** instead of a normal **t-test** ?

Answer 5.3i:

Assignment 5.4: Dependent two-sample t-test in R

To test for a difference in blood pressure depending on their position, 12 people's blood pressure was measured twice, first standing up and then lying down. Researchers want to use this experiment to show with 92.5% confidence that the blood pressure lying down is significantly higher than standing up. They are going to evaluate the results by comparing the **mean** using a **two-sample t-test** .

5.4 a)   Why is this a dependent (also called paired) **t-test** ?

Answer 5.4a:

Run the following code in R:

```
# These are the values for the blood pressure data set
standing <- c(132, 146, 135, 141, 139, 162, 128, 137, 145, 151, 131, 143)
lying <- c(136, 145, 140, 147, 142, 160, 137, 136, 149, 158, 120, 150)
```

5.4 b)   Calculate the **mean** of the **standing** and **lying** data sets, also calculate the differences and the **mean** of the differences.

R code 5.4b:

Answer 5.4b:

Mean standing: _____

Mean lying: _____

Mean difference: _____

5.4 c) Write down the **null hypothesis** H_0 and **alternative hypothesis** H_1 for a one-sided test where the researchers want to show with 92.5% confidence that the **mean** blood pressure is higher lying down than standing up.

Answer 5.4c:

H_0: _____ H_1: _____

Run the following code in R:

```
t.test(x = standing, y = lying, alternative = 'greater', mu = 0 ,
       paired = TRUE, conf.level = 0.925)
```

5.4 d) Check the results of this test. The outcome is a bit strange, can you explain what is wrong here?

Answer 5.4d:

_____

5.4 e) Fix the R code to do a correct test and draw the correct (four part) conclusion.

R code 5.4e:

Answer 5.4e:

_____

_____

_____

_____

_____

_____