2015 AIMMS-MOPTA competition report, Team RensPolymatheIa

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1. Mathematical Formulation for problem without bad luck or with deterministic bad luck

Our approach to model the system is based upon the observation that the durations of all projects and the time span are integral, therefore the time span can be divided into time segments with equal lengths (one month in this case). A binary variable X_{jt} is assigned to Project j at the t^{th} time segment, which indicates whether Project j starts at the beginning of the t^{th} month. This results in a mixed integer model, which contains the following parameters:

$$
\begin{aligned}
c_j &= \{\text{the cost of Project j}\}\\
b_j &= \{\text{the benefit of Project j}\}\\
d_j &= \{\text{the duration of Project j}\}\\
c_j^+ &= \{\text{the added cost to Project j if delayed}\}\\
d_j^+ &= \{\text{the added duration to Project j if delayed}\}\\
T &= \{\text{the number of months in the total time span, integer}\}
\end{aligned}
$$

A binary delay flag q_j is introduced to indicate whether Project j is expected to be delayed, hence the cost and duration of each project under what-if scenarios would be:

$$\tilde{c}_j = c_j + q_j c_j^+, \quad \forall j \in J \tag{1}$$

$$\tilde{d}_j = d_j + q_j d_j^+, \quad \forall j \in J \tag{2}$$

The following constraints are added to the model:

- A project can be started at most once throughout the entire time span:

$$\sum_{t=1}^{T} X_{jt} \leq 1, \forall j \in J \tag{3}$$

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- A project cannot be started if it is too late, that is, we can only start a project if it can be finished before the deadline:

$$\sum_{t \geq T+1-\tilde{d}_j} X_{jt} \ = \ 0, \forall j \in J \tag{4}$$

- For all $i \nsim j$, i cannot be started within d_j units of time after j started, same for j.

$$\sum_{t-\tilde{d}_j+1 \leq t' \leq t+\tilde{d}_i-1} X_{jt'} \ + \ X_{it} \ \leq \ 1, \forall i \nsim j, \forall t \in \{1..T\} \tag{5}$$

- For all $i \succ j$, j can be started only if i has been finished:

$$\sum_{t' \leq t-\tilde{d}_i} X_{it'} \ \geq \ X_{jt}, \forall i \succ j, \forall t \in \{1..T\} \tag{6}$$

- For all $I \ \vdash \ j$, j cannot be started until at least one of the projects in I has been finished:

$$\sum_{i \in I} \sum_{t' \leq t-\tilde{d}_i} X_{it'} \ \geq \ X_{jt}, \forall I \ \vdash \ j, \forall t \in \{1..T\} \tag{7}$$

Apparently Constraint (6) is a special case of Constraint (7), with only Project i rather than multiple elements in Set I, and can thus be generalized into Constraint (7) for simplicity.

The objective function is defined as:

$$NPV_\gamma(S,T) = - \sum_{\substack{j \in S \\ 0 \leq t_j/12 \leq 3}} \gamma^{t_j} \tilde{c}_j + \sum_{\substack{j \in S \\ 0 \leq (t_j+\tilde{d}_j)/12 \leq 3}} \gamma^{t_j+\tilde{d}_j} \tilde{b}_j \tag{8}$$

In the optimization problem, it is written as:

$$NPV_\gamma(S,T) = -\sum_{i,t} \gamma^t \cdot \tilde{c}_i \cdot X(i,t) + \sum_{i,t} \gamma^{t+\tilde{d}_i} \cdot \tilde{b}_i \cdot X(i,t) \tag{9}$$

It is easy to construct the one-to-one correspondence between (S, T) and X, where:

$$S \ = \ \{i | \exists t, \text{ s.t. } X(i,t) = 1\} \tag{10}$$

and for each element t_i in T, which is the starting time of project i in S:

$$t_i \ = \ t, \quad \text{where } X(i,t) = 1 \tag{11}$$

Thus to get the optimal schedule without bad luck or with deterministic bad luck, we can formulate the optimization problem as:

$$\begin{aligned} \max_X \quad & NPV_\gamma(S,T) \\ s.t \quad & (3)-(7) \\ & X_{it} \in \{0, \ 1\} \end{aligned} \tag{12}$$

The above model can be efficiently solved by typical MIP solvers such as Cplex.

2. Scheduling Under Bad Luck

From now on, the problem is to take failures and delays into consideration while determining the project portfolio. A parameter $W \geq 0$ is introduced to represent the budget of bad luck. We define:

$$\begin{aligned} I_f &= \{\text{Project that will be failed by the bad luck}\} \\ I_\delta &= \{\text{Project that will be delayed by the bad luck but eventually will succeed}\} \\ I_{f|\delta} &= \{\text{Project that will be delayed by the bad luck and eventually will fail}\} \end{aligned}$$

Each project has probabilities to delay, to fail, or to delay and then fail. The probabilities can help with calculation of information entropy, which is used to measure the cost of each occurrence of bad luck. The total cost of bad luck is defined as:

$$w(I_f, I_\delta, I_{f|\delta}) := \sum_{j \in I_f} [-log_2(p_{f,j})] + \sum_{j \in I_\delta} [-log_2((1 - p_{f,j})p_{\delta,j})] + \sum_{j \in I_{f|\delta}} [-log_2((1 - p_{f,j}))]$$

Our purpose is to find a schedule that yields the highest profit under certain budget of bad luck. A general formulation is the following maxmin problem:

$$\begin{aligned} & \max_{S,T} \quad \min_{I_f, I_\delta, I_{f|\delta}} \quad NPV'_\gamma(S, T, I_f, I_\delta, I_{f|\delta}) \\ subjectto \quad & i \succ j, \ \forall (i, j) \in E, \\ & i \neq j, \ \forall (i, j) \in E', \\ & I \vdash j, \ \forall (I, j) \in E'', \end{aligned} \tag{13}$$

The net present value of the portfolio is computed by summing up the costs and benefits of all projects in the schedule, discounted appropriately at the time they are incurred:

$$NPV_\gamma(S, T) = \sum_{j \in S, 0 \leq t_j/12 < 3} \gamma^{t_j} \tilde{c}_j + \sum_{j \in S, 0 \leq t_j/12 < 3} \gamma^{t_j + d_j} \tilde{b}_j \tag{14}$$

3. Adaptive Scheduling of Projects

In practice, the original schedule is subject to update once any bad luck occurs. For example, consider a scenario where an intermediate project delays, which could lead to inability of some other projects to be completed in time. Although we can continue with the subsequent projects, a possible better option could be to remove a proportion of projects while updating the schedule. The purpose is to reduce the potential loss based upon information at the current stage. Our analysis will take the update into consideration. At each time $t^{badluck}$ when a bad luck happens, the update must follow the rules:

- Only projects selected in the original schedule can be chosen in the updated schedule.

- The update will take place when a bad luck occurs, and we assume that bad luck only occurs at the scheduled ending time of a project. The cases where projects delay and then fail can be regarded as special occasions where bad luck occurs twice, in which the first time is at the scheduled ending time of the previous plan, when we know the project is delayed, and the second time is the moment it fails at the end of the delay.
- Only the projects that are scheduled at and after the current bad luck can be rescheduled.
- The bad luck that has not happened is unknown. When updating the schedule, we make an optimal choice without assuming that bad luck will take place after $t^{badluck}$.

With the assumptions above, the realized schedule can be obtained by solving a series of optimization problems.We begin with an initial schedule (S, T), which can be the solution in part 1, and at each time t_i when bad luck occurs to a single project, the schedule is updated to (S_i, T_i) based upon current progress and the type of bad luck at t. Let (S_{i-1}, T_{i-1}) be the schedule after last bad luck happens at time t_{i-1} with corresponding decision variable $X'_{i,t}$, and $(I_{tf}, I_{t\delta})$ be the set of projects that has delayed or failed before and at time t. If at time t we know a project has been delayed and failed, we put the project into both set I_{tf} and $I_{t\delta}$. Otherwise, if we only get the information that the project has been delayed, the project will only be selected into the set I_{tf}. We can find an updated schedule(S_i, T_i) after bad luck at time t by solving a problem which has the following constraints with decision variable $X_{i,t}$:

- As in part 1, a single project can only be started at most once.

$$\sum_t X_{it} \leq 1, \forall i \tag{15}$$

- As with part 1, the starting time of a project cannot be too late. To evaluate the effect of delay, since we have already assumed that no bad luck will happen after time t,it's worth to note that no delay except for the projects in set $I_{t\delta}$ will be considered while determining the updated schedule.

$$\sum_{t \geq T+1-d_i} X_{it} = 0, \forall i \tag{16}$$

- The realized schedule, i.e, the schedule prior to time t cannot be changed:

$$X_{i,t} = X'_{i,t}, \forall t < t_i \tag{17}$$

- For any $i \succ j$, if $i \notin I_{tf} \cup I_{t\delta}$, j cannot be started until i has been finished.

$$\sum_{t' \leq t-d_i} X_{it} \geq X_{jt'}, \forall t \tag{18}$$

- For any $i \succ j$, if $i \in I_{tf}$, j cannot be started at any time.

$$\sum_t X_{jt} = 0 \tag{19}$$

- For any $i \succ j$, if $i \in I_{t\delta}$,j cannot be started until i hazs finished.

$$\sum_{t' \leq t - d_i - d_i^+} X_{it} \geq X_{jt'}, \forall t \tag{20}$$

- For all $i \nsim j$, i cannot be started within d_j units of time after j started. The same applies for j.

$$\sum_{t - \tilde{d}_j + 1 \leq t' \leq t + \tilde{d}_i - 1} X_{jt'} + X_{it} \leq 1, \forall t \tag{21}$$

- For any $I \vdash j$, j can be started only if at least one project in set I has been successfully finished.

$$\sum_{p \in (I / \{I_{tf}) \cap I_{t\delta}} \sum_{t' \leq t - d_p - d_p^+} X_{pt'} + \sum_{p \in (I / \{I_{tf}) \cap \bar{I_{t\delta}}} \sum_{t' \leq t - d_p} X_{pt'} \geq X_{jt}, \forall t \tag{22}$$

- Projects that are not selected in the initial schedule (S, T) will not appear in the updated schedule

$$X_{it} = 0, \forall i \notin S_0 \tag{23}$$

Thus given the schedule (S_{i-1}, T_{i-1}), the updated schedule (S_i, T_i) after bad luck happens at time t_i can be acquired by solving the optimization problem:

$$\begin{aligned} \max_X \quad & NPV(S_i, T_i) \\ s.t \quad & (16) - (23) \\ & X_{it} \in \{0,\ 1\} \end{aligned} \tag{24}$$

The algorithm to get the final realization of the original schedule with the given triple of bad luck $(I_f, I_\delta, I_{f|\delta})$ is as follows:

```
**Data**: Initial Schedule $(S_0, T_0)$, Triple of bad luck $(I_f, I_\delta, I_{f|\delta})$
**Result**: Final Realization (S',T')
(S1,T1) = (S,T);
**while** $(I_f, I_\delta, I_{f|\delta})$ *is not empty* **do**
    Choose a bad luck in $(I_f, I_\delta, I_{f|\delta})$ with the earliest time of occurrence;
    Update the set $(I_{tf}, I_{t\delta})$, which is the set of bad luck that we know has happened;
    Solve the problem (24);
    Remove
**end**
Output $(S', T') = (S_n, T_n)$, where n is the last time bad luck occurs.
```

4. Estimation of Worst Case Scenario

In the previous section we developed an algorithm to update the schedule when any bad luck occurs. Given an initial schedule (S, T), we can get a final realization (S', T') after all the bad luck in the triple $(I_f, I_\delta, I_{f|\delta})$ take s place. The impact of bad luck can be quantified by the loss it incurs. The loss is calculated as the difference between the

NPV value of the initial schedule and the realized schedule, which can be expressed as:

$$Loss_{(S,T)}(S',T') = NPV(S',T') - NPV(S,T) \tag{25}$$

The question arises that given a budget of bad luck, which selection of the triple $(I_f, I_\delta, I_{f|\delta})$ will lead to the maximum loss. A brute force approach for assessing the worst selection of the triple is to try every combination of bad luck and choose the one with the largest loss. However, because of our limited source of computing and the size of the problem, the method is computationally infeasible especially when the number of projects grows extremely large, since there will be exponential number of choices of the triple.

An alternative method is the greedy heuristic, which aims at finding a triple $(I_f, I_\delta, I_{f|\delta})$ with a large loss and is also computationally tractable. We begin with an empty triple $(I_f, I_\delta, I_{f|\delta})$ and iterate based upon the current budget of bad luck and bad luck remained. In every iteration, we add a project with certain type of bad luck that yields the most loss to the triple and update the realization of the schedule. The iteration terminates when no bad luck that incurs loss can be added to the triple. The algorithm provides a way to get a bad realization of schedule and helps identify the key components in the schedule, although there's no theoretical proof for the worst. The algorithm is:

```
Data: Initial Schedule (S,T), budget of bad luck W
Result: Triple of bad luck in the worst case $(I_f, I_\delta, I_{f|\delta})$
while 1 do
    for i in S, p in $\{Failure, delay, delayandfailure\}$ with $W_{ip} \leq W$ do
        Find the updated schedule (S',T') by solving the optimization problem if with the triple of bad luck
        contains previously selected bad luck and type p bad luck assigned to project i;
        Compute the loss, which is the total benefit of the plan subject only to the previouly selected bad luck
        minus the result in the last step;
    end
    if There exist some project i with type bad luck that incurs positive loss then
        pick the one that with the most loss, or the highest ratio of loss over $W_{ip}$;
        W=W-$W_{ip}$;
    else
        Jump out of the loop;
    end
end
```

5. Robustification of Schedule

In the last section we apply the greedy heuristic in the last section to identify projects in the worst case scenario, which can be regarded as vulnerable to bad luck and plays an important role in the scheduling network. Given the set of these weak but important components in the schedule, we take several strategies to for robustification based upon the type of bad luck, including:

- If project i fails in the worst case scenario:
    - i is the single project chosen in set I with $I \vdash j$ and j is also in the original schedule. Add another project $k \in I$ to the schedule.

  - Delete project i. This might result in:
    - i and subsequent projects which depend on i will not start
    - other projects which have conflict with i but not selected will start.

- If project i is delayed in the worst case scenario:
  - Move the starting time of i forward if possible.
  - Delete the project i. This may cause the same consequence as with the case of failure.

- if project i is delayed and then failed in the worst case scenario. The treatment can be a combination of that in the case of delay and case of failure.

There are also many other strategies which are not covered in our program, for example, under the condition of adaptive scheduling, the starting time of certain projects, which are prerequisite to many other projects but also highly susceptible to failure, can be moved forward. If they fail, other projects won't start.

Note that the above strategies does not necessarily yield a solution whose worst scenario is better than before. So each time we made a single or a set of changes to the original plan, we need to check whether the worst case of updated one is better than the previous one.

6. Computational Results

6.1. Smaller Dataset

In the second problem, we begin with the solution, where we assumed that no bad luck happens. The greedy heuristic (2) can be applied here. For purpose of illustration we calculate the loss for every possible bad luck happened to a single luck no matter whether the bad luck required is more than the budget. Table 1 gives the loss that incurred by each single occurrence of bad luck when the constant γ is equal to 0.99.

project	Failure	Delay and Failure	Delay
Job1	-12.2333	-12.3193	-12.3193
Job2	-12.2333	-12.3193	-12.3193
Job4	-12.2333	-12.3135	-12.3135
Job6	-12.2333	-12.3211	-0.08775
Job8	-6.11092	-6.30892	-0.198
Job9	-7.77821	-7.95372	-0.1755
Job10	-13.789	-13.9445	-0.15556

Table 1: Loss incurred by each single bad luck occurrence, $\gamma = 0.99$

We study several combinations of γ and W.

- We begin with a simple case $W = 1$ and $\gamma = 0.99$. It is obvious that none of the bad luck will happen, which is shown in Figure 1.

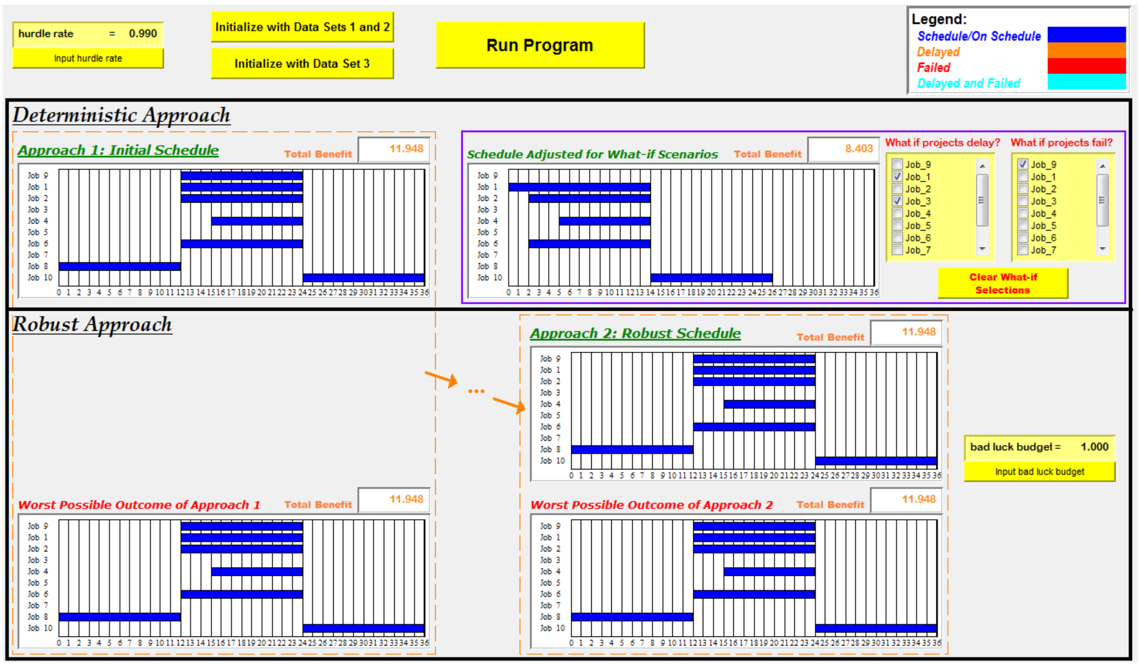


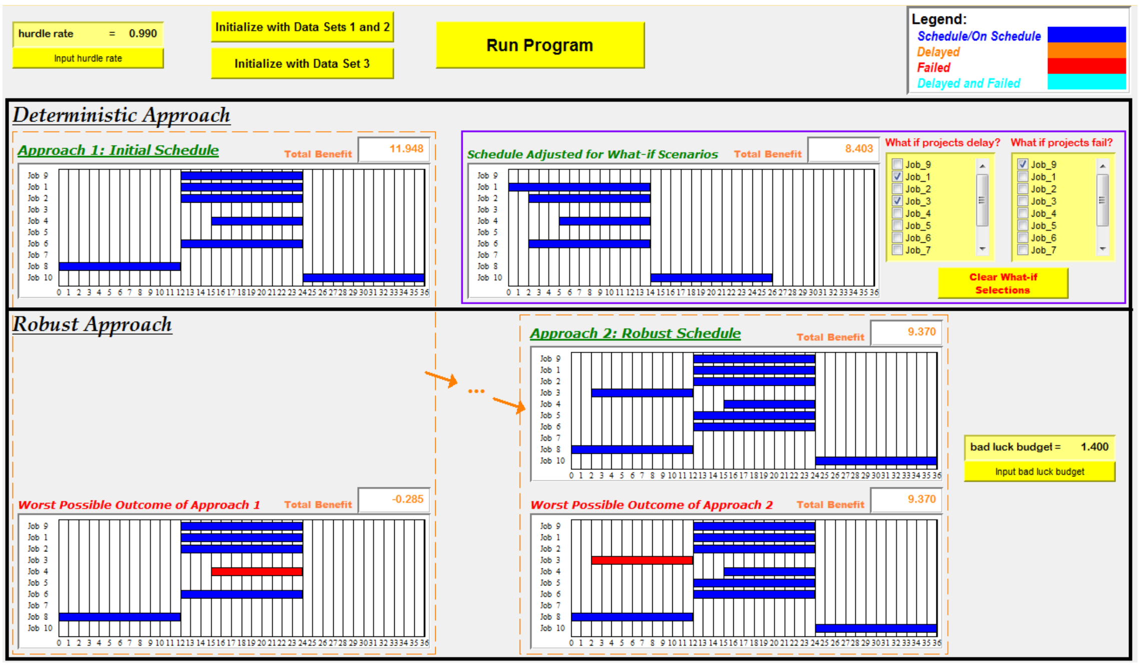
Figure 1: GUI with result, $W = 1.0$, $\gamma = 0.99$

- When the budget $W = 1.4$ and $\gamma = 0.99$. The above table gives the loss of each potential choice of bad luck. When W equals 1.4 it reaches the threshold for Project 4 to fail. In the worst case scenario, Project 4 fails and Project 1 cannot be started. Two alternatives are considered. In the first approach, all decision variable for Project 4 are fixed to be 0, while in the second approach, we'll add Project 3 since $\{3, 4, 5\} \vdash 10$, as the failure of Project 4 leads to failure of Project 1. The worst case scenario of the results of the two alternatives are examined, and worst case in the result of second approach is better than both the previous result and Approach 1. Therefore we add 3 into the plan and update the chart of loss to Table 2 listed below.

project	Failure	Delay and Failure	Delay
Job1	-12.2333	-12.3193	-12.3193
Job2	-12.2333	-12.3193	-12.3193
Job4	-3.55E-15	-0.08016	-0.08016
Job6	-12.2333	-12.3211	-0.08775
Job8	-6.11092	-6.30892	-0.198
Job9	-7.77821	-7.95372	-0.1755
Job10	-13.789	-13.9445	-0.15556
Job3	-12.2333	-6.39589	-6.39589

Table 2: Loss incurred by each single bad luck occurrence, $\gamma = 0.99$

The final schedule is highly resilient to bad luck at the level of W, as is shown in Figure 2.


hurdle rate = 0.990

Approach 1: Initial Schedule — Total Benefit 11.948

Schedule Adjusted for What-if Scenarios — Total Benefit 8.403

What if projects delay? Job_1 ✓, Job_3 ✓

What if projects fail? Job_9 ✓

Approach 2: Robust Schedule — Total Benefit 9.370

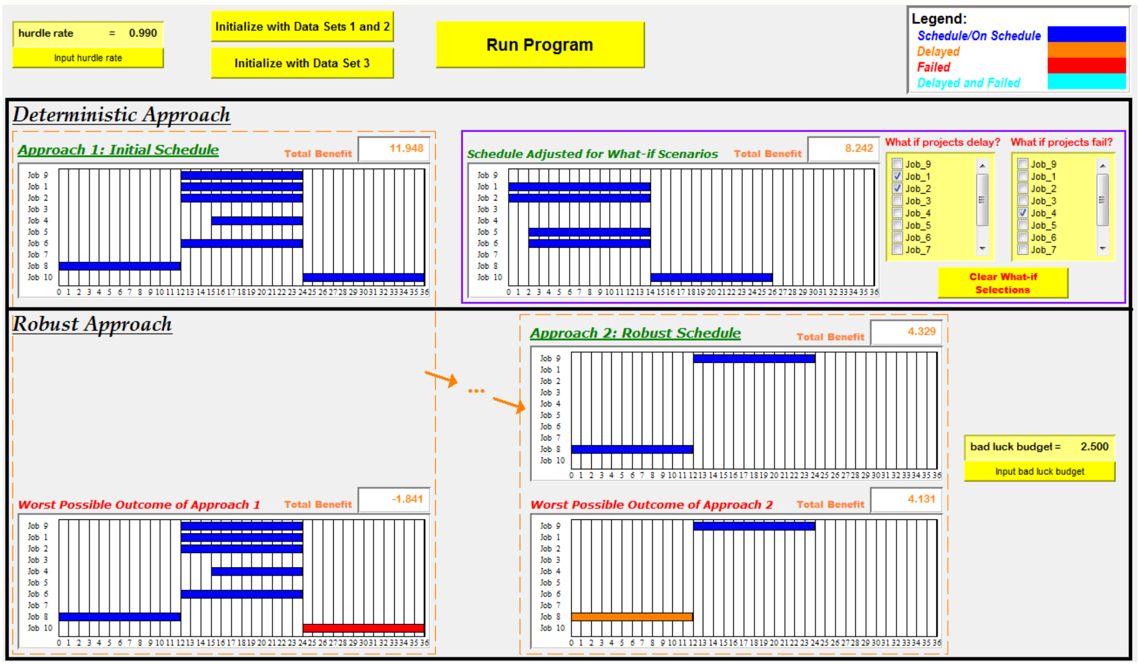
bad luck budget = 1.400

Worst Possible Outcome of Approach 1 — Total Benefit -0.285

Worst Possible Outcome of Approach 2 — Total Benefit 9.370

Figure 2: GUI with result, $W = 1.4$, $\gamma = 0.99$

- When the budget $W = 2.5$ and $\gamma = 0.99$. Under this condition the failure of Project 1 will lead to the most loss. The only possible method for improvement is to restrict decision variable corresponding to Project 1 to be 0. In the resulting scheduling net work we only have the Project 8 and Project 9.


hurdle rate = 0.990

Approach 1: Initial Schedule — Total Benefit 11.948

Schedule Adjusted for What-if Scenarios — Total Benefit 8.242

What if projects delay? Job_1 ✓, Job_2 ✓

What if projects fail? Job_4 ✓

Approach 2: Robust Schedule — Total Benefit 4.329

bad luck budget = 2.500

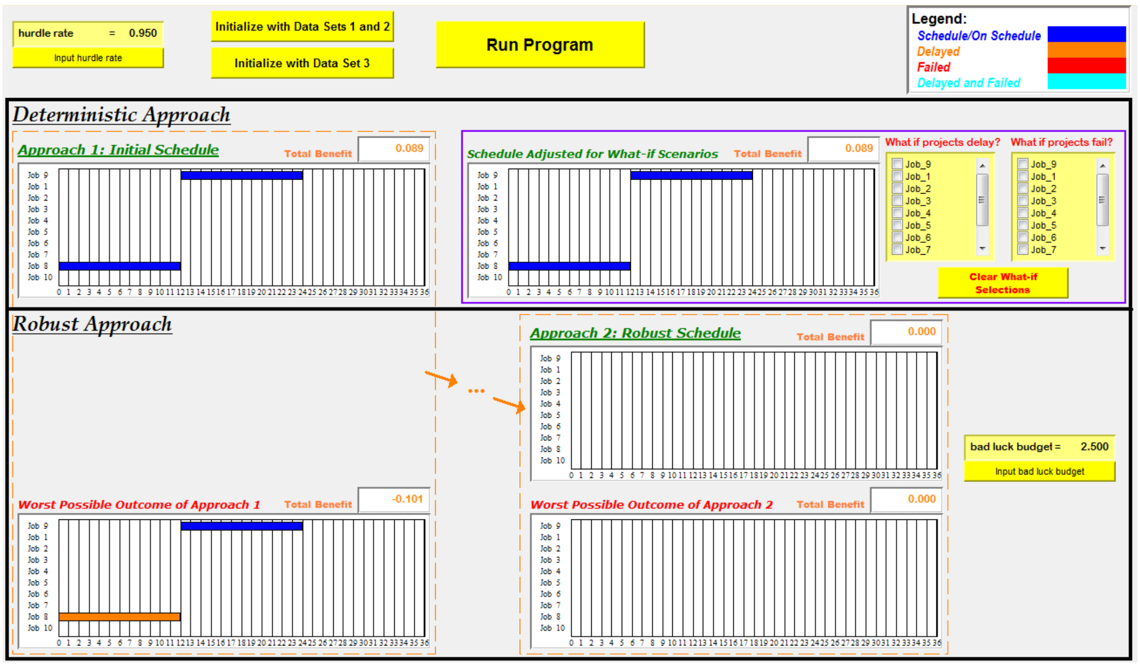
Worst Possible Outcome of Approach 1 — Total Benefit -1.841

Worst Possible Outcome of Approach 2 — Total Benefit 4.131

Figure 3: GUI with result, $W = 2.5$, $\gamma = 0.99$

As W goes larger both Projects 9 and 10 would fail and the best choice would be doing nothing.

- When the budget $W = 2.5$ and $\gamma = 0.95$, we can observe a dramatic change in the intial condition from the case $\gamma = 0.99$. Only Project 8 and Project 9 are selected in the initial schedule. The schedule is highly vulnerable to delay, since the bad luck that will result in the most loss is delay for Project 8. Since Project 8 cannot be moved forward, the only way that may lead to a better solution is to delete Project 8. The resulting schedule contains no projects and of course will no longer be influenced by bad luck.


hurdle rate = 0.950

bad luck budget = 2.500

Approach 1: Initial Schedule — Total Benefit 0.089

Schedule Adjusted for What-if Scenarios — Total Benefit 0.089

Approach 2: Robust Schedule — Total Benefit 0.000

Worst Possible Outcome of Approach 1 — Total Benefit -0.101

Worst Possible Outcome of Approach 2 — Total Benefit 0.000

Figure 4: GUI with result, $W = 2.5$, $\gamma = 0.95$

6.2. Larger Dataset

Similar to the case of the smaller dataset, for the large data set our analysis will begin with the result with Approach 1, which does not take any risk of failure or delay into consideration. Greedy algorithm still plays a central role in acquiring the worst case scenario. However, in the case of the larger dataset, which contains 76 projects, it is time consuming to perform analysis on the loss incurred by every single occurrence of bad luck since the formulated optimization problem has thousands of variables. Two strategies are taken based upon the observation of the initial result from Approach 1:

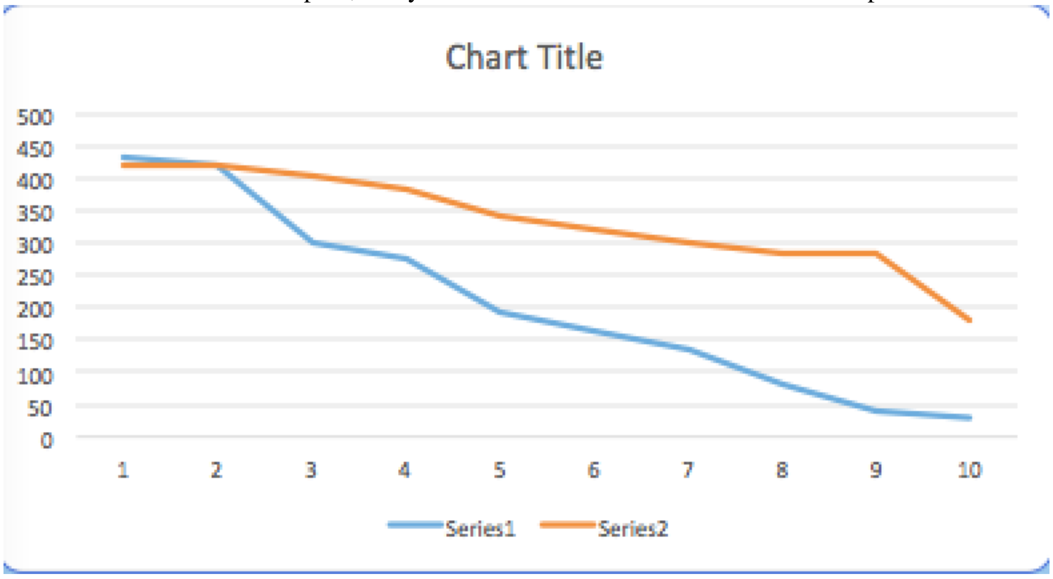
- Most projects starts before the 18th month, which implies that even delay takes place, it is unlikely that there will be any project that cannot be completed in time. And currently we only limit our discussion to the case where the hurdle rate is large,i.e, greater than 0.99, thus the discount effect, together with additional cost of delay is trivial and the resulting loss of delay is rather negligible when compared with that of failure. Another observation is that in many projects, the budget of failure is comparable to that of delay. As a result, in our analysis for the large data set we'll assume that bad luck only happens in the form of failure.

- Instead of giving a precise estimation of the loss of each occurrence of bad luck, we adopted the following algorithm to find a naive estimation of the failure of a single project.

```
Data: Current Realization of Schedule (S',T')
Result: Loss(S,'failure'), each entry represents the naive estimate of loss incured by only failing the
        corresponding entry in S under current realization
$S'' = \{\}$;
while S' is not empty do
    Pick $i \in S'$ with the latest starting time.;
    Loss(i)=$b_i * \gamma^{t_i+d_i}$; if it is not failed, 0 if failed. for $j \in S''$ do
        if $j > i$ then
            Loss(i)=Loss(i)+Loss(j)-$c_j * \gamma^{t_i}$;
        end
        if $j \in S''$ and $I > i$ then
            Loss(i)=Loss(i)+(Loss(j)-$c_j * \gamma^{t_i}$)/ Cardinality($I \cap (S' \cup S'')$);
        end
    end
    Move i from S' to S".
end
```

The general idea of the algorithm is to estimate the loss of the failure of a given project based on loss of other projects that completely or partially depend on the completion of the current project. The algorithm is time efficient at the expense of accuracy. An obvious drawback for this algorithm is that if fails to grasp the interdependence of projects. However, it provided us with insight into components that has high potential to be crucial to the structure of the network.Note that the algorithm can also be extended to evaluate a naive estimation for the loss of delay.

The graph below shows the trend for how the total benefit of worst case scenario changes against W. the blue line is for the worst case for the initial plan, and yellow line for the worst case of the robust plan.



In the robustification of the large data set, we take a conservative approach since our resource of computing is limited. We only consider the case where project i,j fails in the worst scenario, and i is in some set $I \vdash j$. The approach we take is to add other projects in set I that were not previously selected into the plan. Another reason for not deleting failed projects is that the cost for failure does not differ much from project to project, and the deletion will make the remaining projects in the schedule more susceptible to the attack of bad luck. Here we plot how the value in the worst case changes after the robustification for a given budget of W. From the figure we can see that the robust schedule performs much better than the initial schedule in the worst scenario.