

# Package ‘jfa’

December 10, 2019

**Title** A multi-functional R package for auditing

**Version** 1.0.0

**Description** The jfa package provides functions for daily use in auditing, such as calculating sample sizes, sampling, and calculating confidence bounds.

**License** GPL-3 + file LICENSE

**Encoding** UTF-8

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**VignetteBuilder** knitr

**Suggests** testthat, knitr, rmarkdown

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auditPrior	<i>Create a Prior Distribution</i>
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## Description

This function creates a prior distribution according to the audit risk model. The returned object is of class `jfaPrior` and can be used with associated `print()` and `plot()` methods. `jfaPrior` objects can be used as input argument for the `prior` argument in other functions.

## Usage

```
auditPrior(materiality, confidence = 0.95, method = "arm", ir = 1, cr = 1,
            expectedError = 0, likelihood = "binomial", N = NULL)
```

**Arguments**

materiality	a value between 0 and 1 representing the materiality of the audit as a fraction of the total size or value.
confidence	the confidence level desired from the confidence bound (on a scale from 0 to 1). Defaults to 0.95, or 95% confidence.
method	the method by which the prior distribution is constructed. Currently only supports the arm method, which uses the audit risk model (Derks et al., 2019).
ir	the inherent risk probability from the audit risk model. Defaults to 1 for 100% risk.
cr	the inherent risk probability from the audit risk model. Defaults to 1 for 100% risk.
expectedError	a fraction representing the percentage of expected mistakes in the sample relative to the total size, or a number ( $\geq 1$ ) that represents the number of expected mistakes.
likelihood	can be one of binomial, poisson, or hypergeometric.
N	the population size (required for hypergeometric calculations).

**Value**

An object of class `jfaPrior` containing:

method	the method by which the prior distribution is constructed.
likelihood	the likelihood by which the prior distribution is updated.
priorD	the name of the probability density function of the prior distribution.
nPrior	the prior assumed sample size.
kPrior	the prior assumed sample errors
aPrior	the prior parameter alpha.
bPrior	the prior parameter beta.
materiality	the materiality that was used to construct the prior distribution.
N	if specified as input, the population size.

**Author(s)**

Koen Derks, <k.derks@nyenrode.nl>

**References**

Derks, K., de Swart, J., Wagenmakers, E.-J., Wille, J., & Wetzels, R. (2019). JASP for audit: Bayesian tools for the auditing practice.

**See Also**

[planning sampling evaluation](#)

**Examples**

```
library(jfa)
set.seed(1)

# Generate some audit data (N = 1000).
population <- data.frame(ID = sample(1000:100000, size = 1000, replace = FALSE),
                          bookValue = runif(n = 1000, min = 700, max = 1000))

# Specify the materiality, confidence, and expected errors.
materiality <- 0.05 # 5%
confidence <- 0.95 # 95%
expectedError <- 0.025 # 2.5%

# Specify the inherent risk (ir) and control risk (cr).
ir <- 1 # 100%
cr <- 0.6 # 60%

# Create a beta prior distribution according to the Audit Risk Model (arm)
# and a binomial likelihood.
prior <- auditPrior(materiality = materiality, confidence = confidence,
                     method = "arm", ir = ir, cr = cr,
                     expectedError = expectedError, likelihood = "binomial")
print(prior)

# jfa prior distribution for arm method:
#
# Prior sample size:      51
# Prior errors:           1.27
# Prior:                  beta(2.275, 50.725)

# Calculate the sample size according to the binomial distribution with the specified prior.
sampleSize <- planning(materiality = materiality, confidence = confidence,
                       expectedError = expectedError, prior = prior, likelihood = "binomial")
print(sampleSize)

# jfa planning results for beta prior with binomial likelihood:
#
# Materiality:           5%
# Confidence:            95%
# Sample size:           169
# Allowed sample errors: 4.23
# Prior parameter alpha: 2.275
# Prior parameter beta:  50.725

# Draw sample using random monetary unit sampling.
sampleResult <- sampling(population = population, sampleSize = sampleSize,
                        algorithm = "random", units = "mus", seed = 1, bookValues = "bookValue")
print(sampleResult)

# jfa sampling results for random monetary unit sampling:
#
# Population size:       1000
# Sample size:           169
# Proportion n/N:        0.169
# Percentage of value:    16.84%
```

```

# Isolate the sample.
sample <- sampleResult$sample

# For this example, we use the book values as audit values. Implies zero errors at this point.
sample$trueValue <- sample$bookValue

# One overstatement is found.
sample$trueValue[2] <- sample$trueValue[2] - 500

# Evaluate the sample with one partial error using the posterior distribution.
conclusion <- evaluation(sample = sample, bookValues = "bookValue", auditValues = "trueValue",
                        prior = prior, materiality = 0.05)

print(conclusion)

# jfa evaluation results for binomial likelihood with prior:
#
# Materiality:          5%
# Confidence:           95%
# Upper bound:          2.729%
# Sample size:          169
# Sample errors:         1
# Conclusion:           Approve population

```

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BuildIt

*BuildIt Construction financial statements*


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## Description

Fictional data from a construction company in the United States, containing 3500 observations identification numbers, book values, and audit values. The audit values are added for illustrative purposes, as these would need to be assessed by the auditor in the execution stage of the audit.

## Usage

```
data(BuildIt)
```

## Format

A data frame with 3500 rows and 3 variables.

**ID** unique record identification number.

**bookValue** book value in US dollars (\$14.47–\$2,224.40).

**auditValue** true value in US dollars (\$14.47–\$2,224.40).

## References

Derks, K., de Swart, J., Wagenmakers, E.-J., Wille, J., & Wetzels, R. (2019). JASP for audit: Bayesian tools for the auditing practice.

## Examples

```
data(BuildIt)
```

**Description**

This function takes a sample data frame or summary statistics about an evaluated audit sample and calculates a confidence bound according to a specified method. The returned object is of class `jfaEvaluation` and can be used with associated `print()` and `plot()` methods.

**Usage**

```
evaluation(sample = NULL, bookValues = NULL, auditValues = NULL,
  confidence = 0.95, nSumstats = NULL, kSumstats = NULL,
  method = "binomial", materiality = NULL, N = NULL,
  prior = FALSE, nPrior = 0, kPrior = 0,
  rohrbachDelta = 2.7, momentPoptype = "accounts",
  populationBookValue = NULL,
  csA = 1, csB = 3, csMu = 0.5)
```

**Arguments**

<code>sample</code>	a data frame containing at least a column of book values and a column of audit (true) values.
<code>bookValues</code>	the column name for the book values in the sample.
<code>auditValues</code>	the column name for the audit (true) values in the sample.
<code>confidence</code>	the required confidence level for the bound.
<code>nSumstats</code>	the number of observations in the sample. If specified, overrides the <code>sample</code> , <code>bookValues</code> and <code>auditValues</code> arguments and assumes that the data comes from summary statistics specified by <code>nSumstats</code> and <code>kSumstats</code> .
<code>kSumstats</code>	the sum of the errors found in the sample. If specified, overrides the <code>sample</code> , <code>bookValues</code> and <code>auditValues</code> arguments and assumes that the data comes from summary statistics specified by <code>kSumstats</code> and <code>nSumstats</code> .
<code>method</code>	can be either one of <code>poisson</code> , <code>binomial</code> , <code>hypergeometric</code> , <code>stringer</code> , <code>stringer-meikle</code> , <code>stringer-lta</code> , <code>stringer-pvz</code> , <code>rohrbach</code> , <code>moment</code> , <code>direct</code> , <code>difference</code> , <code>quotient</code> , or <code>regression</code> .
<code>materiality</code>	if specified, the function also returns the conclusion of the analysis with respect to the materiality. This value must be specified as a fraction of the total value of the population (a value between 0 and 1). The value is discarded when <code>direct</code> , <code>difference</code> , <code>quotient</code> , or <code>regression</code> method is chosen.
<code>N</code>	the total population size.
<code>prior</code>	whether to use a prior distribution when evaluating. Defaults to <code>FALSE</code> for frequentist evaluation. If <code>TRUE</code> , the prior distribution is updated by the specified likelihood. Chooses a conjugate gamma distribution for the Poisson likelihood, a conjugate beta distribution for the binomial likelihood, and a conjugate beta-binomial distribution for the hypergeometric likelihood.
<code>nPrior</code>	the prior parameter $\alpha$ (number of errors in the assumed prior sample).
<code>kPrior</code>	the prior parameter $\beta$ (total number of observations in the assumed prior sample).

rohrbachDelta	the value of $\Delta$ in Rohrbach's augmented variance bound.
momentPoctype	can be either one of accounts or inventory. Options result in different methods for calculating the central moments, for more information see Dworin and Grimlund (1986).
populationBookValue	the total value of the audit population. Required when method is one of direct, difference, quotient, or regression.
csA	if method = "coxsnell", the $\alpha$ parameter of the prior distribution on the mean taint. Default is set to 1, as recommended by Cox and Snell (1979).
csB	if method = "coxsnell", the $\beta$ parameter of the prior distribution on the mean taint. Default is set to 3, as recommended by Cox and Snell (1979).
csMu	if method = "coxsnell", the mean of the prior distribution on the mean taint. Default is set to 0.5, as recommended by Cox and Snell (1979).

## Details

This section lists the available options for the methods argument.

- **poisson**: The confidence bound taken from the Poisson distribution. If combined with prior = TRUE, performs Bayesian evaluation using a *gamma* prior and posterior.
- **binomial**: The confidence bound taken from the binomial distribution. If combined with prior = TRUE, performs Bayesian evaluation using a *beta* prior and posterior.
- **hypergeometric**: The confidence bound taken from the hypergeometric distribution. If combined with prior = TRUE, performs Bayesian evaluation using a *beta-binomial* prior and posterior.
- **stringer**: The Stringer bound (Stringer, 1963).
- **stringer-meikle**: Stringer bound with Meikle's correction for understatements (Meikle, 1972).
- **stringer-lta**: Stringer bound with LTA correction for understatements (Leslie, Teitlebaum, and Anderson, 1979).
- **stringer-pvz**: Stringer bound with Pap and van Zuijlen's correction for understatements (Pap and van Zuijlen, 1996).
- **rohrbach**: Rohrbach's augmented variance bound (Rohrbach, 1993).
- **moment**: Modified moment bound (Dworin and Grimlund, 1986).
- **coxsnell**: Cox and Snell bound (Cox and Snell, 1979).
- **direct**: Confidence interval using the direct method (Touw and Hoogduin, 2011).
- **difference**: Confidence interval using the difference method (Touw and Hoogduin, 2011).
- **quotient**: Confidence interval using the quotient method (Touw and Hoogduin, 2011).
- **regression**: Confidence interval using the regression method (Touw and Hoogduin, 2011).

## Value

An object of class `jfaEvaluation` containing:

n	the sample size.
k	an integer specifying the number of observed errors.
t	a number specifying the sum of observed taints.

confidence	the confidence level of the result.
popBookvalue	if specified as input, the total book value of the population.
pointEstimate	if method is one of direct, difference, quotient, or regression, the value of the point estimate.
lowerBound	if method is one of direct, difference, quotient, or regression, the value of the lower bound of the interval.
upperBound	if method is one of direct, difference, quotient, or regression, the value of the upper bound of the interval.
confBound	the upper confidence bound on the error percentage.
method	the evaluation method that was used.
materiality	the materiality.
conclusion	if materiality is specified, the conclusion about whether to approve or not approve the population.
N	if specified as input, the population size.
populationK	the assumed total errors in the population. Used for inferences with hypergeometric method.
prior	a logical, indicating whether a prior was used in the analysis.
nPrior	if a prior is specified, the prior assumed sample size.
kPrior	if a prior is specified, the prior assumed sample errors.
multiplicationFactor	if method = "coxsnell", the multiplication factor for the $F$ -distribution.
df1	if method = "coxsnell", the df1 for the $F$ -distribution.
df2	if method = "coxsnell", the df2 for the $F$ -distribution.

#### Author(s)

Koen Derks, <k.derks@nyenrode.nl>

#### References

- Cox, D. and Snell, E. (1979). On sampling and the estimation of rare errors. *Biometrika*, 66(1), 125-132.
- Dworin, L., and Grimlund, R. A. (1986). Dollar-unit sampling: A comparison of the quasi-Bayesian and moment bounds. *Accounting Review*, 36-57.
- Leslie, D. A., Teitlebaum, A. D., & Anderson, R. J. (1979). *Dollar-unit sampling: a practical guide for auditors*. Copp Clark Pitman; Belmont, Calif.: distributed by Fearon-Pitman.
- Meikle, G. R. (1972). *Statistical Sampling in an Audit Context: An Audit Technique*. Canadian Institute of Chartered Accountants.
- Pap, G., and van Zijlen, M. C. (1996). On the asymptotic behaviour of the Stringer bound 1. *Statistica Neerlandica*, 50(3), 367-389.
- Rohrbach, K. J. (1993). Variance augmentation to achieve nominal coverage probability in sampling from audit populations. *Auditing*, 12(2), 79.
- Stringer, K. W. (1963). Practical aspects of statistical sampling in auditing. In *Proceedings of the Business and Economic Statistics Section* (pp. 405-411). American Statistical Association.
- Touw, P., and Hoogduin, L. (2011). *Statistiek voor Audit en Controlling*. Boom uitgevers Amsterdam.

**See Also**

[auditPrior planning sampling](#)

**Examples**

```
library(jfa)

# Generate some audit data (N = 1000)
set.seed(1)
data <- data.frame(ID = sample(1000:100000, size = 1000, replace = FALSE),
                   bookValue = runif(n = 1000, min = 700, max = 1000))

# Using the binomial likelihood, calculates the upper 95% confidence bound for a
# materiality of 5% when 1% full errors are found in a sample (n = 93).
jfaRes <- planning(materiality = 0.05, confidence = 0.95, expectedError = 0.01,
                  likelihood = "binomial")

# Using monetary unit sampling, draw a random sample from the population.
samp <- sampling(population = data, sampleSize = jfaRes, units = "mus",
                bookValues = "bookValue", algorithm = "random")

samp$sample$trueValue <- samp$sample$bookValue
samp$sample$trueValue[2] <- samp$sample$trueValue[2] - 500 # One overstatement is found

# Evaluate the sample using the stringer bound.
conclusion <- evaluation(sample = samp$sample, bookValues = "bookValue",
                      auditValues = "trueValue", method = "stringer", materiality = 0.05)
print(conclusion)

# jfa evaluation results for stringer method:
#
# Materiality:          5%
# Confidence:           95%
# Upper bound:          4.244%
# Sample size:          93
# Sample errors:         1
# Conclusion:           Approve population

# Evaluate the sample using summary statistics (n = 93, k = 1).
conclusion <- evaluation(nSumstats = 93, kSumstats = 1, method = "binomial",
                      materiality = 0.05)
print(conclusion)

# jfa evaluation results for binomial method:
#
# Materiality:          5%
# Confidence:           95%
# Upper bound:          4.999%
# Sample size:          93
# Sample errors:         1
# Conclusion:           Approve population
```



**Description**

This function calculates the required sample size for an audit, based on the poisson, binomial, or hypergeometric likelihood. A prior can be specified to perform Bayesian planning. The returned object is of class `jfaPlanning` and can be used with associated `print()` and `plot()` methods.

**Usage**

```
planning(materiality, confidence = 0.95, expectedError = 0, likelihood = "poisson",
         N = NULL, maxSize = 5000, prior = FALSE, kPrior = 0, nPrior = 0)
```

**Arguments**

<code>materiality</code>	a value between 0 and 1 representing the materiality of the audit as a fraction of the total size or value.
<code>confidence</code>	the confidence level desired from the confidence bound (on a scale from 0 to 1). Defaults to 0.95, or 95% confidence.
<code>expectedError</code>	a fraction representing the percentage of expected mistakes in the sample relative to the total size, or a number ( $\geq 1$ ) that represents the number of expected mistakes.
<code>likelihood</code>	can be one of <code>binomial</code> , <code>poisson</code> , or <code>hypergeometric</code> .
<code>N</code>	the population size (required for hypergeometric calculations).
<code>maxSize</code>	the maximum sample size that is considered for calculations. Defaults to 5000 for efficiency. Increase this value if the sample size cannot be found due to it being too large (e.g., for low materialities).
<code>prior</code>	whether to use a prior distribution when planning. Defaults to <code>FALSE</code> for frequentist planning. If <code>TRUE</code> , the prior distribution is updated by the specified likelihood. Chooses a conjugate gamma distribution for the Poisson likelihood, a conjugate beta distribution for the binomial likelihood, and a conjugate beta-binomial distribution for the hypergeometric likelihood.
<code>kPrior</code>	the prior parameter $\alpha$ (number of errors in the assumed prior sample).
<code>nPrior</code>	the prior parameter $\beta$ (total number of observations in the assumed prior sample).

**Details**

This section elaborates on the available likelihoods and corresponding prior distributions for the likelihood argument.

- `poisson`: The Poisson likelihood is used as a likelihood for monetary unit sampling (MUS). Its likelihood function is defined as:

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

The conjugate  $gamma(\alpha, \beta)$  prior has probability density function:

$$f(x; \alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}$$

- **binomial:** The binomial likelihood is used as a likelihood for record sampling *with* replacement. Its likelihood function is defined as:

$$p(x) = \binom{n}{k} p^k (1-p)^{n-k}$$

The conjugate  $beta(\alpha, \beta)$  prior has probability density function:

$$f(x; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}$$

- **hypergeometric:** The hypergeometric likelihood is used as a likelihood for record sampling *without* replacement. Its likelihood function is defined as:

$$p(x = k) = \frac{\binom{K}{k} \binom{N-K}{n-k}}{\binom{N}{n}}$$

The conjugate  $beta\text{-binomial}(\alpha, \beta)$  prior (Dyer and Pierce, 1993) has probability density function:

$$f(k|n, \alpha, \beta) = \binom{n}{k} \frac{B(k + \alpha, n - k + \beta)}{B(\alpha, \beta)}$$

## Value

An object of class `jfaPlanning` containing:

<code>materiality</code>	the value of the specified materiality.
<code>confidence</code>	the confidence level for the desired population statement.
<code>sampleSize</code>	the resulting sample size.
<code>expectedSampleError</code>	
	the number of full errors that are allowed to occur in the sample.
<code>expectedError</code>	the specified number of errors as a fraction or as a number.
<code>likelihood</code>	the specified likelihood.
<code>errorType</code>	whether the expected errors were specified as a percentage or as an integer.
<code>N</code>	the population size (only returned in case of a hypergeometric likelihood).
<code>populationK</code>	the assumed population errors (only returned in case of a hypergeometric likelihood).
<code>prior</code>	a list containing information on the prior parameters.

## Author(s)

Koen Derks, <k.derks@nyenrode.nl>

## References

Dyer, D. and Pierce, R.L. (1993). On the Choice of the Prior Distribution in Hypergeometric Sampling. *Communications in Statistics - Theory and Methods*, 22(8), 2125 - 2146.

## See Also

[auditPrior sampling evaluation](#)

**Examples**

```
library(jfa)

# Using the binomial distribution, calculates the required sample size for a
# materiality of 5% when 2.5% mistakes are expected to be found in the sample.

# Frequentist planning with binomial likelihood.
planning(materiality = 0.05, confidence = 0.95, expectedError = 0.025,
          likelihood = "binomial")

# Bayesian planning with uninformed prior.
planning(materiality = 0.05, confidence = 0.95, expectedError = 0.025,
          likelihood = "binomial", prior = TRUE)

# Bayesian planning with informed prior (based on 10 correct observations).
planning(materiality = 0.05, confidence = 0.95, expectedError = 0.025,
          likelihood = "binomial", prior = TRUE, kPrior = 0, nPrior = 10)
```

sampling

*Sampling from Audit Populations***Description**

This function takes a data frame and performs sampling according to one of three popular algorithms: random sampling, cell sampling, or fixed interval sampling. Sampling is done in combination with one of two sampling units: records or monetary units. The returned object is of class `jfaSampling` and can be used with associated `print()` and `plot()` methods.

**Usage**

```
sampling(population, sampleSize, bookValues = NULL, units = "records",
          algorithm = "random", intervalStartingPoint = 1, ordered = TRUE,
          ascending = TRUE, withReplacement = FALSE, seed = 1)
```

**Arguments**

<code>population</code>	a data frame containing the population the auditor wishes to sample from.
<code>sampleSize</code>	the number of observations that need to be selected from the population. Can also be an object of class <code>jfaPlanning</code> .
<code>bookValues</code>	a character specifying the name of the column containing the book values (as in the population data).
<code>units</code>	can be either <code>records</code> (default) for record sampling, or <code>mus</code> for monetary unit sampling.
<code>algorithm</code>	can be either one of <code>random</code> (default) for random sampling, <code>cell</code> for cell sampling, or <code>interval</code> for fixed interval sampling.
<code>intervalStartingPoint</code>	the starting point in the interval (used only in fixed interval sampling)
<code>ordered</code>	if <code>TRUE</code> (default), the population is first ordered according to the value of their book values.

ascending	if TRUE (default), order the population in ascending order.
withReplacement	whether sampling should be performed with replacement. Defaults to FALSE.
seed	seed to reproduce results. Default is 1.

## Details

This first part of this section elaborates on the possible options for the `units` argument:

- `records`: In record sampling, each observation in the population is seen as a sampling unit. An observation of \$5000 is therefore equally likely to be selected as an observation of \$500.
- `mus`: In monetary unit sampling, each monetary unit in the population is seen as a sampling unit. An observation of \$5000 is therefore ten times more likely to be selected as an observation of \$500.

This second part of this section elaborates on the possible options for the `algorithm` argument:

- `random`: In random sampling each sampling unit in the population is drawn with equal probability.
- `cell`: In cell sampling the sampling units in the population are divided into a number (equal to the sample size) of intervals. From each interval one sampling unit is selected with equal probability.
- `interval`: In fixed interval sampling the sampling units in the population are divided into a number (equal to the sample size) of intervals. From each interval one sampling unit is selected according to a fixed starting point (`intervalStartingPoint`).

## Value

An object of class `jfaSampling` containing:

<code>population</code>	a data frame containing the input population.
<code>sample</code>	a data frame containing the selected observations.
<code>bookValues</code>	if specified, the name of the specified book value column.
<code>algorithm</code>	the algorithm that was used for sampling.
<code>units</code>	the sampling units that were used for sampling.

## Author(s)

Koen Derks, <k.derks@nyenrode.nl>

## References

Wampler, B., & McEacharn, M. (2005). Monetary-unit sampling using Microsoft Excel. *The CPA journal*, 75(5), 36.

## See Also

[auditPrior planning evaluation](#)

**Examples**

```
library(jfa)

# Generate some audit data (N = 1000).
population <- data.frame(ID = sample(1000:100000, size = 1000, replace = FALSE),
                          bookValue = runif(n = 1000, min = 700, max = 1000))

# Calculate the sample size according to the binomial distribution with zero errors
jfaRes <- planning(materiality = 0.05, confidence = 0.95, expectedError = 0,
                   likelihood = "binomial")

# Draw sample using random record sampling
sampling(population = population, sampleSize = jfaRes, algorithm = "random",
         units = "records", seed = 1)

# Draw sample using random monetary unit sampling
sampling(population = population, sampleSize = jfaRes, algorithm = "random",
         units = "mus", bookValues = "bookValue", seed = 1)
```

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