MOPTA 2015 competition report

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1. Mathematical Formulation for problem without bad luck

Our approach to model the system is based upon the observation that the durations of all projects and the total time are integral, therefore the entire time span can be divided into time segments with equal lengths (one month in this case). A binary variable matrix X_{jt} is assigned to Project j at the t^{th} time segment, which indicates whether the project j is started at the beginning of the t^{th} month. This results in a mixed integer model, which contains the following parameters:

$$
\begin{aligned}
c_j &= \{\text{the cost of Project j}\} \\
b_j &= \{\text{the benefit of Project j}\} \\
d_j &= \{\text{the duration of Project j}\} \\
c_j^+ &= \{\text{the added cost to Project j if delayed}\} \\
d_j^+ &= \{\text{the added duration to Project j if delayed}\} \\
T &= \{\text{the number of months in the total time span, integer}\}
\end{aligned}
$$

A binary delay flag q_j is assigned to each project to indicate whether Project j is expected to be delayed. Therefore the actual cost and duration of each project for optimization is:

$$\tilde{c}_j = c_j + q_j c_j^+, \quad \forall j \in J \tag{1}$$

$$\tilde{d}_j = d_j + q_j d_j^+, \quad \forall j \in J \tag{2}$$

The following constraints are added to the model:

- A project can be started only once throughout the entire time span:

$$\sum_{t=1}^{T} X_{jt} \leq 1, \forall j \in J \tag{3}$$

- A project cannot be started if it is too late. That is, we can only started a project if it can be finished before the deadline:

$$\sum_{t \geq T+1-\tilde{d}_j} X_{jt} = 0, \forall j \in J \tag{4}$$

- For all $i \nsim j$, i cannot be started within d_j units of time after j started, same for j.

$$\sum_{t-\tilde{d}_j+1 \le t' \le t+\tilde{d}_i-1} X_{jt'} + X_{it} \le 1, \forall i \nsim j, \forall t \in \{1..T\} \tag{5}$$

- For all $i \succ j$, j can be started only if i has been finished:

$$\sum_{t' \le t-\tilde{d}_i} X_{it'} \ge X_{jt}, \forall i \succ j, \forall t \in \{1..T\} \tag{6}$$

- For all $I \vdash j$, j cannot be started until at least one of the projects in I has been finished:

$$\sum_{i \in I} \sum_{t' \le t-\tilde{d}_i} X_{it'} \ge X_{jt}, \forall I \vdash j, \forall t \in \{1..T\} \tag{7}$$

Apparently Constraint (6) is a special scenario in Constraint (7), where the only element in I is Project i, and can therefore be generalized into Constraint (7) for simplicity.

The object function is taken as:

$$NPV_\gamma(S,T) = -\sum_{\substack{j \in S \\ 0 \le t_j/12 \le 3}} \gamma^{t_j} \tilde{c}_j + \sum_{\substack{j \in S \\ 0 \le (t_j+\tilde{d}_j)/12 \le 3}} \gamma^{t_j+\tilde{d}_j} \tilde{b}_j \tag{8}$$

In the optimization problem, it is written as:

$$NPV_\gamma(S,T) = -\sum_{i,t} \gamma^t * X(i,t) * c_i + \sum_{i,t} \gamma^{t+d_i} * b_i * X(i,t) \tag{9}$$

It is easy to construct the one to one correspondence between (S, T) and X, where:

$$S = \{i | \exists t, s.t X(i,t) = 1\} \tag{10}$$

and for each element t_i in T, which is the starting time of project i in S:

$$t_i = t \text{ where } X(i,t) = 1 \tag{11}$$

The above model can be efficiently solved by typical MIP solvers such as Cplex.

2. Scheduling Under Bad Luck

In the later parts, the problem is to take failures and delays into consideration while deciding project portfolio. A variable $W \geq 0$ is introduced to measure the budget of bad luck. We define:

$$
\begin{aligned}
I_f &= \{\text{Project that will be failed by the bad luck}\} \\
I_\delta &= \{\text{Project that will be delayed by the bad luck but eventually will succeed}\} \\
I_{f|\delta} &= \{\text{Project that will be delayed by the bad luck and eventually will fail}\}
\end{aligned}
$$

Each project has a probabality to be either delayed, failed or both. The magnitude of each occurence of bad luck is measured by the information entrophy calculated from the probability. The total quantity of bad luck is defined as:

$$
w(I_f, I_\delta, I_{f|\delta}) := \sum_{j \in I_f} [-log_2(p_{f,j})] + \sum_{j \in I_\delta} [-log_2((1 - p_{f,j})p_{\delta,j})] + \sum_{j \in I_{f|\delta}} [-log_2((1 - p_{f,j}))]
$$

Our purpose is to find a schedule that yields the most profit under certain budget of bad luck. A general formulation is the following maximin problem:

$$
\begin{aligned}
&\max_{S,T} \quad \min_{I_f, I_\delta, I_{f|\delta}} \quad NPV'_\gamma(S, T, I_f, I_\delta, I_{f|\delta}) \\
subjectto \quad & i \succ j, \forall (i,j) \in E, \\
& i \neq j, \forall (i,j) \in E', \\
& I \vdash j, \forall (I,j) \in E'',
\end{aligned}
\tag{12}
$$

The net present value of the portfolio is computed by summing up the costs and benefits of the selected projects, discounted appropriately at the time they are incured:

$$
NPV_\gamma(S, T) = \sum_{j \in S, 0 \leq t_j/12 < 3} \gamma^{t_j} \tilde{c}_j + \sum_{j \in S, 0 \leq t_j/12 < 3} \gamma^{t_j + d_j} \tilde{b}_j \tag{13}
$$

3. Adapative Scheduling of Projects

In real world the original schedule is subject to change once any bad luck occurs.The purpose is to reduce the potential loss based upon information at current stage. For example, if one intermediate project is delayed and we can predict that although we can continue with subsequent projects, some other projects cannot completed in time, a possible better option is to remove those projects from our schedule. Our analysis will take the update of schedule into consideration. The update must follow the rules below:

- Only projects selected in the original schedule can be chosen in the updated schedule.
- The update will take place when a bad luck happens, and we assume that bad luck only happens at the scheduled ending time of a project. The case of projects delayed and then failed can be regarded as a special condition where bad luck happens twice. The first time is at the scheduled ending time of the previous plan, when we know the project is delayed, and then the second time is at the moment it fails as the time of delay passes by.

- Only the projects that scheduled at and after the current bad luck can be rescheduled.
- When updating the schedule, an optimal choice is made without assuming that bad luck will take place in the future. In othe words, although we set a total budget for all the bad luck, we assume that the chance of bad luck on a started projects is independent of previous occurence of bad luck.

The impact of bad luck can be quantified by the loss it incurs. Given the original schedule (S, T) and the final realization of schedule (S', T'), the loss is acquired by calculating the difference between the total benefit of Them, which can be expressed as:

$$Loss_{(S,T)}(S', T') = NPV(S', T') - NPV(S, T) \tag{14}$$

With the assumptions above, the final realization of schedule can be obtained by solving a series of optimization problems.We begin with a intitial schedule schedule (S_0, T_0), which can be same as the solution we got in part 1, and at each time t_i when bad luck occurs to a single project, the schedule is updated to (S_i, T_i) based upon current progress and the category of bad luck before and at t. Let (S_{i-1}, T_{i-1}) be the schedule after last bad luck happens at time t_{i-1} with corresponding decision variable $X'_{i,t}$, and $(I_{tf}, I_{t\delta})$ be the set of projects that has been delayed or failed before and at time t. The treatment for the case of delayed and fail is a little tricky. If at time t we know a project has been delayed and failed, then we put it into both set I_{tf} and $I_{t\delta}$, otherwise if we only get the information that the project has been delayed, it will only be selected into the set I_{tf}. We can find an updated schedule(S_i, T_i) after bad luck at time t_i by solving a problem which has the following constraints with decision variable $X_{i,t}$:

- As in part 1, a single project can only be started once.

$$\sum_t X_{it} \leq 1, \forall i \tag{15}$$

- The starting time of a project cannot be too late. This constraint is also the same with that in part 1. Note that here since we made the choice without assuming any delay will happen, it is possible that a project will not be completed in time if it is delayed. However, we don't take the delay in the future into account while deciding whether it should start.

$$\sum_{t \geq T+1-d_i} X_{it} = 0, \forall i \tag{16}$$

- All decision variables in (S_i, T_i) prior to the time t, which can be considered as realized, are fixed to the same value with the decision variable in the previous schedule (S_{i-1}, T_{i-1}):

$$X_{i,t} = X'_{i,t}, \forall t < t_i \tag{17}$$

- For any $i \succ j$, if at time t_i i has failed, or $i \in I_{tf}$, j cannot be started at any time.

$$\sum_t X_{jt} = 0 \tag{18}$$

- For any $i \succ j$, if at time t_i i has been delayed, or $i \in I_{t\delta}$,j cannot be started unless i is finished.

$$\sum_{t' \leq t-d_i-d_i^+} X_{it} \geq X_{jt'}, \forall i \succ j, t \tag{19}$$

- For any $I \vdash j$ and $i \in I$, the starting time of j will depend on other components in the set I.

$$\sum_{p \in (I/\{I_{tf}) \cap I_{t\delta}} \sum_{t' \leq t - d_p - d_p^+} X_{pt'} + \sum_{p \in (I/\{I_{tf}) \cap \bar{I}_{t\delta}} \sum_{t' \leq t - d_p} X_{pt'} \geq X_{jt}, \forall t \tag{20}$$

- Projects that are not selected in the initial schedule will not appear in the updated schedule

$$X_{it} = 0, \forall i \notin S_0 \tag{21}$$

Thus given the schedule (S_{i-1}, T_{i-1}) after bad luck happens at time t_{i-1}, the formulated optimization problem to get the updated schedule after bad luck happens at time t_i is:

$$\begin{aligned} \max_X \quad & NPV(S_i, T_i) \\ s.t \quad & (3) - (21) \\ & X_{it} \in \{0, 1\} \end{aligned} \tag{22}$$

The algorithm to get the final realization of the original schedule with the given triple of bad luck $(I_f, I_\delta, I_{f|\delta})$ is as follows:

Data: Initial Schedule (S_0, T_0), Triple of bad luck $(I_f, I_\delta, I_{f|\delta})$

Result: Final Realization (S',T')

(S1,T1) = (S,T);

while *Not every element in* $(I_f, I_\delta, I_{f|\delta})$ *is selected* **do**

Choose a bad luck in $(I_f, I_\delta, I_{f|\delta})$, which has not be selected before, with the earliest time of occurence;

Update the set $(I_{tf}, I_{t\delta})$, which is the set of bad luck that we know has happened;

Solve the problem (22)

end

Output $(S', T') = (S_n, T_n)$, where n is the last time bad luck occurs.

4. Estimation of Worst Case Scenario

In the previous schedule we developed a way to estimate the loss of a schedule subject to a given set of bad luck in the context of adaptative scheduling. The question arises as to what the worst case would be like given a budget of bad luck. Currently we do not have efficient methods to incorporate adaptative scheduling into a single optimization problem. A brute force approach for assessing the worst case scenario is to try every combination of bad luck, evaluate its total benefit and choose the one with the largest loss. The method is compuationally infeasible especially when the size of scheduling network grows extremely large, since there will be exponetial number of possible combination of bad luck.

Here we adopted a greedy heuristic to assign the bad luck in order to get a relatively bad scenario. We begin with the empty triple $(I_f, I_\delta, I_{f|\delta})$. In every iteration, for each possible assignment of bad luck on a single project, we add it into the triple selected temporarily, calculate the loss incured. At the end of the iteration we pick the assignment that yields the most loss and add it to the triple permanently. The realization of the schedule is updated taking the effect of chosen bad luck into consideration. The iteration terminates when no damage that incur loss can be added to the triple. Although there's no theoretical guarantee for the 'optimal' chocie of bad luck, the algorithm provided a way to get a

comparatively bad case and help identifiy key components in the job scheduling network in a reasonable amount of time. The algorithm is as follows:

Data: Initial Schedule (S,T), budget of bad luck W

Result: Triple of bad luck in the worst case $(I_f, I_\delta, I_{f|\delta})$

while *1* **do**

  for *i in S, p in* $\{Failure, delay, delayandfailure\}$ *with* $W_{ip} \leq W$ **do**

    Find the updated schedule (S',T') by solving the optimization problem if with the triple of bad luck contains previously selected bad luck and type p bad luck assigned to project i;

    Compute the loss, which is the total benefit of the plan subject only to the previouly selected bad luck minus the result in the last step;

  end

  if *There exist some project i with type bad luck that incurs positive loss* **then**

    pick the one that with the most loss, or the highest ratio of loss over W_{ip};

    W=W-W_{ip};

  else

    Jump out of the loop;

  end

end

5. Robustification of Schedule

Our way to find a robust schedule is based upon a 'solve-robustify' approach. We solve the problem in part 1 and use the solution as our initial schedule. Then we apply the greedy heuristic in the last section to identify projects in the worst case scenario, which can be regarded as vulnerable to bad luck and plays an important role in the scheduling network. Given the set of these 'weak' but 'important' components in the scheduling network, we take several strategies to robustify the networks, including:

- If project i fails in the worst case scenario.
  - i is the single project chosen in set I with $I \vdash j$ and j is also in the original schedule. A possible choice is to add another project in I to the schedule, this can be done by replacing the relationship $I \vdash j$ and $k \in I$ by $k \succ j$ and $I/\{k\} \vdash j$ and rerun the optimization problem in part 1.
  - i is the prequisite of some project j in the original schedule. It is obvious that j and subsequent projects that rely on the completion on j cannot be started. A possible way to is to solve an optimization problem with the decision variable X_{it} fixed to be 0 for any t, this might result in:
    - i and its subsequenct projects which depend on i will not be started
    - Same with above. However, it is possible that some other projects, which were not selected in the previous schedule are included in the update schedule. Another possible outcome is that the starting time of some projects, which have conflicts with deleted projects, are moved forward.
- If project i is delayed in the worst case scenario,

  - The approach would be checking whether the starting time of the project can be moved forward. This can be achieved by adding the time of delay to the duration of project i and solve the optimization problem in part 1 again.
  - If some project cannot be completed in time because of the delay of i, delete i by restricting decision variables corresponding to i to be 0 and rerun the optimization problem.

- if project i is delayed and then failed in the worst case scenario. The treatment can be a combination of that in the case of delay and case of failure.

There're also many other strategies which are not covered in our program, for example, under the condition of adapative scheduling, the starting time of certain projects, which is prequiste to many other projects but also highly suspectable to failure, can be moved forward. If they fail then other projects will get the information and won't start. Note that the above strategies does not necessarily yield a solution whose worst scenario is better than before. So each time we made a single or a set of changes to the original plan, we need to check whether the worst cas of updated one is better than the previous one.

we also need to point out that the estimation of final realization for a given triple of bad luck will still rely on the original relation between projects. Thus the scheme for updating needs slight change, for example, some relation between projects will be recovered which is incorporated into our program.

6. Computational Results

6.1. Small Example

In the second problem, we'll begin with the solution, where we assume that no bad luck happens. The greedy heuristic (2) can be applied here. For the purpose of illustration we'll calcuate the loss for every possible bad luck happened to a single luck no matter whether the bad luck required is more than the budget. The chart below gives the loss that incured by each single occurence of bad luck when the constant γ is equal to 0.99:

project	Failure	Delay and Failure	Delay
Job1	-12.2333	-12.3193	-12.3193
Job2	-12.2333	-12.3193	-12.3193
Job4	-12.2333	-12.3135	-12.3135
Job6	-12.2333	-12.3211	-0.08775
Job8	-6.11092	-6.30892	-0.198
Job9	-7.77821	-7.95372	-0.1755
Job10	-13.789	-13.9445	-0.15556

We study several choices of γ and W.

- We begin with a simple case $W = 1$ and $\gamma = 0.99$. It is obvious that none of the bad luck will happen, which is also shown in the numerical experiment.

- When the budget $W = 1.4$ and $\gamma = 0.99$. The above chart gives the loss of each potential choice of bad luck. When W equals 1.4 it reaches the threshold for project 4 to fail. in the worst case scenario, project 4 fails and project 1 cannot be started. Two alternatives are considered. In the first approach, all decision variable for project 4 are fixed to be 0 while in the second approach, since the failure of 4 leads to the failure to 1, we'll add

3 since $\{3, 4, 5\} \vdash 10$. The worst case scenario of the results of the two approaches are examined, and worst case in the result of second approach is better than both the previous result and approach one. Thus we add 3 into the plan and update the chart of loss to table below. The final schedule is highly resillent to bad luck at the level of W.

project	Failure	Delay and Failure	Delay
Job1	-12.2333	-12.3193	-12.3193
Job2	-12.2333	-12.3193	-12.3193
Job4	-3.55E-15	-0.08016	-0.08016
Job6	-12.2333	-12.3211	-0.08775
Job8	-6.11092	-6.30892	-0.198
Job9	-7.77821	-7.95372	-0.1755
Job10	-13.789	-13.9445	-0.15556
Job3	-12.2333	-6.39589	-6.39589

- When the budget $W = 2.5$ and $\gamma = 0.99$. Under this condition the failure of project 1 will lead to the most loss. The only possible method for improvement is to restrict decision variable corresponding to project 1 to be 0. In the resulting scheduling net work we only have the project 8 and project 9. As W goes larger both 9 and 10 will fail and the best choice is doing nothing.

- When the budget $W = 2.5$ and $\gamma = 0.95$, we can observe a dramatic change in the intial condition from the case $\gamma = 0.99$. Only project 8 and project 9 are selected in the initial schedule. The schedule is highly vulnerable to delay, since the bad luck that will result in the most loss is delay for project 8. Since 8 cannot be moved forward, the only way that may lead to a better solution is delete 8. The resulting schedule contains no projects and of course will no longer be influenced by bad luck.

6.2. Large Example

Similar to the case of small data set, for the large data set our analysis will begin with the result in part 1, which does not take any risk of failure or delay into consideration. Greedy algorithm still plays a central role in acquiring the worst case scenario. However, in the case of the larger data set, which contains 76 projects, it is time consuming to perform analysis on the loss incured by every single occurence of bad luck since the formulated optimization problem has thousands of variables. Two strategies are taken based upon the observation of the intial result in part 1:

- Most projects starts before the 18th month, which implies that even delay takes place, it is unlikely that there will be any project that cannot be completed in time. And currently we only limit our discussion to the case where the hurdle rate is large,i.e, greater than 0.99, thus the discount effect, together with additional cost of delay is trivial and the resulting loss of delay is rather neglectable when compared with that of failure. Another oberservation is that in many projects, the budget of failure is comparable to that of delay. As a result, in our analysis for the large data set we'll assume that bad luck only happens in the form of failure.

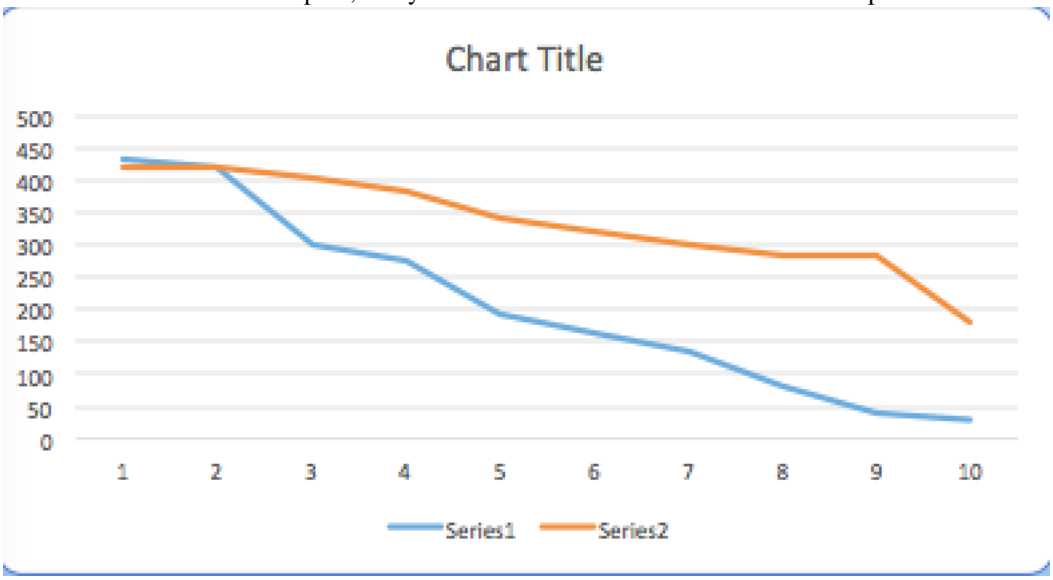
- Instead of giving a precisie estimation of the loss of each occurence of bad luck, we adopted the following algorithm to find a naive estimation of the failure of a single project. The general idea of the algorithm is to estimate the loss of the failure of a given project based on loss of other projects that completely or partially depend on the completion of the current project. The algorithm is time efficient at the expense of accuracy. An

Data: Current Realization of Schedule (S',T')
Result: Loss(S,'failure'), each entry represents the naive estimate of loss incured by only failing the corresponding entry in S under current realization

$S'' = \{\}$;
while *S' is not empty* **do**
  Pick $i \in S'$ with the latest starting time.;
  Loss(i)=$b_i * \gamma^{t_i+d_i}$; if it is not failed, 0 if failed. **for** $j \in S''$ **do**
    if $j \succ i$ **then**
      Loss(i)=Loss(i)+Loss(j)-$c_j * \gamma^{t_i}$;
    end
    if $j \in S''$ *and* $I \succ i$ **then**
      Loss(i)=Loss(i)+(Loss(j)-$c_j * \gamma^{t_i}$)/ Cardinality($I \cap (S' \cup S'')$);
    end
  end
  Move i from S' to S".
end

obvious drawback for this algorithm is that if fails to grasp the interdepence of projects. However, it provided us with insight into components that has high potential to be crucial to the structure of the network.Note that the algorithm can also be extended to evaluate a naive estimation for the loss of delay.

The graph below shows the trend for how the total benefit of worst case scenario changes against W. the blue line is for the worst case for the initial plan, and yellow line for the worst case of the robust plan.



In the robustification of the large data set, we take a conservative approach since our resource of computing is limited. We only consider the case where project i,j fails in the worst scenario, and i is in some set $I \vdash j$. The approach we take is to add other projects in set I that were not previouly selected into the plan. Another reason for not deleting failed projects is that the cost for failure does not differ much from project to project, and the deletion will make the remaining projects in the schedule more suspectable to the attack of bad luck. Here we plot how the value in the worst case changes after the robustification for a given budget of W. From the figure we can see that the robust schedule performs much better than the initial schedule in the worst scenario.